Queen's University
Faculty of Arts and Sciences
Department of Economics
Fall 2005

Economics 250: Introduction to Statistics

Midterm Exam I:
Time allotted: 120 minutes.

Instructions:

READ CAREFULLY. Calculators are permitted. At the end of the exam are several formulae. Answers are to be written in the examination booklet. Do not hand in the question sheet. You are to answer **ALL** questions. **SHOW ALL YOUR WORK**. Most of the marks are awarded for showing how a calculation is done and not for the actual calculation itself!

There are a total of 100 possible marks to be obtained. **Part A** has 12 questions worth 5 marks each for a total of 60 marks. **Part B** has 1 questions worth 40 marks.

PART A: Do all 12 questions (5 Marks each)

A1. A random sample of 60 households are drawn from a small district. The following is the frequency distribution of the number of items donated to local food banks in a week of charity events:

# of items	0	1	2	3	4	5
frequency	8	11	15	20	0	6

(a) Find the relative frequencies for each of the 6 classes. Draw a Histogram. Calculate the sample mean, the sample variance, and the sample standard deviation.

Answer

The items donated is a random variable and we denote it by X. Use the formula for grouped data, we have:

$$
\begin{aligned}
\bar{X} &= \frac{1}{n}\sum_{j=1}^{k}\nu_j f_j = 2.1833, \\
s^2 &= \frac{1}{n-1}\sum_{j=1}^{k} f_j(\nu_j - \bar{X})^2 = 1.9489, \\
s &= \sqrt{1.9489} = 1.3960.
\end{aligned}
$$

(b) Find the median. Is this distribution skewed?

The median is the $(n+1)/2th$ *number in the ordered array. Here it is the* $(60+1)/2 = 30.5th$ *number, or the average of* 2 *and* 2.(some students might try to portion out between 1 and 2 (closer to 2 using a linear extrapolation formula..this is fine)

Since the mean is greater than the median, this distribution is positively skewed, or skewed to the right.

A2. Suppose that the number of pages photocopied with a library card by a university student in one term can be modeled as a random variable X with mean $\mu_X = 50$ and variance $\sigma_X^2 = 30$. The cost of photocopying one page is 10 cents, and there is an annual fee of 100 cents for holding the card. What is the mean and variance of the total cost, C, for using one library card by a typical student? What is the coefficient of variation?

Answer

We can use formulae for the mean and variance of linear functions of random variables.

$$
\begin{aligned}
C &= 100 + 10X, \\
\mu_C &= 100 + 10\mu_X = 100 + 10 \times 50 = 600, \\
\sigma_C^2 &= (10)^2\sigma_X = 100 \times 30 = 3000.
\end{aligned}
$$

For the coefficient of variation, we apply the formula.

$$
\begin{aligned}
\sigma_C &= \sqrt{\sigma_C^2} \approx 54.772, \\
CV &= \frac{\sigma_C}{\mu_C} \times 100\% = \frac{54.772}{600} \times 100\% \approx 9.1287\%
\end{aligned}
$$

A3. There are 8 identically shaped candies in a container that is not transparent. These candies can only be taken out from the container one at a time, without being put back. Suppose there are 6 white ones and 2 orange ones. The color of a candy can only be known after it taken out. Suppose 2 candies are taken out randomly. What is the probability of the event that "one of the two candies is white and the other is orange"? What is the probability of the event that "the first candy is white and the second is orange"?

Answer

The number of basic outcomes in the sample space is C_2^8, *and that in the event "one of the two candies is white and the other is orange"is* $C_1^6 C_1^2$. *Thus the first probability is* $\frac{3}{7}$. *The probability of the event that "the first candy is white and the second is orange"is* $\frac{6}{8} \times \frac{2}{7} = \frac{3}{14}$.

The link between these two probabilities is not hard to see: if, as in the first event, one white candy and one orange candy are drawn, then either the white one is drawn first and the orange one is drawn second (which gives rise to the second event), or the orange one is drawn first and the white one second; these two scenarios are equally likely. Besides, the second event is a subset of the first event. Thus, the second probability is one half of the first.

The way most students will think of this problem is the following: 4 possibilities for the two draws

$$(W_1, W_2) \quad (W_1, O_2) \quad (O_1, W_2) \quad (O_1, O_2)$$

We see the probability

$$\begin{aligned} P((W \cap O) &= P(W_1 \cap O_2) + P(O_1 \cap W_2) \\ &= P(W_1) \times P(O_2|P(W_1) + P(O_1) \times P(W_2|P(O_1) \\ &= \frac{6}{8} \times \frac{2}{7} + \frac{2}{8} \times \frac{6}{7} = .42857 = \frac{3}{7} \end{aligned}$$

For the second question

$$P(W_1 \cap O_2) = \frac{6}{8} \times \frac{2}{7} = .21429 = \frac{3}{14}$$

:

A4. The following table records the number of people in a group of 100 economics students with respect to their philanthropy activities in the previous year:

volunteer/money donation	donated	not donated
volunteered	40	20
not volunteered	30	10

Suppose we are going to randomly find one student from this group. Provide a joint probability table and give the following:

Answer

The joint probability table can be the following. The marginal probabilities need not appear in the answer.

volunteer/money donation	donated	not donated	
volunteered	0.4	0.2	0.6
not volunteered	0.3	0.1	0.4
	0.7	0.3	

(a) P(volunteered and donated)

Denote the event "volunteered "by V and "not volunteered "by $\bar{V}$. Similarly, denote the event "donated "by D and "not donated "by $\bar{D}$.

$$P(V \cap D) = 0.4$$

(b) P(volunteered or donated)

$$P(V \cup D) = P(V) + P(D) - P(V \cap D) = 0.6 + 0.7 - 0.4 = 0.9, \textit{ or}$$

$$P(V \cup D) = 1 - P(\bar{V} \cap \bar{D}) = 1 - 0.1 = 0.9.$$

(c) P(not volunteered)

$$P(\bar{V}) = 0.4$$

(d) P(volunteered|donated)

$$P(V \mid D) = \frac{P(V \cap D)}{P(D)} = \frac{0.4}{0.7} = \frac{4}{7} \approx 0.57143$$

(e) Are volunteering and donation activities independent in this group? Why or why not?

They are not, since the joint probability is not equal to the product of the marginal probabilities for the two activities (that is, for any or all of the event pairs D *and* V*,* D *and* $\bar{V}$*,* $\bar{D}$ *and* V*, and* $\bar{D}$ *and* $\bar{V}$*.). For example, for* V *and* D*, we see that* $P(D \cap V) = 0.4 \neq P(D)P(V) = 0.7 \times 0.6 = 0.42$.

A5. Answer the following.

(a) Give the definition of independence in terms of two events A and B. Is $P(A \cap B) = 0$ if A and B are independent?

Events A *and* B *are statistically independent if and only if* $P(A \cap B) = P(A)P(B)$*. From the multiplication rule, this condition is equivalent to* $P(A \mid B) = P(A)$*, if* $P(B) > 0$*, or* $PB \mid A) = P(B)$*, if* $P(A) > 0$.

If none of $P(A)$ *or* $P(B)$ *is equal to 0, as is usually the case, then statistical independence does not imply* $P(A \cap B) = 0$*. That is, if* $P(A) > 0$*,* $P(B) > 0$*, and* $P(A \cap B) = 0$*, the two events are not independent.* $P(A \cap B) = 0$ *may imply that the two events are mutually exclusive.*

(b) Suppose there are now three events A, B, and C. $P(A) = 0.1$, $P(B) = 0.5$, and $P(C) = 0.2$. A is independent of both B and C. B and C are mutually exclusive. What is $P(A \cap (B \cup C))$?

In this case, $P(A \cap (B \cup C)) = P(A)P(B \cup C) = P(A)(P(B) + P(C)) = 0.1 \times (0.5 + 0.2) = 0.07$*, where the first equality follows since* A *is independent of* B *and* C *and the second equality follows since* B *and* C *are mutually exclusive.*

A6. X and Y are discrete random variables. Their probability distributions are as follows:

x	1	2	3	4
$P(X = x)$	0.2	0.3	0.4	0.1

y	1	2	3
$P(Y = y)$	0.6	0.2	0.2

(a) Construct and graph the (partial) cumulative distribution functions for X and Y, respectively.

Answer

The cumulative distribution functions for X and Y are respectively

$$F(x_1) = 0.2 = P(X \leq 1),$$
$$F(x_2) = 0.5 = P(X \leq 2),$$
$$F(x_3) = 0.9 = P(X \leq 3),$$
$$F(x_4) = 1 = P(X \leq 4)$$

$$F(y_1) = 0.6 = P(Y \le 1),$$
$$F(y_2) = 0.8 = P(Y \le 2),$$
$$F(y_3) = 1 = P(Y \le 3),$$

(b) Suppose X and Y are independent, construct their joint probability table.

Since X and Y are independent, the joint probabilities can be obtained by multiplication

$P(X = x_i, Y = y_j)$	y_1	y_2	y_3
x_1	.12	.04	.04
x_2	.18	.06	.06
x_3	.24	.08	.08
x_4	.06	.02	.02

A7. You are given the following random variable Y such that $Y = a - \frac{X}{b}$, where X is a random variable with $\bar{X} = c$, $S_X = d$, and a, b are nonzero constants. Standardize Y. (Are X and Y independent?)

Answer

Standardized random variable has mean 0 and variance 1. To standardize a random variable, we can subtract its mean from it and then divide the result by its standard deviation.

Let the standardized variable be Z.

$$\bar{X} = c, \qquad S_X = d;$$

$$\bar{Y} = a - \frac{c}{b}, \qquad S_Y^2 = \frac{d^2}{b^2}, \qquad S_Y = \mid \frac{d}{b} \mid;$$

$$Z = \frac{a - \frac{X}{b} - (a - \frac{c}{b})}{\frac{d}{b}} = \frac{c - X}{d}.$$

A8. A light bulb factory has two production lines producing identical bulbs. Line 1 produces 70% of total factory output. 0.5% of the products of line 1 has defect, and only

0.2% of those produced by line 2 has defect. Suppose a quality inspector randomly picks up a light bulb from the factory's output and finds that it has defect. What is the chance that this particular light bulb was produced by line 2?

Answer

Denote the event that light bulb is produced by line 1by I and the event that light bulb is produced by line 2 by II. Denote the event that light bulb has defectby D and the event that light bulb does not have defect by $\bar{D}$.

Thus we are given the following information:

$$P(I) = 0.7, \quad P(II) = 0.3, \quad P(D \mid I) = 0.005, \quad P(D \mid II) = 0.002.$$

What we want to know is $P(II \mid D)$.

We can use the Bayes' Theorem.

$$\begin{aligned} P(II \mid D) &= \frac{P(II \cap D)}{P(D)} = \frac{P(D \mid II)P(II)}{P(D \mid II)P(II) + P(D \mid I)P(I)} \\ &= \frac{0.002 \times 0.3}{0.002 \times 0.3 + 0.005 \times 0.7} \approx 0.14634. \end{aligned}$$

A9. A certain building has two fire alarm systems, A and B. The two separate systems are always both on duty. From past experience, it is know that the probability that system A functions in case of fire is 0.92; and that the probability that system B functions in case of fire is 0.93. Besides, the probability that B functions given that A does not is 0.85, in case of fire.

(a) What is the probability that at least one system functions in case of fire? (Hint: $P(A \cap B) + P(\bar{A} \cap B) = P(B)$.)

Answer

Denote the event that "System A functions "by A and the event that "System A does not function "by $\bar{A}$. Similarly for B and $\bar{B}$.

We have the following information:

$$P(A) = 0.92, \quad P(B) = 0.93, \quad P(B \mid \bar{A}) = 0.85;$$

note that events A and B are not mutually exclusive.

We want to know $P(A \cup B)$, which is equal to $P(A) + P(B) - P(A \cap B)$. $P(A)$ and $P(B)$ are readily available but we need to figure out $P(A \cap B)$. Following the hint, we first try to calculate $P(\bar{A} \cap B)$. Using the definition of conditional probability, we know that

$$P(\bar{A} \cap B) = P(\bar{A})P(B \mid \bar{A}) = (1 - 0.092) \times 0.85 = 0.068$$

. *Thus,*

$$P(A \cap B) = P(B) - P(\bar{A} \cap B) = 0.093 - 0.068 = 0.862.$$

We then know that

$$P(A \cup B) = 0.92 + 0.93 - 0.862 = 0.988$$

.

(b) In case of fire, what is the probability that A functions given B does not?

We want to know $P(A \mid \bar{B})$. We can calculate this probability using result from part (a).

$$\begin{aligned} P(A \mid \bar{B}) &= \frac{P(A \cap \bar{B})}{P(\bar{B})} = \frac{P(A) - P(A \cap B)}{P(\bar{B})} \\ &= \frac{0.92 - 0.862}{0.07} = \frac{0.058}{0.07} \approx 0.829. \end{aligned}$$

A10. Four Books are labeled 1, 2, 3, and 4, respectively. Suppose all of them are put on the shelf together in a row in a random way. What is the probability that they are shelved exactly in the order of "1-2-3-4", from the left to the right? What is the probability that the book labeled "1" is put to the left of the book labeled "3"?

Answer

There is only one way of arrangement (basic outcome) in the event "1-2-3-4", and there are $P_4^4 = 24$ ways of arranging all the books. Thus, the probability is $1/24 = 0.041\dot{6}$.

Most students will likely say: what is the probability of tyhe outcome 1-2-3-4

$$\begin{aligned} P(1-2-3-4) &= P(1) * P(2|1) * P(3|2) * P(4|3) \\ &= \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} = \frac{1}{24} \end{aligned}$$

(**Note: Mark this question eas**y) The most obvious intuition is that there is a symmetry from the left and the right of book 3. That is 50% chance (they cannot be piled on top of each other) Some students might interpret left to mean eaxactly beside on the left. In that case there (1-3-2-4) (1-3-4-2) (2-1-3-4) (4-1-3-2) (2-4-1-3) and (4-2-1-3) with probability now $\frac{1}{4}$

There are at least two ways to approach the second question, one easy and one more standard. The easy way involves realizing that, for each way of arranging books 2 and 4, there are only two possibilities of the **relative positions** *of books 1 and 3—1 to the left of 3 and 3 to the left of 1. Thus, the probability at issue must be* $1/2$*. The more standard way can involve the following reasoning. First fix the positions of books 1 and 3, with book 1 put to the left of book 3. Then put one of the rest two books on the shelf. We know that there are only three possible positions for this book—to the left of 1, between 1 and 3, and to the right of 3. After this book is put down, let us put the fourth book on the shelf. We know that there are four possible positions for this book—to the left of the three already put on, between the left most two of all three already put on, between the right most two of all three already put on, and to the right of all three already put on. Thus, the number of ways to put all books in the event that the book labeled "1" is put to the left of the book labeled "3"is* $3 \times 4 = 12$*. We already know that the total number of ways to put all four books freely is 24. Thus, the probability at issue is* $1/2$.

A11. Suppose we have the following probability distribution for random variable X :

x	0	1	2	3	4
$P(X = x)$	0.1	0.3	0.3	0.2	0.1

Calculate $E[X]$, $E[X+1]$, $E[X^2+1]$, $E[(X+1)^2]$, $E[(X-1)^2]$.

Answer

$$E[X] = \sum_{x_i} x_i P(X = x_i) = 1.9,$$

$$
\begin{aligned}
E[X+1] &= E[X]+1 = 1.9+1 = 2.9, \\
E[X^2+1] &= E[X^2]+1 = \sum_{x_i} x_i^2 P(X=x_i)+1 = 4.9+1 = 5.9, \\
E[(X+1)^2] &= E[X^2+2X+1] = E[X^2]+2E[X]+1 = 5.9+2\times 1.9+1 = 10.7, \\
E[(X-1)^2] &= E[X^2-2X+1] = E[X^2]-2E[X]+1 = 5.9-2\times 1.9+1 = 3.1,
\end{aligned}
$$

A12. A fair coin is flipped and we are recording the number of heads

(a) How many flips would guarantee of 99% chance of at least one head.

Answer

The probability of the event that "there is at least one head in n coin flips "is 1 minus the probability of its complement event that "there is no head in n coin flips ". Thus, we want to know the n such that $1-(\frac{1}{2})^n \geq 0.99$. *This is equivalent to* $(\frac{1}{2})^n \leq 0.01$. *Taking logarithms, we get* $n\ log(0.5) \leq log(0.01)$, *or* $n \geq \frac{log(0.01)}{log(0.5)}$, *remember that the logarithm of a number between 0 and 1 is negative. Using our calculator, we find that* $n \geq 6.6439$. *Thus, there has to be 7 or more flips to guarantee 99% chance of at least one head all the flips. However, full marks is given for brute force method:*

$$
\begin{aligned}
P(H) &= .5 \quad 1flip \\
P(H) &= \frac{3}{4} \quad 2flips \\
P(H) &= 1-(\frac{1}{2})^3 \quad on3flips \\
& andsoon
\end{aligned}
$$

(b) What is the expected number of heads in 1 flip? In 2 flips? In 3 flips. Can you see a pattern between the number of flips and the probability of a single head that gives the expected number of heads in say 50 flips (do not mechanically count this, think of what you have found with 1, 2 and 3 flips!)

For 1 flip, the result may be 1 H with probability 0.5 or T (0 H) with probability 0.5, thus, the expected number of H is $1\times 0.5+0\times 0.5 = 0.5$.

For 2 flips, potential outcome could be one of (H,H), (H,T), (T,H) (T,T), *and each outcome has probability 0.25. The number of of H in each outcome is 2, 1, 1, and 0, respectively. Thus, the expected number of H is* $2.25+1\times 0.25+1\times 0.25_0\times 0.25 = 1$.

For 3 flips, there are 8 potential outcomes, (H,H,H), (H,H,T), (H,T,H), (H,T,T),

(T,H,H), (T,H,T), (T,T,H), (T,T,T), *each with probability 0.125. The number of* H *in each outcome is 3, 2, 2, 1, 2, 1, 1, and 0, respectively. Thus, the expected number of* H *is* $(3+2+2+1+2+1+1+0) \times 0.125 = 1.5$.

We notice that as sample size increases this method becomes less handy. A much better way could be the following. We know that these coin flips are independent with each other. For the ith *flip in a total of* n *flips, let the outcome of* H *or* T *be denoted by a random variable* X_i *which is equal to 1 if* H *shows up and is equal to 0 if* T *shows up. Then, to count the number of* H *in an outcome such as those listed above, we just need to count* $X_1 + X_2 + \cdots + X_n = \sum_{i=1}^{n} X_i$. *Applying the familiar formula for expectation of the sum of several random variables, we know that* $E(\sum_i X_i) = E(X_1 + X_2 + \cdots + X_n) = E(X_1) + E(X_2) + \cdots + E(X_n)$. *However, we know that all* X_i *have identical statistical properties, with* $E(X_i) = 0.5$ *for all* i. *Thus,* $E(X_1) = E(X_2) = \cdots = E(X_n) = 0.5$. *You then know that* $E(\sum_i X_i)$, *the expected number of* H, *is equal to* $n \times 0.5$.

PART B (Long Answer worth 40%.)

B1. A small physicians' clinic has 3 doctors. Dr. Sarabia sees 41% of the patients, Dr. Tran sees 32% and Dr. Jackson the remainder. Dr. Sarabia requests blood tests on 5% of her patients, Dr. Tran requests blood tests on 8% of his patients and Dr. Jackson requests blood tests on 6% of her patients.An auditor randomly selects a patient from the past week and discovers that patient has a blood test as a result of a physician visit.

(a) Explain what is Bayes' theorem, a prior, the posterior in the context of knowing this information, how does this change the probability that the patient saw Dr. Sarabia?

Answer

Denote the event that "patient saw Dr. Sarabia "by S, the event that "patient saw Dr. Tran "by T, and the event that "patient saw Dr. Jackson "by J. Denote the event that "blood test is requested "by t.

In the context of this question, the Bayes' Theorem says that:

$$P(S \mid t) = \frac{P(S \cap t)}{P(t)} = \frac{P(S)P(t \mid S)}{P(S)P(t \mid S) + P(T)P(t \mid T) + P(J)P(t \mid J)}.$$

The prior is the unconditional probability $P(S)$, which can be seen as the initial belief about the likelihood of event S. The posterior is the conditional probability $P(S \mid t)$, which can be seen as the updated belief about the likelihood of event S given that another event, t, has occurred.

We know from the question that:

$$P(S) = 0.41,\ P(T) = 0.32,\ P(J) = 1 - 0.41 - 0.32 = 0.27,$$

$$P(t \mid S) = 0.05,\ P(t \mid T) = 0.08,\ P(t \mid J) = 0.06.$$

Substitute these into the formula above, we get $P(S \mid t) = 0.32905 < P(S) = 0.41$.

(b) For what percentage of all patients at this clinic are blood tests requested?

This percentage is given by the denominator in the formula in part (a), $P(S)P(t \mid S) + P(T)P(t \mid T) + P(J)P(t \mid J) = 0.0623$.

(c) Blood tests come back with false positives (indicate blood problem when there is none) 5% of the time and false negatives (indicate there is no problem when there is one) 1% of the time. Suppose we know that 95% of the people are healthy. Denote the event that "person is healthy " by H and the event that "test is positive " by P. Write out the probability distribution for health and blood testing.

This question is conceptionally easy but numerically more challenging!

Denote the event that "person is not healthy "by $\bar{H}$, and the event that "test is negative "by $\bar{P}$.

We are given this information: $P(H \mid P) = 0.05$, $P(\bar{H} \mid \bar{P}) = 0.01$, $P(H) = 0.95$, *and* $P(\bar{H}) = 0.05$.

The joint probability distribution table for health and blood would take the following form:

	P	$\bar{P}$
H	p_1	p_3
$\bar{H}$	p_2	p_4

where the p's are joint probabilities of the intersections of events, to be calculated soon.

From the information given, we know

$$\begin{aligned} p_1 + p_2 + p_3 + p_4 &= 1 \quad (1) \\ P(H) &= .95 = p_1 + p_3 \quad (2) \\ P(H|P) &= .05 = \frac{P(H \cap P)}{P(P)} = \frac{p_1}{p_1 + p_2} \quad (3) \\ P(\bar{H} \quad &| \quad \bar{P}) = 0.01 = \frac{P(\bar{H} \cap \bar{P})}{P(\bar{P})} = \frac{p_4}{p_3 + p_4} \quad (4) \end{aligned}$$

This is 4 eqautions and 4 unknowns

¿From (2) and 1 we have

$$\begin{aligned} p_2 + p_4 &= .05 \\ p_4 &= .05 - p_2 \\ .05 \times (p_1 + p_2) &= p_1 from(3) \Longrightarrow p_2 = \frac{.95}{.05} p_1 \\ .01 \times (p_3 + p_4) &= p_4 from(4) \Longrightarrow p_4 = \frac{.99}{.01} p_3 \end{aligned}$$

$$\begin{aligned} p_1 + p_2 + p_3 + p_4 &= 1 \\ .95 &= p_1 + p_3 \\ \frac{p_1}{p_1 + p_2} &= .05 \\ \frac{p_4}{p_3 + p_4} &= .01 \end{aligned}$$

Solution is: $[p_1 = 2.1277 \times 10^{-3}, p_2 = 4.0426 \times 10^{-2}, p_3 = 0.94787, p_4 = 9.5745 \times 10^{-3}]$ Obviously this is messy and most marks are given for showing the set-up.

(d) What is the probability that patient in (a) was Dr. Sarabia's **and** had a true negative (did not have the blood problem and the test indicated no problem)?

$$P(S \cap t) \cap (H \cap \bar{P})) =?$$

This is a conceptionally difficult problem!. We need to make an assumption regarding Dr. S and the patients he see., we can assume that $P(H|S) = P(H)$ The doctor does not have control over who walks through the door. But of those coming through Dr. S's door who does he send to testing. The problem is silent on this. There are two easy alternatives:

(i) The easiest way to answer this problem is to assume the doctor randomly selects patients (not related to the health of the patient) to send to testing (not a great doctor I agree) so that

$$P(S \cap t) \cap (H \cap \bar{P}) = P(S \cap t) \times P((H \cap \bar{P}) = .0205 \times 9.5745 \times 10^{-3} = 1.9628 \times 10^{-4}$$

since

$$P(S \cap t) \times P(= P(S) \times P(t|S) = .41 \times .05 = .0205$$

(ii) The other extreme is to assume all of her patients that are sick go for testing (She is an amazing doctor since she sends only $\bar{H}$ people to testing) P(($\bar{H} \cap t)|S) = 1$ In this case

$$P(S \cap t) \cap (H \cap \bar{P})) = 0$$

since there are no healthy people being tested.

Now of course the truth is likely to be some are healthy and some are sick and so the truth would be somewhere between the two answers.

(e) Is there any relationship between the *answers* from the blood testing and the doctor who sends out the tests. That is, what is the probability of Dr. Sarabia's patient given the blood test was a false positive?

There is NO relationship between the answer of the test and who sends it. But there is (hopefully) a relationship about a doctor's ability to detect unhealthy people. I did not expect anyone to really provide a concrete answer to this but rather to be descriptive

For instance in the random assignment case, the actual testing results are irrelevant (provides no new information) so it is the same as (a). In the case where only sick patients are sent to testing the chances of a flase positive is zero (no healthy patients ever go!). In reality, the answer is somewhere in between.