

Economics 250 Mid-Term Test 1

13 February 2012

Instructions: You may use a hand calculator. Do not hand in the question sheet. Answer all **four** questions in the answer booklet provided. Show your work. Formulas and tables are provided after the question pages.

1. Imagine you collect the following data on GDP per capita (in thousands of USD) in a sample of 8 countries: 35, 12, 5, 17, 46, 5, 21, 27.

- (a) Find the mean, median, and mode.
- (b) Find the interquartile range.
- (c) Find the sample standard deviation.

2. Suppose that $x = 1$ is used to denote that a recession occurs and $x = 0$ that no recession occurs, in 2012. The variable y is 1 if there is a recession in 2013 and 0 otherwise. Here is a table of joint probabilities:

		x	
		0	1
y	0	0.6	0.1
	1	0.1	0.2

- (a) Are recessions in the two years independent? (*i.e.* Is y independent of x ?)
- (b) Given that there is a recession in 2012, what is the probability of a recession in 2013?
- (c) Find the conditional expectation $E(y|x = 0)$.

3. Suppose that the return, labelled x , on a domestic bond can take on the following values, with these probabilities:

x	Prob
2	0.3
3	0.5
4	0.2

- (a) Find the expected value and variance of x .
- (b) A foreign bond has a return, labelled y , with mean 4 and variance 2. The correlation between the two returns, x and y , is 0.6. Suppose investors divide their portfolios equally between the two bonds, so that the portfolio return is:

$$r = 0.5x + 0.5y.$$

Find the expected value (mean) and variance of the portfolio return.

4. Researchers study 10 schools to see if reducing class sizes improves students' test scores. Suppose that the probability of an improvement in any school is 0.60.

- (a) What is the probability that exactly 6 schools have improved scores?
- (b) What is the probability that 5 or more schools have improved scores?
- (c) What are the mean and variance of the number of schools with improved scores?

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1. (a) The mean is 21. The median is 19 (the average of 21 and 17). The mode is 5.
(b) The interquartile range is (8.5, 31), averaging the two nearest observations. An equally acceptable answer is (6.75, 33) which runs from position 2.25 to position 6.75.
(c) The sample standard deviation is 14.47 (the square root of the sample variance which is 209.42).

2. (a) No. The probability of a recession in both years is 0.2. That is not equal to the product of the two marginal probabilities, which is 0.09.

(b) $P(y = 1|x = 1) = 0.2/0.3 = 0.67$

(c) Using the two conditional probabilities given $x = 0$:

$$E(y|x = 0) = 1 \times 0.1/0.7 + 0 \times 0.6/0.7 = 0.142$$

3. (a) The mean is 2.9. The variance is 0.49.

(b) The mean of r is

$$E(r) = 0.5(2.9) + 0.5(4) = 3.45$$

and the variance is:

$$\sigma_r^2 = 0.25(0.49) + 0.25(2) + 2(0.5)(0.5)(0.6)(0.7)(1.4142) = 0.1225 + 0.5 + 0.2969 = 0.9194$$

4. (a) The probability that a school fails to improve is 0.4. If 6 improve then 4 fail to do so. Using table 2, that probability is 0.2508. (Or the binomial formula gives the same answer directly.)

(b) This is the probability that 5 or fewer fail to improve, which from table 3 is 0.834.

(c) The mean is 6 and the variance is 2.4.