Queen's University
Faculty of Arts and Sciences
Department of Economics
Economics 250 2005 Final

Instructions: 3 Hours

READ CAREFULLY. Calculators are permitted (no red stickers). At the end of the exam are several formulae and tables for the binomial, normal and t distributions. Answers are to be written in the examination booklet.

Please Note: Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer exam questions as written.

You are to answer **ALL** questions. **SHOW ALL YOUR WORK**. There are a total of 100 possible marks to be obtained. **Part A** has 12 questions worth at total of 60 marks. **Part B** has 1 question worth 40 marks.

PART A (60 marks) : Answer all 12 questions. Five (5) marks each for a total of 60 marks. .

1. There are two independent data such that the population

$$X_{1i} \sim NID(\mu, \sigma^2) with i = 1, \ldots n_1 \quad and X_{2j} \sim NID(\mu, \sigma^2) with j = 1, \ldots n_2$$

We wish to construct an unbiased estimate of the population mean μ , with the sample means $\bar{X}_1$ and $\bar{X}_2$,

$$a\bar{X}_1 + b\bar{X}_2$$

(a) Explain unbiasedness and state what are the restrictions on a and b for an unbiased estimator of μ?

Point estimator $\hat{\theta}$ is said to be an unbiased estimator of the parameter θ if the expected value, or mean, of the sampling distribution of $\hat{\theta}$ is θ; that is, $E(\hat{\theta} = \theta$. For the given estimator to be unbiased, we need

$$E(a\bar{X}_1 + b\bar{X}_2) = \mu.$$

That is,

$$E(a\bar{X}_1 + b\bar{X}_2) = aE(\bar{X}_1) + bE(\bar{X}_2) = a\mu + b\mu = (a + b)\mu = \mu.$$

Thus, we need $a + b = 1$.

(b) Explain minimum variance unbiased estimator, state what are the restrictions on a and b to give the minimum variance estimator.

Suppose there are several unbiased estimators of θ. Then the unbiased estimator with the smallest variance is said to be the most efficient estimator or to be the minimum variance unbiased estimator of θ. It is easy to note that the minimum variance estimator is the sample mean for all observations $n_1 + n_2$. This is obtained by taking the weighted means:

$$\bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$$

and therefore

$$a = \frac{n_1}{n_{1+}n_2} \quad b = \frac{n_2}{n_{1+}n_2}$$

2. Professor Gregory has made a mess of his midterm once again and has grades with the current properties (G_i) for a very large class

$$G_i \sim NID(60, 16)$$

and wants to consider transformations of the kind

$$a + bG_i$$

(a) Suppose Gregory wants the mean of the sampling distribution of the **sample average** of 70 with 100 students but leave the variance of the sample mean unchanged, what should a and b be?

For the mean of the sampling distribution of the sample average to be 70, we need . We know currently that

$$\bar{G}_{100} \sim N(60, \frac{16}{100})$$

Since we want to raise the average but leave the variance unchanged we can simply add 10 to all grades

$$\begin{aligned} 10 + G_i \quad a &= 10 \quad b = 1 \\ G' &\sim N(70, 16) \end{aligned}$$

.

(b) Find the probability of drawing a student at random with an average over 80% with the transformation used in (a).

$$\begin{aligned} P(G' &> 80) = P(\frac{G' - u_{G'}}{\sigma_{G'}} > \frac{80 - 70}{\sqrt{16}}) \\ P(Z &> 2.5) = 1 - F_Z(2.5) = 1 - .9938 = .0062 \end{aligned}$$

(c) Suppose Gregory wants a 70% average but wants to reduce the variance of the sample average by 25% what values should he now choose for a and b then

$$G_i' \sim N(70, 12)$$

we need that

$$\begin{aligned} b &= \sqrt{.75} \\ a + 60b &= 70 \Rightarrow a \approx \mathit{18.0385} \end{aligned}$$

3. A new survey of popularity for the Liberals of 1000 individuals suggest that there current support is 35%

(a) What is the 95% confidence interval for the population currently supporting the Liberals and interpret. State any assumptions that you are making.

Let π denote the population proportion and p the same proportion. Then the confidence interval for the population proportion are obtained from

$$p - Z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}} < \pi < p + Z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}},$$

where, for a 95% confidence interval, $\alpha = 0.5$, *so that*

$$\alpha/2 = 0.025 \qquad Z_{\alpha/2} \approx 1.96$$

from the standard normal distribution. It follows that $n = 1000$, $p = 0.35$, *and* $Z_{\alpha/2} = 1.96$. *Therefore, a 95% confidence interval for the population proportion is*

$$0.35 - 1.96\sqrt{\frac{0.35 \times 0.65}{1000}} < \pi < 0.35 + 1.96\sqrt{\frac{0.35 \times 0.65}{1000}},$$

or $(0.3204, 0.3796)$. *We may say that if we calculate a large number of confidence intervals that 95% of them will bracket the true population proportion* π

Underlying assumptions may include that individuals' opinions are independent from each other and that the sample size is large enough for the central limit theorem to be applicable.

(b) If the liberals need 40% to form the government, would a formal hypothesis test of this confirm they have it at the 5% level of significance? Explain

While we can do a formal hypotheiss test, we see that the 95% confidence interval does not contain the null hypothesis of $\pi = .40$, and therefore this null would be rejected at the 5% level of significance. A formal test is as follows: *The Liberals will be able to form the government if they have no less than 40% support in the population. Thus we formulate the null hypothesis as*

$$H_0: \ \pi \geq \pi_0 = 0.4$$

and the alternative as

$$H_1: \ \pi < 0.4.$$

The decision rule is to reject the null hypothesis in favour of the alternative if

$$\frac{p - \pi_0}{\sqrt{\pi_0(1-\pi_0)/n}} < -Z_\alpha.$$

For this question, the significance level α *is 0.05 and* $Z_\alpha \approx -1.645$. *The test statistic is*

$$\frac{p - \pi_0}{\sqrt{\pi_0(1-\pi_0)/n}} = \frac{0.35 - 0.40}{\sqrt{0.40(1-0.40)/1000}} = -3.227.$$

Since -3.227 is less then -1.645, we reject the null hypothesis and conclude that less than 40% of population supports the Liberals.

(c) With this sample we can say the survey is to within what percentage point 19 times out of 20?

$$B = 1.96\sqrt{\frac{0.35 \times 0.65}{1000}} \times 100\% = 2.956\%$$

4. Suppose a certain car is made only in Japan, Canada and the US, with production percentages of 30%, 20% and 50% respectively. Japan makes 85 percent of its cars with metric mileage and the US makes 90% of its cars with imperial mileage. Canada randomizes imperial measures or metric measures by flipping a fair coin. If we find a metric car

   (a) What is the chance it came from Canada?

   (b) What is the chance it came from the US?

   Denote the event that a car has metric mileage by M and the event that a car has an imperial mileage by I. Similarly, for J, C, and U. We know the following

$$P(J) = 0.3,\ P(C) = 0.2,\ P(U) = 0.5,$$

$$P(M|J) = 0.85,\ P(M|C) = 0.5,\ P(M|U) = 1 - 0.9 = 0.1.$$

   We want to know $P(C|M)$ in part (a) and $P(U|M)$ in part (b).
   Using the Bayes' rule, we know that

$$\begin{aligned} P(C|M) &= \frac{P(C) \times P(M|C)}{P(M)} \\ &= \frac{.1}{.405} = .247 \end{aligned}$$

   $: 0.246\,91 : 2.469\,1 \times 10^{-2}$

$$\begin{aligned} P(M) &= P(J) \times P(M|J) + P(C) \times P(M|C) + P(U) \times P(M|U) \\ &= .3 \times .85 + .2 \times .5 + .5 \times .1 = 405 \end{aligned}$$

   Similarly,

$$P(U|M) = \frac{0.5 * 0.1}{0.405} \approx 0.1235.$$

5. Suppose we have 100 years of data ($n = 100$) on per capita GNP (measured in US dollars) for Canada and the United States

$$\bar{X}_{Can} = \$55.1\ thousand \quad s_{Can} = 10.1 \qquad \bar{X}_{US} = \$65.8\ thousand \quad s_{US} = 25.4$$

(a) Discuss the differences of the sample estimates and their potential implications for the distribution of income in the two countries

The average per capita GNP for Canada over the 100 years is lower than that for the US, and the sample standard deviation is also smaller for the former than for the latter. These could mean that, on average over these 100 years, Canada's per capita GNP is lower but does not fluctuates as much as that of the US. With respect to the fluctuation, if both per capita GNP's are monotonically increasing over the years (that is, both economies are always growing), than the growth measured in per capita GNP terms is higher for the US than for Canada.

(b) Construct an appropriate hypothesis test that per capita income is the same at the 10% level assuming the variances are different. Explain all assumptions

If we assume that the two populations of per capita GNP from which the two samples are drawn are independent and normally distributed, and that the variances are different, we may apply the formula and conduct the following hypothesis test.

$$\begin{aligned} H_o &: u_{can} = u_{us} \\ H_a &: u_{can} \neq u_{us} \end{aligned}$$

The test statistic is

$$\begin{aligned} t &= \frac{\bar{X}_{Can} - \bar{X}_{US} - (u_{can} - u_{us})}{\sqrt{\frac{s^2_{can}}{100} + \frac{s^2_{US}}{100}}} = \frac{55.1 - 65.8}{\sqrt{\frac{10.1^2}{100} + \frac{25.4^2}{100}}} \\ &= -3.914 \end{aligned}$$

We can compare this to

$$t_{100+100-2,.05} \approx Z_{.05} = 1.65$$

Clearly we can rejct the null at the 10% level

6. A test using a one-sided alternative of the population proportion

$$\begin{aligned} H_0 &: \pi \leq .07 \\ H_a &: \pi > .07 \end{aligned}$$

with $n = 100$ yields a $p - value$ (prob-value) = .03.

(a) Would this indicate a rejection of the null hypothesis if you are testing at the 5% level of significance?

Yes, it would, as the p-value is less than the level of significance.

(b) Calculate the sample proportion from the information given.

Denote the population proportion by π_0.

The test statistic is

$$Z = \frac{p - \pi_0}{\sqrt{\pi_0(1-\pi_0)/n}}.$$

The decision rule is

$$Reject\ H_0\ if\ Z > Z_\alpha.$$

Again, by definition, the p-value is the minimum level of significance at which the test statistic will lead to a rejection, we must have

$$P(z > Z_{0.03}) = 0.03.$$

we can calculate that $p = Z_{0.03}\sqrt{\pi_0(1-\pi_0)/n} + \pi_0 \approx 1.89\sqrt{0.07(1-0.07)/100} + 0.07 \approx 0.1182$.

7. This year we have studied a number of continuous distributions and this questions asks you to think of the differences

(a) Suppose X is a uniform distribution over the interval $U[-1.73, 1.73]$. What is its mean and variance (formulae are at the end of the exam). Find two numbers c and d symmetric about the mean of X such that

$$P(c < X < d) = .95$$

(b) Suppose $X \sim N(0,1)$ find two numbers e and f symmetric about the mean of X such that

$$P(e < X < f) = .95$$

(c) Assume $X \sim t_9$ find two numbers g and h symmetric about the mean of X such that

$$P(g < X < h) = .95$$

(d) Explain the differences you have with (c, d), (e, f) and $(g,\ h)$. Use a diagram to assist the explanation.

(a) Using the formulae, we know that

$$E[X] = \frac{a+b}{2} = 0$$

and

$$V[X] = \frac{(b-a)^2}{12} = \frac{(3.46)^2}{12} = 0.9976\dot{3}.$$

Since $0.95 * 1.73 = 1.64375$ *and the distribution is symmetric about the mean,* $c = -1.6435$ *and* $d = 1,6435$.

(b) From the normal table, we find $Z_{0.975} \approx 1.96$. *Thus,* $e \approx -1.96$ *and* $f \approx 1.96$.

(c) From the t distribution table, we find $t_{9,0.025} \approx 2.262$. *Thus,* $g \approx -2.262$ *and* $h \approx 2.262$.

(d) The "probability mass "of the t distribution is less concentrated about the mean than that of the standard normal distribution with the same mean. Figure omitted.

8. One researcher uses a significance level of $\alpha = .075$ and another researcher uses $\alpha = .05$. Suppose the two have independent samples and are lucky enough to test their hypothesis for situations where the null hypotheses **are always true**.

   (a) Find the ratio of the number of tests ($\frac{n_1}{n_2}$) the researchers need to do so that they have the same expected number cases where they do not reject the null hypothesis?

   We are assuming that the null hypothesis is in fact true. Then, for a large number n_1 *of tests by researcher 1, the expected number of cases where researcher 1 does not reject the null hypothesis will be* $(1 - \alpha_1)n_1 = 0.925n_1$. *Similarly, for a large number* n_2 *of tests by researcher 2, this expected number of cases where researcher 2 does not reject the null hypothesis will be* $(1 - \alpha_2)n_2 = 0.95n_2$. *For these two expected number to be equal, we need* $0.925n_1 = 0.95n_2$, *or* $n_1/n_2 = 0.95/0.925 = 1.\dot{0}2\dot{7}$. *That is, the number of tests has to satisfy this ratio.*

   (b) For the same data set and the same null hypothesis which is presumed to be true, what is the chance that the researchers disagree on their conclusion of the null?

   Since the data set and the null hypothesis are the same for the two researchers, they forms of the decision rules used by them will be the same too. Then, they will disagree only if the test statistic is high enough in absolute value to lead to reject for one researcher but not so for the other. This happens when the p-value associated with the test statistic is between the two significance levels. Thus, the probability at issue is $0.075 - 0.05 = 0.025$.

9. At a recent dog show big dogs were judge to be attractive and black dogs we judge to be less attractive. Mixtures of size and colour were always judged by their most attractive feature. Suppose at a recent dog show we had 15 big black dogs, 25 big white dogs, 30 small white dogs, and 10 small black dogs.

   (a) What is the probability of a dog taken at random being judged attractive?

   (b) Are colour and size independent?

   (a) Roughly speaking, we have a 2-by-2 joint distribution. With obvious notations, we want to calculate $P(BIG \cup WHI)$. *This is equal to* $P(BIG) + P(WHI) - P(BIG \cap WHI)$ *or* $1 - P(SMA \cap BLA)$, *and is 0.875.*

(b) Pick any entry in the 2-by-2 joint probability table and check whether the joint probability is equal to the product of the two marginal probabilities. Let us choose $P(SMA \cap BLA) = 0.1$*. The product of marginal probabilities is* $P(SMA)P(BLA) = 0.5 \times 0.3125 = 0.15625 \neq 0.1$*. Thus, the two characteristics are not independent.*

10. Explain the central limit theorem and give an example as to how we use this in statistics.

    Page 227 of textbook. Key points may be "iid observations, as n gets larger, the distribution of the sample mean standardized approaches the standard normal distribution. "

    The example could be the sampling distribution of the sample mean or sample proportion.

11. Consider the following equation

$$R = 2X + 4Y \quad X \sim N(5, 100) \quad Y \sim N(10, 400) \quad \rho_{XY} = -.3$$

   (a) What is the distribution of R and why does it have this distribution?

   (b) Calculate $E[R]$ and $V[R]$.

   (c) What is the

$$P(45 < R < 55)?$$

(a) R is normally distributed, since it is a linear combination of two jointly normally distributed random variables.

(b)

$$\begin{aligned} E[R] &= E[2X + 4Y] = 2E(X) + 4E(Y) = 2 \times 5 + 4 \times 10 = 50. \\ V[R] &= V[2X + 4Y] = 4V(X) + 16V(Y) + 2 \times 2 \times 4 \times Cov[X, Y] \\ but Cov[X, Y] &= \rho_{XY} \times SD[X] \times SD[Y] \end{aligned}$$

$$V[R] = 4 \times 100 + 16 \times 400 - (2 \times 5 \times 4 \times .3 \times 10 \times 20) = 4400.$$

(c) This probability is

$$P(45 < R < 55) = P(\frac{45 - 50}{\sqrt{4400}} < \frac{R - \mu_R}{\sigma_r} < \frac{55 - 50}{\sqrt{4400}})$$

$$\approx P(-0.075 < Z < 0.075) \approx 2 \times 0.0319 = 0.0638.$$

12. About 85% of the people admitted to the bar perform well as lawyers. Suppose that 20 people were recently admitted to the bar

(a) What is the exact probability that 5 to 15 inclusively of recently admitted to the bar performed well as lawyers.

(b) Approximate your answer to (a) using a normal approximation. Be as accurate as you can. Is the approximation any good?

(a) This situation can be dealt with by a Binomial model, with success probability $\pi = 0.15$ *(we define success this way so as to use the cumulative Binomial probability table at the end of the textbook) and number of trials* $n = 20$. *We want to find the exact probability that 15 to 5 persons, inclusive, admitted do not perform well.*

The probability at issue is then

$$P(5 \le x \le 15|n = 20,\ \pi = 0.15)$$

$$= P(x \le 15|n = 20,\ \pi = 0.15) - P(x \le 4|n = 20,\ \pi = 0.15) = 1 - .83 = 0.17.$$

(b) The mean of the approximating normal distribution is $n\pi = 20 \times 0.85 = 17$, *and the variance is* $n\pi(1-\pi) = 2.55$.

Since $n\pi(1-\pi)$ *is even less than 5, the approximation is perhaps not good. To be as accurate as we could, we use the continuity correction factor. Then the probability is*

$$P(4.5 < X < 15.5) \approx P(\frac{4.5 - 17}{\sqrt{2.55}} < \frac{X - n\pi}{\sqrt{n\pi(1-\pi)}} < \frac{15.5 - 17}{\sqrt{2.55}})$$

$$= P(-7.8278 < Z < -0.93934) \approx 0.1736 - 0 = 0.1736.$$

Evidently we did pretty well with this approximation!

PART B : Answer this Question which is worth 40 points

13 As a recent graduate of Econ 250 you are given the following sales data for the company's 9 independent sales districts (measured in millions of dollars)

$$33, 38, 18, 27, 25, 29, 22, 30, 29$$

(a) We wish to test that the population mean is 27 million, set up this test formally

(b) Calculate the test at the 1% level of significance and state your conclusion. Is there any issues with your distributional assumption?

(c) Calculate the 99% confidence interval and interpret.

(d) Approximate the p-value using the normal tables.

(e) Calculate the probability of a type II error at the alternative 28 million (use the normal distribution to approximate this).

(f) Discuss briefly the nature of Type I and II errors and how they relate specifically to this question.

(a) Suppose the population mean is μ_0. Denote the sample mean of 9 observations by $\bar{X}$ and the sample variance by s^2.

In formulating hypotheses, we would set the null hypothesis such that we reject it only if we have strong evidence against it. Based on the question, we can set the null hypothesis as

$$H_0: \ \mu_0 = 27$$

and the alternative hypothesis as

$$H_1: \ \mu_0 \neq 27.$$

This is a two-sided test. Suppose that the significance level is specified to be α. Then the decision rule would be

$$Reject\ H_0\ if\ \frac{\bar{X} - \mu_0}{s/\sqrt{n}} > t_{n-1,\alpha/2}\ or\ if\ \frac{\bar{X} - \mu_0}{s/\sqrt{n}} < -t_{n-1,\alpha/2}.$$

where $t_{n-1,\alpha/2}$ is the number for which

$$P(t_{n-1} > t_{n-1,\alpha/2}) = \alpha/2.$$

(b) The sample mean is $\bar{X} = 27.\dot{8}$. The sample standard deviation is $s = \sqrt{34.611} = 5.8831$. The test statistic is

$$\frac{27.8889 - 27}{5.8831/\sqrt{9}} = 0.45327.$$

On the other hand, $t_{8,0.005} = 3.355$. Thus, we cannot reject the null hypothesis.

The distributional assumption of individual district sales seems to matter, because the sample size is not large.

First, from sections 8.3 and 9.3, we know that the sampling distribution of the sample mean, $\bar{X}$, is an exact normal distribution if the underlying population distribution is normal; under that circumstance, the test statistic $(\bar{X} - \mu_0)/(\sigma/\sqrt{n})$, calculated when the population variance is known, follows the standard normal distribution, and the test statistic $(\bar{X} - \mu_0)/(s/\sqrt{n})$, calculated when the population variance is unknown and has to be substituted for by the sample variance, follows an exact t-distribution.

Second, from the example on page 326 in section 9.3 and the description of central limit theorem in chapter 7, we know that when the sample size is large, the sampling distribution of the sample mean is approximately normal, no matter what the underlying population distribution is. Then the test statistic $(\bar{X} - \mu_0)/(s/\sqrt{n})$ *could be treated as following a t-distribution with* $n - 1$ *degrees of freedom.*

However, when the sample size is not large, the central limit theorem may not apply well, the sampling distribution of the sample mean may not be normal, and the test statistic above may not follow a t-distribution.

Thus, it might be better if we explicitly assume that the population distribution is normal when answering at least this part.

(c) Under the assumption in part (b), the random variable

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

follows a t-distribution with $n - 1$ *degrees of freedom. We have already found in part (b) that* $t_{8,0.005} = 3.355$. *Thus, a 99% confidence interval is given by*

$$\bar{X} - t_{n-1,\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{X} + t_{n-1,\alpha/2}\frac{s}{\sqrt{n}},$$

or

$$27.8889 - 3.355 \times \frac{5.883}{\sqrt{9}} < \mu < 27.8889 + 3.355 \times \frac{5.883}{\sqrt{9}},$$

or $21.3096 < \mu < 34.4682$.

The interpretation could be the following. If, over time, independent, random samples of the sales of these 9 districts are repeatedly taken and confidence intervals for each of these samples constructed, then over a very large number of such exercises, 99% of the confidence intervals will contain the value of the true mean sales. Nevertheless, in practice one does not repeatedly draw such independent samples.

(d) Because the test statistic used in part (a) follows a t-distribution, originally we should refer to the table for t-distribution to find the p-value. But the t-distribution table may not include the quantile corresponding to the computed value of the test statistic and we have to approximate the t_{n-1} *distribution by the normal distribution and use the standard normal table. Recall that the test statistic is 0.45327. We look up in the standard normal table and found that* $1 - 0.6747772 \approx 0.3252$. *Since we are doing a 2-sided test we:* $2\times$ $.3252.= 0.650\,4$

(e) Type II error is the probability of failing to reject a false null hypothesis.

Recall from parts (a) and (b) that our decision rule is

$$\textit{Reject } H_0 \textit{ if } \frac{\bar{X} - \mu_0}{s/\sqrt{n}} > t_{n-1,\alpha/2} \textit{ or if } \frac{\bar{X} - \mu_0}{s/\sqrt{n}} < -t_{n-1,\alpha/2}.$$

This can be written as

$$\text{Reject } H_0 \text{ if } \bar{X} < \mu_0 - t_{n-1,\alpha/2}\frac{s}{\sqrt{n}} \text{ or if } \bar{X} > \mu_0 + t_{n-1,\alpha/2}\frac{s}{\sqrt{n}}.$$

With $\alpha = 0.01$, $t_{8,0.005} = 3.355$, *and "*$H_0: \ \mu_0 = 27$*", the decision rule becomes*

$$\text{Reject } H_0 \text{ if } \bar{X} < 27 - 3.355 \times \frac{5.883}{\sqrt{9}} = 20.4207 \text{ or if } \bar{X} > 27 + 3.355 \times \frac{5.883}{\sqrt{9}} = 33.5793.$$

Under the specific alternative hypothesis "$H_1: \ \mu_0 = 28$*", the sampling distribution of the sample mean is a* t*-distribution that has the same shape (PDF) as that under the null hypothesis* $H_0: \ \mu_0 = 27$ *but is centered around 28 instead of 27. Under this alternative hypothesis, the probability that the sample mean falls within the non-rejection interval* $[20.4207, \ 33.5793]$ *is*

$$P(20.4207 < \bar{X} < 33.5793) = P\left(\frac{20.4207 - 28}{s/\sqrt{n}} < \frac{\bar{X} - 28}{s/\sqrt{n}} < \frac{33.5793 - 28}{s/\sqrt{n}}\right)$$

$$\approx P(-3.8650 < t_8 < 2.8451) \approx 1 - 0.0001 - 0.00225 = 0.99765.$$

(The approximation does not give a reasonable answer, because usually type II error would not exceed 1 minus the significance level, or 0.99 in this question. The excess of 0.00765 seems to come from the approximation.)

(f) When the null hypothesis is true, the test statistic still has some chance to fall outside the acceptance region specified by the decision rule and consequently a true null hypothesis is wrongly rejected. This probability is the type I error and is also called the significance level. In our question, this probability is 0.01 and is the probability that the test statistic $\frac{\bar{X}-\mu_0}{s/\sqrt{n}}$ *falls outside the acceptance region* $[-3.355, \ 3.355]$ *when the population mean is indeed 27.*

When the null hypothesis is false and some specific alternative hypothesis is in fact true, there is still some probability that the test statistic falls within the acceptance region specified by the decision rule and consequently we fail to reject a false null hypothesis. This probability is the type II error, and 1 minus this probability is called the power of the test. In our question, this probability is 0.99765 and is equal to the probability that the test statistic $\frac{\bar{X}-\mu_0}{s/\sqrt{n}}$ *falls within the acceptance region* $[-3.355, \ 3.355]$ *when the population mean is indeed 28.*