

QUEEN'S UNIVERSITY
FACULTY OF ARTS AND SCIENCE

APRIL 2012

ECONOMICS 250
Introduction to Statistics

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Instructions:

The exam is three hours in length.

Do all ten (10) questions.

Be sure to show your calculations and intermediate steps.

Put your student number on each answer booklet.

Formulas and tables are printed at the end of this question paper.

You may use a hand calculator. Allowed calculators include those with blue or gold stickers, the Casio 991, the Sharp EL376S, or other non-programmable calculators. No red-sticker calculators or other aids are allowed.

Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer the exam questions as they are written.

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1. A researcher is studying percentage scores of 30 people who have written an aptitude test. She sees 15 scores of 70%, 9 scores of 76%, and 6 scores of 81%.

- (a) Find the median and mode of these scores.
- (b) Find the sample mean and the sample standard deviation of these scores.

2. A bond can pay a coupon of either \$4 or \$2 in any year. In any year the (marginal or unconditional) probability of each outcome is 0.5. But, if the bond pays \$4 this year, then the probability that it pays \$4 next year is 0.6.

- (a) What is the probability that it pays \$2 next year conditional on paying \$4 this year?
- (b) What is the expected value of next year's coupon payment given that this year's payment is \$4? What would be the expected value of next year's payment if this year's payment were \$2?
- (c) Suppose you forget what happened this year. If you eventually observe that it pays \$4 next year then what is the probability that it paid \$4 this year?
- (d) What is the correlation between the payments in the two years?

3. The growth rate of income per capita, labelled \dot{y} , equals the growth rate of national income, labelled \dot{Y} , minus the growth rate of population, labelled \dot{x} , so:

$$\dot{y} = \dot{Y} - \dot{x}.$$

Suppose national income growth has mean 2 and standard deviation 1. Population growth has mean 1 and standard deviation 0.4. The correlation between national income growth and population growth is 0.5.

- (a) What is the mean of \dot{y} ?
- (b) What is the standard deviation of \dot{y} ?

4. Suppose that waiting times in a medical office are continuously and uniformly distributed between 1 minute and 13 minutes.

- (a) What is the probability of waiting more than 10 minutes?
- (b) What is the probability of waiting exactly 9 minutes?
- (c) What is the standard deviation of the waiting time?
- (d) Suppose you believe that the opportunity cost of a trip to this office consists of a fixed component of \$15 and a variable component of \$0.5 dollars per minute. Find the mean and variance of the cost.

5. Suppose that years of education in a population are normally distributed with mean 12 and variance 1.96.

- (a) Find the probability that an individual has more than 13 years of education.
- (b) Find the probability that a group of 9 people have an average education of more than 13 years.
- (c) An economist estimates that individual income, y , is related to individual education, x , like this:

$$y = 24 + 5(x - 12).$$

What is the variance of income? What is the probability that an individual has income greater than 38?

6. Researchers test a medical treatment on a sample of $n = 100$ people and find that it is successful in 52 cases.

- (a) Test the null hypothesis that the true success rate is 50% against the alternative that it is greater than 50%, with $\alpha = 0.10$.
- (b) What is the p -value for your test statistic?
- (c) If the sample success rate continues to be 52%, how large a sample would you need for the p -value to fall to 10%?

7. Suppose that real income per capita, denoted x , is measured in thousands of dollars. A researcher studies a random sample of 61 people and finds a sample average of $\bar{x} = 30$ and a sample standard deviation of $s = 6$.

- (a) Construct a 95% confidence interval for the population average income.
- (b) A different researcher claims that the population mean income is 31. Test that claim, with $\alpha = 0.05$, against the alternative that average income is less than 31.

8. Economists interview 100 members of the labour force and find that 10 of them are unemployed.

(a) The economists suggest a confidence interval of (0.0349, 0.1651) as an interval estimate of the proportion of the labour force population that is unemployed. What is the confidence level associated with this interval?

(b) At the 10% level of significance, test the null hypothesis that the population unemployment rate is 0.12 against the alternative hypothesis that it is less than 0.12.

(c) Suppose that you test the null hypothesis using the critical value in part (b). But, in fact, the true value of the unemployment rate in the population is an alternative value: 0.09. Find the power of the test and the probability of type II error.

9. Development economists interview 100 people in West Bengal and find that 75 are literate in Hindi. They interview 81 people in Bihar and find that 50 are literate in Hindi.

(a) Find a point estimate of the difference between the two literacy rates.

(b) Construct a 90% confidence interval for the difference in literacy rates between the two populations.

10. Sixteen people write a test and receive an average score, in percentage points, of 60. They then take a training course and re-write the test, receiving an average score of 66. The standard deviation of the improvement in their scores is 4. Find a 90% confidence interval for the average test-score improvement in the population that is caused by the training course.

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1. (a) The mode is 70. The median is 73; the average of the scores in positions 15 and 16.
(b) The sample mean is

$$\bar{x} = 0.5(70) + 0.3(76) + 0.2(81) = 74$$

The sample variance is:

$$\frac{1}{29}[15(70 - 74)^2 + 9(76 - 74)^2 + 6(81 - 74)^2] = 19.655$$

so the sample standard deviation is $s = 4.433$.

2. (a) That probability is $1 - 0.6 = 0.4$.
(b) The expected value is $4(0.6) + 2(0.4) = 3.20$. If, instead, today's payment were \$2 then the expected value of next period's payment would be \$2.80
(c) The joint probability table is completely symmetric so this too is 60%.
(d) A chart of the 4 possible outcomes with joint probabilities shows the two standard deviations are 1. Thus the correlation is equal to the covariance, which is 0.2.

3. (a) The mean is $2 - 1 = 1$.

(b) The variance is:

$$\sigma^2 = 1^2 + 0.4^2 - (2)(0.5)(1)(0.4) = 0.76$$

so the standard deviation is $\sigma = 0.8717$.

4. (a) 0.25 or 25%

(b) 0, because the time is distributed continuously

(c) The variance is 12 so the standard deviation is 3.46 minutes.

(d) We have

$$y = 15 + 0.5x,$$

so the mean cost is $15 + .5(6) = 18.5$, and the variance is $0.5^2(12) = 3$.

5. (a)

$$z = \frac{13 - 12}{1.4} = 0.714,$$

so from table 1 the probability is 0.2389 or 23.89%. (You will get a slightly lower number if you interpolate between 0.71 and 0.72 in table 1.)

(b) Now

$$z = \frac{13 - 12}{1.4/3} = 2.1428,$$

so the probability is 0.0162 or 1.62%.

(c) You can see that $y \sim N(24, 49)$. Thus

$$z = \frac{38 - 24}{7} = 2.00,$$

so the probability is 0.0228 or 2.28%.

6. (a) Our test statistic is

$$z = \frac{0.52 - 0.50}{\sqrt{0.5^2/100}} = 0.4,$$

so we obviously do not reject, since the critical value is 1.282.

(b) The p -value is 0.3446 or 34.46%.

(c) We have:

$$z = \frac{0.52 - 0.50}{\sqrt{0.5^2/n}} = 1.282,$$

which gives $n = 1027.20$ or 1028 people.

7. (a) With $df = 60$ the t -value we need is 2.00, so the confidence interval is:

$$30 \pm 2 \cdot 6/7.810 = (28.463, 31.536)$$

(b) Our test statistic is:

$$t = \frac{30 - 31}{0.768} = -1.30.$$

With 60 degrees of freedom the one-sided critical value is -1.671. We do not reject the null hypothesis.

8. (a) The standard deviation is 0.03. The margin of error is 0.0651. Thus $z_{\alpha/2} = 2.17$. From table 1, that means the confidence level is 97%.

(b) We construct the standard deviation using the hypothesized value of P , which gives 0.03249. Thus the test statistic is

$$z = \frac{0.10 - 0.12}{0.03249} = -0.6155.$$

The one-tailed critical value is -1.28 so we do not reject the null hypothesis.

(c) The critical value is 0.07842. When the true value is 0.09 the standard deviation is 0.0286, so our dividing line in z -space is:

$$z = \frac{0.07842 - 0.09}{0.0286} = -0.404.$$

That means, from table 1, the probability of type II error is 0.655 or 65.5%. The power is 0.345 or 34.5%.

9. (a) The point estimate is $0.75 - 0.61728 = 0.1327$.

(b) The two variances are 0.001875 and 0.002916 so their sum is 0.004791 and the standard deviation of the sum is 0.0692. The 90% confidence interval is:

$$0.1327 \pm 1.645(0.0692) = (0.01886, 0.2465).$$

10. Our CI is:

$$6 \pm 1.753(4/4) = (4.247, 7.753)$$

using table 8 with 15 degrees of freedom.

Weights were 5 per question except 10 on question 2, 5, 6, and 8.