QUEEN'S UNIVERSITY
FACULTY OF ARTS AND SCIENCE

APRIL 2010

ECONOMICS 250
Introduction to Statistics

Instructor: Gregor Smith

Tutors: Michael Haymes, Jier Pan

Instructions:

The exam is three hours in length.

Do all nine (9) questions.

Be sure to show your calculations and intermediate steps.

Put your student number on each answer booklet.

Formulas and tables are printed at the end of this question paper.

You may use a hand calculator. Allowed calculators include those with blue or gold stickers, the Casio 991, the Sharp EL376S, or other non-programmable calculators. No red-sticker calculators or other aids are allowed.

Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer the exam questions as they are written.

1. A random variable X can take on the values 1, 2, 3, 4, or 5 in its population, each with the same probability.

(a) Find the mean and variance of X.

(b) A second variable is given by $Y = 10 - 2X$. Find the mean and variance of Y.

(c) What is the correlation coefficient between X and Y?

2. The probability of being unemployed conditional on being a university graduate is 0.05. The probability of being unemployed conditional on being a non-graduate is 0.15. The probability of anyone graduating from university is 0.45.

(a) What is the probability of being a university graduate and being unemployed?

(b) What is the marginal probability of anyone being unemployed?

(c) If you observe that someone is employed then what is the probability he or she graduated from university?

3. Suppose that competitive cyclists are each tested for banned drugs in 5 independent tests each year. Suppose that the probability of a cyclist taking banned drugs is $P = 0.20$.

(a) What is the probability of no positive tests?

(b) What is the probability of at least two positive tests?

4. Suppose that the exchange rate between the Canadian and US dollars is continuously and uniformly distributed this month with a minimum value of 0.97 US dollars and a maximum value of 1.01 US dollars.

(a) What is the mean value of the exchange rate?

(b) What is the variance of the exchange rate?

(c) What is the probability of an exchange rate greater than 1.00 US dollars?

5. Suppose that the heights of humans in the world are normally distributed with mean 1.5 metres and standard deviation 0.3 metres. Imagine taking repeated samples of 10 people and calculating the mean height $\overline{X}$ in each sample.

(a) What is the distribution of $\overline{X}$?

(b) In a specific sample what is the probability of finding a mean height above 1.6 metres?

6. A survey of 100 voters shows that 10 support the Green Party.

(a) Find a 90% confidence interval for the population proportion that support this party.

(b) Suppose larger surveys also find a sample proportion of 10%. How large would the sample have to be so that the margin of error in the 90% confidence interval is 2% (or 0.02 in terms of the proportion)?

7. A financial economist measures the interest rate on a one-month investment each month for $n = 100$ months. The sample mean is $\overline{X} = 0.5$ and the sample standard deviation is $s = 2$.

(a) Test the hypothesis that the population mean is zero, using a two-tailed test with $\alpha = 0.05$.

(b) What is the p-value associated with your test statistic? (*i.e.* the probability of a statistic this large in absolute value)

8. Suppose that scores on an aptitude test are normally distributed. Twenty (20) students take the test and receive an average grade of 65. They then take a preparatory course before repeating the test. When they write a second test their average grade is 71. The standard deviation of the difference in their grades is 3.

(a) Find the sample mean of the difference between their scores (*i.e.* the grade after the course minus the grade before). Also find the sample standard deviation of the mean difference.

(b) Find a 95% confidence interval for the population difference between the two means.

9. Suppose you sample 100 firms and find that a proportion 0.70 report profits.

(a) Using $\alpha = 0.05$ conduct a lower, one-tailed test of the hypothesis that the population proportion is 0.75 against the alternative hypothesis that it is less than 0.75. (Hint: Remember to find the standard deviation under the null hypothesis.)

(b) What is the critical value of $\hat{p}$ for this test? In other words, what is the value of $\hat{p}$ that corresponds to the critical value z_α you used in part (a)?

(c) Now suppose that the true population proportion is actually 0.72. What is the probability of type II error (failing to reject a false null hypothesis that $p = 0.75$)? In other words, what is the probability of finding a value of $\hat{p}$ above the critical value you found in part (b)?

Economics 250
Winter Term 2010
Final Exam Answer Guide

1. (a) The mean is $\overline{X} = 3$ and the variance is $\sigma_x^2 = 2$.

(b) The mean is $\overline{Y} = 10 - 2(3) = 4$ and the variance is $\sigma_y^2 = 2^2 \cdot 2 = 8$.

(c) There is a perfect negative relationship between the two variables so the correlation is -1.

2. (a)

$$P(U \cap G) = P(U|G)P(G) = 0.05 \cdot 0.45 = 0.0225$$

(b) Using the same method 0.0225 + 0.0825 = 0.105 or 10.5%

(c) Fill in the table of joint probabilities first. Then

$$P(G|NU) = \frac{P(G \cap NU)}{P(NU)} = \frac{0.4275}{0.895} = 0.4776$$

or 47.76%.

3. (a) From the binomial tables or formula the probability is 0.3277.

(b) From the cumulative binomial tables the probability is

$$1 - 0.737 = 0.263$$

4. (a) The mean is the midpoint: 0.99 US dollars.

(b) The variance is 0.000133 dollars or 0.013 cents US.

(c) The probability of a value at 1.00 or above is 0.25.

5. (a)

$$\overline{X} \sim N(1.5, \frac{0.3^2}{10}) \sim N(1.5, 0.009)$$

where we do not need the CLT because the population is normal.

(b)

$$z = \frac{1.6 - 1.5}{.0948} = 1.0548.$$

Using the mid-point of the values for 1.05 and 1.o6 gives 14.6% as the probability of a value above 1.6.

6. (a) The 90% CI is

$$0.10 \pm 1.645\sqrt{\frac{0.10(0.90)}{100}} = 0.10 \pm 1.645(0.03) = 0.10 \pm 0.04935,$$

or (0.05065, 0.14935).

(b) We need:

$$0.02 = 1.645\sqrt{\frac{0.10(0.90)}{n}}$$

so $n = 608.85$ so rounding up gives 609 people.

7. (a) The test statistic is:

$$t = \frac{0.5 - 0}{0.2} = 2.5$$

That is greater than 1.96 so we reject the null hypothesis that $\mu = 0$.

(b) From table 1 when the statistic is 2.5 the probability in the right tail is 0.0062 so the probability in both tails is 0.0124 or 1.24%.

8. (a) The mean difference is 6 and the sample standard deviation of the mean is $3/\sqrt{20} = 0.6708$.

(b) With 19 df the critical value is ± 2.093. The 95% confidence interval is:

$$6 \pm 2.306\frac{3}{\sqrt{20}} = 6 \pm 1.4039 = (4.5960, 7.4039)$$

9. (a) The standard deviation is 0.0433. So the test statistic is:

$$z = \frac{0.70 - 0.75}{0.0433} = -1.1547$$

that is above the critical value of -1.645 (for the one-tailed, 95% test) so we do not reject the null hypothesis that the population proportion is 0.75.

(b) The critical for z is -1.645 so

$$\hat{p}_c = 1.645(0.0433 + +0.75 = 0.6788$$

(c) When the true proportion is 0.72 the sample standard deviation is 0.0449. Standardizing:

$$\frac{0.6788 - 0.72}{.0449} = -0.917$$

so the probability of type II error is 0.8212.