

Economics 250

Counting Formulas

To calculate probabilities we sometimes need to count the number of equally likely things that can happen. Once we have the count, the probability will be 1 divided by that number.

Here are three cases. In each case, imagine the sample space has $n = 4$ elements: A, B, C, and D. And imagine someone is selecting $k = 2$ elements.

Case 1: An item can be chosen twice, and the order matters.

Outcomes: AA, AB, BA, ... DD for a total of 16 cases.

Formula:

$$n^k$$

(The idea is that there are n possibilities in k spots.)

Example: Phone numbers. (These are sometimes called permutations with repetition.)

Case 2: An item cannot be chosen twice, and the order matters.

Outcomes: AB, BA, AC, ... DC for a total of 12 cases.

Formula (permutations):

$${}_n P_k = \frac{n!}{(n-k)!}$$

Example: Awarding ranked prizes randomly.

Case 3: An item cannot be chosen twice, and the order does not matter.

Outcomes: AB, AC, AD, BC, BD, CD for a total of 6 cases.

Formula (combinations):

$${}_n C_k = \binom{n}{k} = \frac{n!}{(n-k)!k!}$$

Example: Choosing a delegation.

A simple reference:

<http://betterexplained.com/articles/easy-permutations-and-combinations/>

A slightly more complicated reference:

<https://www.mathsisfun.com/combinatorics/combinations-permutations.html>