Marriage Matching, Fertility, and Family Labor Supplies: An Empirical Framework*

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Abstract

This paper integrates the marriage matching model of Choo and Siow (2006) with the collective labor supply model of Chiappori (1988, 1992). The marriage matching model and the collective model of labor supply fit together without modification, and can be analyzed independently, as done in previous studies. In addition to marital matching between different types of individuals, the model allows matching that depends on whether the wife works. With information on at least two isolated marriage markets, one can identify the full sharing rule, as well as the preference parameters for single and married couples, from observations on labor supplies for couples in which both partners work. One can also derive a sharing rule from marriage market clearing that is a function of the sex ratio of singles, and the marriage wage premiums for men and women. Thus marriage market clearing introduces an over-identifying restriction on the sharing rule within the collective model for couples in which both partners work. In particular, one can test whether the sharing rule that rationalizes labor supplies in married couples arises as an equilibrium risk sharing outcome in the marriage market. Marriage market clearing is a necessary condition for identification of the sharing rule for couples in which the wife does not work. Finally, we introduce fertility decisions into the model, where agents choose fertility and marital status simultaneously and expenditures on children are a public good within the household.

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1 Introduction

> *Models that analyze bargaining within existing marriages can give only an incomplete picture of the determinants of the well-being of men and women. The marriage market is an important determinant of distribution between men and women. At a minimum, the marriage market determines who marries and who marries whom.*
>
> (Lundberg and Pollak 1996)

That the marriage market affects intra-household allocations is well established (for example, Angrist 2002; Chiappori, Fortin, and Lacroix 2002 (hereafter CFL); Francis 2005; Grossbard-Schechtman 1993, Seitz 2004). As Lundberg and Pollak suggest, empirical models of intra-household allocations in *existing* marriages are well developed. The next step is clear. How do we empirically investigate marriage matching with intra-household allocations?

This paper provides a partial answer to the above question. We develop an empirical framework for analyzing both marriage matching and the intra-household allocation of resources. The empirical framework deliberately minimizes *a priori* restrictions on observed behavior. We establish identification of all structural parameters analytically. In other words, identification is completely transparent. Our answer is partial because we assume that spouses have access to binding marital agreements and we ignore divorce.[1] We also ignore unobserved heterogeneity.

The formulation of the marriage market follows Choo and Siow (2006; hereafter CS). Utility is transferable and equilibrium transfers are used to clear the marriage market. This formulation is consistent with any observed marriage matching pattern in a single marriage market. Our collective model of intra-household allocations follows Chiappori's (1999) spousal risk sharing model and CFL's model of household labor supply. Households are affected by idiosyncratic non-labor income and wage shocks. Full family income is divided between spouses according to a sharing rule to obtain a private budget constraint for each

[1] There is an important literature which studies intra household allocations without binding marital agreements (E.g...).

member. Each spouse maximizes their own utility subject to their own realized private budget constraint. The sharing rule fully shares idiosyncratic risks between the spouses.

The main innovation in this paper is to integrate the collective model with CS. To integrate marriage matching with intra-household allocations, we assume that the equilibrium transfers that clear the marriage market are the sharing rules which determine intra-household allocations. This integration generates several new insights:

1. The two models fit together without modification. The integrated model can be analyzed separately as has been done to date. The CS marriage matching model can be studied without analyzing intra-household allocations and vice versa.

2. With two or more separate marriage markets, we can test whether the equilibrium transfers estimated from the marriage market are consistent with sharing rules estimated from spousal labor supplies for couples in which both spouses work. This over-identifying restriction is not available if the two models are investigated separately. For couples in which only the husband works, the imposition of marriage market clearing is a necessary condition for the identification of the full sharing rule. It is worth emphasizing here that introducing two or more marriage markets allows one to recover the entire sharing rule, which typically is only identified up to an additive constant.[2] Our identification strategy relies upon two assumptions: (i) marriage and labor market conditions, but not preferences, vary across marriage markets, and (ii) agents are exogenously assigned to marriage markets. Assuming common preferences, using multiple segmented markets to identify preference parameters is standard in the empirical hedonic market literature (for example, Brandt and Hosios, 1996; Epple, 1997; Ackerberg and Botticini, 2002).[3]

3. Our framework provides a convenient way to model marriage decisions in combination

[2]Exceptions include Vermeulen (2003), Browning, Chiappori, and Lewbel (2004), and Lise and Seitz (2004), where the entire sharing rule is recovered by imposing restrictions on the degree to which preferences differ across single and married households.

[3]Ekeland, Heckman, and Nesheim (2004) establish conditions under which hedonic models are identified from data on a single market in the case where data on prices are available. In our case, we do not observe prices.

with continuous labor supply decisions for men and both labor supply and participation for women.[4] A natural interpretation of this set-up is that individuals choose whether to enter 'specialized' (non-working wife) or 'non-specialized' (working wife) marriages. We allow preferences and marital production technologies to differ across specialized marriages and non-specialized marriages. The price we pay for this convenience is that we must assume the stochastic components of wages and non-labor income are observed only after marriage and labor force participation decisions are made.[5] Thus the female's participation decision depends on expected wages and non-labor incomes. Conditional on the female's participation decision, the labor supply decisions of all household members depend on actual wages and non-labor incomes.[6]

4. Our way of modelling participation can be used to incorporate other discrete choices in the model. One such decision, which is currently absent in collective models, is fertility. We show how endogenous fertility can be incorporated in the model as part of the marital matching process.

5. Marriage, in our model, serves two purposes. First, it allows for specialization in households where only one spouse works. Second, marriage allows for full income and wage risk sharing between spouses. We provide a characterization of efficient risk sharing over full income. Our characterization builds on that of Chiappori (1999), who considers risk sharing of non-labor and labor income. We show that for efficient *ex-ante* spousal risk sharing over full income, the sharing rule must be a constant fraction of full income.

We are indebted to a large literature. The study of intra-household allocations began with Becker's rotten kid theorem, the early work of Manser and Brown (1980) and McElroy and

[4]Blundell, Chiappori, Magnac and Meghir (2001), Vermeulen (2006), and Lise and Seitz (2004) present collective models in which one or both spouses do not work full time.

[5]In Iyigun (2005), individuals sort by actual wages. In our model, they sort by expected wages. We chose sorting by expected wages to reduce the number of distinct types of individuals in the marriage market. A small number of distinct types avoids the problem of thin cells when estimating marital matching.

[6]This assumption is analogous to the empirical practice of using predicted, as opposed to actual, wages in models of labor supply See, for example, Arrufat and Zabalza (1986) and Hoynes (1996) and ***. MaCurdy et al. (1990) point out that this approach is somewhat problematic as the budget constraints will be miss-specified.

Horney (1981) within a bargaining framework, and Chiappori (1988, 1992) in the collective framework. We also build on static transferable utilities models of the marriage market (Becker 1973, 1974; summarized in Becker 1991). Browning, Chiappori, and Weiss (2003) use it to study marital sorting in the collective framework. Iyigun and Walsh (2004) extend the analysis to include pre-marital investments. Neither model includes labor supply choices.

Two recent papers are closely related to our work. Iyigun (2005) studies marriage matching and family labor supplies in a transferable utilities framework. Chiappori, Iyigun, and Weiss (2005) (CIW hereafter) study matching, labor supply (including the decision to specialize in home production), fertility, and divorce. Our paper differs from this recent work in focus. Our goal is to develop an empirical framework that minimizes *a priori* restrictions on marriage matching and labor supply patterns. Iyigun and CIW are interested in deriving unambiguous predictions for marriage matching and spousal labor supplies by assuming spousal wages (or earnings capacity) are complements in household production. Our empirical framework can be used to test some of the qualitative predictions of Iyigun and CIW's models. Thus, the papers are complementary.

Our static model is restrictive. We assume that the sharing rule is based on expected and not actual wages. That is, we assume spouses have access to binding marital agreements and there is no divorce. There is an active literature studying dynamic intra-household allocations and marital behavior. Davis, Mazzocco, and Yamaguchi (2005) study savings, marriage, and labor supply decisions in a collective framework, in which an individual's weight in the household's allocation process depends on the outside options of each spouse, in this case, divorce. Lundberg and Pollak deal with marriage matching without binding marital arrangements.[7]

The remainder of the paper is organized as follows. In Section 2, we describe our benchmark version of the collective model, which features labor supply, participation decisions, marriage matching, and risk sharing over full income. Section 3 describes the marriage market and the equilibrium. In Section 4, we establish conditions under which the structural parameters of the model (preference parameters and the sharing rule) are identified. A simple example with one marriage market is presented in Section 5. We extend our model to

[7]Other studies include Ayigari, Greenwood, and Guner (2000), Seitz (2004) among others.

incorporate fertility decisions in Section 6. Section 7 concludes.

2 A Collective Model of Household Labor Supply

There are t isolated societies. An important identification assumption, discussed in detail in Section 4, is that variables are society specific while preference parameters are not. Each society has two periods. In the first period, individuals choose whether to marry or not. After their marital choices, they choose labor supplies and consumption in the second period. There are I types of men, $i = 1, .., I$, and J types of women, $j = 1, .., J$. Fertility, whether the woman has children or not, is part of the definition of a woman's type in this version of the model. In Section 6 we extend our model to explicitly incorporate fertility decisions.

Let m_i^t be the number of type i men and f_j^t be the number of type j women in society t. M^t and F^t are the vectors of the numbers of each type of men and women, respectively in society t. If they marry, men and women have to choose their type of spouses. All men and unmarried women have positive hours of work. Married women choose whether to participate in the labor force, and conditional on participation, how many hours to work. The participation status of a wife is known as of the time the marriage decision is made. Thus a marriage is characterized by the quadruplet $\{i, j, p, t\}$ where $p = 1$ if the wife works and $p = 0$ if the wife does not work. One interpretation for this arrangement is that agents choose whether to enter a *specialized* marriage, where one spouse works in the market and one remains at home versus a *non-specialized* marriage, where both spouses work. If a man chooses not to marry, $p = .$ and his spouse is $j = 0$. If a woman chooses not to marry, $p = .$ and her spouse is $i = 0$. If a type i man wants to match with a type j woman in a type p marriage, he must transfer to her τ_{ij}^{pt} units of non-labor income. These transfers are used to clear the marriage market. The equilibrium transfers only depend on $\{i, j, p, t\}$. They do not depend on the particular man or woman in the match. If a man or woman remains unmarried, $\tau_{i0}^{.t} = \tau_{0j}^{.t} = 0$.

Consider the choices that woman G of type j has to make. First she has to decide what type of marriage to enter into, if any. After marriage, she has to decide on her consumption and possibly her labor supply. In order to decide what type of marriage to enter into, she has to evaluate her expected payoffs in marriage from the different choices that are available to her. We start by considering her consumption and labor supply choice in an $\{i, j, p, t\}$ marriage. Much of this part of the analysis is borrowed from Chiappori, Fortin, and Lacroix (2002), and we will be terse in our exposition where possible.

2.1 Preferences

We assume individuals in each society have Stone-Geary utility functions. Consider a woman G in society t. Let C_{ijG}^{pt} be the consumption of woman G of type j matched to a type i man in a type p marriage. H_{ijG}^{pt} is her labor supply, where $H_{ijG}^{0t} = 0$, and L_{ijG}^{pt} is her leisure. Her utility is:

$$U_{ij}^{pt}(C_{ijG}^{pt}, H_{ijG}^{pt}, \varepsilon_{ijG}^{pt}) = (1 - \Delta_{ij}^{p}) \ln \left(\frac{C_{ijG}^{pt} - \Theta_{ij}^{p}}{(1 - \Delta_{ij}^{p})} \right) + \Delta_{ij}^{p} \ln \left(\frac{\Lambda_{ij}^{p} - H_{ijG}^{pt}}{\Delta_{ij}^{p}} \right) + \Gamma_{ij}^{p} + \varepsilon_{ijG}^{pt},$$

where $\Delta_{ij}^{p} > 0$, Θ_{ij}^{p} is her exogenous minimum consumption and Λ_{ij}^{p} her exogenous maximum leisure (i.e. $\Lambda_{ij}^{p} = H_{ijG}^{pt} + L_{ijG}^{pt}$). Notice that Δ_{ij}^{p}, Θ_{ij}^{p} and Λ_{ij}^{p} all depend on (i, j, p), which allows for differences in home production technologies across different types of marriages.[8] Given her individual budget constraint, variations in Θ_{ij}^{p} and Λ_{ij}^{p} will generate systematic differences in labor supplies. Since C_{ijG}^{pt} and H_{ijG}^{pt} must be non-negative, Λ_{ij}^{p} must be positive but Θ_{ij}^{p} may be negative. Since fertility is part of the definition of the type of a woman, we allow women with and without children to make different labor supply choices. Variation in Δ_{ij}^{p}, Θ_{ij}^{p} and Λ_{ij}^{p} across types of marriages allows the model to fit observed labor supply behavior. The parameter Γ_{ij}^{p} shifts her utility by (i, j, p) and allows the model to fit the observed marriage matching patterns in the data. Given her marriage choice, Γ_{ij}^{p} does not have any effect on her consumption and labor supply decisions. Finally, we assume ε_{ijG}^{pt} is a type I extreme value random variable that is realized before marital decisions are made. The realizations of this random variable across different women of type j in the same society will produce different marital choices for different type j women in period one. Given her marital choice, ε_{ijG}^{pt} also has no impact on her consumption and labor supply decisions.

The specification of a representative man's problem is similar to that of women. Let c_{ijg}^{pt} be the consumption of man g of type j matched to a type j woman in a type p marriage in

[8] Following Chiappori, et al. (2002), there is no explicit consideration of the provision of marriage specific public goods or altruistic preferences. See Chiappori, Blundell, and Meghir (2004) and Section 6 of this paper for a collective model with public goods.

society t. Denote his labor supply h^{pt}_{ijg}. If he chooses not to marry, then $p = .$ and $j = 0$. The utility function for males is described by:

$$u^p_{ij}(c^{pt}_{ijg}, h^{pt}_{ijg}, \varepsilon^{pt}_{ijg}) = (1-\delta^p_{ij})\ln\left(\frac{c^{pt}_{ijg} - \theta^p_{ij}}{(1-\delta^p_{ij})}\right) + \delta^p_{ij}\ln\left(\frac{\lambda^p_{ij} - h^{pt}_{ijg}}{\delta^p_{ij}}\right) + \gamma^p_{ij} + \varepsilon^{pt}_{ijg},$$

where θ^p_{ij} is his exogenous minimum consumption and λ^p_{ij} his exogenous maximum leisure ($\lambda^p_{ij} = h^{pt}_{ijg} + l^{pt}_{ijg}$). As is the case for females, γ^p_{ij} allows the males' baseline level of utility to vary by (i, j, p) and ε^{pt}_{ijg} is a type I extreme value random variable which is realized before the marriage decision is made.

2.2 Private budget constraints

We first define full family income for a particular husband g and his wife G in a type $\{i, j, p, t\}$ marriage. Total non-labor family income is A^{pt}_{ijgG}.

$$A^{pt}_{ijgG} = A^{pt}_{ij} \exp \epsilon^{pt}_{ijgG},$$

where ϵ^{pt}_{ijgG} is an *iid* random variable with zero mean and a constant variance σ^2_A. It is realized in period two, after the marital choices occur. The systematic component of per spouse non-labor family income, A^{pt}_{ij}, is known prior to marriage. The wage for a working woman is described by:

$$W^{1t}_{ijG} = W^{1t}_{ij} \exp \xi^{1t}_{ijG},$$

where ξ^{1t}_{ijG} is an *iid* random variable with a zero mean and a constant variance σ^2_W, realized after her marital choice. The systematic component of the wage, W^1_{ij}, is known prior to marriage. Let the covariance of ϵ^{1t}_{ijgG} and ξ^{1t}_{ijG} be σ_{AW}. For families whose wives do not work, $W^{0t}_{ijG} = 0$. The male's wage is determined by:

$$w^{pt}_{ijg} = w^{pt}_{ij} \exp \xi^{pt}_{ijg},$$

where ξ^{pt}_{ijg} has mean zero and a constant variance σ^2_w, covariance with ϵ^{pt}_{ijgG} of σ_{Aw} and covariance with ξ^{1t}_{ijG} of σ_{Ww}. We assume ε^{pt}_{ijg} is realized after marital status is chosen, but w^{pt}_{ij} is known prior to marriage.

We can now define full family income Υ^{pt}_{ijgG}, which is realized in the second period:

$$\Upsilon^{pt}_{ijgG} = A^{pt}_{ijgG} - \theta^p_{ij} - \Theta^p_{ij} + \Lambda^p_{ij} W^{pt}_{ijG} + \lambda^p_{ij} w^{pt}_{ijg}. \tag{1}$$

Full family income is the market value of the endowment of the family in the second period, less minimum consumption $(\theta_{ij}^p + \Theta_{ij}^p)$. We assume that the husband and wife will divide the full family income between them according to the sharing rule, which they take as given at the time they make their labor supply decisions.

Chiappori (1988) shows that, under Pareto efficiency, intra-household allocations may be decentralized by first distributing exogenous non-labor income between the members of the household, according to a pre-determined sharing rule, to obtain a private budget constraint for each member. Here, we assume households fully share risk over full income and intra-household allocations are decentralized by distributing exogenous full income between the members in the same fashion. Let τ_{ij}^{pt} be the pre-determined share of full family income that is allocated to the wife in the second period. The husband then has $(1-\tau_{ij}^{pt})\Upsilon_{ijgG}^{pt}$ of full income available in the second period. In this section of the paper, and in the second period, families take τ_{ij}^{pt} as given. In Section 3, we show how τ_{ij}^{pt} can be derived from marriage market clearing in the first period. If a woman chooses to remain unmarried, $1-\tau_{0j}^{1t} = \theta_{0j}^{1t} = \lambda_{0j}^{1t} = 0$ and if a man chooses to remain unmarried, $\tau_{i0}^{0t} = \Theta_{i0}^{0t} = \Lambda_{i0}^{0t} = 0$. Given her share of full family income, the private budget constraint of the wife is:

$$W_{ijG}^{pt} L_{ijG}^{pt} + C_{ijG}^{pt} \leq \tau_{ij}^{p} \Upsilon_{ijgG}^{pt} + \Theta_{ij}^{p},$$

and the private budget constraint of the husband is:

$$w_{ijg}^{pt} l_{ijg}^{pt} + c_{ijg}^{pt} \leq (1-\tau_{ij}^{pt}) \Upsilon_{ijgG}^{pt} + \theta_{ij}^{p}.$$

Adding the private budget constraints yields the family budget constraint:

$$\begin{aligned} w_{ijg}^{pt} l_{ijg}^{pt} + c_{ijg}^{pt} + W_{ijG}^{pt} L_{ijG}^{pt} + C_{ijG}^{pt} &\leq \Upsilon_{ijgG}^{pt} + \theta_{ij}^{p} + \Theta_{ij}^{p} \\ &\leq A_{ijgG}^{pt} + \Lambda_{ij}^{p} W_{ijG}^{pt} + \lambda_{ij}^{p} w_{ijg}^{pt}. \end{aligned}$$

As long as $\tau_{ij}^{pt} \in (0,1)$, the private budget constraints satisfy the second period family budget constraint. If the husband's wage falls in the second period, the wife's private budget constraint shrinks. If the wife's wage falls in the second period, her husband's private budget constraint also shrinks. The husband and wife thus provide wage insurance for each other. There is full risk-sharing in the household. In Appendix A we show that the household's decisions are ex-ante efficient when husbands and wives share risk over full income.

2.3 Household decision problems in the second period

We can now describe the problem solved by married agents in the second period. The objective of women in $\{i, j, 1, t\}$ marriages, given τ_{ij}^{1t}, is

$$\max_{C_{ijG}^{1t}, L_{ijG}^{1t}} U_{ij}^{1t}(C_{ijG}^{1t}, L_{ijG}^{1t}, \varepsilon_{ijG}^{1t})$$
$$\text{subject to } W_{ijG}^{1t} L_{ijG}^{1t} + C_{ijG}^{1t} \leq \tau_{ij}^{1t} \Upsilon_{ijgG}^{1t} + \Theta_{ij}^{1}. \tag{2}$$

Women in $\{i, j, 0, t\}$ marriages make no decisions after deciding to marry. The objective of men in $\{i, j, p, t\}$ marriages, given τ_{ij}^{pt}, is

$$\max_{c_{ijg}^{pt}, l_{ijg}^{pt}} u_{ij}^{pt}(c_{ijg}^{pt}, l_{ijg}^{pt}, \varepsilon_{ijg}^{pt})$$
$$\text{subject to } w_{ijg}^{pt} l_{ijg}^{pt} + c_{ijg}^{pt} \leq \tau_{ij}^{pt} \Upsilon_{ijgG}^{pt} + \theta_{ij}^{p}. \tag{3}$$

Finally, the objectives of single women and single men are

$$\max_{C_{0jG}^{\cdot t}, L_{0jG}^{\cdot t}} U_{0j}^{\cdot t}(C_{0jG}^{\cdot t}, L_{0jG}^{\cdot t}, \varepsilon_{0jG}^{\cdot t})$$
$$\text{subject to } W_{0jG}^{\cdot t} L_{0jG}^{\cdot t} + C_{0jG}^{\cdot t} \leq \Upsilon_{0jG}^{\cdot t} + \Theta_{0j}^{\cdot} \tag{4}$$

and

$$\max_{c_{i0g}^{\cdot t}, l_{i0g}^{\cdot t}} u_{i0}^{\cdot t}(c_{i0g}^{\cdot t}, l_{i0g}^{\cdot t}, \varepsilon_{i0g}^{\cdot t})$$
$$\text{subject to } w_{i0g}^{\cdot t} l_{i0g}^{\cdot t} + c_{i0g}^{\cdot t} \leq \Upsilon_{i0g}^{\cdot t} + \theta_{i0}^{\cdot}, \tag{5}$$

respectively.

2.4 Spousal labor earnings

Solving her problem of a female in a $i, j, 1, t$ marriage, as outlined above yields the following expression for labor earnings:

$$\begin{aligned} Y_{ijG}^{1t} &= W_{ijG}^{1t} H_{ijG}^{1t} \\ &= W_{ijG}^{1t} \Lambda_{ij}^{1} - \Delta_{ij}^{1} \tau_{ij}^{1t} \Upsilon_{ijgG}^{1t} \\ &= \Delta_{ij}^{1} \tau_{ij}^{1t} (\theta_{ij}^{1} + \Theta_{ij}^{1}) + \Lambda_{ij}^{1} (1 - \Delta_{ij}^{1} \tau_{ij}^{1t}) W_{ijG}^{1t} - \Delta_{ij}^{1} \tau_{ij}^{1t} \lambda_{ij}^{1} w_{ijg}^{1t} - \Delta_{ij}^{1} \tau_{ij}^{1t} A_{ijgG}^{1t}. \end{aligned} \tag{6}$$

The labor earnings for a male g in a $\{i,j,p,t\}$ marriage satisfy:

$$\begin{aligned} y_{ijg}^{pt} &= w_{ijg}^{pt} h_{ijg}^{pt} = w_{ijg}^{pt} \lambda_{ij}^{p} - \delta_{ij}^{p}(1-\tau_{ij}^{pt})\Upsilon_{ijgG}^{pt} \\ &= \delta_{ij}^{p}(1-\tau_{ij}^{pt})(\theta_{ij}^{1}+\Theta_{ij}^{1}) + \lambda_{ij}^{p}(1-\delta_{ij}^{p}(1-\tau_{ij}^{pt}))w_{ijg}^{pt} - \delta_{ij}^{p}(1-\tau_{ij}^{pt})\Lambda_{ij}^{p} W_{ijG}^{pt} \\ &- \delta_{ij}^{p}(1-\tau_{ij}^{pt})A_{ijgG}^{pt}. \end{aligned} \tag{7}$$

It is worth noting that the labor earnings equation are quite flexible. They have a $\{i,j,p,t\}$ specific intercepts, own and spousal wage slopes, and non-labor income slopes. It is also the case that hours are not restricted to be everywhere increasing or decreasing in own wages, and whether labor supply schedule is backward bending or not depends on the marital regime.[9] Hours of work are decreasing in spousal wages and non-labor family income.

2.5 Indirect Utility

In the second period, given $\tau_{ij}^{1t}\Upsilon_{ijgG}^{1t}$ and W_{ijG}^{1t}, a working woman's indirect utility is:

$$\ln \tau_{ij}^{1t} + \ln \Upsilon_{ijgG}^{1t} - \Delta_{ij}^{1} \ln W_{ijG}^{1t} + \Gamma_{ij}^{1} + \varepsilon_{ijG}^{1t}.$$

Let E be the expectations operator. Denote $X_{ij}^{pt} = E[X_{ijgG}^{pt}]$.[10] Since a working woman only observes W_{ij}^{pt}, A_{ij}^{pt}, τ_{ij}^{pt} and ε_{ijG}^{pt} when she chooses her marital status, her expected indirect

[9] For example, female labor supply is upward sloping if $\lambda_{ij}^{1} w_{ijg}^{1t} + A_{ijgG}^{1} > (\theta_{ij}^{1} + \Theta_{ij}^{1})$ and downward sloping otherwise.

[10] For future reference,

$$\begin{aligned} \Upsilon_{ij}^{pt} &= E\Upsilon_{ijgG}^{pt} \\ &= A_{ij}^{pt} - \theta_{ij}^{p} - \Theta_{ij}^{p} + \Lambda_{ij}^{p} W_{ij}^{pt} + \lambda_{ij}^{p} w_{ij}^{pt}, \end{aligned}$$

$$\begin{aligned} \sigma_{\Upsilon_{ij}^{pt}}^{2} = \sigma_{\Upsilon_{ij}^{p}}^{2} =& E(\Upsilon_{ijgG}^{pt} - \Upsilon_{ij}^{pt})^{2} = \sigma_{A}^{2} + (\Lambda_{ij}^{p})^{2}\sigma_{W}^{2} + (\lambda_{ij}^{p})^{2}\sigma_{w}^{2} + 2\lambda_{ij}^{p}\sigma_{Aw} + \\ & 2\Lambda_{ij}^{p}\sigma_{AW} + 2\Lambda_{ij}^{p}\lambda_{ij}^{p}\sigma_{Ww}, \end{aligned}$$

and

$$E(\ln \Upsilon_{ijgG}^{pt}) \simeq \ln \Upsilon_{ij}^{pt} - (\Upsilon_{ij}^{pt})^{-2}\sigma_{\Upsilon_{ij}^{p}}^{2}.$$

The variance of full income, $\sigma_{\Upsilon_{ij}^{p}}^{2}$, of $\{i,j,p,t\}$ couples is independent of t, the society in which the couples are located.

utility from marital choice $\{i,j,1,t\}$ in the first period is:[11]

$$V_{ij}^{1}(\theta_{ij}^{1}+\Theta_{ij}^{1},\tau_{ij}^{1t},A_{ij}^{1t},W_{ij}^{1t},w_{ij}^{1t},\varepsilon_{ijG}^{1t})$$
$$=\ln\tau_{ij}^{1t}+\ln\Upsilon_{ij}^{1t}-\Delta_{ij}^{1}\ln W_{ij}^{1t}+\Gamma_{ij}^{1}+\varepsilon_{ijG}^{1t}.$$

If she chooses to marry and not work, she will obtain an expected indirect utility of:

$$V_{ij}^{0}(\theta_{ij}^{0}+\Theta_{ij}^{0},\tau_{ij}^{0t},A_{ij}^{0t},w_{ij}^{0t},\varepsilon_{ijG}^{0t})=(1-\Delta_{ij}^{0})(\ln\tau_{ij}^{0t}+\ln\Upsilon_{ij}^{0t})+\Delta_{ij}^{0}ln(\Lambda_{ij}^{0}-\Delta_{ij}^{0})$$
$$+\Gamma_{ij}^{0}+\varepsilon_{ijG}^{0t}.$$

Finally, if she chooses to remain unmarried, she will obtain an indirect utility of:

$$V_{0j}^{\cdot}(A_{0j}^{\cdot t},W_{0j}^{\cdot t},\varepsilon_{0jG}^{\cdot t})=\ln\Upsilon_{0j}^{\cdot t}-\Delta_{0j}^{\cdot}\ln W_{0j}^{\cdot t}+\Gamma_{0j}^{\cdot}+\varepsilon_{0jG}^{\cdot t}.$$

In the second period, given $(1-\tau_{ij}^{1t})\Upsilon_{ijgG}^{1t}$ and w_{ijg}^{1t}, the man's indirect utility is:

$$\ln(1-\tau_{ij}^{pt})+\ln\Upsilon_{ijgG}^{pt}-\delta_{ij}^{p}\ln w_{ijG}^{pt}+\gamma_{ij}^{p}+\varepsilon_{ijg}^{pt}.$$

In the first period the man's expected indirect utility from marital choice (i,j,p,t) is:

$$v_{ij}^{p}(\theta_{ij}^{p}+\Theta_{ij}^{p},\tau_{ij}^{pt},A_{ij}^{pt},W_{ij}^{pt},w_{ij}^{pt},\varepsilon_{ijg}^{pt})=\ln(1-\tau_{ij}^{pt})+\ln\Upsilon_{ij}^{pt}-\delta_{ij}^{p}\ln w_{ij}^{pt}+\gamma_{ij}^{p}+\varepsilon_{ijg}^{pt}$$

If he chooses a non-working wife, $W_{ij}^{0t}=0$. If he chooses not to marry, $\Theta_{i0}^{\cdot t}=0$ and $W_{i0}^{\cdot t}=0$.

2.6 Marriage decision problems in the first period

In the first period, agents decide whether to marry and whom to marry given expected wages and non-labor incomes. Given the realizations of all the ε_{ijG}^{pt}, she will choose the marital choice which maximizes her expected utility. She can choose between $I*2+1$ choices. The expected utility from her optimal choice will satisfy:

$$V^{*}(\varepsilon_{0jG}^{t},..,\varepsilon_{ijG}^{0t},..,\varepsilon_{ijG}^{1t},..,\varepsilon_{IjG}^{1t})=$$
$$\max[V_{0j}^{\cdot}(\Theta_{ij}^{\cdot},A_{0j}^{\cdot t},W_{0j}^{\cdot t},\varepsilon_{0jG}^{\cdot t}),..,V_{ij}^{0}(\theta_{ij}^{0}+\Theta_{ij}^{0},\tau_{ij}^{0t},A_{ij}^{0t},w_{ij}^{0t},\varepsilon_{ijG}^{0t}),..,$$
$$V_{Ij}^{1}(\theta_{Ij}^{1}+\Theta_{Ij}^{1},\tau_{Ij}^{1t},A_{Ij}^{1t},W_{Ij}^{1t},w_{Ij}^{1t},\varepsilon_{IjG}^{1t})]. \quad (8)$$

[11] $E(\ln\Upsilon_{ijgG}^{1t})\simeq\ln\Upsilon_{ij}^{1t}-(\Upsilon_{ij}^{1t})^{-2}(\sigma_A^2+(\Lambda_{ij}^{1})^2\sigma_W^2+(\lambda_{ij}^{1})^2\sigma_w^2+2\lambda_{ij}^{1}\sigma_{Aw}+2\Lambda_{ij}^{1}\sigma_{AW}+2\Lambda_{ij}^{1}\lambda_{ij}^{1}\sigma_{Ww})$

The problem facing men in the first stage is analogous to that of women. Given the realizations of all the ε_{ijg}^{pt}, he will choose the marital choice which maximizes his expected utility. He can choose between $J*2+1$ choices. The expected utility from his optimal choice will satisfy:

$$
\begin{aligned}
&v*(\varepsilon_{i0g}^{t},..,\varepsilon_{ijg}^{0t},..,\varepsilon_{ijg}^{1t},..,\varepsilon_{ijg}^{1t}) = \\
&\qquad \max[v_{i0}^{\cdot}(\theta_{ij}^{\cdot},a_{i0}^{\cdot t},w_{i0}^{\cdot t},\varepsilon_{i0g}^{\cdot t}),..,v_{ij}^{0}(\theta_{ij}^{0}+\Theta_{ij}^{0},\tau_{ij}^{0t},a_{ij}^{0t},w_{ij}^{0t},\varepsilon_{i0g}^{0t}),.., \\
&\qquad v_{iJ}^{1}(\theta_{iJ}^{1}+\Theta_{iJ}^{1},\tau_{iJ}^{1t},A_{iJ}^{1t},W_{iJ}^{1t},w_{iJ}^{1t},\varepsilon_{iJg}^{1t})].
\end{aligned}
\tag{9}
$$

3 The Marriage Market

If there are lots of men and women of each type, McFadden (1974) shows that for every type of woman j:

$$
\begin{aligned}
&\ln\overline{\mu}_{ij}^{1t}-\ln\mu_{0j}^{\cdot t} \\
&\quad =(\Gamma_{ij}^{1}-\Gamma_{0j}^{\cdot})+\ln\tau_{ij}^{1t}+\ln\Upsilon_{ij}^{1t}-\Delta_{ij}^{1}\ln W_{ij}^{1t}-(\ln\Upsilon_{0j}^{\cdot t}-\Delta_{0j}^{\cdot}\ln W_{0j}^{\cdot t})\ , \quad i=1,..,I
\end{aligned}
\tag{10}
$$

and

$$
\begin{aligned}
&\ln\overline{\mu}_{ij}^{0t}-\ln\mu_{0j}^{\cdot t} \\
&\quad =(\Gamma_{ij}^{0}-\Gamma_{0j}^{\cdot})+(1-\Delta_{ij}^{0})(\ln\tau_{ij}^{0t}+\ln\Upsilon_{ij}^{0t})-(\ln\Upsilon_{0j}^{\cdot t}-\Delta_{0j}^{\cdot t}\ln W_{0j}^{\cdot t}) \\
&\quad +\Delta_{ij}^{0}ln(\Lambda_{ij}^{0}-\Delta_{ij}^{0})\ , \quad i=1,..,I,
\end{aligned}
\tag{11}
$$

where $\overline{\mu}_{ij}^{pt}$ is the number of (i,j,p,t) marriages supplied by j type females and μ_{0j}^{1t} is the number of type j females who choose to remain unmarried. The right hand side of (10) and (11) may be interpreted as the systematic gain to a random type j female from entering into an (i,j,p,t) marriage relative to remaining unmarried. The expected relative gain for a type j woman who chooses an (i,j,p,t) marriage is larger than for alternative marriages because she is chooses the type of marriage which maximizes her expected utility.

Similarly, if there are lots of men and women of each type, for every type of man i,

$$
\begin{aligned}
&\ln\underline{\mu}_{ij}^{pt}-\ln\mu_{i0}^{\cdot t} \\
&\quad =(\gamma_{ij}^{p}-\gamma_{i0})+\ln(1-\tau_{ij}^{pt})+\ln\Upsilon_{ij}^{pt}-\delta_{ij}^{p}\ln w_{ij}^{pt}-(\ln\Upsilon_{i0}^{\cdot t}-\delta_{i0}^{\cdot}\ln w_{i0}^{\cdot t}), \quad j=1,..,J,
\end{aligned}
\tag{12}
$$

where $\underline{\mu}_{ij}^{pt}$ is the number of (i, j, p, t) marriages demanded by j type males and $\mu_{i0}^{\cdot t}$ is the number of type i males who choose to remain unmarried.

Marriage market clearing requires the supply of wives to be equal to the demand for husbands for each type of marriage:

$$\ln \underline{\mu}_{ij}^{pt} = \ln \overline{\mu}_{ij}^{pt} = \ln \mu_{ij}^{pt}. \tag{13}$$

$\forall (i, j, p, t)$. There is an additional feasibility constraint that the stocks of married and single agents of each gender and type cannot exceed the aggregate stocks of agents of each gender in each society:

$$f_j^t = \mu_{0j}^{\cdot t} + \sum_{i,p} \mu_{ij}^{pt} \tag{14}$$

$$m_i^t = \mu_{i0}^{\cdot t} + \sum_{j,p} \mu_{ij}^{pt} \tag{15}$$

$$F^t = \sum_j f_j^t \tag{16}$$

$$M^t = \sum_i m_i^t. \tag{17}$$

We can now define a rational expectations equilibrium for each society. There are two parts to the equilibrium, corresponding to the two stages at which decisions are made by the agents. The first corresponds to decisions made in the marriage market; the second to the intra-household allocation. In equilibrium, agents make marital status decisions optimally, the sharing rules clear each marriage market, and conditional on the sharing rules, agents choose consumption and labor supply optimally. Formally:

Definition 1. *A rational expectations equilibrium for society t consists of a distribution of males and females across individual type, marital status, and type of marriage $\{\hat{\mu}_{0j}^{\cdot t}, \hat{\mu}_{i0}^{\cdot t}, \hat{\mu}_{ij}^{pt}\}$, a set of decision rules for marriage $\{\widehat{V}(\varepsilon_{0jG}^{t}, .., \varepsilon_{ijG}^{0t}, .., \varepsilon_{ijG}^{1t}, .., \varepsilon_{IjG}^{1t}),$ $\widehat{v}(\varepsilon_{i0g}^{t}, .., \varepsilon_{ijg}^{0t}, .., \varepsilon_{ijg}^{1t}, .., \varepsilon_{iJg}^{1t})\}$ a set of decision rules for consumption and leisure $\{\hat{C}_{ij}^{pt}, \hat{c}_{ij}^{pt}, \hat{L}_{ij}^{pt}, \hat{l}_{ij}^{pt}\}$, and a set of sharing rules $\{\hat{\tau}_{ij}^{pt}\}$ such that:*

1. *The decision rules $\{\widehat{V}^{*}(\varepsilon_{0jG}^{t}, .., \varepsilon_{ijG}^{0t}, .., \varepsilon_{ijG}^{1t}, .., \varepsilon_{IjG}^{1t}), \widehat{v}^{*}(\varepsilon_{i0g}^{t}, .., \varepsilon_{ijg}^{0t}, .., \varepsilon_{ijg}^{1t}, .., \varepsilon_{iJg}^{1t})\}$ solve (8) and (9);*

2. $\{\hat{\tau}_{ij}^{pt}\}$ *clears the* $(i,j,p,t)^{th}$ *market, implying (13), (14), (15), (16), and (17) hold;*

3. *Given* $\{\hat{\tau}_{ij}^{pt}\}$, *the decision rules* $\{\hat{C}_{ij}^{pt}, \hat{c}_{ij}^{pt}, \hat{L}_{ij}^{pt}, \hat{l}_{ij}^{pt}\}$ *solve (2), (3), (4), and (5).*

4 Identification

In this section, we establish the conditions under which preferences and the intra-household allocation process can be recovered. In particular, we show that information on labor supplies, wages, and non-labor incomes from at least two marriage markets (without imposing any restrictions regarding marriage market clearing) allows us to fully recover preferences and the sharing rule for couples in which both spouses work. For couples in which the wife does not work, the restriction that marriage markets clear is necessary for full identification of the model.

4.1 Singles

Recall female G and male g labor earnings equations:

$$Y_{0jG}^{\cdot t} = \Delta_{0j}^{\cdot}\Theta_{0j}^{\cdot} + \Lambda_{0j}^{\cdot}(1-\Delta_{0j}^{\cdot})W_{0jG}^{0t} - \Delta_{0j}^{\cdot}A_{0jG}^{\cdot t}$$

and

$$y_{i0g}^{\cdot t} = \delta_{i0}^{\cdot}\theta_{i0}^{\cdot} + \lambda_{i0}^{\cdot}(1-\delta_{i0}^{\cdot})w_{i0g}^{\cdot t} - \delta_{i0}^{\cdot}A_{i0g}^{\cdot t},$$

respectively. Assume that $Y_{0jG}^{\cdot t}$, $y_{i0g}^{\cdot t}$, $W_{0jg}^{\cdot t}$, $w_{i0g}^{\cdot t}$, $A_{0jG}^{\cdot t}$ and $a_{i0g}^{\cdot t}$ are observed, while $\theta_{i0}^{\cdot}, \Theta_{0j}^{\cdot}$, $\delta_{i0}^{\cdot}$, $\Delta_{0j}^{\cdot}$, λ_{i0} and Λ_{0j}^{1} are unobserved. Consider the following reduced form empirical spousal labor earnings equations:

$$Y_{0jG}^{\cdot t} = B_{0j}^{\cdot t} + B_{0j}^{Wt}W_{0jG}^{\cdot t} + B_{0j}^{At}A_{0jG}^{\cdot t} \tag{18}$$

$$y_{i0g}^{\cdot t} = b_{i0}^{\cdot t} + b_{i0}^{wt}w_{i0g}^{\cdot t} + b_{i0}^{At}A_{i0g}^{\cdot t}. \tag{19}$$

It is straightforward to show that we can estimate all the structural parameters that determine their labor supplies for single women and men as follows:

$$\begin{aligned}
\Delta_{0j}^{\cdot} &= -B_{0j}^{At} \\
\Theta_{0j}^{\cdot} &= \frac{B_{0j}^{\cdot t}}{B_{0j}^{At}} \\
\Lambda_{0j}^{\cdot} &= \frac{B_{0j}^{Wt}}{1 + B_{0j}^{At}} \\
\delta_{i0}^{\cdot} &= -b_{i0}^{At} \\
\theta_{i0}^{\cdot} &= \frac{b_{i0}^{\cdot t}}{b_{i0}^{At}} \\
\lambda_{i0}^{\cdot} &= \frac{b_{i0}^{wt}}{1 + b_{i0}^{At}}.
\end{aligned}$$

4.2 Couples with working wives

Recall the labor earnings equations for husbands and wives in non-specialized marriages:

$$Y_{ijG}^{1t} = \Delta_{ij}^{1}\tau_{ij}^{1t}(\theta_{ij}^{1} + \Theta_{ij}^{1}) + \Lambda_{ij}^{1}(1 - \Delta_{ij}^{1}\tau_{ij}^{1t})W_{ijG}^{1t} - \Delta_{ij}^{1}\tau_{ij}^{1t}\lambda_{ij}^{1}w_{ijg}^{1t} - \Delta_{ij}^{1}\tau_{ij}^{1t}A_{ijgG}^{1t}$$

and

$$\begin{aligned}
y_{ijg}^{1t} &= \delta_{ij}^{1}(1 - \tau_{ij}^{1t})(\theta_{ij}^{1} + \Theta_{ij}^{1}) + \lambda_{ij}^{1}(1 - \delta_{ij}^{1}(1 - \tau_{ij}^{1t}))w_{ijg}^{1t} - \delta_{ij}^{1}(1 - \tau_{ij}^{1t})\Lambda_{ij}^{1}W_{ijG}^{1t} \\
&\quad - \delta_{ij}^{1}(1 - \tau_{ij}^{1t})A_{ijgG}^{1t},
\end{aligned}$$

respectively. Assume that Y_{ijG}^{1t}, y_{ijg}^{1t}, $W_{ijg'}^{1t}$, w_{ijG}^{1t} and A_{ijgG}^{1t} are observed, while τ_{ij}^{1t},θ_{ij}^{1}, Θ_{ij}^{1}, δ_{ij}^{1}, Δ_{ij}^{1}, λ_{ij}^{1} and Λ_{ij}^{1} are unobserved. Consider the following reduced form empirical spousal labor earnings equations:

$$Y_{ijG}^{1t} = B_{ij}^{1t} + B_{ij}^{Wt}W_{ijG}^{1t} + B_{ij}^{wt}w_{ijg}^{1t} + B_{ij}^{At}A_{ijgG}^{1t} \tag{20}$$

$$y_{ijg}^{1t} = b_{ij}^{1t} + b_{ij}^{Wt}W_{ijG}^{1t} + b_{ij}^{wt}w_{ijg}^{1t} + b_{ij}^{At}A_{ijgG}^{1t}. \tag{21}$$

If we do not restrict the equilibrium sharing function τ_{ij}^{1t}, we can identify Λ_{ij}, λ_{ij}, $\Delta_{ij}^{1}\tau_{ij}^{1t}$, $\delta_{ij}^{1}(1-\tau_{ij}^{1t})$ and $(\theta_{ij}^{1}+\Theta_{ij}^{1})$ from estimating the spousal labor earnings equations. In fact, the model is over-identified along this dimension:

$$-\frac{b_{ij}^{1t}}{b_{ij}^{At}} = -\frac{B_{ij}^{1t}}{B_{ij}^{At}} = \theta_{ij}^{1} + \Theta_{ij}^{1} \tag{22}$$

$$\frac{B_{ij}^{wt}}{B_{ij}^{At}} = \frac{b_{ij}^{wt}}{1 + b_{ij}^{At}} = \lambda_{ij}^{1} \tag{23}$$

$$\frac{b_{ij}^{Wt}}{b_{ij}^{At}} = \frac{B_{ij}^{Wt}}{1 + B_{ij}^{At}} = \Lambda_{ij}^{1} \tag{24}$$

Restrictions (22) to (24) on the labor earnings equations hold as long as the equilibrium shares of full family income are determined prior to the realization of wages and non-labor family income. In other words, these restrictions are implied by our version of the collective model of intra-household allocation of resources, not by our model of marriage market clearing.[12] Given estimates of λ_{ij}^{1}, Λ_{ij}^{1}, and $(\theta_{ij}^{1} + \Theta_{ij}^{1})$, we can estimate full family income for each working couple, Υ_{ijgG}^{1t}, using (1). The observation of labor supplies for both members of the household is not sufficient for the separate identification of the transfer (τ_{ij}^{1t}) and the relative weight of consumption versus leisure in preferences for men and women (δ_{ij}^{1} and Δ_{ij}^{1}, respectively). To highlight the identification problem, we have:

$$B_{ij}^{At} = -\Delta_{ij}^{1}\tau_{ij}^{1t} \tag{25}$$

$$b_{ij}^{At} = -\delta_{ij}^{1}(1 - \tau_{ij}^{1t}). \tag{26}$$

This is analogous to the standard result (Chiappori, 1988) that Pareto efficiency and the observation of labor supply, wages, and non-labor incomes allows identification of the sharing rule up to an additive constant, as in this case τ_{ij}^{1t} is a constant.

What is new in our framework are the following observations. First, we show that introducing an additional restriction from the marriage market, namely marriage market clearing, does not solve the above identification problem. Let $\widetilde{\gamma}_{ij}^{1} = \exp(\frac{\gamma_{ij}^{1}}{\gamma_{i0}})$ and $\widetilde{\Gamma}_{ij}^{1} = \exp(\frac{\Gamma_{ij}^{1}}{\Gamma_{0j}})$. Assuming marriage market clearing, the marital demand equation, (12), and the

[12] So for example, if ε_{ijG}^{p} and ε_{ijg}^{p} are not *iid* extreme value random variables, the supply and demand functions in the marriage market will not be of the form described in (10) and (12). But restrictions (22)-(24) have to continue to hold as long as the equilibrium shares of full family income are determined prior to when wages and non-labor family income are realized.

marital supply equation, (10), imply:

$$\frac{\mu_{ij}^{1t}}{\mu_{i0}^{\cdot t}} = (1-\tau_{ij}^{1t})\frac{\widetilde{\gamma}_{ij}^{1}\Upsilon_{ij}^{1t}(w_{ij}^{1t})^{-\delta_{ij}^{1}}}{\Upsilon_{i0}^{\cdot t}(w_{i0}^{\cdot t})^{-\delta_{i0}}} \tag{27}$$

$$\frac{\mu_{ij}^{1t}}{\mu_{0j}^{\cdot t}} = \tau_{ij}^{1t}\frac{\widetilde{\Gamma}_{ij}^{1}\Upsilon_{ij}^{1t}(W_{ij}^{1t})^{-\Delta_{ij}^{1}}}{\Upsilon_{0j}^{\cdot t}(W_{0j}^{\cdot t})^{-\Delta_{0j}}}. \tag{28}$$

Since w_{ij}^{1t} $w_{i0}^{\cdot t}$ W_{ij}^{1t} $W_{0j}^{\cdot t}$ Υ_{ij}^{1t} $\Upsilon_{i0}^{\cdot t}$, and $\Upsilon_{0j}^{\cdot t}$, are observed, the unknowns in (25) to (28) are Δ_{ij}^{1}, δ_{ij}^{1}, τ_{ij}^{1t}, $\widetilde{\gamma}_{ij}^{1}$, $\widetilde{\Gamma}_{ij}^{1}$. We have five unknowns and four equations. So with a single society t, the model is still under identified, even after imposing marriage market clearing. This result is not surprising, as introducing marriage market clearing introduces additional parameters determining the gains to marriage.

Second, incorporating marriage markets introduces provide additional information that does solve the identification problem in the following sense. If we have two societies, x and y, that differ in labor supplies, wages, and non-labor incomes (and thus sharing rules) but not in preferences, then we can identify Δ_{ij}^{1}, δ_{ij}^{1}, τ_{ij}^{1x} and τ_{ij}^{1y} from labor supply as follows:

$$\Delta_{ij}^{1} = \frac{B_{ij}^{Ay}b_{ij}^{1x} - B_{ij}^{Ax}b_{ij}^{1y}}{b_{ij}^{1y} - b_{ij}^{1x}} \tag{29}$$

$$\delta_{ij}^{1} = \frac{B_{ij}^{Ay}b_{ij}^{1x} - B_{ij}^{Ax}b_{ij}^{1y}}{B_{ij}^{1x} - B_{ij}^{1y}} \tag{30}$$

$$\tau_{ij}^{1x} = \frac{B_{ij}^{1x}b_{ij}^{1x} - B_{ij}^{1x}b_{ij}^{1y}}{B_{ij}^{1y}b_{ij}^{1x} - B_{ij}^{1x}b_{ij}^{1y}} \tag{31}$$

$$\tau_{ij}^{1y} = \frac{B_{ij}^{1y}b_{ij}^{1x} - B_{ij}^{1y}b_{ij}^{1y}}{B_{ij}^{1y}b_{ij}^{1x} - B_{ij}^{1x}b_{ij}^{1y}} \tag{32}$$

Since Δ_{ij}^{1}, δ_{ij}^{1}, τ_{ij}^{1x} and τ_{ij}^{1y} are identified from the labor supplies equations, the parameters γ_{ij} and Γ_{ij} are now over-identified, as (12), and the marital supply equation, (10), imply:

$$\widetilde{\gamma}_{ij}^{1} = \frac{\mu_{ij}^{1x}}{\mu_{i0}^{\cdot x}}\frac{\Upsilon_{i0}^{\cdot x}(w_{i0}^{\cdot x})^{-\delta_{i0}}}{\Upsilon_{ij}^{1x}(w_{ij}^{1x})^{-\delta_{ij}^{1}}}\frac{1}{(1-\tau_{ij}^{1x})} = \frac{\mu_{ij}^{1y}}{\mu_{i0}^{\cdot y}}\frac{\Upsilon_{i0}^{\cdot y}(w_{i0}^{\cdot y})^{-\delta_{i0}}}{\Upsilon_{ij}^{1y}(w_{ij}^{1y})^{-\delta_{ij}^{1}}}\frac{1}{(1-\tau_{ij}^{1y})}$$

and

$$\widetilde{\Gamma}_{ij}^{1} = \frac{\mu_{ij}^{1x}}{\mu_{0j}^{\cdot x}}\frac{\Upsilon_{0j}^{\cdot x}(W_{0j}^{\cdot x})^{-\Delta_{0j}}}{\Upsilon_{ij}^{1x}(W_{ij}^{1x})^{-\Delta_{ij}^{1}}}\frac{1}{\tau_{ij}^{1x}} = \frac{\mu_{ij}^{1y}}{\mu_{0j}^{\cdot y}}\frac{\Upsilon_{0j}^{\cdot y}(W_{0j}^{\cdot y})^{-\Delta_{0j}}}{\Upsilon_{ij}^{1y}(W_{ij}^{1y})^{-\Delta_{ij}^{1}}}\frac{1}{\tau_{ij}^{1y}}.$$

Adding marriage matching to the collective model cannot aid identification if there is only a single society because the base gains to marriage matching add additional unknown parameters. Adding additional societies in combination with labor supply data allows us to estimate all the parameters within a marriage pair that determine intra-household allocations. If we have additional societies, labor supplies and marriage matching data, some of the preference parameters will be over-identified. Since transfers are society-specific, τ_{ij}^{1t} is always just identified from observations on labor supply. It is worth emphasizing that we do not need to impose marriage market clearing to identify the sharing rule. Thus, the sharing rule is identified solely off labor supply as long as we have information on more than two markets.

4.2.1 Couples with non-working wives

Recall the husband's g earnings equation in specialized marriages is:

$$y_{ijg}^{0t} = \delta_{ij}^{0}(1-\tau_{ij}^{0t})(\theta_{ij}^{0}+\Theta_{ij}^{0}) + \lambda_{ij}^{0}(1-\delta_{ij}^{0}(1-\tau_{ij}^{0t}))w_{ijg}^{0t} - \delta_{ij}^{0}(1-\tau_{ij}^{0t})A_{ijgG}^{0t}.$$

We observe y_{ijg}^{1t}, w_{ijG}^{1t} and A_{ijgG}^{1t}, while θ_{ij}^{0}, Θ_{ij}^{0}, δ_{ij}^{0}, λ_{ij}^{0} and τ_{ij}^{0t} are unobserved. Consider the following reduced form empirical spousal labor earnings equations:

$$y_{ijg}^{0t} = b_{ij}^{0t} + b_{ij}^{0wt}w_{ijg}^{0t} + b_{ij}^{0At}A_{ijgG}^{0t}. \tag{33}$$

If we do not restrict the equilibrium sharing function τ_{ij}^{0t}, we can identify λ_{ij}^{0}, $\delta_{ij}^{0}(1-\tau_{ij}^{0t})$ and $(\theta_{ij}^{0}+\Theta_{ij}^{0})$ from estimating the husband's labor earnings equations. Given estimates of λ_{ij}^{0} and $(\theta_{ij}^{0}+\Theta_{ij}^{0})$, we can estimate full family income for each working couple, Υ_{ijgG}^{0t}, using (1).[13] As in the case for working couples, it is not possible to separately identify the sharing rule from the relative weight of consumption in preferences for the husband, i.e.:

$$b_{ij}^{0At} = -\delta_{ij}^{0}(1-\tau_{ij}^{0t}).$$

If we have two societies, x and y, then we have:

$$b_{ij}^{0Ax} = -\delta_{ij}^{0}(1-\tau_{ij}^{0x}) \tag{34}$$

$$b_{ij}^{0Ay} = -\delta_{ij}^{0}(1-\tau_{ij}^{0y}) \tag{35}$$

[13]This means that we can also estimate $\sigma_{\Upsilon_{ij}^{0}}^{2}$.

and the model is still not identified from observations of labor supply alone. In this instance, marriage market clearing does aid in identification as follows. Let $\widetilde{\gamma}^0_{ij} = \exp(\frac{\gamma^0_{ij}}{\gamma_{i0}})$ and $\widetilde{\Gamma}^0_{ij} = \exp(\frac{\Gamma^0_{ij}}{\Gamma_{0j}})$. Assuming marriage market clearing, the marital demand equation (12) and the marital supply equation (10) imply:

$$\frac{\mu^{0t}_{ij}}{\mu^{\cdot t}_{i0}} = (1-\tau^{0t}_{ij})\frac{\widetilde{\gamma}^0_{ij}\Upsilon^{0t}_{ij}(w^{0t}_{ij})^{-\delta^0_{ij}}}{\Upsilon^{\cdot t}_{i0}(w^{\cdot t}_{i0})^{-\delta^0_{i0}}} \tag{36}$$

$$\frac{\mu^{0t}_{ij}}{\mu^{\cdot t}_{0j}} = \frac{\widetilde{\Gamma}^0_{ij}\left[\tau^{0t}_{ij}\Upsilon^{0t}_{ij}\right]^{(1-\Delta^0_{ij})}(\Lambda^0_{ij})^{\Delta^0_{ij}}}{\Upsilon^{\cdot t}_{0j}(W^{\cdot t}_{0j})^{-\Delta^0_{0j}}(\Delta^0_{ij})^{\Delta^0_{ij}}}. \tag{37}$$

Here we have six unknowns, Λ^0_{ij}, δ^0_{ij}, Δ^0_{ij}, τ^{0t}_{ij}, $\widetilde{\gamma}^0_{ij}$, and $\widetilde{\Gamma}^0_{ij}$. If we have two societies, x and y, then we have:

$$b^{0Ax}_{ij} = -\delta^0_{ij}(1-\tau^{0x}_{ij}) \tag{38}$$

$$b^{0Ay}_{ij} = -\delta^0_{ij}(1-\tau^{0y}_{ij}) \tag{39}$$

$$\frac{\mu^{0x}_{ij}}{\mu^{\cdot x}_{i0}} = (1-\tau^{0x}_{ij})\frac{\widetilde{\gamma}^0_{ij}\Upsilon^{0x}_{ij}(w^{0x}_{ij})^{-\delta^0_{ij}}}{\Upsilon^{\cdot x}_{i0}(w^{\cdot x}_{i0})^{-\delta^0_{i0}}} \tag{40}$$

$$\frac{\mu^{0x}_{ij}}{\mu^{\cdot x}_{0j}} = \frac{\widetilde{\Gamma}^0_{ij}\left[\tau^{0x}_{ij}\Upsilon^{0x}_{ij}\right]^{(1-\Delta^0_{ij})}(\Lambda^0_{ij})^{\Delta^0_{ij}}}{\Upsilon^{\cdot x}_{0j}(W^{\cdot x}_{0j})^{-\Delta^0_{0j}}(\Delta^0_{ij})^{\Delta^0_{ij}}}. \tag{41}$$

$$\frac{\mu^{0y}_{ij}}{\mu^{\cdot y}_{i0}} = (1-\tau^{0y}_{ij})\frac{\widetilde{\gamma}^0_{ij}\Upsilon^{0y}_{ij}(w^{0y}_{ij})^{-\delta^0_{ij}}}{\Upsilon^{\cdot y}_{i0}(w^{\cdot y}_{i0})^{-\delta^0_{i0}}} \tag{42}$$

$$\frac{\mu^{0y}_{ij}}{\mu^{\cdot y}_{0j}} = \frac{\widetilde{\Gamma}^0_{ij}\left[\tau^{0y}_{ij}\Upsilon^{0y}_{ij}\right]^{(1-\Delta^0_{ij})}(\Lambda^0_{ij})^{\Delta^0_{ij}}}{\Upsilon^{\cdot y}_{0j}(W^{\cdot y}_{0j})^{-\Delta^0_{0j}}(\Delta^0_{ij})^{\Delta^0_{ij}}}. \tag{43}$$

The sharing rule is identified. The solution is recursively given below:

$$\delta_{ij}^{0} = \frac{\ln \mu_{ij}^{0x} \mu_{i0}^{y} b_{ij}^{0Ay} \frac{\Upsilon_{ij}^{0y}}{\Upsilon_{i0}^{\cdot y} w_{i0}^{\cdot y}} - \ln \mu_{i0}^{x} \mu_{ij}^{0y} b_{ij}^{0Ax} \frac{\Upsilon_{ij}^{0x}}{\Upsilon_{i0}^{\cdot x} w_{i0}^{\cdot x}}}{\ln w_{ij}^{0y} - \ln w_{ij}^{0x}} \tag{44}$$

$$\tau_{ij}^{0x} = 1 + \frac{b_{ij}^{0Ax}}{\delta_{ij}^{0}} \tag{45}$$

$$\tau_{ij}^{0y} = 1 + \frac{b_{ij}^{0Ay}}{\delta_{ij}^{0}} \tag{46}$$

$$\widetilde{\gamma}_{ij}^{0} = \frac{\mu_{ij}^{0y} \left(w_{ij}^{0y}\right)^{\delta_{ij}^{0}}}{\mu_{i0}^{\cdot y}(1-\tau_{ij}^{0y}) \frac{\Upsilon_{ij}^{0y}}{\Upsilon_{i0}^{\cdot y} w_{i0}^{\cdot y}}} \tag{47}$$

$$(1-\Delta_{ij}^{0}) = \frac{\ln \mu_{ij}^{0x} \mu_{0j}^{\cdot y} \Upsilon_{0j}^{x} (W_{0j}^{\cdot x})^{-\Delta_{\cdot 0j}} - \ln \mu_{0j}^{\cdot x} \mu_{ij}^{0y} \Upsilon_{0j}^{y} (W_{0j}^{\cdot y})^{-\Delta_{\cdot 0j}}}{\ln \tau_{ij}^{0x} \Upsilon_{ij}^{0x} - \ln \tau_{ij}^{0y} \Upsilon_{ij}^{0y}} \tag{48}$$

$$\widetilde{\Gamma}_{ij}^{0} (\Lambda_{ij}^{0})^{\Delta_{ij}^{0}} = \frac{\mu_{ij}^{0y} \Upsilon_{0j}^{y} (W_{0j}^{\cdot y})^{-\Delta_{\cdot 0j}} (\Delta_{ij}^{0})^{\Delta_{ij}^{0}}}{\mu_{0j}^{\cdot y} \left[\tau_{ij}^{0y} \Upsilon_{ij}^{0y}\right]^{(1-\Delta_{ij}^{0})}}. \tag{49}$$

With two societies, λ_{ij}^{0} and $(\theta_{ij}^{0} + \Theta_{ij}^{0})$ are over-identified from the husband's labor earnings equations, $\widetilde{\Gamma}_{ij}^{0}$ and Λ_{ij}^{0} are not separately identified, and the remaining parameters are just identified.

4.3 Derivation of the sharing rule from marriage market clearing

A primary gain to embedding the collective model in the marriage market is to provide a theoretical rationalization for the origins of the sharing rule. Chiappori, Fortin, and Lacroix (2002), among others, conjecture that the sharing rule in the collective model depends on factors assumed to influence bargaining power within married couples. Such factors typically include the sex ratio and the relative wages of the husband and the wife. We illustrate this point by considering couples in which both spouses work so as to ease comparisons with previous studies. Combining (27) and (28) yields:

$$\mu_{i0}^{\cdot t}(1-\tau_{ij}^{1t}) \frac{\widetilde{\gamma}_{ij}^{1} \Upsilon_{ij}^{1t} (w_{ij}^{1t})^{-\delta_{ij}^{1}}}{\Upsilon_{i0}^{\cdot t} (w_{i0}^{\cdot t})^{-\delta_{i0}}} = \mu_{0j}^{\cdot t} \tau_{ij}^{1t} \frac{\widetilde{\Gamma}_{ij}^{1} \Upsilon_{ij}^{1t} (W_{ij}^{1t})^{-\Delta_{ij}^{1}}}{\Upsilon_{0j}^{\cdot t} (W_{0j}^{\cdot t})^{-\Delta_{\cdot 0j}}}.$$

Then the sharing rule that arises from marriage market clearing can be expressed as:

$$\tau_{ij}^{1t} = \frac{1}{1+\Omega(i,j,1,t)}, \tag{50}$$

where

$$\Omega(i,j,1,t) = \frac{\mu_{0j}^{t}}{\mu_{i0}^{t}} \frac{(W_{0j}^{\cdot t})^{\Delta_{0j}^{\cdot}}}{(W_{ij}^{1t})^{\Delta_{ij}^{1}}} \frac{(w_{ij}^{1t})^{\delta_{ij}^{1}}}{(w_{i0}^{\cdot t})^{\delta_{i0}^{\cdot}}} \frac{\Upsilon_{i0}^{\cdot t}}{\Upsilon_{0j}^{\cdot t}} \frac{\tilde{\Gamma}_{ij}^{1}}{\tilde{\gamma}_{ij}^{1}}.$$

This sharing rule is analogous to the one conjectured by in Chiappori, Fortin, and Lacroix (2002) in the sense that the sharing rule is a function of the sex ratio, and the gender gaps in wages and non-labor incomes of men of type i and of type j. It is:

- increasing in the ratio of single men to women ($\frac{\mu_{i0}^{t}}{\mu_{0j}^{t}}$);
- decreasing in the marriage wage premium for men ($\frac{\hat{w}_{ij}^{1t}}{\hat{w}_{i0}^{\cdot t}}$), and increasing in the marriage wage premium for women ($\frac{\hat{W}_{ij}^{1t}}{\hat{W}_{0j}^{\cdot t}}$);
- increasing in the gender gap in the marriage preference shifters ($\frac{\tilde{\gamma}_{ij}^{1}}{\tilde{\Gamma}_{ij}^{1}}$);
- decreasing in the gender gap in full incomes for singles ($\frac{\Upsilon_{i0}^{\cdot t}}{\Upsilon_{0j}^{\cdot t}}$).

It is clear in this instance that the sex ratio of available men and women is endogenous. This form for the sharing rule cannot be used for policy analysis as changes in, for example $\hat{W}_{0j}^{\cdot t}$ would change the transfer directly but also through $\frac{\mu_{0j}^{t}}{\mu_{i0}^{t}}$. The reduced form transfer will be a function of all of the factors that determine the equilibrium measures of marriages of each type, namely the distributions of wages and non-labor incomes across types, as well as the aggregate stocks of men and women. Factors such as M, F, $w_{i0}^{\cdot t}$, $W_{0j}^{\cdot t}$, $\Upsilon_{i0}^{\cdot t}$, and $\Upsilon_{0j}^{\cdot t}$ are analogous to the distribution factors of Chiappori, Fortin, and Lacroix (2002).

The equilibrium measure of $(i,j,1)$ marriages, as a function of the measures of singles, is:

$$\mu_{ij}^{1t} = \frac{\mu_{i0}^{\cdot t}\mu_{0j}^{\cdot t}A_{ij}^{1t}a_{ij}^{1t}}{\mu_{i0}^{\cdot t}a_{ij}^{1t} + \mu_{0j}^{\cdot t}A_{ij}^{1t}} \tag{51}$$

where

$$A_{ij}^{1t} = \frac{\widetilde{\Gamma}_{ij}^{1}\Upsilon_{ij}^{1t}(W_{ij}^{1t})^{-\Delta_{ij}^{1}}}{\Upsilon_{0j}^{\cdot t}(W_{0j}^{\cdot t})^{-\Delta_{0j}^{\cdot}}},$$

$$a_{ij}^{1t} = \frac{\widetilde{\gamma}_{ij}^{1}\Upsilon_{ij}^{1t}(w_{ij}^{1t})^{-\delta_{ij}^{1}}}{\Upsilon_{i0}^{\cdot t}(w_{i0}^{\cdot t})^{-\delta_{i0}^{\cdot}}}.$$

To solve for the reduced form transfer and the equilibrium measures of singles of each type and marriages of each (i, j, p) combination, we need to solve a system of $(I * J * 2) * 2 + (I + J)$ equations in $(I * J * 2) * 2 + (I + J)$ unknowns, where the equations consist of:

1. $I * J * 2$ supply equations for women (10) and (11)
2. $I * J * 2$ demand equations for men (12)
3. $I + J$ feasibility constraints (14) and (15),

and the unknowns are:

1. $I * J * 2$ equilibrium transfers
2. $I * J * 2$ marriages of type i, j, p
3. $I + J$ singles of types i and j.

In general, there will not be a convenient analytic expression for the transfer. Thus, in Section 5, we provide a simple example for a marriage market with one type of man and one type of woman for illustrative purposes.

For the couples where both spouses work, marriage market clearing provides an over-identifying restriction on the sharing rule within the collective model. In particular, since we can solve a sharing rule from the marriage market that is independent of the sharing rule we derived from labor supplies in Section 4.2, given estimates of $\Delta^{\cdot}_{0j}$, $\Delta^{\cdot}_{0j}$, $\delta^{\cdot}_{i0}$, δ^1_{ij}, $\tilde{\Gamma}^1_{ij}$, and $\tilde{\gamma}^1_{ij}$ we can test whether (50) is consistent with (31). In other words, we can test whether the sharing rule that rationalizes labor supply in marriage couples arises as an equilibrium outcome in the marriage market.

4.4 Summary of identification results

We can summarize our identification results as follows. For couples in which both partners work:

1. From observations on labor supplies in one marriage market we can identify $(\theta_{ij}^{1t}+\Theta_{ij}^{1t})$, λ_{ij}^{1t}, Λ_{ij}^{1t}. Each parameter can be identified separately off male and female labor supplies; thus they are over-identified in non-specialized couples.

2. With the introduction of two marriage markets, along with labor supply, we can identify δ_{ij}^{1t}, Δ_{ij}^{1t}, and τ_{ij}^{1t}. Furthermore, $(\theta_{ij}^{1t}+\Theta_{ij}^{1t})$, λ_{ij}^{1t}, Λ_{ij}^{1t} are over-identified. The imposition of marriage market clearing is not necessary for identification.

3. If we also impose marriage market clearing, we can identify $\widetilde{\gamma}_{ij}^{1t}$ and $\widetilde{\Gamma}_{ij}^{1t}$. These parameters are over-identified with two markets. It will also be the case that τ_{ij}^{1t} is over-identified. We can test whether the sharing rule in the collective model is empirically consistent with the transfer that clears the marriage market.

For specialized couples, in which the wife does not work:

1. From observations on the husband's labor supply in one marriage market, we can identify $(\theta_{ij}^{\cdot t}+\Theta_{ij}^{\cdot t})$ and $\lambda_{ij}^{\cdot t}$.

2. With the introduction of two marriage markets, $(\theta_{ij}^{\cdot t}+\Theta_{ij}^{\cdot t})$ and $\lambda_{ij}^{\cdot t}$ are over-identified, but no additional parameters are identified.

3. If we also impose marriage market clearing, we can identify δ_{ij}^{0t}, $\Delta_{ij}^{\cdot t}$, $\tau_{ij}^{\cdot t}$, and $\widetilde{\gamma}_{ij}^{\cdot t}$. Each parameter is just-identified.

4. We cannot separately identify $\Lambda_{ij}^{\cdot t}$ and $\widetilde{\Gamma}_{ij}^{\cdot t}$.

5 A simple example

In this section, we present a simple example that allows us to derive an expression for the reduced form transfer that clears the marriage market. Suppose we consider a marriage market with one type of woman and one type of man, i.e. $I=J=1$. Suppose further that all agents work positive hours. In this case, the equilibrium sharing rule takes the form:

$$\tau_{11}^{1}=\frac{(M-\mu_{11}^{1})a}{(F-\mu_{11}^{1})A+(M-\mu_{11}^{1})a}$$

where

$$A = \frac{\widetilde{\Gamma}^1_{ij}\Upsilon^{1t}_{ij}(W^{1t}_{ij})^{-\Delta^1_{ij}}}{\Upsilon^{\cdot t}_{0j}(W^{\cdot t}_{0j})^{-\Delta_{\dot{0}j}}},$$

$$a = \frac{\widetilde{\gamma}^1_{ij}\Upsilon^{1t}_{ij}(w^{1t}_{ij})^{-\delta^1_{ij}}}{\Upsilon^{\cdot t}_{i0}(w^{\cdot t}_{i0})^{-\delta_{\dot{i}0}}},$$

and μ^1_{11} is the solution to a quadratic equation of the form $\alpha_a x^2 + \alpha_b x + \alpha_c = 0$ where

$$\begin{aligned}
\alpha_a =& Aa + A + a \\
\alpha_b =& -[AF + aM + Aa(F + M)] \\
\alpha_c =& AaFM.
\end{aligned}$$

There is only one positive root to this quadratic equation; thus the equilibrium stock of marriages is described by:

$$\mu^1_{11} = \frac{[AF + aM + Aa(F + M)] - [(AF + aM + Aa(F + M))^2 - 4(Aa + A + a)AaFM]^{\frac{1}{2}}}{2(Aa + A + a)}. \tag{52}$$

The equilibrium measure of marriages and the equilibrium transfer are complicated functions of the aggregate stocks of men and women, as well as wages and non-labor incomes for men and women when single and married.

6 Endogenous Fertility in the Collective Model

In this section, we show how the model can be extended to incorporate endogenous fertility. The decision to have children is made at the same stage as the labor force participation decision of women. In other words, when deciding whether and whom to marry, agents also decide whether to enter specialized or non-specialized marriages (distinguished by the participation decision of the wife) and whether to have a family (of a particular size) or to be a childless couple. This version of the model allows for differences in home production technologies for families of different sizes. Children are not treated as decision-makers in this version of the model.[14] Parents have preferences over children's consumption. Children's

[14]For a collective model with more than two decision-makers, see Dauphin, El Lahga, Fortin, and Lacroix (2005).

consumption is a public good in the household and parents need not agree on the valuation of this good. This extension of the collective model has been considered by Chiappori, Blundell, and Meghir (2004) (hereafter CBM). CBM establish that this version of the collective model yields efficient outcomes as long as the public good is separable from the private good and leisure in preferences. We use these results here to consider the implications of children for the intra-household allocation of resources. As in Chiappori, Blundell, and Meghir, children are taken as given at the time labor supply decisions are made. Our framework thus adds nothing new in the analysis of the intra-household allocation of resources in the presence of children. Where our framework differs is that endogenize the fertility decision as part of the matching process.

In the extended model, we can describe preferences for women as:

$$\begin{aligned} U_{ij}^{pft}(C_{ijG}^{pft}, H_{ijG}^{pft}, K_{ijgG}^{pft}, \varepsilon_{ijG}^{pft}) = {} & \Delta_{ij}^{fp} \ln\left(\frac{C_{ijG}^{pft} - \Theta_{ij}^{pf}}{\Delta_{ij}^{pf}}\right) + \Phi_{ij}^{pf} \ln\left(\frac{\Lambda_{ij}^{pf} - H_{ijG}^{pft}}{\Phi_{ij}^{pf}}\right) \\ & + (1 - \Delta_{ij}^{fp} - \Phi_{ij}^{fp}) \ln\left(\frac{K_{ijgG}^{pft} - \Psi_{ij}^{pf}}{(1 - \Delta_{ij}^{pf} - \Phi_{ij}^{pf})}\right) + \Gamma_{ij}^{pf} + \varepsilon_{ijG}^{pft}, \end{aligned}$$

where K is total consumption of the kids in the household and f is the number of kids ($f \in \{0, 1, ..., F\}$). Preferences for men can be described by:

$$\begin{aligned} u_{ij}^{pft}(c_{ijg}^{pft}, h_{ijg}^{pft}, K_{ijgG}^{pft}, \varepsilon_{ijg}^{pft}) = {} & \delta_{ij}^{fp} \ln\left(\frac{c_{ijg}^{pft} - \theta_{ij}^{pf}}{\delta_{ij}^{pf}}\right) + \phi_{ij}^{pf} \ln\left(\frac{\lambda_{ij}^{pf} - h_{ijg}^{pft}}{\phi_{ij}^{pf}}\right) \\ & + (1 - \delta_{ij}^{fp} - \phi_{ij}^{fp}) \ln\left(\frac{K_{ijgG}^{pft} - \Psi_{ij}^{pf}}{(1 - \delta_{ij}^{pf} - \phi_{ij}^{pf})}\right) + \gamma_{ij}^{pf} + \varepsilon_{ijg}^{pft}. \end{aligned}$$

Parents are assumed to jointly agree to an efficient level of consumption for their children in the first stage of the two stage budgeting process. Full family income in the second stage of the budgeting process becomes:

$$\Upsilon_{ijgG}^{pft} = A_{ijgG}^{pft} - \theta_{ij}^{pf} - \Theta_{ij}^{pf} + \Lambda_{ij}^{pf} W_{ijG}^{pft} + \lambda_{ij}^{pf} w_{ijg}^{pft} - K_{ijgG}^{pft}.$$

Notice, full income for couples is now net of expenditures on children.

We can now describe the problem solved by married agents in the second period. The objective of women in $\{i, j, 1, t\}$ marriages, given τ_{ij}^{1t} and K_{ijgG}^{pft}, is

$$\max_{C_{ijG}^{1t}, L_{ijG}^{1t}} U_{ij}^{pft}(C_{ijG}^{pft}, H_{ijG}^{pft}, K_{ijgG}^{pft}, \varepsilon_{ijG}^{pft})$$
$$\text{subject to } W_{ijG}^{1t} L_{ijG}^{1t} + C_{ijG}^{1t} \leq \tau_{ij}^{1t} \Upsilon_{ijgG}^{1t} + \Theta_{ij}^{1}$$

as before. Women in $\{i, j, 0, t\}$ marriages make no decisions after deciding to marry and choosing consumption for her children jointly with her spouse. The objective of men in $\{i, j, p, t\}$ marriages, given τ_{ij}^{pt} and K_{ijgG}^{pft}, is

$$\max_{c_{ijg}^{pft}, l_{ijg}^{pft}} u_{ij}^{pft}(c_{ijg}^{pft}, l_{ijg}^{pft}, K_{ijgG}^{pft}, \varepsilon_{ijg}^{pft})$$
$$\text{subject to } w_{ijg}^{pft} l_{ijg}^{pft} + c_{ijg}^{pft} \leq \tau_{ij}^{pft} \Upsilon_{ijgG}^{pft} + \theta_{ij}^{pf}.$$

Finally, the objectives of single women and single men are

$$\max_{C_{0jG}^{\cdot ft}, L_{0jG}^{\cdot ft}, K_{0jG}^{\cdot ft}} U_{0j}^{\cdot ft}(C_{0jG}^{\cdot t}, L_{0jG}^{\cdot t}, K_{0jG}^{\cdot ft}, \varepsilon_{0jG}^{\cdot ft})$$
$$\text{subject to } W_{0jG}^{\cdot ft} L_{0jG}^{\cdot ft} + C_{0jG}^{\cdot ft} + K_{0jG}^{\cdot ft} \leq \Upsilon_{0jG}^{\cdot ft} + \Theta_{0j}^{\cdot f},$$

and

$$\max_{c_{i0g}^{\cdot ft}, l_{i0g}^{\cdot ft}, K_{i0G}^{\cdot ft}} u_{i0}^{\cdot ft}(c_{i0g}^{\cdot ft}, l_{i0g}^{\cdot ft}, K_{i0g}^{\cdot ft}, \varepsilon_{i0g}^{\cdot ft})$$
$$\text{subject to } w_{i0g}^{\cdot ft} l_{i0g}^{\cdot ft} + c_{i0g}^{\cdot ft} + K_{i0g}^{\cdot ft} \leq \Upsilon_{i0g}^{\cdot ft} + \theta_{i0}^{\cdot f},$$

respectively.

Denote $\hat{C}_{ijG}^{pft}$, $\hat{C}_{0jG}^{\cdot ft}$, $\hat{L}_{ijG}^{pft}$, $\hat{L}_{0jG}^{\cdot ft}$, $\hat{c}_{ijg}^{pft}$, $\hat{c}_{i0g}^{\cdot ft}$, $\hat{l}_{ijg}^{pft}$, $\hat{l}_{i0g}^{\cdot ft}$, $\hat{K}_{ijG}^{pft}$, $\hat{K}_{0jG}^{\cdot ft}$, and $\hat{K}_{i0G}^{\cdot ft}$ the solutions to the labor supply and consumption decisions of single and married agents. The associated indirect utilities are described by $V_{0j}^{\cdot f}(\Theta_{ij}^{\cdot f}, A_{0j}^{\cdot ft}, W_{0j}^{\cdot ft}, \varepsilon_{0jG}^{\cdot ft})$,..,$V_{ij}^{0f}(\theta_{ij}^{0f} + \Theta_{ij}^{0f}, \tau_{ij}^{0ft}, A_{ij}^{0ft}, w_{ij}^{0ft}, \varepsilon_{ijG}^{0ft})$,... $V_{Ij}^{1f}(\theta_{Ij}^{1f} + \Theta_{Ij}^{1f}, \tau_{Ij}^{1ft}, A_{Ij}^{1ft}, W_{Ij}^{1ft}, w_{Ij}^{1ft}, \varepsilon_{IjG}^{1ft}$ for women and $v_{i0}^{\cdot f}(\theta_{ij}^{\cdot f}, a_{i0}^{\cdot ft}, w_{i0}^{\cdot ft}, \varepsilon_{i0g}^{\cdot ft})$,..,$v_{ij}^{0}(\theta_{ij}^{0f} + \Theta_{ij}^{0f}, \tau_{ij}^{0ft}, a_{ij}^{0t}, w_{ij}^{0ft}, \varepsilon_{i0g}^{0ft})$,.., $v_{iJ}^{1f}(\theta_{iJ}^{1f} + \Theta_{iJ}^{1f}, \tau_{iJ}^{1ft}, A_{iJ}^{1ft}, W_{iJ}^{1ft}, w_{iJ}^{1ft}, \varepsilon_{iJg}^{1ft})$ for men. Given the realizations of all the ε_{ijG}^{pft}, women will choose the combined marital and fertility choice which maximizes her expected utility. She can choose between $I * 2 * F + F$ choices. The expected utility from her optimal choice will satisfy:

$$V^*(\varepsilon_{0jG}^{ft},..,\varepsilon_{ijG}^{0ft},..,\varepsilon_{ijG}^{1ft},..,\varepsilon_{IjG}^{1ft}) =$$
$$\max[V_{0j}^{\cdot f}(\Theta_{ij}^{\cdot f}, A_{0j}^{\cdot ft}, W_{0j}^{\cdot ft}, \varepsilon_{0jG}^{\cdot ft}),..,V_{ij}^{0f}(\theta_{ij}^{0f} + \Theta_{ij}^{0f}, \tau_{ij}^{0ft}, A_{ij}^{0ft}, w_{ij}^{0ft}, \varepsilon_{ijG}^{0ft}),..,$$
$$V_{Ij}^{1f}(\theta_{Ij}^{1f} + \Theta_{Ij}^{1f}, \tau_{Ij}^{1ft}, A_{Ij}^{1ft}, W_{Ij}^{1ft}, w_{Ij}^{1ft}, \varepsilon_{IjG}^{1ft})]. \quad (53)$$

The problem facing men in the first stage is analogous to that of women. Given the realizations of all the ε_{ijg}^{pft}, he will choose the combined marital and fertility choice which maximizes his expected utility. He can choose between $J*2*F+F$ choices. The expected utility from his optimal choice will satisfy:

$$v*(\varepsilon_{i0g}^{ft},..,\varepsilon_{ijg}^{0ft},..,\varepsilon_{ijg}^{1ft},..,\varepsilon_{ijg}^{1ft}) =$$
$$\max[v_{i0}^{\cdot f}(\theta_{ij}^{\cdot f}, a_{i0}^{\cdot ft}, w_{i0}^{\cdot ft}, \varepsilon_{i0g}^{\cdot ft}),..,v_{ij}^{0f}(\theta_{ij}^{0f} + \Theta_{ij}^{0f}, \tau_{ij}^{0ft}, a_{ij}^{0ft}, w_{ij}^{0ft}, \varepsilon_{i0g}^{0ft}),..,$$
$$V_{iJ}^{1f}(\theta_{iJ}^{1f} + \Theta_{iJ}^{1f}, \tau_{iJ}^{1ft}, A_{iJ}^{1ft}, W_{iJ}^{1ft}, w_{iJ}^{1ft}, \varepsilon_{iJg}^{1ft})]. \quad (54)$$

7 Conclusion

A Efficient risk sharing

A.1 Couples with working wives

Consider a match between a female G and a male g in a match where both spouses work. The problem faced by a social planner in this instance is:

$$\max \int U(C_{gG}, L_{gG}) df(g, G)$$
$$\text{subject to } \int u(c_{gG}, l_{gG}) df(g, G) > \overline{u}$$
$$W_G(T - L_{gG}) + w_g(t - l_{gG}) + A_G + a_g = C_{gG} + c_{gG}, \ \forall G, g,$$

or equivalently:

$$\max \int U(W_G(T - L_{gG}) + w_g(t - l_{gG}) + A_G + a_g - c_{gG}, L_{gG}) df(g, G) \tag{55}$$
$$\text{subject to } \int u(c_{gG}, l_{gG}) df(g, G) > \overline{u}. \tag{56}$$

Let k be the Lagrange multiplier associated with the above reservation utility constraint for the husband, v. The first order conditions with respect to c_{gG}, L_{gG}, and l_{gG} yield:

$$U_c = ku_c \tag{57}$$
$$U_c w = ku_l \tag{58}$$
$$U_c W = U_L. \tag{59}$$

Equations (57) to (59) are the solution to the planner's problem.

Now consider the problem facing wives and husbands within the collective framework. Taking the sharing rule as given, they solve:

$$\max_{C_{gG}, L_{gG}} U(C_{gG}, L_{gG})$$
$$\text{subject to } W_{gG}L_{gG} + C_{gG} \leq \tau_{gG}\Upsilon_{gG}$$

and

$$\max_{c_{gG}, l_{gG}} u(c_{gG}, l_{gG})$$
$$\text{subject to } w_{gG}l_{gG} + c_{gG} \leq \tau_{gG}\Upsilon_{gG},$$

respectively.

The corresponding indirect utility functions can be written as:

$$U(C^*_{gG}, L^*_{gG}) = U(\tau_{gG}\Upsilon_{gG} - W_G L^*_{gG}, L^*_{gG})$$

for women, and

$$V(c^*_{gG}, l^*_{gG}) = V((1-\tau_{gG})\Upsilon gG - w_g l^*_{gG}, l^*_{gG})$$

for men.

Following Chiappori (1999), ex ante efficiency implies the sharing rule is the solution to:

$$max_{\tau_{gG}} \int U(\tau\Upsilon_{gG} - W_{gG}L^*_{gG}, L^*_{gG})df(g,G)$$
$$\text{subject to } \int u((1-\tau_{gG})\Upsilon_{gG} - w_g l^*_{gG}, l^*_{gG})df(g,G) > \overline{u}.$$

Let k be the multiplier. Then:

$$U_C\Upsilon - WU_C L^*_\tau + U_L L^*_\tau - Ku_c\Upsilon - Ku_c w l^*_\tau + Ku_l l^*_\tau = 0$$

Since $U_C W = U_L$ and $u_c w = u_l$, we have

$$U_C = Ku_c.$$

Differentiating indirect utility with respect to full income yields:

$$U_C\tau - U_C W L^*_\Upsilon + U_L L^*_\Upsilon \quad \text{or} \quad U_C\tau$$

for women and

$$u_c(1-\tau) - u_c w l^*_\Upsilon + u_l l^*_\Upsilon \quad \text{or} \quad u_c(1-\tau)$$

for men. Since $\frac{U_C}{u_c} = K$, then $\tau = \frac{1}{1+K}$ if the sharing rule is ex-ante efficient.

A.2 Couples with non-working wives

Consider a match between a female G and a male g in a match where the husband works and the wife does not. The problem faced by a social planner in this instance is:

$$\max \int U(C_{gG}) df(g, G)$$
$$\text{subject to } \int u(c_{gG}, l_{gG}) df(g, G) > \overline{u}$$
$$w_g(t - l_{gG}) + A_G + a_g = C_{gG} + c_{gG}, \ \forall G, g,$$

or equivalently:

$$\max \int U(w_g(t - l_{gG}) + A_G + a_g - c_{gG}) df(g, G)$$
$$\text{subject to } \int u(c_{gG}, l_{gG}) df(g, G) > \overline{u}.$$

Let k be the Lagrange multiplier associated with the above reservation utility constraint for the husband, v. The first order conditions with respect to c_{gG} and l_{gG} yield:

$$U_c = k u_c \tag{60}$$
$$U_c w = k u_l. \tag{61}$$

Equations (60) and (61) are the solution to the planner's problem.

Now consider the problem facing wives and husbands within the collective framework. Taking the sharing rule as given, the husband solves:

$$\max_{c_{gG}, l_{gG}} u(c_{gG}, l_{gG})$$
$$\text{subject to } w_{gG} l_{gG} + c_{gG} \leq \tau_{gG} \Upsilon_{gG}.$$

The corresponding indirect utility functions can be written as:

$$U(C^*_{gG}) = U(\tau_{gG} \Upsilon_{gG})$$

for women, and

$$V(c^*_{gG}, l^*_{gG}) = V((1 - \tau_{gG}) \Upsilon gG - w_g l^*_{gG}, l^*_{gG})$$

for men.

Following Chiappori (1999), ex ante efficiency implies the sharing rule is the solution to:

$$max_{\tau_{gG}} \int U(\tau \Upsilon_{gG}) df(g,G)$$

$$\text{subject to} \int u((1-\tau_{gG})\Upsilon_{gG} - w_g l^*_{gG}, l^*_{gG}) df(g,G) > \overline{u}.$$

Let k be the multiplier. Then:

$$U_C \Upsilon - K u_c \Upsilon - K u_c w l^*_\tau + K u_l l^*_\tau = 0$$

Since $u_c w = u_l$, we have

$$U_C = K u_c.$$

Differentiating indirect utility with respect to full income yields:

$$U_C \tau$$

for women and

$$u_c(1-\tau) - u_c w l^*_\Upsilon + u_l l^*_\Upsilon$$

$$\text{or } u_c(1-\tau)$$

for men. Since $\frac{U_C}{u_c} = K$, then $\tau = \frac{1}{1+K}$ if the sharing rule is ex-ante efficient.

Thus our way of modelling the sharing rule clearing, where τ is independent of the idiosyncratic shocks which affect the family, subsumes risk sharing over full income within the family. Put another way, given τ, there is no other intra-household reallocation of resources which can increase the ex-ante utility of one spouse without making the other spouse worse off.

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