Risk Assessment for Banking Systems*

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Abstract

We propose a new approach to assess the financial stability of an entire banking system using standard tools from modern risk management in combination with a network model of inter-bank loans. Rather than looking at banks individually, we analyze risk at the level of the banking system as a whole. We apply our model to a unique dataset of all Austrian banks. We find that correlation in banks’ asset portfolios dominates contagion as the main source of systemic risk. Contagion occurs rarely but can wipe out a major part of the banking system. Low bankruptcy costs and an efficient crisis resolution policy are crucial to limit the system wide impact of contagious default events. We compute the “value at risk” for a lender of last resort and find the necessary funds to prevent contagion to be surprisingly small. More diversification in the inter-bank market does not necessarily reduce the risk of contagion.

Keywords: Systemic Risk, Financial Stability, Risk Management, Inter-bank Market

JEL-Classification Numbers: G21, C15, C81, E44
1 Introduction

A large scale breakdown of financial intermediation causes huge economic and social costs. Various policy measures have therefore been initiated to improve financial stability by ensuring an appropriate combination of official and market discipline for banks.\(^1\) It has also been a widely held view that official discipline which is implemented by supervision and regulation should, ultimately, be directed towards achieving the overall stability of the financial system. Greenspan (1997) notes, for instance, on the FED’s agenda: “...second only to its macro-stability responsibilities is the central bank’s responsibility to use its authority and expertise to forestall financial crises (including systemic disturbances in the banking system) and to manage such crises once they occur.” Yet the notion of a system (or macroprudential\(^2\)) perspective on supervision has remained rather vague. In this paper we demonstrate that the methods of modern risk management when combined with a careful analysis of financial linkages between banks provide a powerful set of tools to address this issue. Building on these tools our analysis makes a precise suggestion for a system approach to risk assessment of banking systems.

The main challenge is to capture, in a tractable way, the two major sources of systemic risk: first, banks might have correlated exposures and an adverse economic shock may directly result in simultaneous multiple bank defaults; second, troubled banks may default on their inter-bank liabilities and hence cause other banks to default triggering a domino effect. In order to integrate both causes of systemic risk we have to analyze the market- and credit portfolios of all banks simultaneously. At the same time financial linkages and their role in the propagation of shocks in the banking system have to be studied. The innovation of our model is to combine tools from modern financial risk management with a network analysis of the inter-bank market. To analyze the stability of the banking system we determine endogenous default probabilities of individual banks as well as probabilities of joint default. Thus, the model enables us to quantify the probability of a systemic crisis. We study the contribution of contagion to systemic risk by decomposing insolvencies into cases that result from domino effects (contagious defaults) and cases that do not (fundamental defaults). We employ different resolution rules in the case of default to analyze the consequences in terms of contagion. We analyze two

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1 These terms have been used by Crockett (2001).

2 See also Borio (2002).
driving factors of contagion: bankruptcy costs and diversification of banks’ lending in the inter-bank market. The results provide insights into the amount of capital a lender of last resort might have to provide to avoid a crisis. Combining our endogenous default probabilities and endogenous recovery rates can help to fine tune appropriate regulatory capital charges for inter-bank exposures.

We apply the model to a unique dataset provided by the Austrian Central Bank. This dataset gives us detailed information on inter-bank liabilities for the whole banking system. We also have information on the individual banks’ market risk exposures as well as detailed information on the banks’ loan portfolio composition. Such a rich dataset is only available in very few countries and, to the best of our knowledge, this is the first attempt to utilize such a comprehensive dataset for the risk analysis of an entire banking system.

As a result of this analysis we find that among the two sources of systemic risk the correlation in exposures is far more important than financial linkages. We find that the mean default probability is 0.8% but the probability of defaulting due to contagion for the average bank is only 0.0062%. Even though contagious defaults occur rarely, there are scenarios with many contagious defaults. In our simulation we find a scenario where 75% of all defaults are due to contagion. Thus, contagion is a low probability-high impact event. Second, we find that bankruptcy costs play a decisive role for the intensity of domino effects. The probability of contagious defaults depends on bankruptcy costs in a non-linear way. An efficient bankruptcy procedure is therefore crucial in safeguarding financial stability. Third, we find that the reserves to be maintained by a lender of last resort to prevent contagious defaults are relatively small: about 0.0003% of the total assets in the banking system are sufficient to prevent contagion in 99% of the scenarios. However, substantial amounts are required to prevent fundamental defaults. Here, the regulator needs to set aside 0.16% of the total assets for the same confidence level. In this sense, our analysis can be used to pin down a “value-at-risk” for a lender of last resort. Fourth, our framework allows us to vary the structure of linkages in a consistent way and to study the impact on domino effects of bank insolvency. We find that greater diversification in the inter-bank market does not necessarily reduce contagion. Finally, our dataset allows us to compute numbers for bank default probabilities and recovery rates of inter-bank loans. Precise estimates are essential to determine appropriate capital
requirements for inter-bank loans.

Earlier simulation studies analyzing inter-bank exposures such as Humphrey (1986), Angelini, Maresca, and Russo (1996), Furfine (2003), and Upper and Worms (2002) investigate contagious defaults that result from the hypothetical failure of some single institution. Such analyses are able to capture the effect of idiosyncratic bank failures (e.g. because of fraud). This approach can be seen as isolating one source of systemic risk, namely, inter-bank linkages and ignoring the other: correlation in the banks’ exposures. We believe that only the combination of both aspects allows a meaningful risk assessment for the banking system as a whole. It is the actual exposure of banks to economic risks that determines the risk potential concealed in the network of mutual credit exposures among banks. Thus, our model takes these studies an important step further by combining the analysis of inter-bank connections with a simultaneous study of the banking system’s overall risk exposure. Instead of performing banking risk analysis on ad hoc single institution failure scenarios, we study risk scenarios for the banking system which are simulated using standard risk management techniques. Our model can therefore be seen as a comprehensive attempt to judge the risk exposure of the system as a whole.\(^3\)

The paper is organized as follows. Section 2 describes the network model of the inter-bank market and Section 3 details the sample. The two components of the simulation analysis, the structure of the inter-bank liabilities and the generation of economic scenarios are described in Sections 4 and 5, respectively. Section 6 presents the results of the simulation. Conclusions are listed under Section 7.

2 A Network Model of the Inter-bank Market

The conceptual framework we use to describe the system of inter-bank credits has been introduced to the literature by Eisenberg and Noe (2001) who have studied a centralized static clearing mechanism for a financial system with exogenous income positions and a given structure of bilateral nominal liabilities. We build on this model and extend it to include uncertainty.

\(^3\)Such a ‘system perspective’ on banking supervision has for instance been actively advocated by Hellwig (1997).
Consider a finite set $\mathcal{N} = \{1, \ldots, N\}$ of banks. Each bank $i \in \mathcal{N}$ is characterized by a given value $e_i$ net of inter-bank positions and nominal liabilities $l_{ij}$ against other banks $j \in \mathcal{N}$ in the system. The entire banking system is thus described by an $N \times N$ matrix $L$ and a vector $e \in \mathbb{R}^N$. We denote this system by the pair $(L, e)$.

If for a given pair $(L, e)$ the total net value of a bank becomes negative, the bank is insolvent. In this case it is assumed that creditor banks are rationed proportionally. Following Eisenberg and Noe (2001) we can formalize proportional rationing in case of default as follows: denote by $d \in \mathbb{R}^N_+$ the vector of total obligations of banks towards the rest of the system i.e., we have $d_i = \sum_{j \in \mathcal{N}} l_{ij}$. Proportional sharing of value in case of insolvency is described by defining a new matrix $\Pi \in [0, 1]^{N \times N}$ which is derived from $L$ by normalizing the entries by total obligations.

$$\pi_{ij} = \begin{cases} \frac{l_{ij}}{d_i} & \text{if } d_i > 0 \\ 0 & \text{otherwise} \end{cases}$$ (1)

We describe a financial system as a tuple $(\Pi, e, d)$ for which we define a so called clearing payment vector $p^*$ that respects limited liability of banks and proportional sharing in case of default. It denotes the total payments made by the banks under the clearing mechanism.

**Definition 1** A clearing payment vector for the system $(\Pi, e, d)$ is a vector $p^*$ such that for all $i \in \mathcal{N}$

$$p^*_i = \min \left[ d_i, \max \left( \sum_{j=1}^{N} \pi_{ji} p^*_j + e_i, 0 \right) \right]$$ (2)

Thus, the clearing payment vector directly gives us two important insights: for a given structure of liabilities and bank values $(\Pi, e, d)$ it tells us which banks in the system are insolvent ($p^*_i < d_i$) and it tells us the recovery rate for each defaulting bank ($\frac{p^*_i}{d_i}$).

To find a clearing payment vector we employ the fictitious default algorithm developed by Eisenberg and Noe (2001) where they prove that under mild regularity conditions a unique clearing payment vector for $(\Pi, e, d)$ always exists. These results extend - with slight modifications - to our framework as well.\footnote{In Eisenberg and Noe (2001) the vector $e$ is in $\mathbb{R}_+^N$ whereas in our case the vector is in $\mathbb{R}^N$.}
From the solution of the clearing problem, we can gain additional economically important information with respect to systemic stability. Default of bank $i$ is called fundamental if bank $i$ is not able to honor its promises under the assumptions that all other banks honor their promises, \(^5\), i.e.

$$\sum_{j=1}^{N} \pi_{ji}d_j + e_i - d_i < 0.$$ 

A contagious default occurs, when bank $i$ defaults only because other banks are not able to keep their promises, i.e.

$$\sum_{j=1}^{N} \pi_{ji}d_j + e_i - d_i \geq 0 \text{ but } \sum_{j=1}^{N} \pi_{ji}p^*_j + e_i - d_i < 0.$$ 

To use the model for risk analysis, we extend it to an uncertainty framework by assuming that $e$ is a random variable. \(^6\) As there is no closed form solution for the distribution of $p^*$, given the distribution of $e$, we have to resort to a simulation approach where each draw is called a scenario. By the theorem of Eisenberg and Noe (2001) we know that there exists a (unique) clearing payment vector $p^*$ for each scenario. \(^7\) Thus from an ex-ante perspective we can assess expected default frequencies from inter-bank credits across scenarios as well as the expected severity of losses from these defaults given that we have an idea about the distribution of $e$. Furthermore, we are able to decompose insolvencies across scenarios into fundamental and contagious defaults. An example illustrating the procedure is given in Appendix B.

To pin down the distribution of $e$ we choose the following approach: assume that there are two dates: $t = 0$ which is the \textit{observation date} and $t = 1$ which is a hypothetical clearing date where all inter-bank claims are settled according to the clearing mechanism. At

\(^5\)Note that our setup implicitly contains a seniority structure of different debt claims of banks. By interpreting $e_i$ as net value from all bank activities except the inter-bank business we assume that inter-bank debt claims are junior to other claims, like deposits or bonds. However inter-bank claims have absolute priority in the sense that the owners of the bank get paid only after all debts have been paid. In reality the legal situation is much more complicated and the seniority structure might very well differ from the simple procedure we employ here. For our purpose it gives us a convenient simplification that makes a rigorous analysis of inter-bank defaults tractable.

\(^6\)One could also allow for a stochastic matrix $L$. In our analysis we take the nominal face value of inter-bank debt as fixed.

\(^7\)Our system $(L, e)$ fulfills all the necessary conditions in each scenario.
$t = 0$ the portfolio holdings of each bank are observed. The inter-bank related exposures constitute the matrix $L$. The remaining portfolio holdings consist of loans, bonds, stocks on the asset side and of liabilities to non banks on the liabilities side. These positions are exposed to market and/or credit risk. We assume that the portfolio holdings remain constant for the time horizon under consideration. Hence the value of the portfolio at $t = 1$ depends solely on the realization of the relevant risk factors. To generate a scenario we draw a realization of these risk factors from their joint distribution and revalue the portfolio. Given this scenario the system is cleared and a clearing vector $p^*$ is determined.

An application of the network model for the assessment of credit risk from inter-bank positions therefore requires the determination of $L$ from the data and the definition of a framework to create meaningful risk scenarios.

3 Data

Our main sources of data are bank balance sheet and supervisory data from the monthly reports (MAUS) to the Austrian Central Bank (OeNB) and the database of the OeNB major loans register (Großkreditevidenz, GKE). We also use data on default frequencies in certain industry groups from the Austrian rating agency Kreditschutzverband von 1870. Finally we use market data from Datastream.

3.1 Bank Data

Banks in Austria file monthly reports on their business activities to the central bank. In addition to balance sheet data, MAUS contains a fairly extensive assortment of other data that are required for supervisory purposes. They include among others numbers on capital adequacy statistics, off-balance exposures, times to maturity and foreign exchange exposures with respect to different currencies.

In our analysis we use a cross section from the MAUS database of all 881 independent banks for September 2002 which we take as our observation period. We use these data to determine the matrix $L$ as well as the portfolio holdings that are not related to the inter-bank business. The data on inter-bank exposures is not as detailed as required for
our purposes. Yet a particular institutional feature of the Austrian banking system helps us with the estimation of bilateral inter-bank exposures. Austrian banks are grouped into sectors for historic reasons.\textsuperscript{8} Three out of the seven sectors have a multi-tier structure with head institutions. Banks have to break down their MAUS reports on claims and liabilities with other banks according to the different banking sectors, head institution, central bank, and foreign banks. This practice of reporting on balance inter-bank positions reveals some structure of the $L$ matrix. Over the years, this sectoral organization has lost relevance with few banks confined to specific lines of business.

### 3.2 Credit Exposure Data

The major loans register of OeNB (GKE) provides us with detailed information on the banks’ loan portfolios to non-banks. This database contains all loans exceeding a volume of 350,000 Euro on a loan by loan basis.\textsuperscript{9} They are classified into 59 industry sectors according to the NACE standard, plus 3 aggregate foreign non bank sectors grouped by industrialized countries, non industrialized countries, and Eastern Europe.\textsuperscript{10} Since only loans above a threshold volume are reported, we introduce domestic and foreign residual categories computed from the difference between the total loan volume numbers in the banks’ balance sheets and the volume numbers of the major loan register.

Combining this information with data from the Austrian rating agency Kreditschutzverband von 1870 (KSV) we can estimate the riskiness of a loan in a particular industry. The KSV database gives us time series of default rates for the different NACE branches. From these statistics we estimate the average default frequency and its standard deviation for each NACE branch. These data serve as input to the credit risk model.

For that part of the loans which we cannot allocate to particular industry sectors, we have no default statistics and do not know the number of loans. To construct insolvency statistics for the residual sector we take averages from the data that are available. To

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\textsuperscript{8}Banks belong to one of seven sectors: joint stock banks, savings banks, state mortgage banks, Raiffeisen banks, Volksbanken, building associations, savings and loan associations, and special purpose banks.

\textsuperscript{9}The GKE database covers 64.7\% of all loans of Austrian banks.

\textsuperscript{10}Because of the close economic links between Austria and Central and Eastern European Countries we chose to define them as a separate category in the loan portfolio.
arrive at an estimate of the number of loans for the residual sector, we assume that the loan numbers in industry and in the residual sector are proportional to the share of loan volumes between these sectors. We should also note that the insolvency series is very short. Data are available on a semi-annual basis beginning with January 1997.

3.3 Market Data

Certain assets in the banks’ portfolios are subject to market risk. Daily market data corresponding to the exposure categories are collected for the twelve years from September 1989 to September 2002 from Datastream. These data are used for the simulation of scenarios. Specifically we collect exchange rates of USD, JPY, GBP and CHF to the Euro (Austrian Schilling before 1999) to compute exchange rate risk. As the reported equity holdings are only divided into domestic and international equity exposure we include the Austrian index ATX and the MSCI-world index in our analysis. To account for interest rate risk, we compute zero bond prices for maturities of three months, one, five and ten years, using zero rates in EUR, USD, JPY, GBP and CHF.\textsuperscript{11}

4 Estimating Inter-bank Liabilities from Partial Information

Partial information about the inter-bank liability matrix $L$ is readily available. The bank by bank record of assets and liabilities with other banks gives us the column and row sums of the matrix $L$. Further, we possess some structural information. For instance, it is obvious that the diagonal of $L$ must contain only zeros since banks do not have claims and liabilities against themselves. We exploit the sectoral structure of the Austrian banking system as described in Section 3 to determine many of the entries in $L$ exactly. This pins down $72\%$ of all entries of the matrix $L$.

We estimate the remaining $28\%$ of the entries of $L$ by optimally exploiting the available information. Our ignorance about the unknown parts of the matrix should be reflected

\textsuperscript{11}Sometimes a zero bond series is not available for the length of the period required for our exercise. In these cases swap rates have been employed.
in the fact that these entries are treated homogeneously in the reconstruction process. The procedure should be adaptable to include any new information that might become available in the process of data collection. We formulate the reconstruction of the unknown parts of the $L$ matrix as an entropy optimization problem.

Intuitively, this procedure finds a matrix that treats all entries as balanced as possible and satisfies all known constraints. This can be formulated as minimizing a suitable measure of distance between the estimated matrix and a matrix that reflects our a priori knowledge on large parts of bilateral exposures. The so called cross entropy measure is a suitable concept for this task (see Fang, Rajasekera, and Tsao (1997) or Blien and Graef (1997)). A detailed description is provided in Appendix C.

In the present case, the application of entropy optimization is not straightforward, due to data inconsistencies. For instance, the liabilities of all banks in sector $k$ against all banks in sector $l$ do typically not equal the claims of all banks in sector $l$ against all banks in sector $k$.\footnote{Some of the inconsistencies seem to suggest that the banks assign some of their counterparties to the wrong sectors.} A proposed workaround to this problem is described in detail in Appendix C.

We see two main advantages of this method to deal with the incomplete information problem encountered. First, the method is fairly flexible with respect to the inclusion of additional information we might gather from different sources. Second, there exist computational procedures that are easy to implement and that can deal efficiently with very large datasets (see Fang, Rajasekera, and Tsao (1997)). Thus, problems similar to ours can be solved efficiently and quickly on an ordinary personal computer, even for very large banking systems.

5 Generating Scenarios

Our model of the banking sector uses different scenarios to model uncertainty. In each scenario banks face gains and losses from FX and interest rate changes as well as from equity price changes and losses from loans to non-banks. Some banks may fail. This possibly triggers failures of other banks, as modeled in our network clearing framework.
Figure 1. The figure shows the basic structure of the model. Banks are exposed to shocks from credit risk and market risk according to their respective exposures. Due to these economic shocks some banks may default. Inter-bank credit risk is endogenously explained by the network model. The clearing of the inter-bank market determines the solvency of other banks and defines endogenous default probabilities for banks as well as the respective recovery rates.

Hence, the credit risk in the inter-bank network is modeled endogenously while all other risks are reflected in the position $e$. We examine the consequences of different scenarios for $e$ on the banking system. Figure 1 shows the basic structure of our model.

We choose a standard risk management framework to model the shocks to banks. To simulate scenario losses that are due to exposures to market risk, we conduct a historical simulation, where we expose all banks simultaneously to the same realizations of the risk factors. To capture losses from loans to non-banks we use a credit risk model.

Table 1 lists the balance sheet items included in our analysis and the corresponding model for the risk exposure. Market risk (stock price changes, interest rate movements and FX rate shifts) is captured by a historical simulation approach (HS) for all items...
Table 1. The table summarizes how risk of the different balance sheet positions is covered in our scenarios. HS is short for historic simulation.

except other assets and other liabilities, which include long term major equity stakes in not-listed companies, non financial assets like property and office equipment and cash on the asset side and equity capital and provisions on the liability side. Credit losses from non-banks are modeled using a credit risk model. The credit risk from bonds is not included since most banks hold only government bonds. The credit risk in the inter-bank market is determined endogenously.

### 5.1 Market Risk: Historical Simulation

We use a historical simulation approach as it is documented in the standard risk management literature (Jorion (2000)) to assess the market risk of the banks in our system. This methodology has the advantage that we do not have to specify a certain parametric distribution for our returns. Instead we can use the empirical distribution of past observed returns and thus capture also extreme changes in market risk factors. By this procedure we capture the joint distribution of the market risk factors and thus take correlation structures between interest rates, stock markets, and FX markets into account.

To estimate shocks on bank capital stemming from market risk, we include positions in foreign currency, equity, and interest rate sensitive instruments. For each bank, we collect

<table>
<thead>
<tr>
<th>Assets</th>
<th>Interest rate/stock price risk</th>
<th>Credit risk</th>
<th>FX risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>short term government bonds and receivables</td>
<td>Yes (HS)</td>
<td>No</td>
<td>Yes (HS)</td>
</tr>
<tr>
<td>loans to other banks</td>
<td>Yes (HS)</td>
<td>endogenous by clearing</td>
<td>Yes (HS)</td>
</tr>
<tr>
<td>loans to non banks</td>
<td>Yes (HS)</td>
<td>credit risk model</td>
<td>Yes (HS)</td>
</tr>
<tr>
<td>bonds</td>
<td>Yes (HS)</td>
<td>no as mostly government</td>
<td>Yes (HS)</td>
</tr>
<tr>
<td>stock holdings</td>
<td>Yes (HS)</td>
<td>No</td>
<td>Yes (HS)</td>
</tr>
<tr>
<td>other assets</td>
<td>No</td>
<td>No</td>
<td>No</td>
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</tbody>
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<table>
<thead>
<tr>
<th>Liabilities</th>
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<td>liabilities other banks</td>
<td>Yes (HS)</td>
<td>endogenous by clearing</td>
<td>Yes (HS)</td>
</tr>
<tr>
<td>liabilities non banks</td>
<td>Yes (HS)</td>
<td>No</td>
<td>Yes (HS)</td>
</tr>
<tr>
<td>securitized liabilities</td>
<td>Yes (HS)</td>
<td>No</td>
<td>Yes (HS)</td>
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<tr>
<td>other liabilities</td>
<td>No</td>
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foreign exchange exposures for USD, JPY, GBP, and CHF only, as no bank in our sample has open positions of more than 1% of total assets in any other currency at the observation date. From the MAUS database we get exposures to foreign and domestic stocks, which are equal to the market value of the net position held in these categories. The exposure to interest rate risk can not be read directly from the banks’ monthly reports. Information on net positions in all currencies combined for different maturity buckets (up to 3 months but not callable, 3 months to 1 year, 1 to 5 years, more than 5 years) is available. These maturity bands allow only a coarse assessment of interest rate risk.

Nevertheless, the available data allow us to estimate the impact of changes in the term structure of interest rates. To calculate the interest rate exposure for each of the five currencies EUR, USD, JPY, GBP and CHF we split the aggregate exposure according to the relative weight of foreign currency assets in total assets. This procedure gives us a vector of 26 exposures, 4 FX, 2 equity, and 20 interest rate, for each bank. Thus we get a $N \times 26$ matrix of market risk exposure.

We collect daily market prices over 3,220 trading days for the risk factors as described in Subsection 3.3. From the daily prices of the 26 risk factors we compute daily returns. We rescale these to monthly returns assuming twenty trading days and construct a $26 \times 3219$ matrix $R$ of monthly returns.

For the historical simulation we draw 10,000 scenarios from the empirical distribution of returns. To illustrate the procedure: let $R_s$ be one such scenario, i.e. a column vector from the matrix $R$. Then the profits and losses that arise from a change in the risk factors as specified by the scenario are simply given by multiplying them with the respective exposures. Let the exposures that are directly affected by the risk factors in the historical simulation be denoted by $a$. The vector $aR_s$ then contains the profits or losses each bank realizes under the scenario $s$. Repeating the procedure for all 10,000 scenarios, we get a distribution of profits and losses due to market risk.
5.2 Credit Risk: Calculating Loan Loss Distributions

We employ one of the standard modern credit risk models, CreditRisk+ for the modeling of loan losses.\(^{13}\) While CreditRisk+ is designed to deal with a single loan portfolio, we have to deal with a *system* of portfolios since we have to consider all banks simultaneously. For the purpose of our analysis the correlation between loan losses across banks is important. The adaptation of the model to deal with such a system of loan portfolios turns out to be quite straightforward.

The loan loss distribution in the CreditRisk+ model is driven by two sources of uncertainty. First, economic uncertainty affects all loans. This models business cycle effects on average industry defaults: default frequencies increase in a recession and decrease in booms. Second, for a given economic shock, defaults are assumed to be conditionally independent. The conditional loss distribution can be derived analytically using an iterative algorithm. Throughout our calculations we assume a recovery rate of 50\% for corporate loans.

We construct the bank loan portfolios by grouping the loans to non banks by different industry sectors according to the information from the major loan register. The rest is summarized in a residual position as described in Section 3 above. Using the KSV insolvency statistics for each of the 59 industry branches and the three foreign sectors and the proxy insolvency statistics for the residual sectors, we can assign an unconditional expected default frequency and a standard deviation of this frequency to each loan. In line with the CreditRisk+ specification, we aggregate these numbers for each bank.

Figure 2 illustrates the procedure for scenario generation in our extended CreditRisk+ framework. We follow a four step procedure to generate scenarios for the whole banking system.\(^{14}\) In step one of the simulation we compute the distribution of each bank’s average default frequencies.\(^{15}\) Then we draw for each bank a realization from the bank’s individual

\(^{13}\)A recent overview on different standard approaches to model credit risk is Crouhy, Galai, and Mark (2000). CreditRisk+ is a trademark of Credit Suisse Financial Products (CSFP). It is described in detail in Credit Suisse (1997).

\(^{14}\)For a single loan portfolio these four stages can be combined and the unconditional loss distribution can be derived analytically. Our model is identical to the CreditRisk+ model in this case.

\(^{15}\)In CreditRisk+ the distribution of the expected default frequency is specified as a gamma distribution. The parameters of the gamma distribution can be determined by the average number of defaults in the loan portfolio and its standard deviation.
1. Calculate distribution of average default frequency

2. Draw same quantile for all banks (common economic shock)

3. Compute loan loss distributions

4. Draw independent quantile for each bank (idiosyncratic shock)

**Figure 2.** Computation of Credit loss scenarios following an extended CreditRisk+ model. Based on the composition of the individual bank’s loan portfolio we estimate the distribution of the mean default rate for each bank (step 1). Reflecting the idea of a common economic shock we draw the same quantile from each bank’s mean default rate distribution (step 2). Conditional on this draw, we can compute each bank’s individual loan loss distribution (step 3). The scenario loan losses are then drawn independently for each bank to reflect an idiosyncratic shock (step 4). 10,000 scenarios are drawn repeating steps 2 to 4.

distribution of average default frequencies. To model this as an economy wide shock, we draw the same quantile for all banks in the banking system (step 2). Given this realization of the average default frequency, defaults are assumed to be conditionally independent. We can then calculate a conditional loss distribution for each bank (step 3). Finally (step 4) we draw loan losses.\(^\text{16}\)

\(^{16}\)We apply standard variance reduction techniques in our Monte Carlo simulation. We go through the quantiles of the distribution of average default frequencies at a step length of 0.01. Thus, we draw hundred economy wide shocks from each of which we draw 100 loan loss scenarios, yielding a total number of 10,000 scenarios.
5.3 Combining Market Risk, Credit Risk, and the Network Model

The credit losses across scenarios are combined with the results of the historic simulations to create $e_i$ for each bank $i$. By the network model the inter-bank payments for each scenario are then calculated (see Figure 1). Thus we get endogenously a distribution of clearing vectors, default frequencies, recovery rates and statistics on the decomposition into fundamental and contagious defaults.

6 Results

The network model generates a distribution of clearing vectors $p^*$ and therefore also a distribution of insolvencies for each individual bank across scenarios: Whenever a component in $p^*$ is smaller than the corresponding component in $d$ the bank has not been able to honor its inter-bank promises. The relative frequency of default across scenarios is then interpreted as a default probability.

To discuss the effects of risk scenarios on the banking system it is useful to impose additional assumptions reflecting the time horizon we have in mind. Although, technically the network model works with a fixed future date at which all claims are cleared simultaneously we can model time horizons by imposing assumptions on details of the clearing process. We can model a short run perspective by assuming that there will be no inter-bank payments after netting following a bank default. We model a long run perspective by assuming that the residual value of an insolvent bank can be fully transferred to the creditor institutions up to some bankruptcy costs according to the rules of the clearing mechanism. The long run perspective analyzes shocks by assuming that an immediate stop of all payments after netting, as we assume under the short run perspective, can be prevented by crises management and an efficient bankruptcy procedure can eventually transfer the whole value of the insolvent institutions to its creditors.

Clearly both situations are of interest for a regulator. The short run assumptions help estimate the amounts that have to be ready immediately for emergency intervention to prevent wide spread contagion of defaults. The long run assumptions can then give an
estimate of the eventual costs of a shock to the system. Crises intervention decisions can be evaluated along these hypothetical situations.

We organize the discussion of our main results along the lines of the five most important lessons that can be drawn from the analysis: first, we find that correlated portfolio exposures of banks are the main source of systemic risk and that domino effects only occur rarely. However, contagion is a low probability high impact event. Second, bankruptcy costs play an important role for the system wide impact of contagious default events. Third, analyzing the “value at risk” for a lender of last resort reveals that the funds that are needed to stop contagious defaults can be surprisingly small. Fourth, we show that more diversification in the inter-bank market does not necessarily reduce the risk of contagion. Finally we quantify default probabilities and inter-bank recovery rates.

6.1 The two Sources of Systemic Risk: Correlated Exposures and Domino-effects

Perhaps one of the most interesting features of our method is that it allows a decomposition of bank insolvency cases into those resulting directly from macroeconomic shocks and those that are consequences of a domino effect. Bank defaults may be driven by losses from market and credit risk (fundamental default). Bank defaults may however also be initiated by contagion: as a consequence of other bank failures in the system (contagious default).

Our model when combined with our unique data set allows a quantification of these different cases and gives us a decomposition into fundamental and contagious defaults. The quantification of contagious default events using a real dataset is interesting since the empirical importance of domino effects in banking has been controversial in the literature.\textsuperscript{17}

Table 2 summarizes the probabilities of fundamental and contagious defaults derived from our data. These probabilities are grouped by the number of fundamentally defaulting banks. In 88.18\% of all scenarios 0 to 10 banks are facing fundamental default. In 87.03\% of all cases these defaults do not trigger any contagious defaults in the short run. In

\textsuperscript{17}See in particular the detailed discussion in Kaufman (1994).
### Table 2. Probabilities of fundamental and contagious defaults in the short run and in the long run.

A fundamental default is due to the losses arising from exposures to market risk and non-bank credit risk, while a contagious default is triggered by the default of another bank that cannot fulfill its promises in the inter-bank market. The short run analysis is under the assumption that insolvent banks pay nothing in the inter-bank market after netting, whereas zero bankruptcy costs are assumed in the long run simulation. Banks are grouped by fundamental defaults. The probability of occurrence of fundamental defaults alone and concurrently with contagious defaults is observed.

In the long run there is only one scenario where the fundamental default of up to 10 banks causes contagious defaults. If between 91 and 100 banks were to face fundamental default (0.19% of all scenarios), we see that in the short run this leads to contagious defaults in all cases. The results confirm the intuition that contagion is relatively more likely in scenarios where many banks simultaneously face fundamental default.

Comparing the short run and the long run simulation we see that the latter consists predominantly of scenarios with fundamental defaults and that domino effects do not play a particularly prominent role. However, in the short run, the incidence of contagion can reach fairly high levels (about 7% of all scenarios) and might be a cause for concern. We therefore cannot conclude from our analysis that contagion might be safely ignored for all practical purposes.

The importance of contagion in crisis scenarios can be seen in Table 3, where we compute several percentiles of the ratio of contagious defaults to total defaults. In the short run case contagious defaults can account for up to 3/4 of all defaults, i.e. there exists a scenario where 3/4 of the defaults are contagious. Although this is not a representative
Panel A: Short Run Simulation:

<table>
<thead>
<tr>
<th>Total Defaults</th>
<th>Prob. of State</th>
<th>Minimum</th>
<th>10%-Quantile</th>
<th>Median</th>
<th>90%-Quantile</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 10</td>
<td>87.14%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>50.00%</td>
</tr>
<tr>
<td>11 to 20</td>
<td>4.80%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>9.09%</td>
</tr>
<tr>
<td>21 to 30</td>
<td>1.91%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>3.70%</td>
<td>20.00%</td>
</tr>
<tr>
<td>31 to 40</td>
<td>1.60%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>2.63%</td>
<td>44.12%</td>
<td>63.33%</td>
</tr>
<tr>
<td>41 to 50</td>
<td>1.45%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>2.44%</td>
<td>32.50%</td>
<td>40.00%</td>
</tr>
<tr>
<td>51 to 60</td>
<td>0.63%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>1.96%</td>
<td>25.09%</td>
<td>30.77%</td>
</tr>
<tr>
<td>61 to 70</td>
<td>0.55%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>2.99%</td>
<td>21.54%</td>
<td>25.38%</td>
</tr>
<tr>
<td>71 to 80</td>
<td>0.27%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>13.51%</td>
<td>22.54%</td>
<td>23.61%</td>
</tr>
<tr>
<td>81 to 90</td>
<td>0.25%</td>
<td>0.00%</td>
<td>1.16%</td>
<td>11.11%</td>
<td>17.24%</td>
<td>20.48%</td>
</tr>
<tr>
<td>91 to 100</td>
<td>0.15%</td>
<td>1.02%</td>
<td>1.09%</td>
<td>4.35%</td>
<td>15.46%</td>
<td>16.84%</td>
</tr>
<tr>
<td>More</td>
<td>1.25%</td>
<td>0.00%</td>
<td>1.17%</td>
<td>13.33%</td>
<td>41.54%</td>
<td>73.30%</td>
</tr>
</tbody>
</table>

Panel B: Long Run Simulation

<table>
<thead>
<tr>
<th>Total Defaults</th>
<th>Prob. of State</th>
<th>Minimum</th>
<th>10%-Quantile</th>
<th>Median</th>
<th>90%-Quantile</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 10</td>
<td>87.40%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>12.50%</td>
</tr>
<tr>
<td>11 to 20</td>
<td>4.87%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>7.69%</td>
</tr>
<tr>
<td>21 to 30</td>
<td>2.44%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>3.57%</td>
</tr>
<tr>
<td>31 to 40</td>
<td>1.44%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>3.13%</td>
</tr>
<tr>
<td>41 to 50</td>
<td>1.04%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>4.08%</td>
</tr>
<tr>
<td>51 to 60</td>
<td>0.67%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>1.77%</td>
<td>2.00%</td>
</tr>
<tr>
<td>61 to 70</td>
<td>0.42%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>1.54%</td>
<td>1.67%</td>
</tr>
<tr>
<td>71 to 80</td>
<td>0.33%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>1.37%</td>
<td>2.82%</td>
</tr>
<tr>
<td>81 to 90</td>
<td>0.34%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>2.36%</td>
<td>3.53%</td>
</tr>
<tr>
<td>91 to 100</td>
<td>0.22%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>2.16%</td>
<td>3.03%</td>
</tr>
<tr>
<td>More</td>
<td>0.83%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>1.57%</td>
<td>7.53%</td>
<td>10.07%</td>
</tr>
</tbody>
</table>

All banks 100.00% 0.00% 0.00% 0.00% 0.00% 12.50%

Table 3. Probability of bank defaults and proportion of contagious to total defaults. Scenarios are grouped by total bank defaults (fundamental and contagious). For each group the probability of that state is shown as well as quantiles of the fraction of contagious defaults to total defaults. In the long run analysis we assume zero bankruptcy costs, whereas in the short run simulation insolvent banks are assumed to default completely and pay nothing.
Table 4. Total and contagious default probabilities of individual banks in the short run and the long run, grouped by quantiles of size of total assets and for the entire banking system. A bank defaults contagiously because other banks do not fully honor their promises. Small banks are defined to be in the first quartile of the total asset distribution; medium banks are defined as banks between the lower quartile and the 90% quantile of the total asset distribution. Large banks are defined as institutions in the top decile of the total asset distribution. The short run analysis (Panels A and C) is under the assumption that insolvent banks pay nothing in the inter-bank market; in the long run (Panels B and D) the residual value of the bank is proportionally shared among claimants assuming zero bankruptcy costs.

<table>
<thead>
<tr>
<th></th>
<th>Panel A: Short Run Simulation</th>
<th>Panel B: Long Run Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10%-Quant.</td>
<td>Median</td>
</tr>
<tr>
<td>Small</td>
<td>0.00%</td>
<td>0.44%</td>
</tr>
<tr>
<td>Medium</td>
<td>0.00%</td>
<td>0.12%</td>
</tr>
<tr>
<td>Large</td>
<td>0.00%</td>
<td>0.07%</td>
</tr>
<tr>
<td>All banks</td>
<td>0.00%</td>
<td>0.16%</td>
</tr>
</tbody>
</table>

Scenario (the median of the fraction is in a range of 0% to 14% depending on the number of total defaults), it is important in terms of the stability of the system. It indicates that despite contagion being a rare event, there are potential situations where it accounts for a major part of defaults. This result illustrates the low probability high impact nature of contagion. As we have already seen, contagion is a minor problem in the long run case (Panel B of Table 3).

The impact of contagion on an individual bank’s default probability can perhaps be better understood by directly looking at Table 4. We see the 10 and 90 percent quantiles as well as the median of the distribution of individual bank default probabilities grouped by bank size. The last line shows these probabilities for the entire banking system. Banks are sorted into three groups by the size of total assets. Banks are defined as small if they are in the first quartile of the total assets distribution. Banks between the first quartile and the 90% quantile are defined as medium whereas banks in the highest decile are defined as large. The probability of a contagious default of an individual bank is remarkably low.
when compared to the total default probability. Thus, most bank defaults are fundamental
defaults. In the long-run 90% of the banks face a contagious default in at most 1 scenario
out of 10,000. The mean contagious default probability (not shown in the table) is only
0.0062%. In the short run this probability is 0.09%. In the upper part of the table total
default probabilities are displayed for both the long and the short run simulation.

The lesson we draw from these results is that among the sources of systemic risk the
direct effects from correlated exposures of banks are far more important than contagion.
However, contagion is a problem not to be ignored in the short run. Though it is not
very likely to occur, once contagion emerges it can wipe out major parts of the banking
system. The concern for contagion and domino effects can therefore not be deleted from
a regulator’s agenda. However, the analysis shows that the preoccupation with domino-
effects should not distract regulators from the most important source of systemic risk:
the correlation structure in the banks’ asset portfolios.

6.2 Bankruptcy Costs and Contagion

The amount of contagion in the short run simulation is substantially higher than in
the long run analysis. These two simulations can be seen as polar cases with respect
to possible recoveries from defaulted counterparties. This raises the question on the
relation between contagion and the magnitude of bankruptcy costs. We use our data
to estimate the consequences of different bankruptcy costs (measured as percent of the
value of total assets) measured by the number of contagious defaults. Figure 3 shows the
impact of bankruptcy costs on contagious defaults. For each level of bankruptcy costs we
compute the distribution of contagious defaults across scenarios. Since we are interested
in crisis scenarios, we plot the tail of the distribution, i.e., how many banks fail in the bad
scenarios.

The lesson that we can draw from this analysis is that efficient bankruptcy resolu-
tion is crucial to restrict contagion. More specifically, the relation is nonlinear, and once
bankruptcy costs exceed a certain threshold, the number of contagious defaults increases
quite dramatically. We see little contagion for low bankruptcy costs and very high con-
tagion for levels above 30%. The jump in the maximum number of contagious defaults
clearly shows that financial stability can be enhanced when bankruptcy costs are kept
low. This highlights the importance of a lender of last resort and an efficient crisis resolution policy. When regulators are able to support efficient bankruptcy resolutions they can effectively keep contagion on a low level.

### 6.3 Value at Risk for the Lender of Last Resort

A relevant aspect of our model for the regulator is that it can be used to estimate the cost of crises intervention. We can estimate the funds that would have to be available to avoid contagious defaults or even fundamental defaults.\(^\text{18}\) Table 5 reports our results for the short run and the long run analysis.

It is remarkable that the amounts that have to be ready to prevent only contagious defaults are not very high. For the data set analyzed here, the amount is 46 million Euro

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\(^\text{18}\)Our model can also be used to determine the optimal size of a deposit insurance fund. For this purpose we would have to include the amount of insured deposits in our analysis.
Table 5. Costs of avoiding fundamental and contagious defaults: In the first row we give estimates for the 90, 95, 99 and 100 percentile of the avoidance cost distribution across scenarios for fundamental defaults. The lower part of the table shows the amounts necessary to avert contagious defaults once fundamental defaults have occurred. Costs are in million Euro.

for the short run case. This is less than 0.01% of the banking system’s total assets. In the long run case about 20 million Euro would suffice. The reserves required to avoid fundamental defaults in our simulation completely add up to about 0.98% of the banking system’s total assets. A less ambitious regulator who is satisfied with preventing fundamental defaults in 99% of all scenarios would need approximately 0.16% of the total assets.

Thus we see that the model provides estimates of the funds a lender of last resort has to stand ready to inject into the system to keep the probability of fundamental or contagious defaults below a certain limit. For our dataset the amount that has to be ready to prevent contagious defaults can be remarkably small. If we think about this result in the light of the previous discussion about bankruptcy costs and contagion we see that a lender of last resort has an important role to play for short run crises intervention. It can be an effective instrument in preventing the progression of bank defaults into a systemic crisis.

6.4 The Role of the Network Structure

In the theoretical literature on contagion and banking crises Allen and Gale (2000) have suggested that the pattern of linkages in the inter-bank market is critical for financial fragility. In particular they contrast two kinds of inter-bank market structures, which are called complete and incomplete. A complete structure refers to a network topology of the inter-bank market where all banks are connected with each other by claims or liabilities. An incomplete structure is one in which banks are only partially connected. In a liquidity
insurance framework they analyze the risk allocation and fragility properties of a banking system comprising four banks which face different liquidity shocks but could enter risk sharing agreements to achieve better allocations of liquidity than without an inter-bank market. The authors show that the structure of the inter-bank market is decisive whether an aggregate liquidity shock leads to contagion and financial crises or not. They conclude that a complete structure is more robust than an incomplete structure and leads to a better distribution of the risk of an aggregate liquidity shock. Therefore, the costly liquidation of long term assets is not necessary. An incomplete structure may however lead to large effects of an aggregate liquidity shock because banks can only turn on their neighboring region for liquidity and may enforce premature liquidation with a knock-on effect to the other region.

The analysis by Allen and Gale (2000) raises the question of whether a complete inter-bank market structure can, in general, make a banking system more resilient to aggregate shocks. We can use our framework to provide some empirical evidence. For the construction of the inter-bank matrix $L$ we exploit the structural information about the multi-tier architecture of the Austrian banking system. This leads to a network topology where many banks are only connected to the central institution in their sector. The inter-bank market is far from complete. Ignoring the structural information we estimate by entropy maximization a new inter-bank matrix $L^*$ which exhibits a complete structure.\footnote{The sum of the inter-bank liabilities of each bank is unchanged. Hence, fundamental defaults are not affected by the way the matrix is estimated. In the revised estimate all banks are considered as counterparties and we might see different results in terms of contagion only.}

The simulation results with the complete structure and the long run simulation are reported in Table 6. Analogous to Table 2, bank failure scenarios are grouped by the number of fundamental defaults. For each group, we compute probabilities that only fundamental and both fundamental and contagious defaults occur.

The striking finding here is that contrary to the example discussed in Allen and Gale (2000) the complete market structure in our case leads to an increase in scenarios with contagious defaults by roughly three percentage points. The conclusion drawn from this exercise is that the classification into complete and incomplete structures does not give the whole story if we want to understand the role played by the network topology of inter-bank linkages for financial fragility of a banking system.
<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>No Contagion</th>
<th>Contagion</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-10</td>
<td>88.18%</td>
<td>87.93%</td>
<td>0.25%</td>
</tr>
<tr>
<td>11-20</td>
<td>4.45%</td>
<td>4.22%</td>
<td>0.23%</td>
</tr>
<tr>
<td>21-30</td>
<td>2.23%</td>
<td>1.68%</td>
<td>0.55%</td>
</tr>
<tr>
<td>31-40</td>
<td>1.46%</td>
<td>0.95%</td>
<td>0.51%</td>
</tr>
<tr>
<td>41-50</td>
<td>0.96%</td>
<td>0.60%</td>
<td>0.36%</td>
</tr>
<tr>
<td>51-60</td>
<td>0.62%</td>
<td>0.28%</td>
<td>0.34%</td>
</tr>
<tr>
<td>61-70</td>
<td>0.42%</td>
<td>0.07%</td>
<td>0.35%</td>
</tr>
<tr>
<td>71-80</td>
<td>0.33%</td>
<td>0.03%</td>
<td>0.30%</td>
</tr>
<tr>
<td>81-90</td>
<td>0.34%</td>
<td>0.01%</td>
<td>0.33%</td>
</tr>
<tr>
<td>91-100</td>
<td>0.19%</td>
<td>0.00%</td>
<td>0.19%</td>
</tr>
<tr>
<td>more</td>
<td>0.82%</td>
<td>0.03%</td>
<td>0.79%</td>
</tr>
<tr>
<td>Total</td>
<td>100.00%</td>
<td>95.80%</td>
<td>4.20%</td>
</tr>
</tbody>
</table>

Table 6. Probabilities of fundamental and contagious defaults assuming zero bankruptcy costs and a complete market structure, i.e., banks diversify their inter-bank business as much as possible. Bank failure scenarios are grouped by the number of fundamental defaults. For each group, the probability that only fundamental defaults and that both fundamental and contagious defaults occur are shown. A fundamental default is due to the losses arising from exposures to market risk and credit risk to the corporate sector, while a contagious default is triggered by the default of another bank which is unable to fulfill its promises in the inter-bank market.

6.5 Estimating Parameters to determine Capital Charges for Inter-bank Loans

The recent developments in capital adequacy regulation have made substantial progress to impose regulatory capital charges on banks that are closely in line with their actual risk positions. For inter-bank exposures these risk-adequate charges are particularly difficult to determine because the credit risk from inter-bank linkages is hard to assess. Our framework allows us to address this issue. We have computed estimates of the probability of default and inter-bank recovery rates. These are important for determining risk-adequate capital charges for inter-bank loans. For this task our procedure has an advantage over an analysis relying entirely on historical bank defaults. Our analysis explicitly addresses and quantifies the threat of a systemic crisis which may be underestimated when relying solely on a history of observed bank defaults. Apart from the concern that historical bank default rates are distorted because many troubled banks might be saved by regulatory intervention, defaults due to a systemic crisis are rarely observed.

The banking system’s overall stability as measured by the median default probability (Table 2) across the entire banking system is high both in the short run and in the long
Figure 4. The figure shows the share in total defaults of banks in each decile of the total asset distribution (long run case). Small banks account for a large fraction of total defaults. Banks in the fourth, fifth, and sixth decile account for a less than proportional share of defaults.

The mean default probability (not shown in the table) in the long run is 0.8%. Median default probabilities decline when we move from the short run to the long run perspective. This decrease is more pronounced for large banks. This finding remains more or less unchanged if one introduces the assumption that some resources are lost during a bankruptcy procedure. Adopting James (1991) estimates of bankruptcy costs of 10% of total assets, we find that the median default probability in the long run rises to 0.12% for the entire banking system.

Prima facie the default risk seems to decrease with bank size. This picture can be refined if we group banks by deciles of the distribution of total assets and plot them against their shares of the number of total defaults across banks and scenarios (Figure 4 for the long run case). The relation between bank size and default probability appears now more ambiguous. The default probability is high for small banks (the lowest three deciles account for approximately 50% of all defaults). It decreases sharply for the next three deciles and than increases again.
Table 7. Recovery rates from inter-bank exposures in the long run grouped by quantiles of size of total assets and for the entire banking system. Small Banks are defined as institutions in the first quartile of the total asset distribution; large banks are defined as institutions in the top decile of the total asset distribution.

<table>
<thead>
<tr>
<th>Size</th>
<th>10%-Quantile</th>
<th>Median</th>
<th>90%-Quantile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>0.00%</td>
<td>23.02%</td>
<td>81.06%</td>
</tr>
<tr>
<td>Medium</td>
<td>20.17%</td>
<td>76.36%</td>
<td>95.96%</td>
</tr>
<tr>
<td>Large</td>
<td>10.86%</td>
<td>89.84%</td>
<td>98.16%</td>
</tr>
<tr>
<td>Banking system</td>
<td>0.00%</td>
<td>65.83%</td>
<td>95.54%</td>
</tr>
</tbody>
</table>

The model also allows us to give an estimate of the value recoverable from defaulted counterparties in the inter-bank market. This information is interesting in its own right because relatively little is known about recoveries from inter-bank credits. This estimate is, of course, only relevant for the long run since we assume that the recovery rate in the short run is zero apart from netting.

The following numbers refer to the long run assumption that the residual value of an insolvent institution is fully transferred to the creditors. In each scenario where bank $i$ defaults, a recovery rate is calculated by dividing $p_{i}^*$, the amount paid by bank $i$ under the clearing mechanism, by $d_{i}$, the amount initially owed by bank $i$. These rates for bank $i$ are averaged across scenarios where bank $i$ defaults. The values of recovery rates are reported in Table 7.\(^{20}\)

These recovery rates are implied by the network model. In practice, recovery rates might be higher because some of the exposures would be collateralized. We have included no assumptions about collateral since we have no appropriate information. What is remarkable in these numbers is that the median recovery rate for the large institutions is high. This indicates that if they fail, the losses for the counterparties will be moderate. The median recovery rate taken over the entire banking system is 66%. It should also be noted that these recovery rates drop sharply if we deduct bankruptcy costs of 10% of total assets as estimated by James (1991). In this case the median recovery rate goes down to zero and reaches a value of about 46% in the 90% quantile of the recovery rate distribution across the entire banking system.

\(^{20}\)To calculate the recovery rates we clear the system without netting. In the case of clearing with netting we could clearly not simply calculate recovery rates as $p_{i}^*/d_{i}$.\)
7 Conclusions

In this paper we have developed a new framework for the risk assessment of a banking system. The innovation is that we judge risk at the level of the entire banking system rather than at the level of an individual institution.

Conceptually it is possible to take this perspective by a systematic analysis of the impact of a set of macroeconomic risk factors on banks in combination with a network model of mutual credit relations. The framework can easily be employed empirically as the input data is available with the regulatory authority, exactly the institution for which an assessment method of the type suggested here is of crucial interest.

Since our method is a first step, there are several issues that have to be discussed to judge the reliability of the assessments generated with the help of our model. We would like to point out the main advantages of our general approach. The system perspective can uncover exposures to aggregate risk that are invisible for banking supervision which relies on the assessment of single institutions only. We can distinguish defaults directly caused by a macroeconomic shock from those triggered by defaults of other banks in the inter-bank market. We believe that our framework can divert the discussion about systemic risk in banking from continuous refinements and extensions of capital adequacy regulation for individual banks to the crucial issue of how much risk is actually borne by the entire banking system. Evidently, the model does not rely on a sophisticated theory of economic behavior. In fact, the model is really a tool to interpret data in a particular way. The consequences of a given liability and asset structure in combination with realistic shock scenarios are uncovered in terms of implied technical insolvencies of institutions. The model is designed to exploit existing data sources. Although these sources are not ideal, our approach shows that we can start to think about financial stability at the system level with available data.

We have learned five main lessons from the application of our model to a unique and comprehensive Austrian bank data set. First, we find that correlated portfolio exposures of banks are the main source of systemic risk and that domino effects only occur rarely. However, contagion is a low probability high impact event. Second, bankruptcy costs play an important role for the system wide impact of contagious default events. Third, analyzing the “value at risk” for a lender of last resort reveals that the funds that are
needed to stop contagious defaults can be surprisingly small. Fourth, we show that more diversification in the inter-bank market does not necessarily reduce contagion. Finally we quantify default probabilities and inter-bank recovery rates.

We hope that our work would be useful for regulators and central bankers by offering a practicable way to interpret the data they have available with them in the light of aggregate risk exposure of the banking system. We therefore hope to have given a perspective of how a “macroprudential” approach to banking supervision could proceed. We also do hope that our paper contributes to theoretical work in financial stability and banking as well, and that the questions it raises would contribute in a fruitful way to the debate about the system approach to banking supervision and risk assessment.
References


Table 8. Summary statistics for Austrian banks. The table shows total assets (TA), book value of equity to total assets, inter-bank assets and liabilities as fraction of total assets as well as deposits by non-banks to total assets. The last line summarizes the regulatory capital ratio as tier two capital over required capital times 8%, i.e., a bank just fulfilling minimum capital requirements has a regulatory capital ratio of 8%.

### The Austrian Banking System

The Austrian banking system has a sectoral organization for historic reasons. Banks belong to one of the seven sectors: joint stock banks, savings banks, state mortgage banks, Raiffeisen banks, Volksbanken, building associations, and special purpose banks. The savings banks and the Volksbanken sector are organized in a two tier structure with a sectoral head institution. The Raiffeisen sector is organized as a three tier structure, with a head institution for every federal state of Austria. The federal state head institutions have a central institution, Raiffeisenzentralbank (RZB) which is at the top of the Raiffeisen structure. From the viewpoint of banking activities the sectoral organization is today not particularly relevant. The activities of the sectors differ only slightly and only a few banks are specialized in specific lines of business. The 881 independent banks in our sample are to the largest extent universal banks.

Total assets in the Austrian banking sector in September 2002 were 575,315 million Euro (GDP 2002: 216,600 million Euro). The sectoral shares are: 22% joint stock banks, 35% savings banks, 6% state mortgage banks, 21% Raiffeisen banks, 5% Volksbanken, 3% building associations, and 8% special purpose banks.

The banking system is dominated by a few big institutions: 50% of total assets are concentrated with the 6 largest banks (57% with the 10 biggest banks). The Gini coefficient is 0.8845. The share of total inter-bank liabilities in total assets of the banking system is 33%. Only six banks, which hold 35% of all the assets, have issued common shares to the public.
B  An Illustrative Example

Let us illustrate the concepts introduced in section 2 by an example: Consider a system with three banks. The inter-bank liability structure is described by the matrix

\[
L = \begin{pmatrix}
0 & 0 & 2 \\
3 & 0 & 1 \\
3 & 1 & 0
\end{pmatrix}
\]

Bank 3 has – for instance – liabilities of 3 with bank 1 and liabilities of 1 with bank 2. It has of course no liabilities with itself. The total inter-bank liabilities for each bank in the system is given by a vector \( d = (2, 4, 4) \). With actual balance sheet data the components of the vector \( d \) correspond to the position due to banks for bank 1, 2 and 3 respectively. If we alternatively look at the column sum of \( L \) we get the position due from banks. Assume that we can summarize the net wealth of the banks that is generated from all other activities by a vector \( e = (1, 1, 1) \). This vector corresponds to the difference of asset positions such as bonds, loans and stock holdings and liability positions such as deposits and securitized liabilities.

The normalized liability matrix \( \Pi \) is given by

\[
\Pi = \begin{pmatrix}
0 & 0 & 1 \\
\frac{3}{4} & 0 & \frac{1}{4} \\
\frac{3}{4} & \frac{1}{4} & 0
\end{pmatrix}
\]

Applying the fictitious default algorithm to the situation where \( e = (1, 1, 1) \) yields a clearing payment vector of \( p^* = (2, \frac{28}{15}, \frac{52}{15}) \). It is easy to check that bank 2 is fundamentally insolvent whereas bank 3 is “dragged into insolvency” by the default of bank 2.

Suppose \( e \) is uncertain whereas \( L \) is deterministic. Assume that we draw two realizations from the distribution of \( e \). Let these two scenarios be the vectors \( e_1 = (1, 1, 1) \) and \( e_2 = (1, 3, 2) \). Given the matrix \( L \), in the first scenario banks 2 and 3 default whereas in the second scenario no bank is insolvent. The clearing payment vectors and the network structure for this example are illustrated in Figure 5.

The ex-ante expected number of bank defaults, of fundamental and contagious defaults as well as expected average recovery rates from inter-bank credits are the averages over the scenarios.
C Estimating the $L$ matrix

Assume that we have, in total, $K$ constraints that include all constraints on row and column sums as well as on the value of particular entries. Let us write these constraints as

$$
\sum_{i=1}^{N} \sum_{j=1}^{N} a_{kij}l_{ij} = b_k 
$$

for $k = 1, \ldots, K$ and $a_{kij} \in \{0, 1\}$.

We seek to find the matrix $L$ that has the least discrepancy to some a priori matrix $U$ with respect to the (generalized) cross entropy measure

$$
C(L, U) = \sum_{i=1}^{N} \sum_{j=1}^{N} l_{ij} \ln \left( \frac{l_{ij}}{u_{ij}} \right) 
$$

among all the matrices satisfying (4) with the convention that $l_{ij} = 0$ whenever $u_{ij} = 0$ and $0 \ln(0)$ is defined to be 0.

The constraints for the estimations of the matrix $L$ are not always consistent. For instance the liabilities of all banks in sector $k$ against all banks in sector $l$ do typically not equal the claims of all banks in sector $l$ against all banks in sector $k$. We deal with this problem by applying a two step procedure.
In a first step we replace an a priori matrix \( U \) reflecting only possible links between banks by an a priori matrix \( V \) that takes actual exposure levels into account. As there are seven sectors we partition \( V \) and \( U \) into 49 sub-matrices \( V^{kl} \) and \( U^{kl} \) which describe the liabilities of the banks in sector \( k \) against the banks in sector \( l \) and our a priori knowledge. Given the bank balance sheet data we define \( u_{ij} = 1 \) if bank \( i \) belonging to sector \( k \) might have liabilities against bank \( j \) belonging to sector \( l \) and \( u_{ij} = 0 \) otherwise. The (equality) constraints are that the liabilities of bank \( i \) against the sector \( l \) equal the row sum of the sub-matrix and that the claims of bank \( j \) against the sector \( k \) equal the column sum of the sub-matrix, i.e.

\[
\sum_{j \in l} v_{ij} = \text{liabilities of bank } i \text{ against sector } l \quad (6)
\]

\[
\sum_{i \in k} v_{ij} = \text{claims of bank } j \text{ against sector } k \quad (7)
\]

For the matrices describing claims and liabilities within a sector (i.e. \( V^{kk} \)) which has a central institution we get further constraints. Suppose that bank \( j^* \) is the central institution. Then

\[
v_{ij^*} = \text{liabilities of bank } i \text{ against central institution} \quad (8)
\]

\[
v_{j^*i} = \text{claims of bank } i \text{ against central institution} \quad (9)
\]

Though these constraints are inconsistent given our data, we use the information to get a revised matrix \( V \) which reflects our a priori knowledge better than the initial matrix \( U \). Contrary to \( U \) which consists only of zeroes and ones, the entries in \( V \) are adjusted to the actual exposure levels.\(^{21}\)

In a second step we recombine the results of the 49 approximations \( V^{kl} \) to get an entire \( N \times N \) improved a priori matrix \( V \) of inter-bank claims and liabilities. Now we replace the original constraints by just requiring that the sum of all (inter-bank) liabilities of each bank equals the row sum of \( L \) and the sum of all claims of each bank equals the column sum of \( L \).

\[
\sum_{j=1}^{N} l_{ij} = \text{liabilities of bank } i \text{ against all other banks} \quad (10)
\]

\[
\sum_{i=1}^{N} l_{ij} = \text{claims of bank } j \text{ against all other banks} \quad (11)
\]

\(^{21}\)Note that the algorithm that calculates the minimum entropy entries does not converge to a solution if data are inconsistent. Thus to arrive at the approximation \( V \) we terminate after 10 iterations immediately after all row constraints are fulfilled.
Again we face the problem that the sum of all liabilities does not equal the sum of all claims but corresponds to only 96% of them. By scaling the claims of each bank by 0.96 we enforce consistency.\textsuperscript{22} Given these constraints and the prior matrix $V$ we estimate the matrix $L$.

Finally we can use the information on claims and liabilities with the central bank and with banks abroad. By adding two further nodes and by appending the rows and columns for these nodes to the $L$ matrix, we get a closed (consistent) system of the inter-bank network.

\textsuperscript{22}The remaining 4% of the claims are added to the vector $e$. Hence they are assumed to be fulfilled exactly.