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More generally, when there are p dimensions, we can write

$$\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 \ldots + \beta_p x_p = 0. \tag{2}$$

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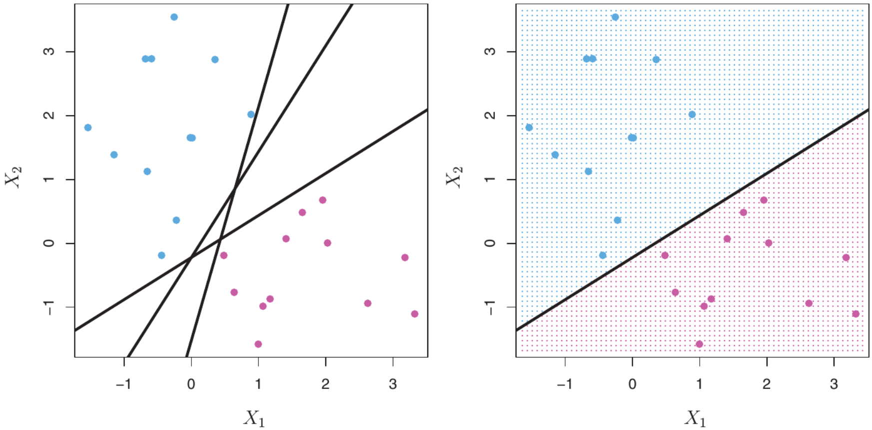
If one separating hyperplane exists, then many of them exist, and we want to choose the best one. See Figure 23.1.

More commonly, no separating hyperplane exists. If so, we attempt to find a hyperplane that comes as close as possible.

Figure 23.1 — Separating Hyperplanes



Three separating hyperplanes (left panel) and the decision rule associated with a separating hyperplane (right panel).

Let the training observations be denoted y_i and $\boldsymbol{x}_i$, where y_i contains the class labels, which are -1 and 1.

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If a separating hyperplane exists, it must have the property that

$$\begin{aligned} \beta_0 + \boldsymbol{x}_i^\top \boldsymbol{\beta} &> 0 \quad \text{if } y_i = 1 \\ \beta_0 + \boldsymbol{x}_i^\top \boldsymbol{\beta} &< 0 \quad \text{if } y_i = -1 \end{aligned} \tag{4}$$

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More compactly, we can write

$$y_i(\beta_0 + \boldsymbol{x}_i^\top \boldsymbol{\beta}) > 0 \quad \text{for all } i = 1, \ldots, n. \tag{5}$$

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When there exists a separating hyperplane, it provides a **perfect classifier**. We simply classify based on the sign of $\beta_0 + \boldsymbol{x}_i^\top \boldsymbol{\beta}$.

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The **maximal margin hyperplane**, or **optimal separating hyperplane**, is the one that is farthest from the training observations.

Intuitively, this seems like the best choice.

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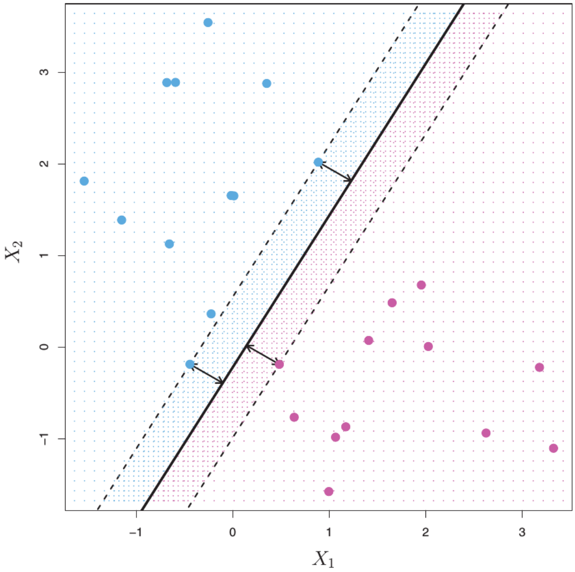
The three support vectors (observations) are shown in the figure. Notice what would happen if any of them changed.

Figure 23.2 — Maximal Margin Classifier



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We need to maximize a scalar M with respect to M itself, β_0, and $\boldsymbol{\beta}$ subject to the constraints

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For numerical solution, it is easier to rewrite the optimization problem so that M has vanished and there is no constraint on the β coefficients.

The new optimization problem is

$$\min_{\beta_0, \boldsymbol{\beta}} (\beta_0^2 + \boldsymbol{\beta}^\top \boldsymbol{\beta}) \quad \text{subject to} \tag{8}$$

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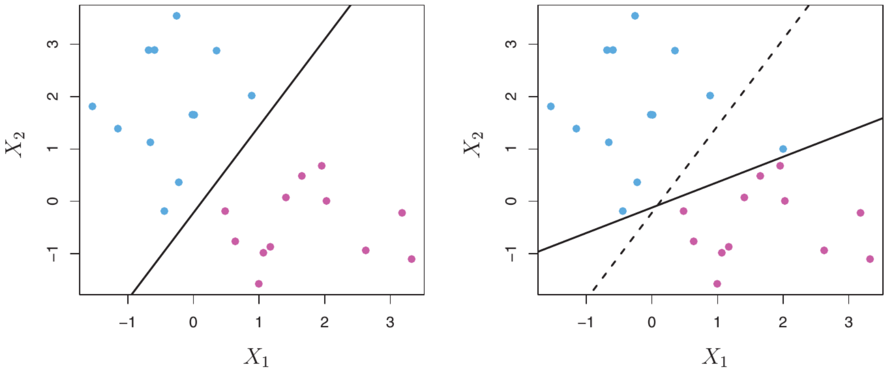
- The solution to this optimization problem depends on just a few support vectors.
- Changing the data slightly can greatly change the location of the maximal margin hyperplane, or cause it to vanish.
- In Figure 23.3, adding one observation has changed two of the three support vectors and greatly changed the slope of the maximal margin hyperplane.

Figure 23.3 — Sensitivity of the Maximal Margin Classifier



In the right panel, one observation is added to the ones in the left panel. The maximal margin hyperplane changes greatly.

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where $\epsilon_i \geq 0$ and $\sum_{i=1}^n \epsilon_i \leq C$, with $C > 0$ a tuning parameter.

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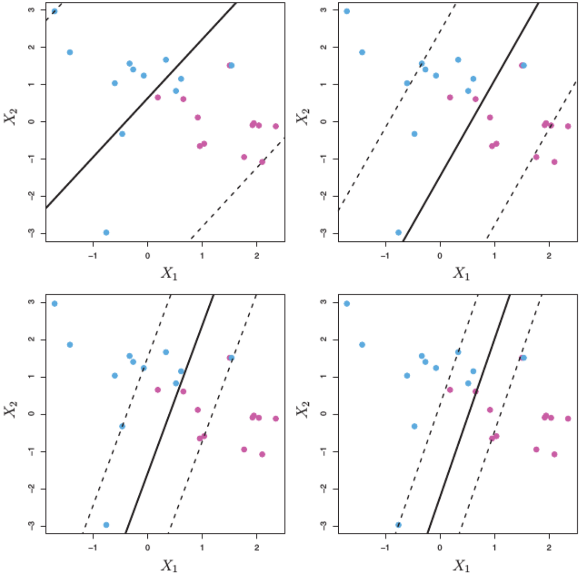
- For $C > 0$, no more than C observations can be on the wrong side of the hyperplane, because $\epsilon_i > 1$ for every such observation.
- Since every violation of the margin increases the sum of the ϵ_i, we can afford more violations when C is large than when it is small. Thus M will almost surely increase with C.

Figure 23.4 — Effects of the Tuning Parameter C



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- When C is large, the margin is wide, and many observations are allowed to violate it. There will tend to be low variance, because there are many support vectors, but high bias.
- Notice that the slope of the (non-separating) hyperplane becomes considerably steeper as C shrinks.

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As C diminishes from upper left to lower right, the margin shrinks.

- When C is large, the margin is wide, and many observations are allowed to violate it. There will tend to be low variance, because there are many support vectors, but high bias.
- Notice that the slope of the (non-separating) hyperplane becomes considerably steeper as C shrinks.
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The SV classifier is totally insensitive to observations on the correct side of the margin, and therefore (for a wide margin) on observations that are on the correct side of the hyperplane by far.

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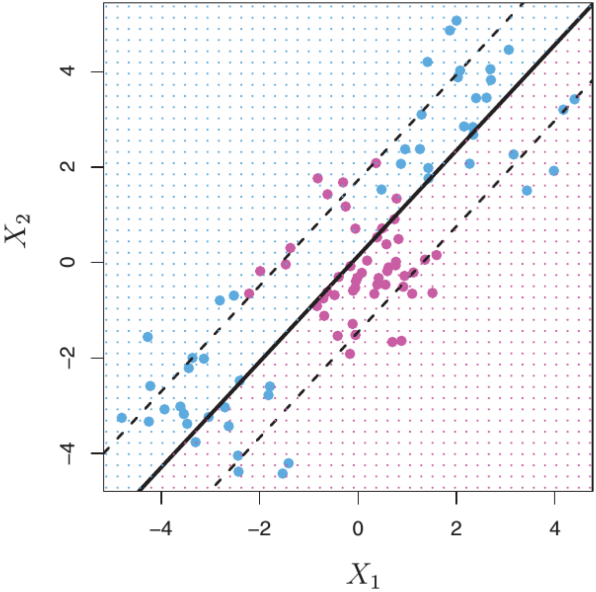
Consider Figure 23.5. Here the boundaries between the two classes are very nonlinear.

The linear boundary found by the SV classifier works poorly.

Figure 23.5 — Support Vector Classifier Works Badly



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However, it turns out that $\alpha_i = 0$ if $\boldsymbol{x}_i$ is not a support vector.

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We will of course get different results by using different kernels.

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The value of γ, which is a positive constant, may be chosen in advance or by cross-validation.

The radial kernel has very local behavior. When a test observation $\boldsymbol{x}_0$ is far from the training observation $\boldsymbol{x}_i$, then

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One advantage of using kernels instead of simply adding functions of the original inputs is that the data continue to affect the results only through the $n(n-1)/2)$ distinct values of $K(\boldsymbol{x}_i, \boldsymbol{x}_{i'})$.

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SVM with a well-chosen kernel can work well. See Figure 23.6, which is taken from ESL, and Figure 23.7, which is ISLR/ISLP Figure 9.9.

Figure 18.7 — Support Vector Classifier with Radial Kernel

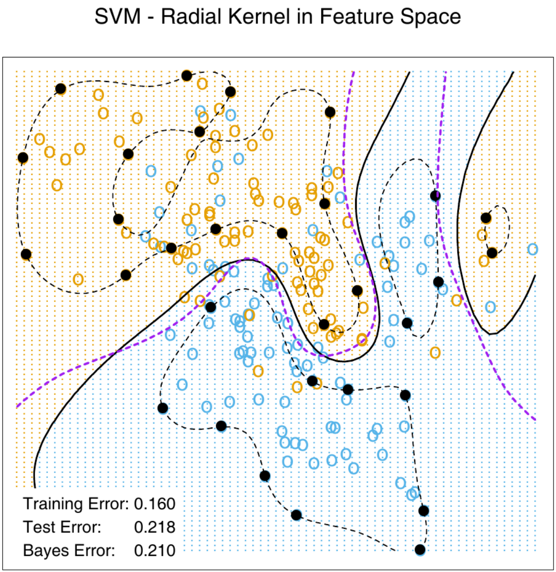
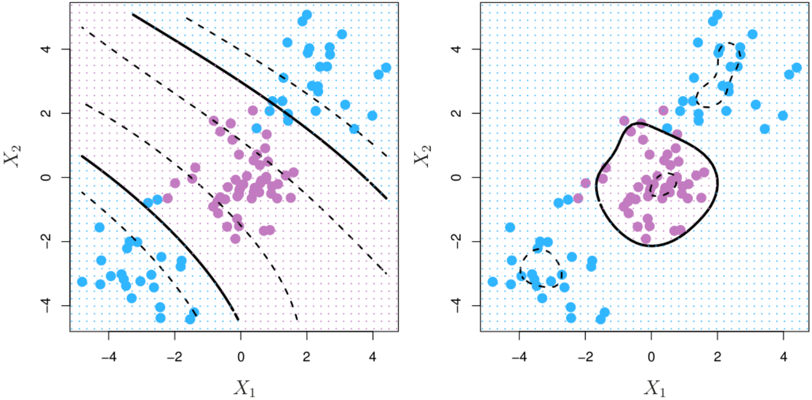


Figure 23.7 — Support Vector Classifiers with Two Kernels



Left panel uses polynomial kernel; right panel uses radial kernel.

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`svm` can also perform support vector regression, it can deal with more than two classes, and it can output fitted values which can be used to plot ROC curves.

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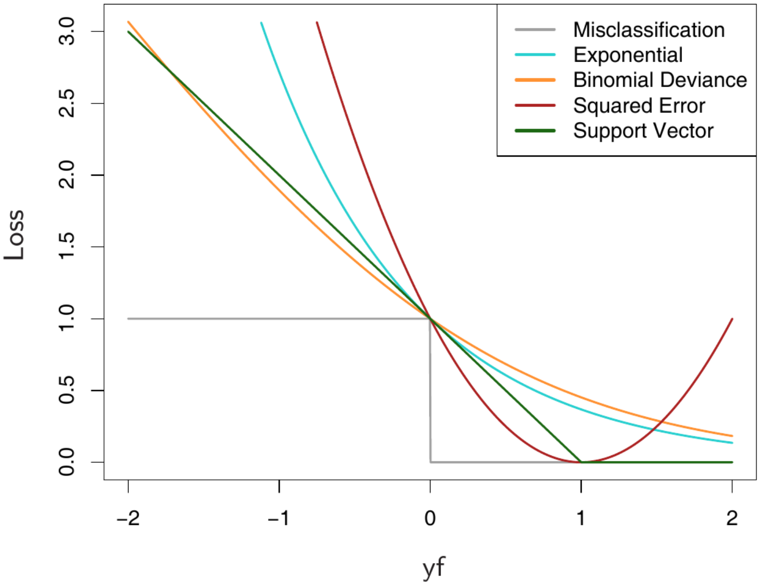
- If an observation is on the correct side of the margin, the loss is 0. If it is on the wrong side, the loss is linear with a slope of 1.

Figure 23.8 — Loss Functions for Binary Classification



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Other features of Figure 23.8 are also interesting.

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- We could perfectly well use nonlinear transformations with (regularized) logistic regression.
- The fact that SVMs routinely use nonlinear transformations (kernels) and logistic regression does not mainly reflects history and software availability.