Housing Liquidity, Mobility and the Labour Market

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Abstract

We study the interactions among geographical mobility, unemployment and home-ownership in an economy with heterogeneous locations, endogenous construction and search frictions in the markets for both labour and housing. The decision of home-owners to accept job offers from other cities depends on how quickly they can sell their houses (i.e. their *liquidity*), which in turn depends on local labour market conditions. Consequently, home-owners accept job offers from other cities at a lower rate than do renters, generating a link between home-ownership and aggregate unemployment. When calibrated to match aggregate U.S. statistics on mobility, housing and labour flows, our model predicts that the effect of home-ownership on aggregate unemployment is small. When unemployment is high, however, changes in the rate of home-ownership can have economically significant effects.

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1 Introduction

In this paper, we study the relationships among geographical mobility, unemployment and the value of owner-occupied housing in an environment characterized by frictions in the markets for both labour and houses. The price of a house reflects its *liquidity* — *i.e.* the speed with which it can be transferred from one home-owner to another — and this in turn affects both mobility and labour market outcomes. Our model is consistent both with recent micro-evidence on the relationship between ownership and unemployment across cities and with large observed differences in mobility between renters and owners. Nevertheless, we find that the impact of home-ownership on aggregate unemployment is likely to be economically significant only if both unemployment and average mobility are high. This combination is inconsistent with calibration targets based on either the U.S. or continental European nations.

In our economy a growing population of *ex ante* identical households may choose to live in any of a large (finite) number of cities. Cities are of two types which differ in regard to the productivity of jobs. Households are at any point in time resident in one particular city, and are either employed or unemployed. Irrespective of their current employment status, households receive randomly job offers in both their city of current residence and in other cities. In order to take a job in a different city, a household must move to that location.

Households require housing in their city of residence and may either rent in a competitive market or purchase in a market characterized by a search friction. Migrating home-owners put their houses up for sale and initially rent in their new city while searching for a house. New housing of each type is constructed in response to anticipated demand. In this environment, we compute a stationary equilibrium (which is unique within a class) characterized by constant relocation, housing market activity, and construction of new housing units.

The willingness of a worker to accept a job in another city depends on the offered wage, their current employment status, rents, the price of houses in the city where the offered job is located, and the market value of their current house, if they own one. As vacant houses have an opportunity cost, the latter depends on how quickly a buyer can be found. Thus, the liquidity of housing affects the distribution of households across cities and unemployment both at the city level and in the aggregate. At the same time, the frequency with which households choose to relocate affects the liquidity of housing in all cities.

There is considerable evidence that home-owners move less frequently than renters, even after controlling for both household and locational characteristics (see, for example, Rohe and Stewart, 1996, or Boheim and Taylor, 2002). Recently, it has been argued that because of its relationship with mobility, home-ownership creates frictions in the labour market that may lead to inefficient outcomes.[1] Indeed, it has been conjectured that differences in home-ownership rates across countries may be a leading factor driving differences in unemployment. For example, using cross-country regressions Oswald (1997, 2009) estimates the effect of reducing home-ownership by ten percent to be a reduction of unemployment by between 1.7 and 2 percentage points. Similarly, Nickell (1998) estimated the effect of a ten percent *increase* in home-ownership to be a rise in unemployment of 1.3 percentage points.[2]

It is difficult to reject this conjecture using micro-data. Although unconditionally the unemployment rate among renters is significantly *higher* than that among owners, this largely reflects the differing characteristics of households in the two groups. Home-owners tend, for example, to be more educated, older and more likely to be married than renters. People with these characteristics are also less likely to be unemployed, independent of any direct effect of ownership on mobility. Moreover, there may be factors that operate in the opposite direction. For example, home-owners may search harder locally for jobs or be willing to accept lower wages than renters, so that their unemployment duration is shorter.[3]

Our framework allows us to isolate the effects of home-ownership *per se* on unemployment through its effect on mobility. We model all households as *ex ante* identical and so in our theory, home-ownership affects mobility, rather than the reverse. Ownership, *ceteris paribus,* increases the likelihood of unemployment for an individual because, while job separation and offer rates are the same for all, only unemployed home-owners turn down job offers in equilibrium. There is, however, a second effect which offsets the impact of home-ownership on unemployment in particular cities. While high-wage cities have a lower vacancy rate and thus a higher rate of home-ownership, they also have higher rents, and so are unattractive to the unemployed. These households may remain in or move to a low-wage city without a job offer, just to take advantage of lower rent. In contrast, employed renters in low-wage cities move to high-wage (and high rent) cities as long as the wage premium is sufficient. Unemployed home-owners never re-locate to a high-wage city without an offer. These factors combine to generate higher unemployment in low-wage cities, where home-ownership is also

[1] See, for example, Blanchflower (2007) and Harford (2007).

[2] Partridge and Rickman (1997), Pehkohnen (1999) and Cochrane and Poot (2007). Rouwendal and Nijkamp (2007) critically review some of this work.

[3] Munch et al. (2006) find evidence of the former effect using Danish data. On the other hand, in a French dataset, Brunet and Lesueur (2009) find that, once controls for search intensity are included, homeowners have lower exit rates from unemployment. Coulson and Fisher (2009) find no evidence that owners accept lower wages in the US.

lower. This is consistent with the findings of Coulson and Fisher (2009), who find that across U.S. metropolitan areas home-ownership rates are correlated positively with average wages and negatively with unemployment.

In a version of our model calibrated to match aggregate U.S. labour market flows and mobility rates for both owners and renters, the equilibrium fraction of unemployed home-owners that turn down offers of jobs in different cities is substantial: over 23% in low-wage cities and 14% overall. Consequently, in accordance with the empirical evidence, home-owners are significantly less mobile than are renters. Nevertheless, we find that the impact of home-ownership on aggregate unemployment is small: A ten percent reduction in the home-ownership rate is associated with a reduction of unemployment of only one-third of a percentage point. Effects in the range of those estimated by Oswald (2009) and Nickell (1998) can be obtained in equilibrium, but only if average unemployment is raised significantly *and* relatively high mobility is maintained. These assumptions are consistent neither with our Baseline calibration (based on U.S. targets) nor with calibrations to European economies, which typically exhibit much lower mobility rates.

Others have developed theories of the impact of home-ownership on labour market outcomes. For example, Dohmen (2005) and Munch, Rosholm and Svarer (2006) present models of labour market search in which home-owners and renters are assumed to behave differently. Coulson and Fisher (2009) present a theory based on endogenous job creation that is consistent with the cross-city evidence on unemployment, but implies counter-factually that home-owners receive lower wages than renters as a result of their immobility. All of these theories, however, abstract from both housing choice and transactions in the housing market. Owners are either simply assumed to be immobile or to face higher moving costs than renters. Here, because the price of housing is endogenously determined, the relative degree of mobility depends on labour and housing market conditions.

Rupert and Wasmer (2009) develop a theory of the relationship between unemployment and housing market frictions which focusses on the trade-off between commuting time and locational decisions within a single labour market. In contrast, our work, focusses on the role of housing markets in generating frictions *between* labour markets.[4] Also, they do not distinguish between ownership and renting, a trade-off which plays a significant role in our results. As such, we view our paper as complementary to theirs.

Albrecht, Anderson, Smith, and Vroman (2007) consider a search model in which the

[4] In our calibration, we allow for within-city relocation following Wheaton (1990), who develops a model of housing markets but considers neither linkages to labour markets nor cross-city re-location.

flow values of search to buyers and sellers change over time and Caplin and Leahy (2008) study the role of trading frictions (due to mis-match) on house price dynamics. These papers are related to ours but study neither mobility nor the interactions between the labour and housing markets.

Recently, several authors have also studied spatial aspects of housing prices. Van Nieuwerburgh and Weill (2010) study the long-run connection between wages and house prices in a multi-city model with free mobility, stochastic productivity and endogenous construction. Consistent with our analysis, they also find a long run relationship between wages and prices across cities. Their model, however, does not distinguish between owning and renting, labour is assumed to be perfectly mobile and there are no search frictions in housing markets. Consequently, there is no notion of housing liquidity, no differences between the mobility of owners and renters and no unemployment. Ortalo-Magne and Prat (2010) distinguish between the spatial (where to live) and quantity decision (how much to invest) dimensions of housing choice and study the implications of this for portfolio allocation theory. We distinguish only between owned and rented housing and abstract from the quantity decision in order to focus on market frictions.

Since we focus on households' location decisions and abstract from most forms of risk by studying a stationary equilibrium, we conjecture that financing constraints are of secondary importance for our analysis.[5] For this reason we abstract from them entirely and also do not consider direct moving costs in order to focus on mobility *per se*. There is, however, a substantial literature which considers these issues. Using life-cycle a model, Ortalo-Magne and Rady (2006) find important effects of financial constraints. Kiyotaki, Michaelides and Nikolov (2011) find much smaller effects, but consider mortgages financed by equity, rather than debt. Halket and Vasudev (2009) study the use of housing as a form of savings in a model of uninsured idiosyncratic risk based on Aiyagari (1994). Their work, along with that of Favilukas, Ludvigson, and Nieuwerburg (2011) studies the role of housing in determining consumption, wealth, and aggregate economic activity in settings with borrowing constraints. Much of this literature devotes considerable attention to movements in house prices and their implications for aggregate wealth and consumption. In contrast, we focus on the effects of the illiquidity of housing on labour market outcomes for individual households.

A substantial literature also focuses on the relationship between the length of residence

[5] We investigate this conjecture in a crude way by considering a case in which unemployed households are unable to buy houses, a restriction that can be viewed as a very stark form of borrowing constraint given the price of a house relative to income. In our environment, this makes little difference for the phenomena we study.

spells (which tend to be higher for home-owners than for renters) and investments in social capital (*e.g.* see Rossi and Weber (1996) and DiPasquale and Glaeser (1999)). Empirically, Coulson, Hwang, and Imai (2002) find that the fraction of home-owners in a neighborhood is associated with higher property values. While our model is consistent with this observation (as home-ownership and house prices are both higher in the high-wage city) as well as the fact that home-owners remain in a city longer than renters, in this paper we abstract from investment in social capital.

This remainder of the paper is organized as follows: Section 2 describes the environment. Section 3 defines a symmetric stationary equilibrium and establishes its existence and uniqueness within a class. Section 4 characterizes the equilibrium and considers the implications of the theory for the relationships among ownership, mobility and unemployment at the individual, city and aggregate levels. Section 5 calibrates the economy to match several aspects of the U.S. economy. Section 6 assesses the theory's quantitative implications and considers their robustness to changes to various parameters. Section 7 summarizes and describes future work. Proofs, longer derivations, details of some of the extensions, statistics associated with robustness exercises, and a description of the data used in the calibration are contained in appendices.

2 Model Environment

Time is continuous. At each point in time, t, the economy is populated by a measure, $\bar{N}(t)$, of infinitely lived, *ex ante* identical, risk-neutral *households* who discount the future at rate ρ. The population grows at rate ν.

There are multiple locations, called *cities*, which are of two types indexed by $i \in \{1, 2\}$. There are a finite number, s_i, of cities of type i and all cities of a given type are identical. Households must, at any point in time, reside in exactly one city. They are free, however, to move between cities instantaneously at no direct cost. Each city contains two types of residential dwellings. Let $R_i(t)$ and $H_i(t)$ denote the stocks of *rental* and *owner-occupied* housing in cities of type i, respectively. Let π^R and $\pi^H > \pi^R$ denote the flow utilities received by households which are renters and home-owners, respectively, any city.[6]

Firms in cities of type i produce output, y_i, of a single consumption good using labour,

[6] There are several possible motivations for this difference in flow utility. For example, it could be associated with the size differential between owned and rented houses, or with the non-monetary value individuals place on being an owner.

l_i, according to the following technology:

$$y_i = A_i l_i, \tag{1}$$

where A_i is a constant city type-specific productivity parameter, and $A_2 > A_1$.

Each city has a labour market which functions much like that considered by Lucas and Prescott (1974). For a household to enter this market in any city, they must first receive an *offer*. Firms enter and hire workers to operate the production technology. Inside each labour market, firms and workers take the wage, w_i, as given, and the market clears in Walrasian fashion. A household may work only in their city of current residence. At any point in time, each household is either employed, by which we mean present in a labour market and receiving flow income equal to w_i, or unemployed, in which case they receive flow consumption z.

All households, regardless of their employment status, randomly receive offers of jobs both in their city of residence and in the other cities. Let μ and μ^*denote the Poisson rates at which households receive offers from within and outside their city of residence, respectively. The rates at which households receive offers are symmetric across cities and $\mu > \mu^*$. Let fraction β_i of "outside offers" (*i.e.* those from other cities) received by households in a city of type i come from another city of type i. Thus, the fraction of outside offers that come from a city of type $j \neq i$ is $1 - \beta_i$. A household (employed or unemployed) which receives an employment offer must either accept or reject it immediately. Employed households in all cities lose their jobs at Poisson rate δ. Loss of a job does not affect a household's residency status. New households enter the economy unemployed and immediately rent.

There are also in the economy a large number of specialized firms called *real estate managers* (REM's). These firms are owned by households and perform three functions: First, they build houses of both types using a development technology described below. Second, they rent housing in cities of type i to households in a competitive market at rental rate κ_i. Third, they intermediate between buyers and sellers in city-specific markets for owner–occupied housing.

Only REM's hold vacant houses. Home-owners may sell their houses at any time to a REM in a competitive market specific to their city of residence. Let V_i (the value of a vacant house) denote the price at which such transactions take place in cities of type i. REM's receive no direct service flow from houses and hold them only for the purposes either of re-sale or conversion into rental units. An REM can convert a formerly owner–occupied unit into a rental unit at a fixed per unit cost $d^R 0 \geq 0$. Similarly, an REM can convert a rental unit to an owner–occupied one at cost $d^H \geq 0$.

Vacant houses are matched randomly with potential *home buyers*, the stock of whom is comprised simply of all current renters within the city. Let ϕ_i denote the ratio of the measure of buyers to the stock of vacant houses available for sale in City i. We assume that the rate at which buyers find houses is constant and given by λ. It follows that the rate at which houses match with potential buyers is given by

$$\gamma_i = \lambda \phi_i \qquad i = 1, 2. \tag{2}$$

We adopt this specification for expositional simplicity. Our main results, however, are robust to generalizations of the matching function.[7]

When a potential home buyer matches with a vacant house, we assume that the buyer and the REM which owns the house share the aggregate match surplus (provided it is positive) according to a simple bargaining rule, with σ denoting the buyer's share of the total surplus. Let P_i^W and P_i^U denote the prices paid for houses in cities of type i by employed and unemployed households respectively.[8]

Given the matching process, it takes time for owner-occupied houses to be transferred from one household to another. This results in houses being to some extent *illiquid*, in the sense that their value depends on the speed with which a buyer can be found for a vacant house. To see this, note that the value of a vacant house in cities of type i satisfies:

$$\rho V_i = \gamma_i \mathrm{E}\left[\max\{P_i^j - V_i, 0\}\right] \qquad i = 1, 2; \qquad j = W, U. \tag{3}$$

In every city there are at any time at most four types of households, as each may be either employed or unemployed and may either rent or own a house. The measures of households in City i that are employed-owners, employed-renters, unemployed-owners and unemployed-renters are given by N_i^{WH}, N_i^{WR}, N_i^{UH} and N_i^{UR} respectively. The values associated with being in each of these states are given, in the same order, by W_i^H, W_i^R, U_i^H and U_i^R.

[7] Here we assume that unemployed and employed renters find houses at the same rate. Differential matching rates depending on employment status, however, make no difference for our qualitative results, and have minimal quantitative effect as well. These results are omitted for brevity, but are available on request.

[8] Since they earn zero profits when purchasing a previously owned house, the role played by REM's in intermediating transactions is virtually equivalent to assuming that households which wish to move continue to own their vacant house until they match with and sell to another household. Assuming that this function is performed by REM's greatly simplifies our analysis, however, because it rules out the possibility of a household moving from one city to another, and then returning to its previous location and moving back into its old house before selling. While in principle this would make little difference, and in practice it would happen very infrequently in equilibrium, allowing for it would expand the number of household states and complicate the analysis substantially, while adding nothing significant to our results.

Additions to the stocks of each type of housing may be made using construction technologies which we represent by cost functions. The unit cost of producing additional rental housing is given by

$$C_i^R(R_i, \bar{N}) = c_0 + c_i^R \frac{R_i}{\bar{N}}, \tag{4}$$

and that of providing owner-occupied housing is

$$C_i^H(H_i, \bar{N}) = c_0 + c_i^H \frac{H_i}{\bar{N}}, \tag{5}$$

where the c's are positive constants. This linear specification is similar to that assumed by Glaeser *et al.* (2010). Under this specification, at each point in time, the cost of providing an additional unit of either type of dwelling consists of a common constant, c_0, and a city–specific component which depends linearly on the existing stock of that particular type of housing relative to the population. We interpret c_0 as the cost of constructing the house, which we assume to be the same across cities and dwelling types. We interpret the second component as reflecting the cost of land. The elasticities of unit costs with respect to housing stocks are allowed to varying by city and by type of dwelling. Unit construction costs are higher the larger is the existing stock of housing, partly reflecting the rising cost of land. Note that we divide these stocks by the aggregate population $\bar{N}(t)$ in order to obtain stationarity. This is analogous to assuming that the supply of residential land increases with the population (see Davis and Heathcote, 2005). Recall that once produced, existing houses can, in principle, be converted from rental to owner occupied and *vice versa* at constant unit costs d^H and d^R, respectively.

3 Stationary Equilibrium

We consider equilibria which are symmetric in that all households of a given type behave in the same way and stationary in that the fractions of the total population in each household state remain constant. In such an equilibrium there is constant construction of both rental and owner-occupied houses, provided that conversion costs are sufficient to guarantee that REM's do not convert previously constructed houses from one use to the other. Below, we report the minimum conversion costs that ensure this outcome for our calibration.

In a stationary equilibrium, the values of households in a city of type i satisfy

$$\begin{aligned}
\rho W_i^R &= w_i + \pi^R - \kappa_i + \delta\left(U_i^R - W_i^R\right) + (1-\beta_i)\mu^* \max\left\{W_j^R - W_i^R, 0\right\} & (6)\\
&\quad +\lambda \max\left\{W_i^H - P_i^W - W_i^R, 0\right\} \\
\rho U_i^R &= z + \pi^R - \kappa_i + (\mu + \beta_i\mu^*)\left(W_i^R - U_i^R\right) + (1-\beta_i)\mu^* \max\left\{W_j^R - U_i^R, 0\right\} & (7)\\
&\quad +\lambda \max\left\{U_i^H - P_i^U - U_i^R, 0\right\} \\
\rho W_i^H &= w_i + \pi^H + \delta\left(U_i^H - W_i^H\right) + (1-\beta_i)\mu^* \max\left\{W_j^R + V_i - W_i^H, 0\right\} & (8)\\
&\quad +\beta_i\mu^* \max\left\{W_i^R + V_i - W_i^H, 0\right\} \\
\rho U_i^H &= z + \pi^H + \mu\left(W_i^H - U_i^H\right) + (1-\beta_i)\mu^* \max\left\{W_j^R + V_i - U_i^H, 0\right\} & (9)\\
&\quad +\beta_i\mu^* \max\left\{W_i^R + V_i - U_i^H, 0\right\}
\end{aligned}$$

where the subscript $j \neq i$ indexes cities of the other type.

A *stationary symmetric equilibrium* for this economy is a collection of ten values, eight for the different types of households, W_i^R, W_i^H, U_i^R, and U_i^H, and two for vacant houses in each city type, V_i; rental rates, κ_i; house prices, P_i^W and P_i^U; wages, w_i and employment levels, l_i; ratios of buyers to houses for sale, ϕ_i; and measures of households in each of the eight states, N_i^{WR}, N_i^{UR}, N_i^{WH}, and N_i^{UH}, for $i = 1, 2$, such that:

i. Given wages, firms choose employment levels l_i to maximize profits. Free entry into production implies zero profits so that:

$$w_i = A_i. \tag{10}$$

ii. House purchase prices (P_i^W,P_i^U) in each city are consistent with the surplus sharing rules.

iii. The value of vacant housing satisfies (2) and (3)

iv. Given prices and the value of vacant houses in each city, the values of households in each state satisfy (6)-(9).

v. The rental rates, $\kappa_i \geq 0$, clear the markets for rental housing in each city:

$$N_i^{WR} + N_i^{UR} = R_i \qquad i = 1, 2. \tag{11}$$

vi. The distribution of households over states and cities is consistent with the population:

$$\sum_{i=1,2} s_i \left[N_i^{WR} + N_i^{UR} + N_i^{WH} + N_i^{UH}\right] = \bar{N}. \tag{12}$$

vii. The paths of the housing stocks in each city are consistent with profit maximizing construction and free entry on the part of REM's,

$$\frac{\kappa_i}{\rho} = c_0 + c_i^R \frac{R_i}{\bar{N}}, \quad \dot{R}_i \geq 0 \tag{13}$$

$$V_i = c_0 + c_i^H \frac{H_i}{\bar{N}}, \quad \dot{H}_i \geq 0, \tag{14}$$

where the option to convert housing from one type to another is not exercised:

$$d^H \geq V_i - \kappa_i/\rho \geq -d^R. \tag{15}$$

viii. Market tightness, ϕ_i, equals the ratio of buyers to houses for sale where, given house prices (P_i^W,P_i^U) and the values of being in each state, buyers consist of all those renters who wish to buy and houses for sale consist of all those vacant houses that REMs wish to sell if given the opportunity.

Depending on parameters, stationary equilibria exhibit a range of characteristics with regard to the functioning of labour and housing markets. Our approach will be to concentrate on equilibria with particular characteristics because, as we will show below, our calibration will give rise to an equilibrium with these features, which we refer to as our *benchmark configuration*. Note that in each city the equilibrium wage is proportional to local productivity and is unaffected by conditions in the housing market. From now on we will therefore refer to city types 1 and 2 as low and high-wage cities, respectively.

3.1 Benchmark Equilibrium Configuration

We begin by supposing that a stationary equilibrium with the following features exists, derive certain conditions that the economy must satisfy for this to be so, and then show that in these circumstances the equilibrium, if it exists, must be unique. We then compute the equilibrium and verify that it does indeed have the following characteristics:[9]

1. There are positive measures of unemployed renters in all cities. Because these households are mobile, this restriction requires that in equilibrium they are indifferent with regard to their city of residence. That is,

$$U_1^R = U_2^R = U^R. \tag{16}$$

[9] Under different conditions, the model has equilibria with different characteristics. We omit here a complete analysis of all these possibilities for brevity and because they will not be relevant to our quantitative analysis below. A complete analysis of the model is contained in a separate technical appendix.

2. Employed renters in low-wage cities (Type 1) always accept employment offers from high-wage cities (Type 2), but not vice versa:

$$W_2^R > W_1^R. \tag{17}$$

3. Employed home-owners do not move if offered a job in any other city:

$$W_i^H - V_i > \max\left[W_i^R, W_j^R\right] \tag{18}$$

4. A fraction $\theta_i \in (0,1)$ of the unemployed home-owners in cities of type i that receive an offer of employment in cities of type $j \neq i$ accept that offer and move. We may think of θ_i as the probability with which an individual unemployed homeowner re-locates to the other city in response to an offer. In order for this probability to be interior, it must be that the home-owner is indifferent. That is

$$U_1^H - V_1 = W_2^R \tag{19}$$

$$U_2^H - V_2 = W_1^R. \tag{20}$$

Under the assumption that at least one equilibrium with these characteristics exists, we now derive several other features that it must necessarily have. A first implication is that:

Lemma 1: *All renters, whether employed or not, buy houses when they get the chance. That is, the surplus from a meeting between a renter and an REM is always positive.*

$$W_i^H - W_i^R > V_i \qquad \text{and} \qquad U_i^H - U_i^R > V_i \qquad\qquad i = 1, 2. \tag{21}$$

It follows that:

Lemma 2: *The value of a vacant house in cities of type i is given by*

$$V_i = \frac{(1-\sigma)\gamma_i}{\rho + (1-\sigma)\gamma_i}\left[\alpha_i\left(W_i^H - W_i^R\right) + (1-\alpha_i)\left(U_i^H - U_i^R\right)\right] \tag{22}$$

where $\alpha_i = N_i^{WR}/R_i$ represents the fraction of renters in type i cities that are employed.

Given this and the four requirements that define our benchmark configuration, the Bellman equations for home-owners, (8) and (9), simplify to

$$\rho W_1^H = w_1 + \pi^H + \delta\left(U_1^H - W_1^H\right) \tag{23}$$
$$\rho W_2^H = w_2 + \pi^H + \delta\left(U_2^H - W_2^H\right) \tag{24}$$
$$\rho U_1^H = z + \pi^H + \mu\left(W_1^H - U_1^H\right) \tag{25}$$
$$\rho U_2^H = z + \pi^H + \mu\left(W_2^H - U_2^H\right) + \beta_2\mu^*\left(W_2^R + V_2 - U_2^H\right) \tag{26}$$

The first two expressions reflect the fact that employed owners do not accept outside offers from any other city. Unemployed owners in cities of both types accept offers of employment in their own city and are indifferent regarding offers from cities of the other type. Unemployed owners in low-wage cities, however, always turn down offers from other low-wage cities, whereas those in high-wage cities always accept offers of employment from other high wage cities.

Since we require that unemployed renters are indifferent with respect to their location (16) and employed renters do not move from the high-wage city to the low-wage one (17), we may express the Bellman equations for renters as:

$$\begin{aligned}\rho W_1^R &= w_1 + \pi^R - \kappa_1 + \delta\left(U_1^R - W_1^R\right)\\ &\quad + (1-\beta_1)\,\mu^*\left(W_2^R - W_1^R\right) + \lambda\left(W_1^H - P_1^W - W_1^R\right)\end{aligned} \tag{27}$$

$$\begin{aligned}\rho U^R &= z + \pi^R - \kappa_1 + (\mu + \beta_1\mu^*)\left(W_1^R - U^R\right)\\ &\quad + (1-\beta_1)\,\mu^*\left(W_2^R - U^R\right) + \lambda\left(U_1^H - P_1^U - U^R\right)\end{aligned} \tag{28}$$

$$\rho W_2^R = w_2 + \pi^R - \kappa_2 + \delta\left(U_2^R - W_2^R\right) + \lambda\left(W_2^H - P_2^W - W_1^R\right) \tag{29}$$

$$\begin{aligned}\rho U^R &= z + \pi^R - \kappa_2 + (\mu + \beta_2\mu^*)\left(W_2^R - U^R\right)\\ &\quad + (1-\beta_2)\,\mu^*\left(W_1^R - U^R\right) + \lambda\left(U_2^H - P_2^W - U^R\right).\end{aligned} \tag{30}$$

We then have that

Lemma 3: *The value of a job to a renter in each city is given by*

$$W_1^R - U^R = \Gamma_1 = \left(\frac{\rho+\delta+\mu+\sigma\lambda}{\rho+\delta+\mu+\mu^*+\sigma\lambda}\right)\left(\frac{w_1 - z}{\rho+\delta+\mu}\right) \tag{31}$$

$$W_2^R - U^R = \Gamma_2 = \left(\frac{w_2 - z}{\rho+\delta+\mu+\beta_2\mu^*}\right) - \frac{\left[(\rho+\delta+\mu)(1-\beta_2) - \sigma\lambda\beta_2\right]\mu^*\Gamma_1}{(\rho+\delta+\mu+\beta_2\mu^*)(\rho+\delta+\mu+\sigma\lambda)}. \tag{32}$$

Using these expressions, we can now state the first main implication of the restrictions imposed by our benchmark configuration:

Proposition 1. *: The net benefit to an employed renter of living and working in a high-wage city as opposed to in a low-wage one is proportional to the wage differential:*

$$W_2^R - W_1^R = \Gamma_2 - \Gamma_1 = \frac{w_2 - w_1}{\rho + \delta + \mu + \beta_2 \mu^*}. \tag{33}$$

Note that equating (28) and (30), and using (27) and (29), it is also apparent that the rent differential between high- and low-wage cities is proportional to the wage differential:

$$\kappa_2 - \kappa_1 = \left(\frac{\mu - (1 - \beta_1 - \beta_2)\, \mu^* - \sigma\lambda}{\rho + \delta + \mu + \beta_2 \mu^*} \right) (w_2 - w_1)\,. \tag{34}$$

The sales price differential across city types is also proportional to the wage differential and is given by

$$\rho\,(V_2 - V_1) = \left(\frac{\rho + \mu + \beta_2 \mu^*}{\rho + \delta + \mu + \beta_2 \mu^*} \right) (w_2 - w_1)\,. \tag{35}$$

While the *levels* of rents and prices in all cities depend on the housing stocks, neither the rent nor sales price *differential* does.

Proposition 2. *: The dispersion in house values across cities exceeds that in the present discounted value of rents. That is*

$$V_2 - V_1 > \frac{\kappa_2 - \kappa_1}{\rho}.$$

This result stems from the frictions in both the labour market and the housing market. The rent differential is determined by unemployed renters who can move costlessly between cities even if they do not receive an offer. In contrast, the differential in the value of houses is determined by the marginal (unemployed) home-owners who must first receive an outside offer and then incur the endogenous liquidity cost of selling their house. That the dispersion in the present discounted value of rents across U.S. cities is indeed less than that for prices is documented by van Nieuwerburgh and Weill (2010). Our model offers one possible explanation for this observation, based on the interaction of labour and housing market frictions. However, as we will see below, in our baseline calibration the difference turns out to be quantitatively small.

Since in each city renters constitute the potential buyers in the housing market, we may write the ratios of buyers to sellers as:

$$\phi_i = \frac{N_i^{WR} + N_i^{UR}}{H_i - N_i^{WH} - N_i^{UH}} \qquad i = 1, 2. \tag{36}$$

Combining (11), (12) and (36) we can then derive the following expression implied by rental market clearing aggregation:

$$\frac{s_1 R_1}{\phi_1} + \frac{s_2 R_2}{\phi_2} = s_1 R_1 + s_2 R_2 + s_1 H_1 + s_2 H_2 - \bar{N}. \tag{37}$$

Stationarity of the equilibrium requires that stocks of both types of housing *per capita* must be constant. Equation (37) can then be written:

$$\frac{s_1 r_1}{\phi_1} + \frac{s_2 r_2}{\phi_2} = s_1 r_1 + s_2 r_2 + s_1 h_1 + s_2 h_2 - 1 \tag{38}$$

where $r_i = R_i/\bar{N}$ and $h_i = H_i/\bar{N}$. For given housing stocks, this condition yields a locus of combinations of matching rates, ϕ_1 and ϕ_2, consistent with market clearing and aggregation. This locus is depicted in Figure 1 as the downward sloping curve labelled AM.[10]

The flow of households between states in a stationary equilibrium under the benchmark configuration is described by (11), (36) and the following equations:

$$\begin{aligned}
\dot{N}_1^{WR} &= (\mu + \beta_1 \mu^*) N_1^{UR} + \frac{s_2}{s_1}(1-\beta_2)\mu^* \left(N_2^{UR} + \theta_2 N_2^{UH}\right) \qquad (39)\\
&\quad - (\delta + (1-\beta_1)\mu^* + \lambda) N_1^{WR}\\
\dot{N}_1^{UH} &= \delta N_1^{WH} + \lambda N_1^{UR} - (\mu + (1-\beta_1)\mu^* \theta_1) N_1^{UH} \qquad (40)\\
\dot{N}_1^{WH} &= \lambda N_1^{WR} + \mu N_1^{UH} - \delta N_1^{WH} \qquad (41)\\
\dot{N}_2^{WR} &= (\mu + \beta_2 \mu^*) N_2^{UR} + \frac{s_1}{s_2}(1-\beta_1)\mu^* \left(N_1^{UR} + N_1^{WR} + \theta_1 N_1^{UH}\right) \qquad (42)\\
&\quad - (\delta + \lambda) N_2^{WR}\\
\dot{N}_2^{UH} &= \delta N_2^{WH} + \lambda N_2^{UR} - (\mu + (1-\beta_2)\mu^* \theta_2 + \beta_2 \mu^*) N_2^{UH} \qquad (43)\\
\dot{N}_2^{WH} &= \lambda N_2^{WR} + \mu N_2^{UH} - \delta N_2^{WH}. \qquad (44)
\end{aligned}$$

>From (39) we have that the increase in the number of employed renters in a given low-wage city must equal the measure that *become* employed renters in that city minus the measure of agents that *cease* being employed renters in the city. Those that become employed renters in a given low wage city consist of

[10]As we demonstrate below, one consequence of Lemma 1 is that $\phi_2 > \phi_1$. Thus, without loss of generality, the equilibria that we study here are all associated with combinations of matching rates located on the segment of the AM curve above the 45^o line.

1. unemployed renters in that city who receive an offer: μN_1^{UR}

2. unemployed renters in other low-wage cities who receive an offer from this city: In each of the $s_1 - 1$ other low-wage cities, $\beta_1 \mu^* N_1^{UR}/(s_1 - 1)$ unemployed renters receive offers from this city.

3. unemployed renters in high wage cities that receive an offer from this city: In each of the s_1 high-wage cities, $(1 - \beta_2)\mu^* N_2^{UR}/s_1$ unemployed renters receive offers from this city.

4. unemployed owners in high-wage cities that receive and accept job offers in low-wage cities: $s_2(1 - \beta_2)\mu^* \theta_2 N_2^{UH}/s_1$.

Agents cease being employed renters in a low-wage city by losing their job, δN_1^{WR}, by accepting an offer in a high-wage city, $(1 - \beta_1)\mu^* N_1^{WR}$, or by buying a house, λN_1^{WR}.

Similarly, from (40) we have that the change in the measure of unemployed home-owners in a given low-wage city equals the difference between the flows into and out of that state, and from (41) we have the same for *employed* home-owners in that city. Equations (42)-(44) represent the analogous conditions for high-wage cities.[11]

Equations (11), (36), and (39)-(44) can be used to derive a relationship between the ratios of buyers to sellers in the low and high wage cities, $\phi_1$and ϕ_2, making use of the requirements that both unemployed renters with no job offer and unemployed home-owners with an offer from a city of the other type be indifferent between locations (that is, the first and third conditions for the benchmark configuration, (16) and (18)). Fixing the stocks of housing in all cities, this relationship determines a locus of combinations of matching rates consistent with these restrictions. We depict this relationship with locus VV in Figure 1, and have the following result, proved in Appendix B.

Proposition 3. *: Under the restrictions imposed by the benchmark configuration, for fixed housing stocks there is a positive relationship between the ratios of buyers to sellers in low- and high-wage cities, ϕ_1 and ϕ_2. That is, the VV locus is upward sloping.*

To understand this result, consider an increase in the matching rate in low-wage cities, $\phi_{1.}$. *Ceteris paribus,* this is associated with a higher house sale price in these cities, and

[11] There are two asymmetries: (1) employed renters move from low-wage to high-wage cities, when offered a job but not vice versa, and (2) unemployed owners in high-wage cities accept offers in other high-wage cities, but those in low-wage cities do not accept offers in other low-wage cities.

thus lowers the cost of moving for an unemployed home-owner who receives a job offer in a high-wage city. Since this home-owner must be indifferent to moving, this lower cost of relocation must be offset by a higher rental rate in high-wage cities, κ_2. As unemployed renters with no job offers must also be indifferent between cities, this in turn must be offset by a higher rental rate in low-wage cities, κ_1. A higher rental rate in low-wage cities discourages migration from high-wage cities, and the resulting reduction in the number of vacant houses raises the matching rate in those cities, ϕ_2.

In a stationary equilibrium, the growth rates of the measures of households in each state are all equal to ν so that the fractions of the population in each state remain constant. Letting $n_i^{WR} = N_1^{WR}/\bar{N}$, etc. we can re-write (39)-(44):

$$\begin{aligned}
(\delta + (1-\beta_1)\mu^* + \lambda + \nu) n_1^{WR} &= (\mu + \beta_1\mu^*)\, n_1^{UR} \\
&\quad + (1-\beta_2)\frac{s_2}{s_1}\mu^* \left(n_2^{UR} + \theta_2 n_2^{UH}\right) && (45)\\
(\mu + (1-\beta_1)\mu^*\theta_1 + \nu)\, n_1^{UH} &= \delta n_1^{WH} + \lambda n_1^{UR} && (46)\\
(\delta + \nu)\, n_1^{WH} &= \lambda n_1^{WR} + \mu n_1^{UH} && (47)\\
(\delta + \lambda + \nu)\, n_2^{WR} &= (\mu + \beta_2\mu^*)\, n_2^{UR} \\
&\quad + (1-\beta_1)\frac{s_1}{s_2}\mu^* \left(n_1^{UR} + n_1^{WR} + \theta_1 n_1^{UH}\right) && (48)\\
(\mu + (1-\beta_2)\mu^*\theta_2 + \beta_2\mu^* + \nu)\, n_2^{UH} &= \delta n_2^{WH} + \lambda n_2^{UR} && (49)\\
(\delta + \nu)\, n_2^{WH} &= \lambda n_2^{WR} + \mu n_2^{UH}. && (50)
\end{aligned}$$

Similarly, (36) can be expressed as

$$n_i^{WH} + n_i^{UH} = h_i - \frac{r_i}{\phi_i} \qquad i = 1, 2. \tag{51}$$

and rental market clearing (11) can be expressed as

$$n_i^{UR} + n_i^{WR} = r_i \qquad i = 1, 2. \tag{52}$$

Solving this system of equations yields the stationary measure of agents in each state plus the fractions of unemployed owners who move in response to offers from a city of a different type. To ensure that this solution is indeed consistent with a stationary equilibrium under the benchmark configuration, two conditions (which we have to this point assumed to hold but not imposed) must be checked.[12] First, the measure of employed renters must be strictly less than the stock of rental housing in each city: $n_i^{WR} < r_i$, $i = 1, 2$. If this were not the

[12] This is in addition to the conditions that the option to convert between dwelling types is not exercised.

case, then unemployed renters would strictly prefer one city to the other, violating (16). Second, the fractions of unemployed owners who move from each city must be interior, *i.e.* $\theta_i \in (0,1)$, $i = 1, 2$, as these are conditions (19) and (20).

In Figure 1, the points $X = (\phi_1^X, \phi_2^X)$ and $Y = (\phi_1^Y, \phi_2^Y)$ represent the combinations of ϕ_1 and ϕ_2 at which $\theta_1 = 1$ and $\theta_2 = 1$, respectively, and rental markets clear.

Lemma 4: *The pairs (ϕ_1^X, ϕ_2^X) and (ϕ_1^Y, ϕ_2^Y) are unique.*

Provided (16), (19), and (20) hold, for given housing stocks a combination of matching rates in the two cities consistent with a stationary equilibrium under the benchmark equilibrium is represented by the intersection of the VV and AM loci in ϕ_1-ϕ_2 space (Figure 1). Only (ϕ_1, ϕ_2) pairs along the AM curve between X and Y yield interior solutions. Since the VV locus is monotone increasing and the AM locus is decreasing, if they intersect, then they do so only once. The following result establishes conditions sufficient to identify a unique intersection point of the AM and VV curves which satisfies the requirements of the benchmark configuration.

Lemma 5: *For given housing stocks per capita in all cities, the steady-state identified by (ϕ_1^*, ϕ_2^*) is the unique equilibrium under the benchmark configuration if*
(1)

$$\frac{(1-\beta_2)\mu^* + \lambda}{\delta + (1-\beta_1)\mu^* + \lambda + \nu} < \frac{s_1 r_1}{s_2 r_2} < \frac{\delta + \lambda + \nu}{(1-\beta_2)\mu^* + \lambda}. \tag{53}$$

(2)

$$\phi_1^X > \phi_1^* > \phi_1^Y, \qquad \phi_2^X < \phi_2^* < \phi_2^Y. \tag{54}$$

Subject to these conditions, for a given vector of housing stocks and the associated matching rates, (ϕ_1^*, ϕ_2^*), there exist unique pairs of equilibrium rental rates (κ_1^*, κ_2^*) and vacant house values (V_1^*, V_2^*), where $\kappa_i^* = \kappa_i(r_1, r_2, h_1, h_2)$ and $V_i^* = V_i(r_1, r_2, h_1, h_2)$.

We now consider the determination of the housing stocks in equilibrium. To find an equilibrium, we must find optimal paths for the stocks of rental and owner-occupied housing in cities of both types which give rise to steady-state matching rates that satisfy the conditions given in Lemma 5. In any stationary equilibrium, all housing stocks must grow at rate ν. Thus, it follows that in any such equilibrium:

$$\frac{\kappa_i}{\rho} = c_0 + c_i^R r_i \tag{55}$$

$$V_i = c_0 + c_i^H h_i. \tag{56}$$

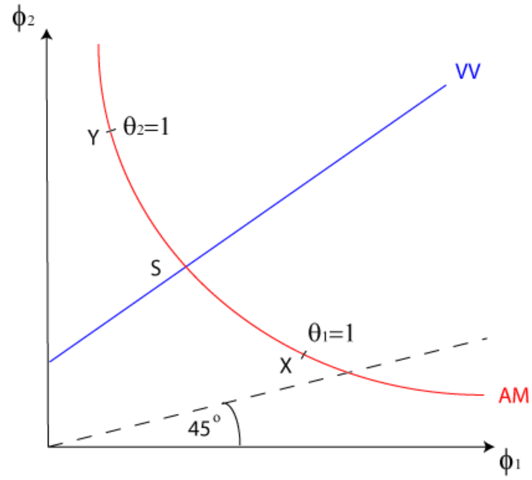

Figure 1: Stationary Equilibrium

We may think of (55) and (56) as stationary "housing supply curves" for each city type. Recall that independent of the housing stocks, rental rates and prices are all positively and linearly related to each other under the benchmark configuration, (34) and (35). Thus, given (55) and (56), in any stationary equilibrium of such an economy the stocks of different types of housing must all be linearly and positively related to each other.

Consequently, the equilibrium system of equations may be reduced to a single equation relating any one of the four housing prices (rental and house sale prices in each city type) to the four housing stocks. It is straightforward to show that this price is decreasing in the value of an unemployed renter, U^R. Since this is the only component of the equation that depends on the housing stocks, we have the following:

Proposition 4. *If* $U^R(r_1, r_2, h_1, h_2)$ *is increasing in all of its arguments, then if there exists a stationary equilibrium under the benchmark configuration, it is unique.*

To this point, we have derived conditions under which a stationary equilibrium, if one exists, 1) satisfies the conditions of the benchmark configuration, and 2) is unique. Below, we compute directly a stationary equilibrium for our calibrated economy and confirm that it satisfies these conditions.

4 Mobility, Home Ownership and Unemployment in the Stationary Equilibrium

In this section we present several results which describe qualitatively the stationary equilibrium of our economy under the benchmark configuration. These results help provide intuition for the quantitative characteristics of our calibrated economy in Section 6.

First, we prove that houses sell more quickly in high-wage cities:

Proposition 5. *If the wage differential across city types, $w_2 - w_1$, is sufficiently large then market tightness is highest in high-wage cities: $\phi_2 > \phi_1$.*

In general we find that the minimum wage differential that is sufficient for this result to hold is tiny.

In equilibrium, the home-ownership rate in cities of type i can be expressed as a function of the matching rate:

$$\Omega_i(\phi_i) = \frac{n_i^{UH} + n_i^{WH}}{r_i + n_i^{UH} + n_i^{WH}} = \frac{\frac{h_i}{r_i} - \frac{1}{\phi_i}}{1 + \frac{h_i}{r_i} - \frac{1}{\phi_i}}. \tag{57}$$

The rate of home-ownership is increasing in the ratio of the stock of owned to rental housing. Moreover, since h_i is increasing in γ_i it follows that

Corollary 1. *If the ratio of the stocks of owned and rental houses is the same in both types of city, then home-ownership is greatest in high-wage cities: $\Omega_2 > \Omega_1$.*

Since home-owners and renters receive offers and are separated from jobs at the same rates, they differ only with regard to the likelihood with which they accept offers. As only home-owners turn down jobs in equilibrium, the following result is not surprising:

Proposition 6. *The unemployment rate among home-owners exceeds that among renters who are not new entrants to the labour force.*

Note the emphasis on renters who are not new entrants to the labour force. Because households in our model are *ex ante* identical, Proposition 6 is not in conflict with the empirical observation that unemployment is higher among *all* renters than among home-owners. Since households enter the population as unemployed renters, this biases upwards the unemployment rate among renters in a way that does not affect that of owners. The unemployment rate among all renters can be expressed as

$$\tilde{u}^R = \underbrace{\frac{\delta}{\delta+\mu+\mu^*+\lambda+\nu}}_{\text{non-entrants}} + \underbrace{\frac{\nu}{(\delta+\mu+\mu^*+\lambda+\nu)(s_1 r_1 + s_2 r_2)}}_{\text{new labour force entrants}} \tag{58}$$

To isolate the effect of ownership, we can remove this bias by considering only the unemployment rate of those renters who are not new entrants, which is the first term on the right-hand side of (58).

The following proposition characterizes the tendency of unemployed renters to remain in or move to the low-wage city:[13]

Proposition 7. *If*
(a) $\beta_2 \geq \beta_1$,
(b) relative stocks of ownable to rented housing are similar across cities, $h_1/r_1 \simeq h_2/r_2$, *and*
(c) the relative number of renters in small cities is sufficient large, $\frac{s_1 r_1}{s_2 r_2} > \xi$ *for* $\xi \in (0,1)$,
then the fraction of renters who are employed is greatest in high wage cities: i.e. $\alpha_2 > \alpha_1$.

In any stationary equilibrium, the majority of households (all renters and some homeowners) resident in low-wage cities that receive high-wage job offers move to accept them. In contrast, only unemployed renters and some fraction of unemployed home-owners resident in high-wage cities migrate to a low-wage city to accept a job offer. This asymmetry tends to drive up the rent differential between high- and low-wage cities. This in turn may induce unemployed households with no job offer to remain in or move to a low-wage (and low rent) city. Consequently, the proportion of renters who are unemployed tends to be higher in low-wage cities. Proposition 7 establishes that this is true unless rental housing in low-wage cities is sufficiently scarce.

The unemployment rate in a city of type i can conveniently be expressed as

$$u_i = \underbrace{\left(\frac{\delta+\nu}{\delta+\mu+\nu}\right)\Omega_i}_{\text{home-ownership effect}} + \underbrace{\left[1 - \left(\frac{\delta+\mu+\nu+\lambda}{\delta+\mu+\nu}\right)\alpha_i\right](1-\Omega_i)}_{\text{rent differential effect}} \tag{59}$$

where Ω_i is given by (57). The first term reflects the positive impact of home-ownership to the city's unemployment rate due to the fact that some unemployed home-owners turn down job offers rather than relocate. The second reflects the fact that there is typically a

[13]Recall that β_i denotes the fraction of outside offers received by households cities of type i that come from other cities of type i.

higher concentration of unemployed renters (represented in (59) by $1 - \alpha_i$) in cities with lower rent (*i.e.* low-wage ones). The home-ownership effect is typically larger in high-wage cities while the rent differential effect is typically higher in low-wage cities, as some unemployed households move to these cities to take advantage of relatively low rent. Overall, the relationship between unemployment and home-ownership in cities of either type depends on which of the two effects dominate.

Proposition 8. *: If households never recieve outside offers from cities of the same type as that in which they currently live (i.e. if $\beta_1 = \beta_2 = 0$), then the aggregate unemployment rate can be expressed as*

$$\bar{u} = A + B\bar{\Omega}$$

where $\bar{\Omega}$ is the aggregate home-ownership rate and $A > 0$ and $B > 0$ are constants depending only on parameters.

When β_1 and β_2 are positive, the relationship becomes more complicated and $\bar{u}$ no longer depends only on $\bar{\Omega}$. As we will see below, however, in our calibrated economies the positive relationship remains.

5 Calibration

Before calibrating, it is useful to introduce three generalizations of the basic model that greatly improve our ability to map the economy's parameters into characteristics of the data. These generalizations do introduce new parameters, but affect neither the existence of a unique stationary equilibrium under the benchmark configuration nor any of the qualitative results presented in Section 4. Full details of how these changes affect the equilibrium are provided in Appendix C.

5.1 Intra–city Relocation

To this point we have abstracted from housing transactions among households who do not migrate, but remain within a city. If all owner-occupied houses within a city are identical, there is no reason for a home-owner to sell one house in order to move to another within the same city. Empirically, however, most actual movement of home-owners is within rather than between cities, and most intra-city moves are not job-related (Rupert and Wasmer, 2009). Moreover, intra-city relocation affects inter-city migration in the model quantitatively, through its effect on the liquidity of housing. In order to account simultaneously for inter-city mobility and the levels of house prices, we now modify our model to allow for intra-city movement of home-owners.

Following Wheaton (1990), we assume home-owners experience *housing taste shocks* at rate ψ. On experiencing a shock, the service flow a home-owner receives from their *current* house falls permanently to $\pi^H - \varepsilon$, while that potentially available to them from other houses remains π^H. All such *mismatched* owners immediately become potential buyers, search for a new house, and match with vacant houses via the same technology as renters. Once they find a new house, they immediately sell their old house to an REM at the market price. The REM sells them the new house at a price which reflects the usual bargaining outcome:

$$P_i^{WH} = (1-\sigma)\left(W_i^H - \tilde{W}_i^H\right) + V_i \qquad P_i^{UH} = (1-\sigma)\left(U_i^H - \tilde{U}_i^H\right) + V_i \qquad i = 1, 2. \tag{60}$$

where $\tilde{W}_i^H$ and $\tilde{U}_i^H$ denote the values of being a mismatched owner who is employed and unemployed, respectively. Note that while the surplus split is assumed to be the same regardless of whether an REM is bargaining with a renter or a mismatched home-owner, the exchange prices will differ in these situations as the home buyers' outside options differ across the two cases.

As in the basic model we restrict attention to equilibria which are stationary and symmetric. We also restrict attention to the case in which the marginal home-owner in all cities is unemployed and not mismatched with their current house (since this is the case that arises for our benchmark calibration). We impose the following additional restrictions on our benchmark configuration and confirm that they hold in the equilibrium of our calibrated economy:

5. Mismatched owners do not become renters

$$\tilde{W}_i^H - V_i > W_i^R \quad \text{and} \quad \tilde{U}_i^H - V_i > U_i^R \qquad i = 1, 2. \tag{61}$$

6. All mismatched owners buy houses when they get the chance.

7. Mismatched unemployed owners in cities of type i accept offers from other cities of the same type i:

$$\tilde{U}_i^H - V_i < W_i^R \qquad i = 1, 2 \tag{62}$$

These conditions together imply that employed home-owners (matched and mismatched) are also unwilling to move from high wage cities to low wage ones:

$$W_2^H - V_2 > \tilde{W}_2^H - V_2 > W_2^R > W_1^R. \tag{63}$$

Let $\tilde{n}_i^{WH}$ and $\tilde{n}_i^{UH}$ denote the stocks *per capita* of mismatched employed and unemployed owners in city i, respectively. Since the stock of potential buyers now includes mismatched owners as well as renters, it follows that the ratio of buyers to sellers in city i's housing market is given by

$$\tilde{\phi}_i = \frac{r_i + \tilde{n}_i^{WH} + \tilde{n}_i^{UH}}{h_i - n_i^{WH} - n_i^{UH} - \tilde{n}_i^{WH} - \tilde{n}_i^{UH}} \qquad i = 1, 2. \tag{64}$$

The value of a vacant house in a type i city now satisfies

$$\rho V_i = \lambda \tilde{\phi}_i (\bar{P} - V_i) \tag{65}$$

where the average house price is

$$\bar{P}_i = \eta_i \left[\alpha_i P_i^{WR} + (1 - \alpha_i) P_i^{UR} \right] + (1 - \eta_i) \left[\zeta_i P_i^{WH} + (1 - \zeta_i) P_i^{UH} \right].$$

Here $\eta_i = r_i / \left(r_i + \tilde{n}_i^{WH} + \tilde{n}_i^{UH} \right)$ denotes the fraction of buyers that are renters and $\zeta_i = \tilde{n}_i^{WH} / \left(\tilde{n}_i^{WH} + \tilde{n}_i^{UH} \right)$ the fraction of mismatched owners that are employed.

This generalization introduces two new parameters: ε and ψ. We restrict attention to cases in which the stationary equilibrium that we compute remains the unique one under the benchmark configuration. For ε sufficiently small, in our calibration the marginal home-owner in each city is indeed unemployed and satisfied with their current house.[14] Consequently, unemployed households in either city that become dissatisfied with their home strictly prefer to move if offered a job in any other city.

5.2 Inter-city relocation for non-employment related reasons

Although moves between cities are more likely to be job related than moves within cities, it is still the case that many inter-city moves occur for reasons other than to obtain employment or a higher wage. To allow for such moves we assume that all households are subject to exogenous relocation shocks which cause them to move even if the net benefits (as measured above) of doing so would otherwise be negative. Specifically, we assume that a fraction χ of all those households that receive job offers from other cities also receive a relocation shock. We assume that the shock to utility is just enough to induce the household to move.[15]

[14] For some parameter configurations, the marginal home owner could be one dissatisfied with their current match. Thus, this extension of the environment introduces several additional equilibrium confugurations. A full analysis of all the possible cases is omitted for brevity.

[15] This assumption greatly simplifies things because it avoids us having to calibrate a utility shock for every possible state.

Random relocation leads to more households of each type moving than would in the basic model. In addition, its effects are not symmetric across households in different states. For example, a fraction χ of employed owners will now move, whereas none do in the basic model. There is, however, no increase in the fraction of employed renters in low-wage cities that move to high-wage cities (they would have moved anyway). *Ceteris paribus*, an increase in χ tends to increase the mobility of owners relative to renters.

5.3 Mortgage Interest Deductibility

There are potentially many distortions in housing markets that could drive a wedge between the price that a buyer pays and the price that a seller receives. Although such wedges will not typically affect the our qualitative results, they could have quantitative implications. For the U.S., a particularly significant distortion is the deductibility of mortgage interest payments from taxable income. Following Gervais (2002), we assume that when households buy a house they pay a down-payment, dP, (which could be zero) and then take out an infinite mortgage on the remainder, paying $\rho P(1-d)$ per period, where P is the price received by the seller. We assume that taxable income is computed *net* of these payments for home-owners, so that the effective price paid by the buyer is $(1-\tau(1-d))P$, where τ is the income tax rate. Since labour supply is exogenous in our economy and we assume that revenues are transferred back to households as lump-sum transfers, variation in the tax rate affects *only* the cost of purchasing a house relative to renting, and in our computational experiments we will interpret variations in τ as capturing changes to the deductibility of mortgage interest.

To see how mortgage interest deductibility works in the model let $T = \tau(1-d)$. In a transaction between an employed renter and a seller, the seller receives $(1-T)P_i^{WR} - V_i$ and the buyer gets $W_i^H - W_i^R - (1-T)P_i^{WR}$. The total surplus in this transaction is therefore $W_i^H - W_i^R - V_i$. It follows that the price paid by an employed renter satisfies $(1-T)P_i^{WR} = \sigma\left(W_i^H - W_i^R\right) + (1-\sigma)V_i$. In a transaction between a employed mismatched owner and a seller, the seller receives $(1-T)P_i^{WH} - V_i$ and the buyer gets $W_i^H - \tilde{W}_i^H - (1-T)P_i^{WH} - V_i$. The total surplus is therefore $W_i^H - \tilde{W}_i^H$. It follows that the price paid by an employed (mismatched) owner satisfies $(1-T)P_i^{WH} = V_i + \sigma\left(W_i^H - \tilde{W}_i^H\right)$. Similar implications hold for unemployed renters and mismatched owners, respectively.

5.4 Parameter Choices: Baseline Calibration

Table 1 — Parameter Choices (Baseline Calibration)

Parameter Symbol	Parameter Value	Target and Source	Target Value
ρ	0.0033	Annual discount factor (Shimer, 2005)	0.96
ν	0.001	Annual population growth, 1990-2000 (USCB)	1.2%
z/w_1	0.71	Flow value of non-work (Hall & Milgrom, 2008)	0.71
w_2/w_1	1.097	Large-small city real wage ratio (USCB & ACCRA)	1.097
τ	0.29	Average income tax rate (Gervais, 2002)	29%
d	0.2	% mortgage required as down-payment (Gervais, 2002)	20%
χ	0.70	Fraction who move counties for non-job reasons (CPS)	70%
c_0	2.2	Construction as % of house price (Davis & Palumbo, 2008)	71%
β_1	0.285	% population in small cities (USCB)	28.5%
β_2	0.715	% population in large cities (USCB)	71.5%
δ	0.0225	Unemployment rate (Shimer, 2005)	5.7%
μ	0.431	Monthly hiring rate (Shimer, 2005)	0.44
μ^*	0.0095	Annual mobility of renters between counties (USCB)	12%
λ	0.0074	Annual mobility of owners between counties (USCB)	3.6%
ψ	0.0185	% of owner-moves within county (USCB)	57%
σ	0.379	Average price–average income ratio (USCB/ACCRA)	3.08
$\pi^H - \pi^R$	0.009	Average rent–average income ratio (USCB/NIPA/ACCRA)	0.14
ε	0.0045	Half the utility difference between owning and renting	0.0045
$c_1^R r_1$	23.1	Rental units in small cities per household (USCB)	0.09
$c_1^H h_1$	30.3	Owned units in small cities per household (USCB)	0.23
$c_2^R r_2$	50.0	Rental units in large cities per household (USCB)	0.20
$c_2^H h_2$	66.3	Owned units in large cities per household (USCB)	0.49

Notes: USCB = United States Census Bureau
NIPA = National Income and Product Accounts
ACCRA = American Chamber of Commerce Research Association

We choose parameters so that the stationary equilibrium is consistent with several observed aspects of the U.S. economy. The parameter values and the relevant targets are given in Table 1. The first nine parameters are set directly to match the associated targets. The remaining eleven parameters jointly determine the extent to which the equilibrium matches the remaining targets as a group. For illustrative purposes, however, in the table we associate each parameter with a specific target for which it is particularly relevant. We base our calibration on monthly data where possible.

Target values for the discount rate, the unemployment rate, the hiring rate and the separation rate are taken from Shimer (2005). The relative flow value of non-work, z, which includes unemployment benefits and the estimated relative utility of leisure, is taken from Hall and Milgrom (2008). The average income tax rate and downpayment are taken from Gervais (2002) and the fraction of households that move between counties for non job-related reasons, χ, is based on a survey conducted as part of the Current Populations Survey (CPS).[16]

We take housing market statistics is mainly from the 2000 U.S. Census, which provides information regarding housing markets by Metropolitan Statistical Area (MSA). Using the pre-2006 definitions, there are 279 MSAs listed in the Census. We remove the three Puerto Rican MSAs, Anchorage and Honolulu and divide the remaining 274 cities into two groups based on population size. Specifically we divide them into cities with more and with less than one million inhabitants in 2000. This results in a group of 49 large and typically "high-wage" Type 2 MSAs and a group of 225 smaller and typically "low-wage" Type 1 MSAs.

For all cities we have data on nominal incomes, house prices and rents. We deflate all of these using the non-shelter Cost of Living Index (COLI) for each MSA (see Appendix A). In nominal terms, median household income for the large cities is roughly 23% higher on average than that of the smaller cities. Once adjusted for the cost of living (other than housing), however, we find that real incomes for large cities are on average 9.7% higher than those for small cities.[17] Similarly, real house prices and real rents are, respectively, 40% and 20% higher.

One problem for studying housing and rental markets in some areas (e.g. Manhattan and central Los Angeles) is the existence of stringent rent control laws.[18] That this distortion is significant is demonstrated by very low home-ownership rates in these locations relative to other counties within the same MSA and to other cities in their respective population group. Our view (which is consistent with the evidence of Glaeser and Luttmer, 2003) is that rent control in the central areas of these cities generates (both intra– and inter– city) immobility, especially among poorer households. Indeed, it is likely that this immobility is even more acute than for home-owners: when these households move they cannot sell their "claim"

[16] http://www.census.gov/hhes/migration/data/cps.html

[17] There is evidence that wage differentials arise in part from difference in the compositions of the workforce across cities. After controlling for educational and occupational differences Glaeser and Mare (2001) estimate a dense metropolitan wage premium for cities with more than 500,000 inhabitants of 0.24 log points and a non-dense metropolitan premium of 0.14 log points. In our baseline calibration we consider a real income differential of 9.7%. We also, however, consider the sensitivity of our results to variation in the wage differential across city types.

[18] Although rent control has decreased over the last decade, because our quantitative analysis is largely based on the 2000 census, we expect that it could have an significant effect.

to the low rent housing. Since our model does not allow for these effects, in computing our calibration targets we treat some proportion of renters living in these particular areas as being equivalent to home-owners with respect to mobility. We do this by using only the relative stock of ownable to rented housing in the rest (*i.e.* the other counties) of the relevant MSA.

To compute the stocks of owned housing, we divide the number of currently owner-occupied houses by one minus the reported owned-housing vacancy rate for each MSA. For the stocks of rental units we use only occupied rental units which we take to be the "effective" rental stock.[19] The stocks are aggregated across MSAs and divided by the total number of households. They imply an average vacancy rate for owner–occupied housing of 1.55%, which is similar to the average for MSAs reported by the U.S. Census Bureau between 1986 and 2005.[20] Interestingly, the ratios of owned housing to rental housing for the two groups of cities are virtually identical *(i.e.* $h_1/r_1 \simeq h_2/r_2$) and so differences in ownership rates across city groups arise almost entirely from differences in vacancy rates.

Estimates of housing construction costs by city do not generally distinguish between owned and rental housing and are generally hard to come by (see Glaeser et al., 2010). Rather than measuring costs directly, we calibrate the parameters of the housing supply functions, so that the *per capita* stocks of each type of housing in each city match the averages in the data (from the 2000 U.S. Census) for each of our two MSA groups. In our Baseline calibration we assume that construction costs are constant across cities and dwelling types. Following Davis and Heathcote (2007) and Saiz (2011), we choose c_0 so that land accounts for 29% of the price of a house on average across the entire economy. We then select the remaining four parameters, $\{c_i^R, c_i^H\}_{i=1}^2$, so as to match the housing stocks. These imply residential land costs in large cities which are more than twice as high as those of smaller cities.

The average annual mobility rate (the % of the population that change address in a given year) is provided by the U.S. Census Bureau. Although more than 15% of the U.S.

[19] In our model, we abstract entirely from frictions in the rental market. In reality, vacancy rates for rental units are often higher than for owner-occupied units, so one may wonder whether this would affect the nature of our results. The key issue, however, is whether vacancies in the rental market are associated with costs to households of moving and therefore affect mobility. Vacancies in the rental market are more likely to be symptomatic of the fact that, once a rental unit is vacated, it may not immediately be available to the rental market. For example, maintenance and decorating may be needed before it is ready to be rented again.

[20] See http://www.census.gov/hhes/www/housing/hvs/hvs.html. Over this period home-owner vacany rates were quite stable, varying between 1.4 and 1.7%. However, since 2005 vacancy rates have risen as high as 2.8%. We do not include the last few years since they appear to be associated with the recent housing crisis.

population changes address each year, this includes people who move short distances within a county. For our purposes, a more appropriate estimate of mobility is that between labour markets. We therefore use as a target the component of the mobility rate associated with people who move between counties, which is 6.4%.[21] According to the U.S. Census Bureau, in 2000 the fraction of moving owners who moved between counties was about 43% with the remaining 57% moving locally.

We jointly choose the values of δ, μ, μ^*, λ and ψ so that the output of the model matches the target estimates of (1) the average monthly hiring rate from the unemployed workforce (Shimer, 2005), (2) the average unemployment rate, (3) the cross-county annual mobility rate of renters, (4) the cross-county annual mobility rate of owners and (5) the fraction of owners who move but remain within a county. We assume that the probability with which a household in any city who receives an outside offer from a city of a particular type is proportional to the share of total labour force located in cities of that type; thus $\beta_2 = 1 - \beta_1$. The population ratio of large to small cities is therefore equal to $\beta_2/\beta_1 = 2.54$.

Given the other parameter values, we set the bargaining parameter, σ, so that the ratio of the average price to the average income in the model matches that for the U.S. economy. Average income in the model includes both income from employment and unemployment in both cities. The average price is computed by weighting $\bar{P}_i$ by the the number of housing transactions in each city. To match the target requires that the buyer's share must equal approximately 38% of the surplus.

The net utility from ownership, $\pi^H - \pi^R$, is set so that the ratio of average rent to average income matches that for the U.S. economy. Note that, the income of the average renter in the U.S. is less than half of that of the average owner, reflecting the fact that the characteristics of owners and renters differ systematically. On average, a renter in the U.S. allocates 24% of his after-tax income to rent (see Davis and Ortalo-Magne, 2009). Since in our model all agents are homogeneous, we target the ratio of rent to the average income of owners and renters, which is somewhat lower at about 14% (see Appendix A). Finally, we set ε equal to one-half of the difference between π^H and π^R.[22]

[21] We investigate the robustness of our findings to lower overall mobility in our quantitative analysis below.

[22] Our results are largely insensitive to the exact value of ε, provided it is less that $\pi^H - \pi^R$. Throughout this range, the stationary equilibrium reflects the benchmark configuration and dissatisfied home-owners always buy new houses when they get the chance.

6 The Quantitative Relationship between Home Ownership and Unemployment

6.1 Baseline Calibration

We compute numerically the unique stationary equilibrium consistent with the benchmark configuration. Table 2 describes the distribution of the total population over locations, jobs and housing tenure in this equilibrium. High-wage cities have larger populations than low-wage ones, and this includes having proportionately more employed renters, employed owners and unemployed owners. Low-wage cities, in contrast, have proportionally more unemployed renters, reflecting the incentive of these households to live in cities with lower rent.

Table 2: Allocation of households by location, job and housing status

	Low-wage cities		High-wage cities		
	Renters	Owners	Renters	Owners	Total
Employed	0.084	0.179	0.222	0.458	0.943
Unemployed	0.008	0.011	0.010	0.028	0.057
Total	0.092	0.190	0.232	0.486	1.000

Table 3.1 contains statistics on mobility and unemployment for the U.S. economy and a series of artificial economies, beginning with the Baseline calibration (column 2). Recall that the mobilities of renters and owners in the U.S. economy are calibration targets. With regard to unemployment, the aggregate rate, 5.7%, is also a calibration target, but the breakdown across cities is endogenous. As in the data, low-wage cities have higher rates of unemployment than high-wage ones, although the model overstates the difference, due mainly to its prediction of a high rate of joblessness among renters in low-wage cities. The overall unemployment rate among owners is more than one percentage point higher than that for renters who are not new entrants to the labour force. This reflects the fact that 14% of unemployed owners turn down opportunities to relocate for employment reasons. The unemployment rate for *all* renters, however, is only slightly less than that for owners. Thus, although renters are much more mobile than owners, they are not unemployed at a significantly lower rate.

Table 3.1 – Mobility and Unemployment

	U.S. Data	Baseline	Owned -10%	Rental only	No m.i.d.	Increased wage diff.
Mobility rate	0.064	0.064	0.075	0.090	0.073	0.060
– of renters	0.120	0.120	0.099	0.090	0.110	0.103
– of owners	0.036	0.036	0.057	—	0.048	0.039
Population ratio	2.54	2.54	2.54	2.52	2.21	10.25
Unemployment rate	0.0570	0.0570	0.0536	0.0506	0.0549	0.0569
– in low-wage cities	0.0608	0.0679	0.0599	0.0548	0.0629	0.0843
– in high-wage cities	0.0566	0.0527	0.0512	0.0490	0.0512	0.0542
– for all renters	0.082	0.0545	0.0528	0.0506	0.0533	0.0545
– for non-new entrant renters	–	0.0477	0.0477	0.0485	0.0477	0.0477
– for owners	0.044	0.0582	0.0542	—	0.0558	0.0581
Rejection rate	–	0.1407	0.1064	—	0.1246	0.1136
– in low-wage cities	–	0.2305	0.1903	—	0.2072	0.2172
– in high wage cities	–	0.1055	0.0736	—	0.0875	0.1043

Table 3.2 – Housing market statistics

		U.S. Economy	Baseline	Owned -10%	Rental only	No m.i.d.	Increased wage diff.
	% ownable	68.0%	68.0	58.0	—	61.6	65.7
	Rent	0.125	0.086	0.111	0.086	0.111	0.035
Low	Price	2.54	2.15	2.77	—	2.13	0.813
wage	$\kappa_1 - \rho V_1$	-	0.0164	0.0164	—	0.0154	0.0503
cities	Months to sell	-	3.74	3.32	0.0	3.17	13.28
	Ownership rate	67.6%	67.4	57.2	0.0	60.9	63.3
	Vacancy Rate	1.78%	2.78%	3.08	—	2.71	9.96
	% ownable	68.0%	68.0	58.0	—	61.8	67.9
	Rent	0.153	0.178	0.202	0.177	0.203	0.225
	Price	3.55	4.50	5.11	—	3.93	5.68
High	$\kappa_2 - \rho V_2$	-	-0.0163	-0.0163	—	-0.0163	-0.0163
wage	Months to sell	-	1.42	1.41	—	1.35	1.13
cities	Ownership rate	67.7%	67.8	57.7	0.0	61.5	67.7
	Vacancy Rate	1.47%	1.06%	1.31%	0.0	1.16	0.85

The rate at which unemployed home-owners reject outside offers is much higher in low-wage cities than in high-wage ones. This largely reflects the fact that unemployed owners in low-wage cities do not accept offers in other low-wage cities, whereas those in high-wage cities do accept offers from other high-wage cities. Effectively, low house values impose a relatively high cost of moving on home-owners in low-wage cities. Recalling (59), however, the home-ownership effect on unemployment is small even in low-wage cities. In contrast, the rent differential effect is very large, as rent is more than twice as high in high-wage cities as in low-wage ones. Overall, this results in significantly higher unemployment in low-wage cities (two full percentage points). Thus, as observed by Coulson and Fisher (2009), the baseline calibration implies a negative relationship between unemployment and home-ownership across cities and a positive one between wages and home-ownership.

Table 3.2 contains selected housing market statistics for the data on the same cities and for the same computed examples as Table 3.1. The calibration targets the economy-wide averages of home-ownership rates, the ratio of house prices to annual income, and the ratio of rent to annual income. In the table rents and prices, both in the data and the model are normalized by the average income in low wage cities. The theory accounts reasonably well for the facts that house prices are higher and vacancies lower in high-wage cities than in low-wage ones. In both cases, however, the calibrated economy overstates somewhat the differences across cities. Qualitatively, the model is also consistent with the fact that the rents are higher in high wage cities. The difference, however, is much bigger in the model than in the data.

The model predicts significantly longer time on the market for houses in low-wage cities. This is the sense in which low-wage city homes are less liquid than high-wage ones, and is a driving force behind the employment and mobility results in Table 3.1. On average it takes about two months to sell a house in Baseline calibration. This is, in fact, close to the estimated time taken to sell a typical house provided by the National Association of Realtors. It should be noted, however, that there is considerable uncertainty surrounding this estimate. One reason is that houses may sometimes be strategically de-listed and quickly re-listed in order to reset the "days on market" field in the MLS listing. In their detailed analysis of the housing market in 34 Cook county (Illinois) suburbs in and around Chicago over the period 1992-2002, Levitt and Syverson (2008) compute time-to-sale by "summing across all of a house's listing periods that are separated by fewer than 180 days." They estimate that the average time on the market for a house that eventually sells is 94 days (3.07 months).[23]

We have assumed that the costs of conversion are sufficiently high that no conversion takes place in the stationary equilibrium. It is straightforward to determine the mini-

[23] In their sample of 127,000 houses, 22% of houses put up for sale never sell.

mum conversion costs necessary to support this. In low-wage cities, since $\kappa_1 - \rho V_1 > 0$, it is never profitable to convert rental to ownable housing. Provided $d^R > (\kappa_1 - \rho V_1)/\rho = 0.0164/0.04 = 0.410$, it is not profitable to convert ownable to rental housing, either. Conversely, in the high wage city, we require only that $d^H > 0.408$. That is, one-time conversion costs (d^R, d^H) both equal to half the average monthly wage are sufficient.

Another possibility is for REMs to put rented houses up for sale, and convert them to owner-occupied houses only once they have matched with a buyer.[24] The flow value of renting out a house which is currently for sale in a type i city is $\rho\tilde{V}_i = \kappa_i + \gamma_i(\bar{P}_i - d^H - \tilde{V}_i)$. It follows that

$$\tilde{V}_i = \frac{\kappa_i + \gamma_i(\bar{P}_i - d^H)}{\rho + \gamma_i} \tag{66}$$

In a stationary equilibrium, the value of an *unrented* vacant house satisfies

$$V_i = \frac{\gamma_i \bar{P}_i}{\rho + \gamma_i} \tag{67}$$

It follows that the REM will not rent temporarily as long as $V_i > \tilde{V}_i$ or if $d^H > \kappa_i/\gamma_i$. In our baseline example, in low-wage cities where such an action would be most profitable, a conversion cost approximately one-third of the average monthly wage is sufficient to prevent REM's from putting rented houses up for sale.[25]

Overall, in the baseline calibration, our economy is broadly consistent with the cross-city evidence on unemployment, mobility, house prices, rental rates, and vacancy rates. Houses are significantly less liquid in low-wage cities and this is reflected in both house prices and the frequency with which home-owners turn down offers of employment in other cities. The fact that the model cannot reproduce exactly the quantitative differences in housing market statistics across cities is not surprising. The *only* sources of cross-city heterogeneity are the wage and the housing stocks. In principle, there could be other sources of heterogeneity across city groups (e.g. ameneties, worker flows). However, we do not have direct observations on these factors by MSA. Another likely source of divergence between the cross-city difference in the model and the data is the linearity of preferences. Although, this allows for a tractable analysis of equilibrium with search in two markets, it imposes strong requirements on the relationship between wages, rent and price differentials across cities.

We now conduct a series of experiments in order to examine the relationships between housing and both mobility and unemployment. In particular, we are interested in whether

[24] We exclude the possibility of the REM selling the house immediately to the current renter.

[25] In the benchmark case, $\kappa_1 = 0.086$, $\gamma_1 = 0.267$, $\kappa_2 = 0.178$ and $\gamma_2 = 0.704$.

and by how much home-ownership affects mobility and unemployment, both in the aggregate and city by city. The results of these experiments are reported in columns three to six of Tables 3.1 and 3.2.

6.1.1 A reduction in the (fixed) supply of owner occupied housing

We first consider an exogenous reduction in the stock of owner-occupied housing in all cities by ten percentage points.[26] We may think of this as being accomplished by changing the costs of construction for owner occupied and rental housing so as to support an equilibrium with the same quantity of housing overall as in the baseline calibration, but with ten percent less of it being owner-occupied. Overall, this change in the environment lowers home-ownership rates and increases the matching rates for houses in all cities in the stationary equilibrium. Houses become more liquid everywhere, time on the market falls, and both house prices and rents rise in all cities. The increase in the liquidity of housing results in substantially increased mobility overall (from 6.4% to just under 7.5%), which is driven by a significant reduction in the rejection rates of offers by home-owners. Indeed, mobility *falls* for renters.

In spite of the significant increase in mobility, a reduction in the stock of owner-occupied housing (and specifically the associated lower rate of home-ownership) has only a small effect on aggregate unemployment, which falls by only one-third of one percentage point. This effect is roughly one-quarter the size of that estimated by Nickell (1998) and even smaller relative to the estimates of Oswald (2009). Unemployment falls by more (eight-tenths of a percentage point) in low-wage cities, owing to a relatively large increase in the mobility of home-owners in these cities. Both home-owners and renters experience unemployment at lower rates, but while for the former this results from an reduced rate of rejection of job offers, for the latter it is due only to a composition effect: A smaller fraction of renters are now new entrants to the economy who are by construction unemployed at a high rate.

6.1.2 Complete elimination of owner occupied housing

We now consider the effect of eliminating owner-occupied housing entirely (calculations associated with this modification to the environment are straightforward but lengthy, and so are relegated to Appendix C). In this experiment all households rent competitively in their city of residence. To make sense of this, suppose that developers build only rental units (facing the same unit costs as in the benchmark). As before, we focus on an interior equilibrium

[26] A ten percentage point increase in the stock of owner-occupied housing has essentially symmetric effects in the opposite directions.

in which the unemployed continue to be indifferent between locations: $U_1^R = U_2^R = U^R$.[27] In this case, the unemployed accept offers of employment from all other cities, whereas only the employed in low-wage cities move and only in response to offers from high-wage cities.

Eliminating owner-occupied housing in this way effectively eliminates the trading friction in the housing markets[28]. Thus, we may think of this experiment as a consideration of the case in which housing is perfectly liquid. The fourth column of Table 3.1 contains statistics on mobility and unemployment for this case and the (very sparse) fourth column of Table 3.2 contains the only relevant housing market statistic for this economy, the rent-income ratio in each city.

The elimination of home-ownership has no significant effect on the distribution of the population across city-types, as the rent differential adjusts to equate the values of being unemployed in either city. Although wages are nearly 10% higher in large cities, this is offset by the fact that in our calibrated economy unit land costs are twice as high. Since moving is costless, households never turn down job offers from a high-wage city and mobility is maximized, rising from 6.4% to 9% overall. Unemployment falls relative to the baseline in all cities and falls in the aggregate by roughly two-thirds of a percentage point.

As in the case of a partial reduction in home-ownership, unemployment falls by more in low-wage cities, reducing the unemployment gap across city types. Again, this happens as low-wage city home-owners are significantly less mobile than their high-wage city counterparts in the base case. Moreover, the increased mobility of low-wage city residents more than offsets the incentive of unemployed households to live in the low-wage cities in order to take advantage of lower rent.

6.1.3 Elimination of Mortgage Interest Deductibility

As noted above, home-ownership *per se* contributes to aggregate unemployment, although in our calibration the effect is relatively small. We now consider the elimination of mortgage interest deductibility (m.i.d.). Effectively, such a change to the tax code reduces the return to home-ownership and thus affects the willingness both of households and REM's to purchase and to construct houses, respectively. In this experiment, we allow the housing stocks to adjust in response. Thus, this experiment can be expected to result in an endogenous

[27] For the parameter choices described above, this is the nature of the equilibrium that pertains. For other, more extreme parameter values, it is possible to have non-interior corner equilibria with all the unemployed in one city.

[28] There are only two differences: the utility of housing (i.e. π^H versus π^R) and the unit cost of production (i.e. C_i^H versus C_i^R).

reduction of the ownership rate, whereas in the previous experiments, home-ownership was reduced exogenously.

The fourth columns of Tables 3.1 and 3.2 contain labour and housing market statistics for this experiment. The elimination of m.i.d. results in a reduction of home-ownership by approximately 6.4 percentage points. Rent rises and house prices fall in all cities. While the housing markets in all cities become smaller, the effect on time on the market differs across cities, with houses becoming slightly more and less liquid in low- and high-wage cities, respectively. Mobility rises overall, both because of a reduction in the rejection rates of offers and because of a drop in the ownership rate. Unemployment falls by roughly two-tenths of one percentage point in the aggregate. Again, the reduction in unemployment is strongest in low-wage cities, and again this is because the effect on mobility through an increase in the liquidity of housing is strongest there.

6.1.4 An increase in the city wage differential

We now consider an increase in the wage differential across cities from 9.7% to 20%, holding fixed construction costs so that, again, the housing stocks adjust. This results in a major movement of households to high-wage cities, and affects housing markets significantly as well. Houses become much less liquid in low-wage cities, and their prices fall dramatically. Similarly, rents fall significantly as population shifts to high-wage cities. Aggregate mobility is reduced (albeit only slightly) in spite of the fact that job offers are rejected less frequently, because of the higher population concentration in high-wage cities. With regard to labour market outcomes, aggregate unemployment remains essentially unchanged relative to the baseline. This masks, however, significant changes at the city level. A dramatic increase in the share of the population residing in high-wage cities, however, coupled with an increase of unemployment exceeding two percentage points in low-wage cities results in a greatly increased unemployment differential accross cities.

6.2 A High Unemployment Calibration

Our baseline calibration targets an unemployment rate of 5.7%, which was the average U.S. rate between 1950 and 2005. Long-run unemployment rates in many continental European countries have tended to be significantly higher and in recent years the U.S. has experienced similarly high rates. Recently, Elsby, Hobijn and Sahin (2011) have documented that continental European labour markets with high rates of unemployment tend to have substantially lower inflows to and outflows from unemployment than Anglo-Saxon ones. In this section we therefore consider an alternative calibration of the model with aggregate unemployment of 10% and a monthly hiring rate of 0.1. We leave all other calibration targets unchanged relative to the baseline. To attain the higher unemployment rate, the job separation rate, δ, is reduced to 0.0065 and several other parameters must be adjusted to continue to match other targets.[29] Thus, higher unemployment in this calibration is attributed to a more sclerotic labour market, in line with what might be considered reasonable for a typical continental European economy.

Table 4 contains selected statistics for the "High Unemployment" calibration (column one) and for two experiments, reducing the stock of owner-occupied housing by 10% (column two) and eliminating mortgage interest deductability (column three). Overall, the results of these experiments are qualitatively similar in the High Unemployment case to what they are under the Baseline calibration. Quantitatively, however, there are both similarities and significant differences. . Rents, prices and vacancy rates across cities are in the same ballpark. The the unemployment gap across cities, however, is substantially magnified. Moreover, the difference between the unemployment rates of owners and renters is now much larger.

The effect of home-ownership on aggregate unemployment is now stronger, with an exogenous ten percentage point reduction in the stock of owner–occupied housing resulting in a reduction of aggregate unemployment by 1.2 percentage points (as opposed to one-third of a percentage point in the Baseline calibration). This effect is almost as strong as that estimated by Nickell (1998) based on cross country regressions, and is arguably economically significant. When a reduction in home-ownership of similar magnitude is induced by removing mortgage interest deductibility (rather than imposed exogenously) aggregate unemployment is reduced by two-thirds of a percentage point. This again contrasts with the Baseline calibration, in which the analogous experiment reduces unemployment by only two-tenths of a percentage point.

Comparing the High Unemployment and Baseline calibration illustrates that the extent to which housing market frictions affect unemployment, both across locations and in the aggregate, depends on the level of aggregate unemployment. This suggests that for economies with high unemployment relative to that of the U.S. (*e.g.* many European economies) home-ownership may indeed be a significant factor in generating unemployment. A conclusion along these lines is somewhat suspect, however, because European countries with high levels of unemployment, tend to have mobility rates much lower than observed for the U.S.[30] For

[29]The new parameter values are: $\mu^* = 0.0104$, $\lambda = 0.0049$, $\psi = 0.043$, $\pi^H - \pi^R = 0.04$ and $\sigma = 0.711$.

[30]Mobility rates have also declined in the U.S. during the last few years.

example, Rupert and Wasmer (2009) document average cross–regional mobility rates in Europe of approximately 2%. When we adjust the parameters of our High Unemployment case so that mobility is 2% (holding all other targets the same), the effect of a ten percentage point reduction in the stock of owner-occupied housing on aggregate unemployment is cut in half, to 0.6 percentage points as opposed to 1.2.

Table 4: High Unemployment Calibration

		Base	Owned -10%	Zero m.i.d.
Mobility rate		0.064	0.074	0.071
– of renters		0.120	0.109	0.115
– of owners		0.036	0.048	0.041
Unemployment		0.1000	0.0881	0.0929
– low-wage		0.1613	0.1331	0.1404
– high-wage		0.0759	0.0704	0.0716
– all renters		0.0793	0.0727	0.0750
– non entrants		0.0528	0.0528	0.0529
– owners		0.1099	0.0995	0.1041
Rent	– low-wage	0.096	0.12	0.121
	– high wage	0.180	0.21	0.204
Price	– low-wage	2.18	2.85	2.16
	– high wage	4.58	5.24	3.99
Months to sell	– low-wage	4.68	4.27	4.04
	– high wage	1.77	1.83	1.74
Vacancy rate	– low-wage	2.78	3.06	2.67
	– high wage	1.06	1.32	1.17

6.3 Robustness

We now consider briefly the robustness of our quantitative results to two more deviations from the baseline calibration. For each of the alternative calibrations we consider here, stationary equilibria continue to conform to the benchmark configuration. Moreover, since these variations in parameters have little effect on most of the statistics presented in Tables 3.1 and 3.2 above, here we report results (Table 5) only for the effects of parameter changes on mobility and unemployment (by city type and in the aggregate).[31]

[31] Full results are contained in Appendix C.

Table 5: Robustness

	High wage differential			Low mobility		
	Base	Owned -10%	Zero m.i.d.	Base	Owned -10%	Zero m.i.d.
Mobility rate	0.064	0.075	0.076	0.050	0.0591	0.0571
Unemployment	0.0570	0.0537	0.0547	0.0570	0.0543	0.0553
– low-wage	0.0680	0.0600	0.0630	0.0655	0.0589	0.0614
– high-wage	0.0527	0.0512	0.0506	0.0537	0.0525	0.0525

6.3.1 High wage differential

In both the Baseline and High Unemployment calibrations, the cross-city type wage differential is set at 9.7% to capture differences in cost of living between our two categories of MSA's, even though the unconditional wage difference between these city groups is significantly higher. Above, we considered the implications of increasing the wage differential to 20%, holding all other *parameters* fixed at their levels in the Baseline calibration. We now consider an alternative calibration with the wage differential at 15% rather than 9.7%, and other parameters adjusted to maintain the *targets* of the Baseline calibration.[32]

The resulting economy is for the most part similar to that considered in the previous high-wage differential experiment. Moreover, with regard to the two experiments conducted, the economy responds similarly to the Baseline calibration. Thus, we conclude that our quantitative results are robust to changes in the wage differential across city types that continue to support a stationary equilibrium under the benchmark configuration.

6.3.2 Low mobility

In the Baseline calibration, the target for average mobility is the frequency of moves across county lines. We suspect that this measure overstates to some extent the frequency of moves associated with changes in employment. Because many MSA's are comprised of multiple counties, a certain number of these cross-county moves are of fairly short distance and thus may not be associated with a change of job. For this reason, we consider the implications of lowering average mobility from 6.4% to 5%, maintaining all other targets of the Baseline calibration. Again we compute a baseline version and conduct two experiments. Overall,

[32] We could recalibrate to a wage differential of 20%. However, in this case, reducing the ownable housing stock by 10% results in a violation of one of the conditions necessary for the equilibrium configuration on which we are focussing.

our quantitative results are robust to variations in mobility of this magnitude. A reduction in mobility has little effect on unemployment in either type of city or in the aggregate.

6.4 Summary

Overall, we find that due to its illiquidity, owner–occupied housing significantly affects mobility in response to changes in employment status and opportunities. Moreover, the illiquidity of housing results in significant differences in house prices and rents across cities and may generate significant differences in unemployment rates as well. Nevertheless, home-ownership and the illiquidity of housing typically has a fairly small effect on aggregate unemployment. In our Baseline calibration, a ten percentage point reduction of home-ownership reduces unemployment by only one third of a percentage point, less than one quarter of that estimated by Nickell (1998) and Oswald (2009). Moreover, a total elimination of home-ownership would only double this effect. Eliminating mortgage interest deductability would encourage mobility and reduce home-ownership but again the effect on unemployment, would be small (two-tenths of a percentage point in our experiment).

There are two main reasons why the effect of home-ownership on aggregate unemployment is small in our calibrated economies despite the large difference between owners and renters with regard to mobility:

1. In order to match inter-city mobility, the rate at which households receive offers from other cities, μ^*, is small relative to the rate at which they receive offers from their own city, μ. Thus, only a small fraction of the overall flow out of unemployment is associated with households moving between cities. Consequently, the friction associated with the time taken to sell does not have much affect on overall unemployment. If we were to increase μ^* and hold mobility constant by reducing the matching parameter λ, the effect of ownership on unemployment would rise. This, however, would imply a both a longer average time to sell and a lower average house price than we observe.

2. The illiquidity of housing affects unemployment through its influence on the decisions of the unemployed, who make up only 5.7% of households in the Baseline. As average unemployment rises, the effect of home-ownership rises in part simply because home-ownership affects the mobility decisions of a larger share of the population.

In a version of the economy with high rates of *both* unemployment and mobility, the affect of ownership on aggregate unemployment can be large (as illustrated by our High

Unemployment calibration). Such a configuration of parameters appears, however, to be counter-factual: countries which have high rates of mobility tend to have low average unemployment (e.g. the U.S.), and countries with low rates of mobility tend to have relatively high rates of average unemployment (e.g. continental Europe).

7 Conclusion

We have developed a multi-city model that allows for interactions between search frictions in both housing and labour markets. A house's liquidity—the time it would take to sell it to an appropriate buyer—determines its value in the event that its owner would like to sell it so that he/she can move to a different city. In equilibrium, willingness to move affects cities' populations and rates of both home-ownership and unemployment. These, in turn, determine vacancy rates and, hence, the liquidity of housing in each city.

In equilibrium, home-owners turn down job offers in certain circumstances, even if they are currently unemployed or are offered a higher wage than their current one, because the illiquidity of their house renders migration not worthwhile. The likelihood of unemployment for home-owners exceeds that for otherwise identical renters. Nevertheless, unemployment is negatively related to ownership rates across cities because unemployed renters tend to live disproportionately in low rent (low wage) cities, where home-ownership is also lower.

A version of the model calibrated to match U.S. labour market flows and mobility rates for both home-owners and renters, generates plausible cross city home-ownership rates and qualitatively reasonable cross-city differences in unemployment, house prices, rents, and time-on-the market. We find, however, that the impact of home-ownership on aggregate unemployment is small. Moreover, this quantitative finding is robust to variations in economy parameters in several dimensions. If parameters are such that both unemployment and mobility are high, the effects of ownership on aggregate unemployment can be economically significant. However, such a combination appears to be counter-factual.

Given that the only sources of cross-city heterogeneity are wages and housing stocks, we find that the model performs reasonably well in matching qualitative cross–city differences in price, rents, unemployment rates, vacancy rates and ownership rates. However, its quantitative success in matching these differences is limited for two main reasons. First, the assumption of risk-neutrality on the part of households imposes strong restrictions on the relationships between wage, rent and price differentials. Secondly, worker flow rates and other parameters are assumed to be identical across cities. Weakening these assumptions might allow for a better fit of the model. However, moving away from linear preferences in a spatial search equilibrium would significantly complicate the analysis because household decisions would depend on their current wealth. Moreover, we have not been able to find the data necessary to compute city-specific worker flows. These remain challenges for future research.

8 Appendix A: Data Sources

All population, income and housing data are taken from the U.S. Decennial Census 2000 Summary File 3 (see http://factfinder.Census.gov/). The universe of cities are the 279 Metropolitan Statistical Areas minus the three Puerto Rican MSA's, Anchorage and Honolulu.

• City populations are taken from Table P1.

• For income in each city we use median household income (in dollars) from Table P53. Household income for cities of type i is then computed as the population-weighted average of these incomes.

• For house prices we use the median value (in dollars) for specified owner–occupied houses from Table H76. This category includes only one-family houses on less than ten acres without a business or medical office on the property. The house price for cities of type i is then the average of these prices weighted by the owned-housing stock.

• To compute rents for each city we start with median contract rents (in dollars) from Table H56. Contract rent is the monthly rent agreed to or contracted for and may or may not include utilities. To correct for the inclusion of utilities, we compute the fraction, α_j, of rents that include utilities in city j computed from Table H68. From NIPA Table 2.4.5, the ratio of total expenditures on household utilities (line 55) in the U.S. to total expenditures on rent (line 50) is 0.43. We therefore compute rent in city j, not including utilities, according to

$$\text{Rent}_j = \frac{(\text{Contract Rent})_j}{1 + 0.43\alpha_j}.$$

The rent for cities of type i is then computed as the average of these rents weighted by the rental stock.

• Effective ownable housing stocks for each city are computed as the sum of owner-occupied units (Table H7) and vacant units that are "for sale only" (Table H8).

• Effective rental housing stocks are assumed to equal the occupied rental units from Table H7.
• Home-owner vacancy rates for each city are computed by dividing the number of vacant units "for sale only" (Table H8) by the effective ownable housing stocks.

Average mobility rates by tenure are from the Current Population Survey and are provided on-line by the Census Bureau (http://www.census.gov/population/www/socdemo/migrate.html). To be consistent with our housing data, we have used the rates from 1999-2000 CPS. These are close to the average rates over the whole period that data are available, 1987-2009. There does, however, appear to be a slight downward trend in mobility rates overall.

Unemployment rates by MSA are from the 2000 Census. Unemployment rates of owners and renters are from Bernstein (2009) and are based on the CPS between 2005 and 2008. We adjust them using the fractions of households that are owners and renters, so that the implied weighted–average unemployment rate is 5.7%, but the difference between them is maintained.

To convert income, rents and prices into real terms, we deflate using the non-shelter Cost of Living Index for 2000 produced by the American Chamber of Commerce Research Association. Unfortunately, it is only possible to obtain this index for 222 of the MSAs. In particular, although we were able to obtain it for the 49 large cities, some of the smaller cities are missing. For the small city group, we compute real values in two ways. First we compute weighted averages of real values using only the cities for which we have COLIs. Second, we compute a nominal weighted average for all the cities and divide by a weighted average of the COLIs for the cities which had them. Both methods yield similar real values for all three series. Note also that whether we use the full sample of 274 cities or the smaller sample of 222 cities, makes no difference for the average housing stocks and vacancy rates.

9 Appendix B: Proofs and Extended Derivations

Proof of Lemma 1: Equation (18) implies that $W_i^H - W_i^R > V_i$. Equations (19), (20) and the fact that $W_i^R > U^R$ imply $U_i^H - V_i > U^R$.■

Proof of Lemma 2: Given the Nash bargaining assumption, the price paid by employed households is given by

$$P_i^W = (1-\sigma)\left(W_i^H - W_i^R\right) + \sigma V_i \tag{68}$$

and that paid by unemployed households is

$$P_i^U = (1-\sigma)\left(U_i^H - U_i^R\right) + \sigma V_i. \tag{69}$$

Substituting these values into (3) yields

$$\rho V_i = \gamma_i(1-\sigma)\left[\alpha_i(W_i^H - W_i^R - V_i) + (1-\alpha_i)(U_i^H - U^R - V_i)\right]. \tag{70}$$

Re-arranging yields (22).■

Proof of Lemma 3: First observe that subtracting (25) from (23) and (26) from (24) we get

$$W_1^H - U_1^H = \frac{w_1 - z}{\rho + \delta + \mu} \tag{71}$$

$$W_2^H - U_2^H = \frac{w_2 - z}{\rho + \delta + \mu} - \frac{\beta_2\mu^*\left(W_2^R - W_1^R\right)}{\rho + \delta + \mu}. \tag{72}$$

Subtracting (28) from (27) and using (68) and (69) yields

$$(\rho + \delta + \mu + \mu^* + \sigma\lambda)\left(W_1^R - U^R\right) = w_1 - z + \lambda\sigma\left(W_1^H - U_1^H\right). \tag{73}$$

Substituting for $W_1^H - U_1^H$ using (71) and re-arranging yields (31). Similarly, subtracting (30) from (29) and using (68) and (69) yields

$$\begin{aligned}(\rho + \delta + \mu + \beta_2\mu^* + \sigma\lambda)\left(W_2^R - U^R\right) &= w_2 - z + \sigma\lambda\left(W_2^H - U_2^H\right) \\ &\quad - (1-\beta_2)\,\mu^*\left(W_1^R - U^R\right).\end{aligned} \tag{74}$$

Substituting for $W_2^H - U_2^H$ using (72) and re-arranging yields (32).■

Proof of Proposition 1: Subtracting (31) from (32) we get

$$\Gamma_2 - \Gamma_1 = \left(\frac{w_2 - z}{\rho + \delta + \mu + \beta_2\mu^*}\right) - \left[\frac{(\rho + \delta + \mu)(\rho + \delta + \mu + \mu^* + \sigma\lambda)}{(\rho + \delta + \mu + \beta_2\mu^*)(\rho + \delta + \mu + \sigma\lambda)}\right]\Gamma_1. \tag{75}$$

Substituting for Γ_1 using (31) and cancelling terms yields (33).■

Proof of Proposition 2: Using (34) and (35) we have

$$\rho\left(V_2 - V_1\right) - \left(\kappa_2 - \kappa_1\right) = \left(\rho + \left(1-\beta_1\right)\mu^* + \sigma\lambda\right)\left(\frac{w_2 - w_1}{\rho + \delta + \mu + \beta_2\mu^*}\right) > 0.■$$

Derivation 1 — The equilibrium allocation of workers: The solution to the ten equation system described by (45) – (52) can be expressed recursively as

$$\begin{aligned}
n_2^{WH}\left(\phi_1,\phi_2\right) &= \frac{\Psi_2}{b_{22}^W - b_{22}^U} && (76)\\
n_1^{WH}\left(\phi_1,\phi_2\right) &= \frac{\Psi_1}{b_{11}^W - b_{11}^U} - \left(\frac{b_{12}^W - b_{12}^U}{b_{11}^W - b_{11}^U}\right) n_2^{WH}\left(\phi_1,\phi_2\right) && (77)\\
n_1^{UH}\left(\phi_1,\phi_2\right) &= h_1 - \frac{r_1}{\phi_1} - n_1^{WH}\left(\phi_1,\phi_2\right) && (78)\\
n_2^{UH}\left(\phi_1,\phi_2\right) &= h_2 - \frac{r_2}{\phi_2} - n_2^{WH}\left(\phi_1,\phi_2\right) && (79)\\
n_1^{WR}\left(\phi_1,\phi_2\right) &= \left(\frac{\delta+\mu+\nu}{\lambda}\right) n_1^{WH}\left(\phi_1,\phi_2\right) - \frac{\mu}{\lambda}\left(h_1 - \frac{r_1}{\phi_1}\right) && (80)\\
n_2^{WR}\left(\phi_1,\phi_2\right) &= \left(\frac{\delta+\mu+\nu}{\lambda}\right) n_2^{WH}\left(\phi_1,\phi_2\right) - \frac{\mu}{\lambda}\left(h_2 - \frac{r_2}{\phi_2}\right) && (81)\\
n_1^{UR}\left(\phi_1,\phi_2\right) &= r_1 - n_1^{WR}\left(\phi_1,\phi_2\right) && (82)\\
n_2^{UR}\left(\phi_1,\phi_2\right) &= r_2 - n_2^{WR}\left(\phi_1,\phi_2\right) && (83)\\
\theta_1\left(\phi_1,\phi_2\right) &= \frac{\lambda r_1 - \nu\left(n_1^{UH}\left(\phi_1,\phi_2\right) + n_1^{WH}\left(\phi_1,\phi_2\right)\right)}{\left(1-\beta_1\right)\mu^* n_1^{UH}\left(\phi_1,\phi_2\right)} && (84)\\
\theta_2\left(\phi_1,\phi_2\right) &= \frac{\lambda r_2 - \left(\beta_2\mu^* + \nu\right) n_2^{UH}\left(\phi_1,\phi_2\right) - \nu n_2^{WH}\left(\phi_1,\phi_2\right)}{\left(1-\beta_2\right)\mu^* n_2^{UH}\left(\phi_1,\phi_2\right)} && (85)
\end{aligned}$$

where

$$\begin{aligned}
\Psi_1 &= \left(\mu + \beta_1\mu^*\right) r_1 + \frac{s_2}{s_1}\left((1-\beta_2)\mu^* + \lambda\right) r_2 - b_{11}^U\left(h_1 - \frac{r_1}{\phi_1}\right) - b_{12}^U\left(h_2 - \frac{r_2}{\phi_2}\right) && (86)\\
\Psi_2 &= \left(\mu + \beta_2\mu^*\right) r_2 + \frac{s_1}{s_2}\left((1-\beta_1)\mu^* + \lambda\right) r_1 - b_{21}^U\left(h_1 - \frac{r_1}{\phi_1}\right) - b_{22}^U\left(h_2 - \frac{r_2}{\phi_2}\right) && (87)
\end{aligned}$$

and

$$\begin{aligned}
b_{11}^{W} &= (\delta+\mu+\mu^{*}+\lambda+\nu)\left(\frac{\delta+\nu}{\lambda}\right) && (88)\\
b_{11}^{U} &= -(\delta+\mu+\mu^{*}+\lambda+\nu)\frac{\mu}{\lambda} && (89)\\
b_{12}^{W} &= \frac{s_2}{s_1}\left[(1-\beta_2)\mu^{*}\left(\frac{\delta+\nu}{\lambda}\right)+\nu\right] && (90)\\
b_{12}^{U} &= \frac{s_2}{s_1}\left[(\beta_2\mu^{*}+\nu)-(1-\beta_2)\mu^{*}\frac{\mu}{\lambda}\right] && (91)\\
b_{21}^{W} &= b_{21}^{U}=\frac{s_1}{s_2}\nu && (92)\\
b_{22}^{W} &= (\delta+\mu+\beta_2\mu^{*}+\lambda+\nu)\left(\frac{\delta+\nu}{\lambda}\right) && (93)\\
b_{22}^{U} &= -\left(\delta+\mu+\beta_2\mu^{*}+\lambda+\nu\right)\frac{\mu}{\lambda}. && (94)
\end{aligned}$$

From the above we can obtain the following derivatives:

$$\frac{dn_2^{WH}}{d\phi_1}<0,\ \frac{dn_2^{WH}}{d\phi_2}>0,\ \frac{dn_2^{WR}}{d\phi_1}<0,\ \frac{dn_2^{WR}}{d\phi_2}=0. \qquad (95)$$

Also, if $\beta_2 < \frac{\delta+\mu+\nu}{\delta+\mu+\nu+\lambda}$ then

$$\frac{dn_1^{WH}}{d\phi_1}>0,\ \frac{dn_1^{WH}}{d\phi_2}<0,\ \frac{dn_1^{WR}}{d\phi_1}>0,\ \frac{dn_1^{WR}}{d\phi_2}<0. \qquad (96)$$

Derivation 2 — The VV Curve: The house prices in low-wage cities must satisfy

$$\left(\rho+(1-\sigma)\gamma_1\right)V_1=(1-\sigma)\gamma_1\left[\alpha_1\left(W_1^H-W_1^R\right)+(1-\alpha_1)\left(U_1^H-U^R\right)\right]. \qquad (97)$$

Using (19) to substitute out V_1 and rearranging we get

$$\rho U^R=\rho U_1^H-(1-\sigma)\gamma_1\alpha_1\left(W_1^H-U_1^H\right)+(1-\sigma)\gamma_1\alpha_1\Gamma_1-\left(\rho+(1-\sigma)\gamma_1\right)\Gamma_2. \qquad (98)$$

Substituting using (25) and (71) yields

$$\begin{aligned}
\rho U_1^R(\phi_1,\phi_2) &= z+\pi^H+(\mu-(1-\sigma)\gamma_1\alpha_1)\left(\frac{w_1-z}{\rho+\delta+\mu}\right)\\
&\quad+(1-\sigma)\gamma_1\alpha_1\Gamma_1-\left(\rho+(1-\sigma)\gamma_1\right)\Gamma_2. && (99)
\end{aligned}$$

Similarly, for high wage cities we have

$$\begin{aligned}\rho U_2^R(\phi_1, \phi_2) &= z + \pi^H + (\mu - (1-\sigma)\gamma_2\alpha_2)\left(\frac{w_2 - z}{\rho + \delta + \mu}\right) + (1-\sigma)\gamma_2\alpha_2\Gamma_2 \\ &\quad - (\rho + (1-\sigma)\gamma_2)\,\Gamma_1 + \left(\frac{\rho + \delta + (1-\sigma)\gamma_2\alpha_2}{\rho + \delta + \mu}\right)\beta_2\mu^*\left(\Gamma_2 - \Gamma_1\right). \quad (100)\end{aligned}$$

Equating $U_1^R(\phi_1, \phi_2) = U_2^R(\phi_1, \phi_2)$ yields the VV curve.

Proof of Proposition 3: Using (31) we can re-write (99) as

$$\begin{aligned}\rho U_1^R(\phi_1, \phi_2) &= z + \pi^H + \mu\left(\frac{w_1 - z}{\rho + \delta + \mu}\right) - (\rho + (1-\sigma)\lambda\phi_1)\,\Gamma_2 \\ &\quad -(1-\sigma)\lambda\phi_1\alpha_1\left(\phi_1, \phi_2\right)\left(\frac{\mu^*}{\rho + \delta + \mu + \mu^* + \sigma\lambda}\right)\left(\frac{w_1 - z}{\rho + \delta + \mu}\right). \quad (101)\end{aligned}$$

From (96) we have $\frac{d\alpha_1}{d\phi_1} = \frac{1}{r_1}\frac{dn_1^{WR}}{d\phi_1} > 0$ and $\frac{d\alpha_1}{d\phi_2} = \frac{1}{r_1}\frac{dn_1^{WR}}{d\phi_2} < 0$. It is immediate that $\rho U_1^R(\phi_1, \phi_2)$ is decreasing in ϕ_1 and increasing in ϕ_2. Substituting out Γ_2 from (100) using (32) and re-arranging yields

$$\begin{aligned}\rho U_2^R(\phi_1, \phi_2) &= z + \pi^H + \mu\left(\frac{w_2 - z}{\rho + \delta + \mu}\right) - \frac{(1-\sigma)\lambda\phi_2\alpha_2(\phi_1)\mu^*\Gamma_1}{\rho + \delta + \mu + \sigma\lambda} \\ &\quad - (\rho + (1-\sigma)\lambda\phi_2)\,\Gamma_1 + \left(\frac{\rho + \delta}{\rho + \delta + \mu}\right)\beta_2\mu^*\left(\Gamma_2 - \Gamma_1\right). \quad (102)\end{aligned}$$

Using (95) it is apparent that $\frac{d\alpha_2}{d\phi_1} = \frac{1}{r_2}\frac{dn_2^{WR}}{d\phi_1} < 0$. It follows that $\rho U_2^R(\phi_1, \phi_2)$ is increasing in ϕ_1 and decreasing in ϕ_2. The VV curve is defined by $U_1^R(\phi_1, \phi_2) = U_2^R(\phi_1, \phi_2)$. Totally differentiating yields

$$\left.\frac{d\phi_2}{d\phi_1}\right|_{VV} = \frac{\frac{\partial U_2^R}{\partial \phi_1} - \frac{\partial U_1^R}{\partial \phi_1}}{\frac{\partial U_1^R}{\partial \phi_2} - \frac{\partial U_2^R}{\partial \phi_2}} > 0.\blacksquare \quad (103)$$

Proof of Lemma 4: Using (78) and (84) $\theta_1 < 1$ implies that (ϕ_1, ϕ_2) pairs satisfy

$$\lambda r_1 - \nu\left(h_1 - \frac{r_1}{\phi_1}\right) < (1 - \beta_1)\,\mu^*\left(h_1 - \frac{r_1}{\phi_1} - n_1^{WH}\left(\phi_1, \phi_2\right)\right). \quad (104)$$

Similarly, using (79) and (85) $\theta_2 < 1$ implies that (ϕ_1, ϕ_2) pairs satisfy

$$\lambda r_2 - (\beta_2\mu^* + \nu)\,n_2^{UH}\left(\phi_1, \phi_2\right) - \nu n_2^{WH}\left(\phi_1, \phi_2\right) < (1 - \beta_2)\,\mu^* n_2^{UH}\left(\phi_1, \phi_2\right). \quad (105)$$

The boundaries associated with conditions (104) and (105) can be expressed as

$$\lambda s_1 r_1 - \nu\left(s_1 h_1 - \frac{s_1 r_1}{\phi_1}\right) = (1-\beta_1)\,\mu^*\left(s_1 h_1 - \frac{s_1 r_1}{\phi_1} - s_1 n_1^{WH}(\phi_1,\phi_2)\right) \tag{106}$$

$$\lambda s_2 r_2 - \nu\left(s_2 h_2 - \frac{s_2 r_2}{\phi_2}\right) = \mu^*\left(s_2 h_2 - \frac{s_2 r_2}{\phi_2} - s_2 n_2^{WH}(\phi_1,\phi_2)\right), \tag{107}$$

respectively. It is straightforward to show that these conditions are linear in $s_1 h_1 - \frac{s_1 r_1}{\phi_1}$ and $s_2 h_2 - \frac{s_2 r_2}{\phi_2}$, which are themselves monotone functions of ϕ_1 and ϕ_2, respectively. The AM curve can be written as

$$s_2 h_2 - \frac{s_2 r_2}{\phi_2} = 1 - s_1 r_1 - s_2 r_2 - \left(s_1 h_1 - \frac{s_1 r_1}{\phi_1}\right), \tag{108}$$

which is also linear in these terms. It follows that the intersection points (ϕ_1^X, ϕ_2^X) and (ϕ_1^Y, ϕ_2^Y), corresponding to X and Y in Figure 1, must be unique.■

Proof of Lemma 5: Fix the stocks of each type of housing in each city. First observe that since AM is downward sloping and VV is upward sloping, if an equilibrium under the benchmark configuration exists, it must be unique. Note first that the benchmark configuration requires $n_i^{WR} < r_i$ and $\theta_i < 1$, $i = 1, 2$.

(1) Condition (53) implies $n_i^{WR} < r_i$ and may be derived using (76)-(94).

(2) Conditions (54) are illustrated in Figure 1, and imply $\theta_i < 1$, $i = 1, 2$..

Proof of Proposition 4: For given housing stocks, Lemma 5 implies there is a unique relationship between the value of being an unemployed renter and the various housing stocks given by $U^R(r_1, r_2, h_1, h_2)$. Using (28) and (69) we can express the rental rate in city 1 for given stocks of housing as

$$\kappa_1 = K - \rho U^R(r_1, r_2, h_1, h_2) \tag{109}$$

where $K = z + \pi^R + (\mu + \beta_1\mu^*)\,\Gamma_1 + ((1-\beta_1)\,\mu^* + \sigma\lambda)\,\Gamma_2$ depends only on parameters. From (55) and (56), in a stationary equilibrium r_1, r_2, h_1 and h_2 are linearly and positively related to κ_1, κ_2, ρV_1 and ρV_2, respectively. Moreover, since they can all be expressed in the same form as (109), κ_1, κ_2, ρV_1 and ρV_2 are positively and linearly related to each other. We can therefore express the rental rate in city 1 as

$$\kappa_1 = K - \rho U^R(r_1(\kappa_1), r_2(\kappa_1), h_1(\kappa_1), h_2(\kappa_1))$$

where $r_1(\kappa_1)$, $r_2(\kappa_1)$, $h_1(\kappa_1)$ and $h_2(\kappa_1)$ are positive, linear functions of κ_1. A sufficient condition for this equation to imply a unique value of κ_1 is that U^R is increasing in all of its arguments.■

Proof of Proposition 5: Equating (99) and (100) and re–arranging yields

$$
\begin{aligned}
& \mu\left(\frac{w_2-w_1}{\rho+\delta+\mu}\right)+(1-\sigma)\gamma_1\left(\frac{\alpha_1\left(w_1-z\right)}{\rho+\delta+\mu}-\alpha_1\Gamma_1+\Gamma_2\right)+\left(\rho+\left(\frac{\rho+\delta}{\rho+\delta+\mu}\right)\beta_2\mu^*\right)\left(\Gamma_2-\Gamma_1\right) && (110)\\
= \; & (1-\sigma)\gamma_2\left[\frac{\alpha_2\left(w_2-z\right)}{\rho+\delta+\mu}-\alpha_2\Gamma_2+\Gamma_1-\left(\frac{\alpha_2\beta_2\mu^*}{\rho+\delta+\mu}\right)\left(\Gamma_2-\Gamma_1\right)\right]. && (111)
\end{aligned}
$$

If and only if $\gamma_2 > \gamma_1$ then

$$
\begin{aligned}
& \mu\left(\frac{w_2-w_1}{\rho+\delta+\mu}\right)+(1-\sigma)\gamma_1\left(\frac{\alpha_1\left(w_1-z\right)}{\rho+\delta+\mu}-\alpha_1\Gamma_1+\Gamma_2\right)+\left(\rho+\left(\frac{\rho+\delta}{\rho+\delta+\mu}\right)\beta_2\mu^*\right)\left(\Gamma_2-\Gamma_1\right) > \\
& (1-\sigma)\gamma_1\left[\frac{\alpha_2\left(w_2-z\right)}{\rho+\delta+\mu}-\alpha_2\Gamma_2+\Gamma_1-\left(\frac{\alpha_2\beta_2\mu^*}{\rho+\delta+\mu}\right)\left(\Gamma_2-\Gamma_1\right)\right]. && (112)
\end{aligned}
$$

Re-arranging yields

$$
\begin{aligned}
& \mu\left(\frac{w_2-w_1}{\rho+\delta+\mu}\right)+\left(\rho+\left(\frac{\rho+\delta}{\rho+\delta+\mu}\right)\beta_2\mu^*\right)\left(\Gamma_2-\Gamma_1\right) > && (113)\\
& (1-\sigma)\gamma_1\left[\alpha_2\left(\frac{w_2-z}{\rho+\delta+\mu}-\Gamma_2-\frac{\beta_2\mu^*}{\rho+\delta+\mu}\left(\Gamma_2-\Gamma_1\right)\right)-\alpha_1\left(\frac{w_1-z}{\rho+\delta+\mu}-\Gamma_1\right)-\left(\Gamma_2-\Gamma_1\right)\right].
\end{aligned}
$$

The left hand side of this expression is clearly positive. It follows that a sufficient condition for $\gamma_2 > \gamma_1$ is that the term in square brackets is negative. Re-arranging this term and substituting out Γ_1 and $\Gamma_2 - \Gamma_1$ using (31) and (33) yields

$$
\frac{w_2-w_1}{\rho+\delta+\mu+\beta_2\mu^*} > \frac{\left(\alpha_2-\alpha_1\right)\mu^*}{\rho+\delta+\mu+\mu^*+\sigma\lambda}\left(\frac{w_1-z}{\rho+\delta+\mu}\right). \tag{114}
$$

Since $\alpha_2 - \alpha_1$ is bounded above by 1, the result follows.■

Proof of Proposition 6: The change in the number of unemployed renters is

$$
s_1\dot{N}_1^{UR}+s_2\dot{N}_2^{UR}=\delta\left(s_1N_1^{WR}+s_2N_2^{WR}\right)+\nu\bar{N}-\left(\mu+\mu^*+\lambda\right)\left(s_1N_1^{UR}+s_2N_2^{UR}\right) \tag{115}
$$

In the steady-state, flows into and out of the pool of unemployed renters must satisfy

$$
\begin{aligned}
\left(\mu+\mu^*+\lambda+\nu\right)\left(s_1n_1^{UR}+s_2n_2^{UR}\right) &= \delta\left(s_1n_1^{WR}+s_2n_2^{WR}\right)+\nu && (116)\\
&= \delta\left(s_1r_1-s_1n_1^{UR}+s_2r_2-s_2n_2^{UR}\right)+\nu.
\end{aligned}
$$

Dividing through by $(\delta+\mu+\mu^*+\lambda+\nu)\left(s_1r_1+s_1r_2\right)$, it follows that the unemployment rate among renters is given by (58). The second term in (58) represents the new entrants into the labour force each period who are unemployed by construction. The fraction of non-new entrant renters that are unemployed is given by

$$u^R = \frac{\delta}{\delta + \mu + \mu^* + \lambda + \nu}. \tag{117}$$

The steady-state flows into and out of the pool of unemployed owners must satisfy

$$\begin{aligned} & (\mu + \nu)\left(s_1 n_1^{UH} + s_2 n_2^{UH}\right) + (1-\beta_1)\mu^* \theta_1^{UH} s_1 n_1^{UH} + (1-\beta_2)\mu^* \theta_2^{UH} s_2 n_2^{UH} + \beta_2 \mu^* s_2 n_2^{UH} \\ = \; & \delta\left(s_1 n_1^{WH} + s_2 n_2^{WH}\right) + \lambda\left(s_1 n_1^{UR} + s_2 n_2^{UR}\right). \end{aligned} \tag{118}$$

Using the fact that $s_1 n_1^{WH} + s_2 n_2^{WH} = 1 - s_1 r_1 - s_2 r_2 - s_1 n_1^{UH} - s_2 n_2^{UH}$, dividing through by $(\delta + \mu + \mu^* + \lambda + \nu)(1 - s_1 r_1 - s_2 r_2)$ and re-arranging yields the rate of unemployment among home-owners

$$u^H = \frac{s_1 n_1^{UH} + s_2 n_2^{UH}}{1 - s_1 r_1 - s_2 r_2} = u^R + \frac{\Upsilon}{(\delta + \mu + \mu^* + \lambda + \nu)(1 - s_1 r_1 - s_2 r_2)}, \tag{119}$$

where

$$\begin{aligned} \Upsilon = \; & \mu^*\left((1 - (1-\beta_1)\theta_1^{UH}) s_1 n_1^{UH} + \left(1 - (1-\beta_2)\theta_2^{UH} - \beta_2\right) s_2 n_2^{UH}\right) + \\ & \lambda\left(s_1 n_1^{UR} + s_2 n_2^{UR} + s_1 n_1^{UH} + s_2 n_2^{UH}\right). \end{aligned} \tag{120}$$

Υ must be positive since $(1-\beta_1)\theta_1^{UH} \leq 1$ and $(1-\beta_2)\theta_2^{UH} + \beta_2 \leq 1$. It follows that $u^H > u^R$.■

Proof of Proposition 7: Using (80) and (81) to substitute for n_1^{WH} and n_2^{WH} in (77) yields

$$n_1^{WR} + \frac{\mu}{\lambda}\left(h_1 - \frac{r_1}{\phi_1}\right) = \left(\frac{\delta + \mu + \nu}{\lambda}\right)\frac{\Psi_1}{b_{11}^W - b_{11}^U} - \left(\frac{b_{12}^W - b_{12}^U}{b_{11}^W - b_{11}^U}\right)\left(n_2^{WR} + \frac{\mu}{\lambda}\left(h_2 - \frac{r_2}{\phi_2}\right)\right). \tag{121}$$

Given that $\alpha_1 = \frac{n_1^{WR}}{r_1}$ and $\alpha_2 = \frac{n_2^{WR}}{r_2}$, this can be expressed as

$$\alpha_1 + \frac{\mu}{\lambda} x_1 = \left(\frac{\delta + \mu + \nu}{\lambda}\right)\frac{1}{r_1}\frac{\Psi_1}{b_{11}^W - b_{11}^U} - \left(\frac{b_{12}^W - b_{12}^U}{b_{11}^W - b_{11}^U}\right)\frac{r_2}{r_1}\left(\alpha_2 + \frac{\mu}{\lambda} x_2\right), \tag{122}$$

where $x_i = \frac{h_i}{r_i} - \frac{1}{\phi_i}$. Using (86) and (88)-(91) we have

$$\begin{aligned} \alpha_1 = \; & \frac{\mu + \beta_1 \mu^* + ((1-\beta_2)\mu^* + \lambda)\frac{s_2 r_2}{s_1 r_1}}{\delta + \mu + \mu^* + \lambda + \nu} - \left(\frac{(1-\beta_2)\mu^* - \beta_2 \mu^* \frac{\mu}{\delta + \mu + \nu}}{\delta + \mu + \mu^* + \lambda + \nu}\right)\frac{s_2 r_2}{s_1 r_1}\alpha_2 \\ & - \left(\frac{\beta_2 \mu^*\left(\frac{\delta + \nu}{\delta + \mu + \nu}\right) + \nu}{\delta + \mu + \mu^* + \lambda + \nu}\right)\frac{s_2 r_2}{s_1 r_1} x_2. \end{aligned} \tag{123}$$

If $\alpha_2 > \alpha_1$ it follows that

$$\alpha_2 > \frac{\mu + \beta_1\mu^* + ((1-\beta_2)\mu^* + \lambda)\frac{s_2r_2}{s_1r_1} - \left(\beta_2\mu^*\left(\frac{\delta+\nu}{\delta+\mu+\nu}\right) + \nu\right)\frac{s_2r_2}{s_1r_1}x_2}{\delta + \mu + \mu^* + \lambda + \nu + \left((1-\beta_2)\mu^* - \beta_2\mu^*\frac{\mu}{\delta+\mu+\nu}\right)\frac{s_2r_2}{s_1r_1}}. \tag{124}$$

Dividing (81) by r_2 and using (76), (87), (93) and (94) we have

$$\alpha_2 = \frac{\mu + \beta_2\mu^* + \frac{s_1r_1}{s_2r_2}\left((1-\beta_1)\mu^* + \lambda\right) - \nu\frac{s_1r_1}{s_2r_2}x_1}{\delta + \mu + \beta_2\mu^* + \lambda + \nu}. \tag{125}$$

Since $\beta_2\mu^* < \mu^* + \left((1-\beta_2)\mu^* - \beta_2\mu^*\frac{\mu}{\delta+\mu+\nu}\right)\frac{s_2r_2}{s_1r_1}$, we can use (124) and (125) to derive a sufficient condition for $\alpha_2 > \alpha_1$:

$$(\beta_2 - \beta_1)\,\mu^* + \left[((1-\beta_1)\mu^* + \lambda) - \nu x_1\right] Z > \frac{((1-\beta_2)\mu^* + \lambda) - \left(\beta_2\mu^*\left(\frac{\delta+\nu}{\delta+\mu+\nu}\right) + \nu\right)x_2}{Z}. \tag{126}$$

where $Z = \frac{s_1r_1}{s_2r_2}$. If $Z = 1$:

$$2\,(\beta_2 - \beta_1)\,\mu^* > -\beta_2\mu^*\left(\frac{\delta+\nu}{\delta+\mu+\nu}\right)x_2 - \nu\,(x_2 - x_1)\,. \tag{127}$$

If (a) $\beta_2 \geq \beta_1$ and (b) $h_2/r_2 \simeq h_1/r_1$ (which implies that $x_2 > x_1$), this condition must hold. Since the left hand side of (126) is increasing in Z and the right hand side is decreasing, (126) must hold for all $Z > 1$. For $Z < 1$, (126) holds as long as Z does not become too small.■

Derivation 3 — City–level unemployment rate: The unemployment rate in a city of type i is

$$u_i = \frac{n_i^{UH} + n_i^{UR}}{r_i + n_i^{UH} + n_i^{WH}} = \frac{r_i - n_i^{WR} + \left(\frac{\delta+\nu}{\delta+\mu+\nu}\right)\left(h_i - \frac{r_i}{\phi}\right) - \frac{\lambda}{\delta+\mu+\nu}n_i^{WR}}{r_i + h_i - \frac{r_i}{\phi}}. \tag{128}$$

where the second equality uses (76)-(94). Dividing through by r_i and re–arranging yields

$$u_i = \frac{1 - \left(1 + \frac{\lambda}{\delta+\mu+\nu}\right)\alpha_i}{1 + \frac{h_i}{r_i} - \frac{1}{\phi_i}} + \left(\frac{\delta+\nu}{\delta+\mu+\nu}\right)\left(\frac{\frac{h_i}{r_i} - \frac{1}{\phi_i}}{1 + \frac{h_i}{r_i} - \frac{1}{\phi_i}}\right). \tag{129}$$

Re-arranging and using (57) yields (59).

Proof of Proposition 8: The aggregate unemployment rate is

$$\bar{u} = n_1^{UR} + n_2^{UR} + n_1^{UH} + n_2^{UH} \tag{130}$$

Using (76)-(94) this can be expressed as

$$\bar{u} = s_1 r_1 + s_2 r_2 + \left(\frac{\delta+\nu}{\delta+\mu+\nu}\right)(1 - s_1 r_1 - s_2 r_2) - \left(1 + \frac{\lambda}{\delta+\mu+\nu}\right)\left(s_1 n_1^{WR} + s_2 n_2^{WR}\right). \tag{131}$$

But

$$\begin{aligned} s_1 n_1^{WR} + s_2 n_2^{WR} &= \left(\frac{\delta+\mu+\nu}{\lambda}\right)\left(s_1 n_1^{WH} + s_2 n_2^{WH}\right) - \frac{\mu}{\lambda}\left(s_1 h_1 - \frac{s_1 r_1}{\phi_1} + s_2 h_2 - \frac{s_2 r_2}{\phi_2}\right) \\ &= \left(\frac{\delta+\mu+\nu}{\lambda}\right)\left(s_1 n_1^{WH} + s_2 n_2^{WH}\right) - \frac{\mu}{\lambda}(1 - s_1 r_1 - s_2 r_2), \end{aligned} \tag{132}$$

and

$$s_1 n_1^{WH} + s_2 n_2^{WH} = s_1 \frac{\Psi_1}{b_{11}^W - b_{11}^U} + \left(s_2 - s_1\left(\frac{b_{12}^W - b_{12}^U}{b_{11}^W - b_{11}^U}\right)\right)\frac{\Psi_2}{b_{22}^W - b_{22}^U}. \tag{133}$$

If $\beta_1 = \beta_2 = 0$ then

$$\begin{aligned} s_1 n_1^{WH} + s_2 n_2^{WH} &= \frac{\mu+\mu^*+\lambda}{(\delta+\mu+\mu^*+\lambda+\nu)\left(\frac{\delta+\mu+\nu}{\lambda}\right)}(s_1 r_1 + s_2 r_2) \\ &\quad + \left(\frac{\frac{\mu}{\lambda}(\delta+\mu+\mu^*+\lambda+\nu) - \upsilon}{(\delta+\mu+\mu^*+\lambda+\nu)\left(\frac{\delta+\mu+\nu}{\lambda}\right)}\right)(1 - s_1 r_1 - s_2 r_2). \end{aligned} \tag{134}$$

and so substituting and rearranging yields

$$\begin{aligned} \bar{u} &= \frac{(\delta+\mu+\nu)(\delta+\nu) - \lambda(\mu+\mu^*+\lambda)}{(\delta+\mu+\mu^*+\lambda+\nu)(\delta+\mu+\nu)} + \\ &\quad \left(\frac{(\delta+\nu)(\mu^*+\lambda) + (\delta+\mu+\nu+\lambda)\upsilon + \lambda(\mu+\mu^*+\lambda)}{(\delta+\mu+\mu^*+\lambda+\nu)(\delta+\mu+\nu)}\right)\bar{\Omega}. \end{aligned} \tag{135}$$

In this case, $\bar{u}$ and $\bar{\Omega} = 1 - s_1 r_1 - s_2 r_2$ are positively and linearly related.■

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10 Appendix C: Not for publication

10.1 Details of Generalized Model

Here we provide details of the full model, generalized to allow for (1) intra-city relocation, (2) inter-city relocation for non-employment reasons and (3) mortgage interest deductability.

In each city there are six types of households, as each may be either employed or unemployed, either rent or own a house and, if they are owners, may either be matched or mis-matched with their house. The measures of households in city i that are matched employed-owners, mis-matched employed-owners, employed-renters, matched unemployed-owners, mis-matched unemployed-owners and unemployed-renters are given by n_i^{WH}, $\tilde{n}_i^{WH}$, n_i^{WR}, n_i^{UH} $\tilde{n}_i^{UH}$ and n_i^{UR} respectively. The values associated with being in each of these states are given by W_i^H, $\tilde{W}_i^H$, W_i^R, U_i^H, $\tilde{U}_i^H$ and U_i^R, respectively. We let P_i^{WR}, P_i^{UR}, P_i^{WH} and P_i^{UH} denote the prices paid for houses in City i by employed and unemployed renters and by employed and unemployed owners respectively.

10.1.1 Household Flows

The steady–state flow of households between states is described by (11), (36) and the following 10 equations:

$$(\delta + (1-\beta_1)\mu^*(1-\chi) + \lambda + \nu + \mu^*\chi)n_1^{WR} = (\mu + \beta_1\mu^*(1-\chi))\, n_1^{UR} + \beta_1\mu^*(1-\chi)\tilde{n}_1^{UH} + \frac{\beta_1}{s_1}\mu^*\chi \tag{136}$$

$$+ (1-\beta_2)\frac{s_2}{s_1}\mu^*(1-\chi)\left(n_2^{UR} + \tilde{n}_2^{UH} + m_2\right)$$

$$(\mu + \psi + \nu + \mu^*\chi)\, n_1^{UH} + (1-\beta_1)\mu^*(1-\chi)m_1 = \delta n_1^{WH} + \lambda\left(n_1^{UR} + \tilde{n}_1^{UH}\right) \tag{137}$$

$$(\delta + \psi + \nu + \mu^*\chi)\, n_1^{WH} = \lambda\left(n_1^{WR} + \tilde{n}_1^{WH}\right) + \mu n_1^{UH} \tag{138}$$

$$(\mu + \mu^*(1-\chi) + \lambda + \nu + \mu^*\chi)\, \tilde{n}_1^{UH} = \psi n_1^{UH} + \delta\tilde{n}_1^{WH} \tag{139}$$

$$(\delta + \lambda + \nu + \mu^*\chi)\, \tilde{n}_1^{WH} = \psi n_1^{WH} + \mu\tilde{n}_1^{UH} \tag{140}$$

$$\left(\delta+\lambda+\nu+\mu^{*}\chi\right)n_{2}^{WR}=\left(\mu+\beta_{2}\mu^{*}(1-\chi)\right)n_{2}^{UR}+\beta_{2}\mu^{*}(1-\chi)\tilde{n}_{2}^{UH}+\frac{\beta_{2}}{s_{2}}\mu^{*}\chi$$
$$+(1-\beta_{1})\frac{s_{1}}{s_{2}}\mu^{*}(1-\chi)\left(n_{1}^{UR}+n_{1}^{WR}+\tilde{n}_{1}^{UH}+m_{1}\right) \tag{141}$$
$$\delta n_{2}^{WH}+\lambda\left(n_{2}^{UR}+\tilde{n}_{2}^{UH}\right)=\left(\mu+\psi+\beta_{2}\mu^{*}(1-\chi)+\nu+\mu^{*}\chi\right)n_{2}^{UH}+(1-\beta_{2})\mu^{*}m_{2} \tag{142}$$
$$\left(\delta+\psi+\nu+\mu^{*}\chi\right)n_{2}^{WH}=\lambda\left(n_{2}^{WR}+\tilde{n}_{2}^{WH}\right)+\mu n_{2}^{UH} \tag{143}$$
$$\psi n_{2}^{UH}+\delta\tilde{n}_{2}^{WH}=\left(\mu+\mu^{*}(1-\chi)+\lambda+\nu+\mu^{*}\chi\right)\tilde{n}_{2}^{UH} \tag{144}$$
$$\psi n_{2}^{WH}+\mu\tilde{n}_{2}^{UH}=\left(\delta+\lambda+\nu+\mu^{*}\chi\right)\tilde{n}_{2}^{WH} \tag{145}$$

where $m_i=\theta_i n_i^{UH}$. This system of 12 linear equations can be solved for the 12 unknowns as a function of $(\{r_i,h_i,\phi_i\}_{i=1}^{2})$.

Using (139), (140), (144) and 145) we can derive

$$\tilde{n}_{i}^{UH} = a^{UU}n_{i}^{UH}+a^{UW}n_{i}^{WH} \tag{146}$$
$$\tilde{n}_{i}^{WH} = a^{WU}n_{i}^{UH}+a^{WW}n_{i}^{WH} \tag{147}$$

where $a^{UU}=\psi(\delta+\lambda+\nu+\mu^{*}\chi)/D$, $a^{UW}=\psi\delta/D$, $a_{i}^{WU}=\psi\mu/D$,
$a_{i}^{WW}=\psi\left(\mu+\mu^{*}(1-\chi)+\lambda+\nu+\mu^{*}\chi\right)/D$ and
$D=\left(\delta+\lambda+\nu+\mu^{*}\chi\right)\left(\mu+\mu^{*}(1-\chi)+\lambda+\nu+\mu^{*}\chi\right)-\mu\delta$.

Using (11) we can reduce the system to

$$(\delta+\mu+\mu^{*}(1-\chi)+\lambda+\nu+\mu^{*}\chi)n_{1}^{WR}=\left(\mu+\beta_{1}\mu^{*}(1-\chi)\right)r_{1}+\frac{\beta_{1}}{s_{1}}\mu^{*}\chi+\beta_{1}\mu^{*}(1-\chi)\tilde{n}_{1}^{UH}$$
$$+(1-\beta_{2})\frac{s_{2}}{s_{1}}\mu^{*}(1-\chi)\left(r_{2}-n_{2}^{WR}+\tilde{n}_{2}^{UH}+m_{2}\right) \tag{148}$$
$$\left(\mu+\psi+\nu+\mu^{*}\chi\right)n_{1}^{UH}+(1-\beta_{1})\mu^{*}(1-\chi)m_{1}=\delta n_{1}^{WH}+\lambda\left(r_{1}-n_{1}^{WR}+\tilde{n}_{1}^{UH}\right) \tag{149}$$
$$\left(\delta+\psi+\nu+\mu^{*}\chi\right)n_{1}^{WH}=\lambda\left(n_{1}^{WR}+\tilde{n}_{1}^{WH}\right)+\mu n_{1}^{UH} \tag{150}$$

$$\left(\delta+\mu+\beta_{2}\mu^{*}(1-\chi)+\lambda+\nu+\mu^{*}\chi\right)n_{2}^{WR}=\left(\mu+\beta_{2}\mu^{*}(1-\chi)\right)r_{2}+\frac{\beta_{2}}{s_{2}}\mu^{*}\chi+\beta_{2}\mu^{*}(1-\chi)\tilde{n}_{2}^{UH}$$
$$+(1-\beta_{1})\frac{s_{1}}{s_{2}}\mu^{*}(1-\chi)\left(r_{1}+\tilde{n}_{1}^{UH}+m_{1}\right) \tag{151}$$
$$\left(\mu+\psi+\beta_{2}\mu^{*}(1-\chi)+\nu+\mu^{*}\chi\right)n_{2}^{UH}=\delta n_{2}^{WH}+\lambda\left(r_{2}-n_{2}^{WR}+\tilde{n}_{2}^{UH}\right)-(1-\beta_{1})\mu^{*}(1-\chi)m_{2} \tag{152}$$
$$\left(\delta+\psi+\nu+\mu^{*}\chi\right)n_{2}^{WH}=\lambda\left(n_{2}^{WR}+\tilde{n}_{2}^{WH}\right)+\mu n_{2}^{UH} \tag{153}$$

Substituting out m_i, n_i^{WR} $\tilde{n}_i^{UH}$ and $\tilde{n}_i^{WH}$ $i\in\{1,2\}$ yields

$$c_{11}^U n_1^{UH} + c_{11}^W n_1^{WH} + c_{12}^U n_2^{UH} + c_{12}^W n_2^{WH} = \mu r_1 + (\mu^*(1-\chi)+\lambda)\, r_2 + \frac{\beta_1}{s_1}\mu^*\chi \tag{154}$$

$$c_{21}^U n_1^{UH} + c_{21}^W n_1^{WH} + c_{22}^U n_2^{UH} + c_{22}^W n_2^{WH} = (\mu^*(1-\chi)+\lambda)\, r_1 + \mu r_2 + \frac{\beta_2}{s_2}\mu^*\chi \tag{155}$$

where

$$\begin{aligned}
c_{11}^W &= (\delta+\mu+\mu^*(1-\chi)+\lambda+\nu+\mu^*\chi)\left(\frac{\delta+\psi+\nu+\mu^*\chi}{\lambda} - a^{WW}\right) \qquad (156)\\
&\quad -\beta_1\mu^*(1-\chi)a^{UW}\\
c_{11}^U &= -(\delta+\mu+\mu^*(1-\chi)+\lambda+\nu+\mu^*\chi)\left(\frac{\mu}{\lambda}+a^{WU}\right) - \beta_1\mu^*(1-\chi)a^{UU} \qquad (157)\\
c_{12}^W &= \frac{s_2}{s_1}\left[\begin{array}{c} \psi+\nu+\mu^*\chi+(1-\beta_2)\mu^*(1-\chi)\left(\frac{\delta+\psi+\nu+\mu^*\chi}{\lambda}\right) \\ -\left((1-\beta_2)\mu^*(1-\chi)+\lambda\right)\left(a^{WW}+a^{UW}\right) \end{array}\right] \qquad (158)\\
c_{12}^U &= \frac{s_2}{s_1}\left[\begin{array}{c} \psi+\beta_2\mu^*+\nu+\mu^*\chi-(1-\beta_2)\mu^*(1-\chi)\frac{\mu}{\lambda} \\ -\left((1-\beta_2)\mu^*(1-\chi)+\lambda\right)\left(a^{WU}+a^{UU}\right) \end{array}\right] \qquad (159)\\
c_{21}^W &= \frac{s_1}{s_2}\left[\psi+\nu+\mu^*\chi-\left((1-\beta_1)\mu^*(1-\chi)+\lambda\right)a^{UW}-\lambda a^{WW}\right] \qquad (160)\\
c_{21}^U &= \frac{s_1}{s_2}\left[\psi+\nu+\mu^*\chi-\left((1-\beta_1)\mu^*(1-\chi)+\lambda\right)a^{UU}-\lambda a^{WU}\right] \qquad (161)\\
c_{22}^W &= (\delta+\mu+\beta_2\mu^*(1-\chi)+\lambda+\nu+\mu^*\chi)\left(\frac{\delta+\psi+\nu+\mu^*\chi}{\lambda} - a^{WW}\right) \qquad (162)\\
&\quad -\beta_2\mu^*(1-\chi)a^{UW}\\
c_{22}^U &= -\left(\delta+\mu+\beta_2\mu^*(1-\chi)+\lambda+\nu+\mu^*\chi\right)\left(\frac{\mu}{\lambda}+a^{WU}\right) - \beta_2\mu^*(1-\chi)a^{UU} \qquad (163)
\end{aligned}$$

From (36) we have

$$\phi_i n_i^{UH} + \phi_i n_i^{WH} + (1+\phi_i)\tilde{n}_i^{UH} + (1+\phi_i)\tilde{n}_i^{WH} = \phi_i h_i - r_i \tag{164}$$

substituting for $\tilde{n}_i^{UH}$ and $\tilde{n}_i^{WH}$ using (146) and (147), and re-arranging yields

$$\begin{aligned}
n_1^{UH} &= \frac{\phi_1 h_1 - r_1}{b_1^U} - \frac{b_1^W}{b_1^U} n_1^{WH} \qquad (165)\\
n_2^{UH} &= \frac{\phi_2 h_2 - r_2}{b_2^U} - \frac{b_2^W}{b_2^U} n_2^{WH} \qquad (166)
\end{aligned}$$

where

$$\begin{aligned}
b_1^W &= \phi_1 + (1+\phi_1)\left(a^{UW}+a^{WW}\right) \qquad (167)\\
b_1^U &= \phi_1 + (1+\phi_1)\left(a^{UU}+a^{WU}\right) \qquad (168)\\
b_2^W &= \phi_2 + (1+\phi_2)\left(a^{UW}+a^{WW}\right) \qquad (169)\\
b_2^U &= \phi_2 + (1+\phi_2)\left(a^{UU}+a^{WU}\right) \qquad (170)
\end{aligned}$$

It follows that

$$\begin{bmatrix} n_1^{WH} \\ n_2^{WH} \end{bmatrix} = \Phi^{-1}\Psi \tag{171}$$

where

$$\Phi = \begin{bmatrix} c_{11}^W - c_{11}^U \frac{b_1^W}{b_1^U} & c_{12}^W - c_{12}^U \frac{b_2^W}{b_2^U} \\ c_{21}^W - c_{21}^U \frac{b_1^W}{b_1^U} & c_{22}^W - c_{22}^U \frac{b_2^W}{b_2^U} \end{bmatrix}$$

$$\Psi = \begin{bmatrix} (\mu + \beta_1\mu^*)\, r_1 + \frac{s_2}{s_1}\left((1-\beta_1)\mu^* + \lambda\right) r_2 + \frac{\beta_1}{s_1}\mu^*\chi - \frac{c_{11}^U(\phi_1 h_1 - r_1)}{b_1^U} - \frac{c_{12}^U(\phi_2 h_2 - r_2)}{b_2^U} \\ (\mu + \beta_2\mu^*)\, r_2 + \frac{s_1}{s_2}\left((1-\beta_2)\mu^* + \lambda\right) r_1 + \frac{\beta_2}{s_2}\mu^*\chi - \frac{c_{21}^U(\phi_1 h_1 - r_1)}{b_1^U} - \frac{c_{22}^U(\phi_2 h_2 - r_2)}{b_2^U} \end{bmatrix}.$$

The measure of employed renters in each city can then be derived from

$$n_1^{WR} = \left(\frac{\delta + \psi + \nu + \mu^*\chi}{\lambda}\right) n_1^{WH} - \frac{\mu}{\lambda} n_1^{UH} - \tilde{n}_1^{WH} \tag{172}$$

$$n_2^{WR} = \left(\frac{\delta + \psi + \nu + \mu^*\chi}{\lambda}\right) n_2^{WH} - \frac{\mu}{\lambda} n_2^{UH} - \tilde{n}_2^{WH} \tag{173}$$

and the measure of owners who move in each city from

$$m_1 = \frac{\lambda\left(r_1 + \tilde{n}_1^{UH} + \tilde{n}_1^{WH}\right) - (\psi + \nu + \mu^*\chi)\left(n_1^{UH} + n_1^{WH}\right)}{(1-\chi)\mu^*(1-\beta_1)} \tag{174}$$

$$m_2 = \frac{\lambda\left(r_2 + \tilde{n}_2^{UH} + \tilde{n}_2^{WH}\right) - (\psi + \beta_2\mu^* + \nu + \mu^*\chi)\, n_2^{UH} - (\psi + \nu + \mu^*\chi)\, n_2^{WH}}{(1-\chi)\mu^*(1-\beta_2)} \tag{175}$$

Using (64) the generalized AM curve can be expressed as

$$\begin{aligned} s_1h_1 + s_2h_2 + s_1r_1 + s_2r_2 - 1 \;=\; & \frac{1}{\phi_1}\left(s_1r_1 + s_1\tilde{n}_1^{WH}(\phi_1,\phi_2) + s_1\tilde{n}_1^{UH}(\phi_1,\phi_2)\right) \\ & + \frac{1}{\phi_2}\left(s_2r_2 + s_2\tilde{n}_2^{WH}(\phi_1,\phi_2) + s_2\tilde{n}_2^{UH}(\phi_1,\phi_2)\right). \end{aligned} \tag{176}$$

10.1.2 Household Values

The flow values of owners are given by

$$\rho W_i^H = w_i + \pi^H + \delta\left(U_i^H - W_i^H\right) + \psi(\tilde{W}_i^H - W_i^H) \tag{177}$$

$$\rho U_i^H = z + \pi^H + \mu\left(W_i^H - U_i^H\right) + \psi(\tilde{U}_i^H - U_i^H) \tag{178}$$

$$\rho \tilde{W}_i^H = w_i + \pi^H - \varepsilon + \delta\left(\tilde{U}_i^H - \tilde{W}_i^H\right) + \sigma\lambda\left(W_i^H - \tilde{W}_i^H\right) \tag{179}$$

$$\begin{aligned} \rho \tilde{U}_i^H = {} & z + \pi^H - \varepsilon + \mu\left(\tilde{W}_i^H - \tilde{U}_i^H\right) + \sigma\lambda\left(U_i^H - \tilde{U}_i^H\right) \\ & + (1-\beta_i)\mu^*\left(W_j^R + V_i - \tilde{U}_i^H\right) + \beta_i\mu^*\left(W_i^R + V_i - \tilde{U}_i^H\right). \end{aligned} \tag{180}$$

Using the fact that $U_i^H = V_i + W_j^R$ the last Bellman equation can be simplified to

$$\begin{aligned} \rho \tilde{U}_i^H &= z + \pi^H - \varepsilon + \mu \left(\tilde{W}_i^H - \tilde{U}_i^H \right) + (\mu^* + \sigma\lambda) \left(U_i^H - \tilde{U}_i^H \right) \\ &\quad + \beta_i \mu^* \left(W_i^R - W_j^R \right) \end{aligned} \tag{181}$$

Solving this system yields

$$\Omega_1^W = W_1^H - \tilde{W}_1^H = \frac{(\rho + \delta + \mu + \psi + \mu^* + \sigma\lambda)\,\varepsilon + 1\delta\beta_1\mu^* \left(W_2^R - W_1^R \right)}{\Delta_1} \tag{182}$$

$$\Omega_1^U = U_1^H - \tilde{U}_1^H = \frac{(\rho + \delta + \mu + \psi + \sigma\lambda)\,\varepsilon + (\rho + \delta + \psi + \sigma\lambda)\,\beta_1\mu^* \left(W_2^R - W_1^R \right)}{\Delta_1} \tag{183}$$

$$\Omega_2^W = W_2^H - \tilde{W}_2^H = \frac{(\rho + \delta + \mu + \psi + \mu^* + \sigma\lambda)\,\varepsilon - \delta\beta_2\mu^* \left(W_2^R - W_1^R \right)}{\Delta_2} \tag{184}$$

$$\Omega_2^U = U_2^H - \tilde{U}_2^H = \frac{(\rho + \delta + \mu + \psi + \sigma\lambda)\,\varepsilon - (\rho + \delta + \psi + \sigma\lambda)\,\beta_2\mu^* \left(W_2^R - W_1^R \right)}{\Delta_2} \tag{185}$$

where $\Delta_i = (\rho + \mu + \psi + \mu^* + \sigma\lambda)(\rho + \delta + \psi + \sigma\lambda) - \mu\delta > 0$, and

$$W_i^H - U_i^H = \frac{w_i - z - \psi(\Omega_i^W - \Omega_i^U)}{\rho + \delta + \mu} \tag{186}$$

$$\rho W_i^H = w_i + \pi^H - \delta \left(W_i^H - U_i^H \right) - \psi \Omega_i^W \tag{187}$$

$$\rho U_i^H = z + \pi^H + \mu \left(W_i^H - U_i^H \right) - \psi \Omega_1^U \tag{188}$$

The values of renters is given by

$$\rho W_1^R = w_1 + \pi^R - \kappa_1 + \delta(U^R - W_1^R) + \mu^*(W_2^R - W_1^R) + \lambda\sigma \left(W_1^H - W_1^R - V_1 \right) \tag{189}$$

$$\rho U^R = z + \pi^R - \kappa_1 + \mu(W_1^R - U^R) + \mu^*(W_2^R - U^R) + \lambda\sigma \left(U_1^H - U^R - V_1 \right) \tag{190}$$

$$\rho W_2^R = w_2 + \pi^R - \kappa_2 + \delta(U^R - W_2^R) + \lambda\sigma \left(W_2^H - W_2^R - V_2 \right) \tag{191}$$

$$\rho U^R = z + \pi^R - \kappa_2 + \mu(W_2^R - U^R) + \mu^*(W_1^R - U^R) + \lambda\sigma \left(U_2^H - U^R - V_2 \right) \tag{192}$$

Subtracting (190) from (189)

$$(\rho + \delta + \mu + \mu^* + \sigma\lambda) \left(W_1^R - U^R \right) = w_1 - z + \lambda\sigma \left(W_1^H - U_1^H \right)$$

Substituting for $W_1^H - U_1^H$ using (186) and re-arranging yields

$$\Gamma_1 = W_1^R - U^R = \left(\frac{\rho + \delta + \mu + \sigma\lambda}{\rho + \delta + \mu + \mu^* + \sigma\lambda} \right) \left(\frac{w_1 - z}{\rho + \delta + \mu} \right). \tag{193}$$

Similarly, subtracting (192) from (191)

$$(\rho+\delta+\mu+\sigma\lambda)\left(W_2^R-U^R\right)=w_2-z+\lambda\sigma\left(W_2^H-U_2^H\right)-\mu^*(W_1^R-U^R)$$

Substituting for $W_2^H-U_2^H$ using (186) and re-arranging yields

$$\begin{aligned}\Gamma_2 &= \frac{(\rho+\delta+\mu+\sigma\lambda)(w_2-z)-\sigma\lambda\frac{\psi\mu^*\varepsilon}{\Delta_2}}{(\rho+\delta+\mu)(\rho+\delta+\mu+\beta_2\mu^*+\sigma\lambda)+\sigma\lambda\frac{\psi(\rho+\psi+\sigma\lambda)\beta_2\mu^*}{\Delta_2}} \\ &\quad+\left[\frac{\sigma\lambda\frac{\psi(\rho+\psi+\sigma\lambda)\beta_2\mu^*}{\Delta_2}-(\rho+\delta+\mu)(1-\beta_2)\mu^*}{(\rho+\delta+\mu)(\rho+\delta+\mu+\beta_2\mu^*+\sigma\lambda)+\sigma\lambda\frac{\psi(\rho+\psi+\sigma\lambda)\beta_2\mu^*}{\Delta_2}}\right]\end{aligned} \tag{194}$$

Subtracting and simplifying yields

$$W_2^R-W_1^R = \Gamma_2-\Gamma_1 \tag{195}$$

$$= \frac{(\rho+\delta+\mu+\sigma\lambda)(w_2-w_1)-\sigma\lambda\frac{\psi\mu^*}{\Delta_2}\varepsilon}{(\rho+\delta+\mu)(\rho+\delta+\mu+\beta_2\mu^*+\sigma\lambda)+\sigma\lambda\frac{\psi(\rho+\psi+\sigma\lambda)\beta_2\mu^*}{\Delta_2}} \tag{196}$$

The value of a vacant house in city i satisfies

$$\begin{aligned}\rho V_i &= \gamma_i\left\{\begin{array}{c}\eta_i\left[\alpha_i((1-T)P_i^{WR}-V_i)+(1-\alpha_i)((1-T)P_i^{UR}-V_i)\right] \\ +(1-\eta_i)\left[\zeta_i((1-T)P_i^{WH}-V_i)+((1-T)P_i^{UH}-V_i)\right]\end{array}\right\} \\ &= \gamma_i(1-\sigma)\left\{\begin{array}{c}\eta_i\left[\alpha_i(W_i^H-W_i^R-V_i)+(1-\alpha_i)(U_i^H-U^R-V_i)\right] \\ +(1-\eta_i)\left[\zeta_i(W_i^H-\tilde{W}_i^H)+(U_i^H-\tilde{U}_i^H)\right]\end{array}\right\}\end{aligned}$$

Solving for V_i yields

$$V_i=\frac{\gamma_i(1-\sigma)}{\rho+\eta_i\gamma_i(1-\sigma)}\left\{\eta_i\left[\alpha_i(W_i^H-W_i^R)+(1-\alpha_i)\left(U_i^H-U^R\right)\right]+(1-\eta_i)\left[\zeta_i\Omega_i^W+(1-\zeta_i)\,\Omega_i^U\right]\right\} \tag{197}$$

In city 1 this can be expressed as

$$V_1=\frac{\gamma_1(1-\sigma)}{\rho+\eta_1\gamma_1(1-\sigma)}\left\{\eta_1\left[\alpha_1(W_1^H-\Gamma_1)+(1-\alpha_1)U_1^H-U^R\right]+(1-\eta_1)\left[\zeta_1\Omega_1^W+(1-\zeta_1)\,\Omega_1^U\right]\right\} \tag{198}$$

Since, in this equilibrium, matched home-owners in city 1 are indifferent between staying or leaving we have that

$$U_1^H-V_1=\Gamma_2+U^R \tag{199}$$

Substituting for V_1 yields

$$\begin{aligned}(\rho+\eta_1\gamma_1(1-\sigma))\left(U_1^H-\Gamma_2-U^R\right) &= \gamma_1(1-\sigma)\left\{\begin{array}{c}\eta_1\left[\alpha_1(W_1^H-\Gamma_1)+(1-\alpha_1)U_1^H-U^R\right] \\ +(1-\eta_1)\left[\zeta_1\Omega_1^W+(1-\zeta_1)\,\Omega_1^U\right]\end{array}\right\} \\ (\rho+\eta_1\gamma_1(1-\sigma))\left(U_1^H-\Gamma_2-U^R\right) &= \gamma_1(1-\sigma)\eta_1\left[(1-\alpha_1)U_1^H-U^R\right] \\ &\quad+\gamma_1(1-\sigma)\left\{\eta_1\alpha_1\left(W_1^H-\Gamma_1\right)+(1-\eta_1)\left[\zeta_1\Omega_1^W+(1-\zeta_1)\,\Omega_1^U\right]\right\}\end{aligned}$$

Solving for ρU^R we get

$$\begin{aligned}\rho U_1^R(\phi_1,\phi_2) &= (\rho+\alpha_1\eta_1\gamma_1(1-\sigma))\,U_1^H-(\rho+\eta_1\gamma_1(1-\sigma))\,\Gamma_2 \\ &\quad -\gamma_1(1-\sigma)\left\{\eta_1\alpha_1\left(W_1^H-\Gamma_1\right)+(1-\eta_1)\left[\zeta_1\Omega_1^W+(1-\zeta_1)\,\Omega_1^U\right]\right\}\end{aligned} \tag{200}$$

Similarly, for city 2 we have

$$\begin{aligned}\rho U_2^R(\phi_1,\phi_2) &= (\rho+\alpha_2\eta_2\gamma_2(1-\sigma))\,U_2^H-(\rho+\eta_2\gamma_2(1-\sigma))\,\Gamma_1 \\ &\quad -\gamma_2(1-\sigma)\left\{\eta_2\alpha_2\left(W_2^H-\Gamma_2\right)+(1-\eta_2)\left[\zeta_2\Omega_2^W+(1-\zeta_2)\,\Omega_2^U\right]\right\}\end{aligned} \tag{201}$$

Equating $U_1^R(\phi_1,\phi_2)=U_2^R(\phi_1,\phi_2)$ yields a generalized VV curve. The intersection of this with the generalized AM curve yields the equilibrium values of ϕ_1 and ϕ_2 given (r_1,r_2,h_1,h_2).

10.2 Equilibrium in an Economy with Rental Housing Only

The flow values of employed and unemployed renters in each city are given by

$$\rho W_1^R = w_1+\pi^R-\kappa_1+\delta\left(U_1^R-W_1^R\right)+(1-\beta_1)\,\mu^*\left(W_2^R-W_1^R\right) \tag{202}$$

$$\rho U^R = z+\pi^R-\kappa_1+(\mu+\beta_1\mu^*)\left(W_1^R-U^R\right)+(1-\beta_1)\,\mu^*\left(W_2^R-U^R\right) \tag{203}$$

$$\rho W_2^R = w_2+\pi^R-\kappa_2+\delta\left(U_2^R-W_2^R\right) \tag{204}$$

$$\rho U^R = z+\pi^R-\kappa_2+(\mu+\beta_2\mu^*)\left(W_2^R-U^R\right)+(1-\beta_2)\,\mu^*\left(W_1^R-U^R\right) \tag{205}$$

Using these it is straightforward to show that the implied rent–differential between high and low wage cities is given by

$$\kappa_2-\kappa_1=\frac{\mu(w_2-w_1)}{\rho+\delta+\mu+\beta_2\mu^*} \tag{206}$$

The steady state supply curves for rental housing (55) imply that in steady state a positive relationship must exist between rental stocks for a given rent-differential:

$$r_2=\frac{\kappa_2-\kappa_1}{\rho c_2^R}+\frac{c_1^R}{c_2^R}r_1 \tag{207}$$

Aggregate rental market clearing requires that

$$s_1r_1+s_2r_2=1 \tag{208}$$

which implies a negative relationship between rental stocks. It follows that provided $\rho c_2^R > s_2\left(\kappa_2-\kappa_1\right)$, there exists a unique pair $(r_1^*,\,r_2^*)$ satisfying both (207) and (208).

The flows of workers in this stationary equilibrium satisfy (52) and the following:

$$(\delta+(1-\beta_1)\mu^*(1-\chi)+\nu+\mu^*\chi)n_1^{WR} = (\mu + \beta_1\mu^*(1-\chi))\, n_1^{UR}+(1-\beta_2)\frac{s_2}{s_1}\mu^*(1-\chi)n_2^{UR}+\frac{\beta_1}{s_1}\mu^*\chi \tag{209}$$

$$(\delta+\nu+\mu^*\chi)n_2^{WR} = (\mu + \beta_2\mu^*(1-\chi))\, n_2^{UR}+(1-\beta_1)\frac{s_1}{s_2}\mu^*(1-\chi)\left(n_1^{UR} + n_1^{WR}\right)+\frac{\beta_2}{s_2}\mu^*\chi \tag{210}$$

Provided that the solution to these equations imply positive values for the numbers of workers in each state and $\kappa_1 > 0$, a unique interior stationary equilibrium exists.

10.3 Complete Results of Robustness Experiments

Table C1 – Robustness: Mobility and Unemployment

	High wage differential w_2/w_1= 1.15			Low Mobility 5%		
	Base	Owned -10%	Reduced m.i.d.	Base	Owned -10%	Reduced m.i.d.
Mobility rate	0.064	0.075	0.076	0.050	0.0591	0.0571
– of renters	0.120	0.099	0.114	0.095	0.0767	0.086
– of owners	0.036	0.057	0.050	0.028	0.0460	0.040
Population ratio	2.55	2.55	2.05	2.54	2.54	2.21
Unemployment	0.0570	0.0537	0.0547	0.0570	0.0543	0.0553
– low-wage	0.680	0.0600	0.0630	0.0655	0.0589	0.0614
– high-wage	0.0527	0.0512	0.0506	0.0537	0.0525	0.0525
– all renters	0.0546	0.0529	0.0533	0.0561	0.0544	0.0549
– non entrants	0.0478	0.0478	0.0478	0.0493	0.0493	0.0493
– owners	0.0582	0.0543	0.0556	0.0575	0.0542	0.0555
Rejection rate	0.1402	0.1058	0.1216	0.1307	0.1009	0.1170
– low-wage	0.2297	0.1890	0.1973	0.2223	0.1877	0.2018
– high-wage	0.1052	0.0735	0.0868	0.0984	0.0669	0.0790

Table C2 – Robustness: Housing Market Statistics

	High wage differential w_2/w_1= 1.15			Low Mobility 5%		
Low-wage cities	Base	Owned -10%	Reduced m.i.d.	Base	Owned -10%	Zero m.i.d.
% ownable	68.0	58.0	59.5	68.0	58.0	61.5
Rent	0.059	0.067	0.0823	0.0843	0.107	0.109
Price	1.90	2.10	1.92	2.1415	2.70	2.12
$\kappa_1 - \rho V_1$	-0.1911	-0.1911	-0.1935	-0.0002	-0.0002	-0.0014
Months to sell	4.30	3.99	3.36	4.64	4.11	3.89
Ownership rate	67.3	57.1	58.8	67.4	57.2	60.8
Vacancy Rate	3.18	3.69	3.01	2.76	3.09	2.69
High-wage cities						
% ownable	68.0	58.0	61.6	68.0	58.0	61.8
Rent	0.200	0.207	0.223	0.176	0.198	0.201
Price	5.54	5.73	4.71	4.49	5.04	3.92
$\kappa_2 - \rho V_2$	-0.2488	-0.2488	-0.2488	-0.0267	-0.0267	-0.0267
Months to sell	1.20	1.14	1.08	1.78	1.73	1.66
Ownership rate	67.7	57.7	61.3	67.8	57.7	61.5
Vacancy Rate	0.90	1.07	0.93	1.07	1.31	1.15