Search and the Dynamics of House Prices and Construction

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Abstract

We characterize the dynamics of relative house prices, construction rates and population growth across US cities. We find that, in response to shocks to relative incomes, construction rates and population growth exhibit quite different short-run dynamics and that price appreciation exhibits considerable short-run serial correlation and long-run mean reversion. We develop a competitive search model of the housing market in which construction, the entry of buyers, house prices and rents are determined in equilibrium. Our theory generates dynamics that are qualitatively consistent with our empirical observations. We calibrate the model to match key long-run features of the housing market in a typical US city and assess the extent to which the model can quantitatively account for the key moments in the data.

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1 Introduction

In this article we explore the consequences of search and matching frictions for the dynamics of house prices and construction. We consider an environment in which both the entry of new buyers and the construction of new houses are determined endogenously in equilibrium. In the presence of search frictions, in our environment both construction and house prices may exhibit short term *momentum* (*i.e.* serially correlated growth rates), even if housing dividends are strictly mean–reverting. In the long run, both variables return to their long-run values, as the construction of new homes eventually reduces the ratio of buyers to houses for sale. A version of the model calibrated to data on U.S. cities (with housing dividends represented by local incomes), accounts for substantial shares of the observed momentum and variance of house prices. While the calibrated model also accounts for a significant share of the observed momentum in construction (measured by permits) it cannot account for the variance.

Housing market dynamics in US cities can be characterized by several key stylized facts. Firstly, most time-series variation in house prices is local in nature, not national.[1] This has motivated researchers to use local factors such as income, regulations and construction costs to account for price movements. Secondly, houses prices are very volatile when compared with per capita incomes and rents. This appears to be true both at the in relative terms across cities and in aggregate. A third key observation is that there is strong positive serial autocorrelation in house price appreciation over the short term, but mean reversion in prices over longer periods.[2] Finally, as Glaeser et al. (2010) highlight, there is strong short run persistence of construction rates with long–run weak mean-reversion, and high volatility of construction levels within markets.

While the movements in house prices are reasonably well documented, Capozza, Hendershott and Mack (2004) point out that a well–developed behavioural theory to account for them has proved difficult to construct. Since the work of Case and Shiller (1989) and Cutler, Poterba and Summers (1991), it has been recognized that movements in house prices (like those of many other assets) pose a challenge to strict versions of the efficient markets view. In particular, the fact that the strong positive autocorrelation of house price appreciation does not appear to be explained by fundamentals suggests that a simple asset–pricing approach

[1] This has been noted in the US by Abraham and Hendershott (1996) and Del Negro and Otrok (2006), but also in Canada by Allen, Amano, Byrne and Gregory (2007).

[2] See Abraham and Hendershott (1996), Capozza, Mack and Mayer (1997), Malkpezzi (1999) and Meen (2002).

alone may be of limited value.[3] Many authors have gone further to argue that to explain housing market dynamics, one must introduce aspects of irrationality and/or rule–of–thumb behaviour.

Several authors have argued that their are good reasons to suspect that search and matching frictions play an important role in housing markets. For example, the observed positive comovement of prices and sales (Rios-Rull and Sanchez Marcos, 2007) and the fact that prices and sales are negatively correlated with average time on the market (Krainer, 2008). As first noted by Peach (1983) and more recently documented by Caplin and Leahy (2008), there is significant negative correlation between vacancies and price appreciation. Diaz and Jerez (2010) suggest that movements in the division of surplus between buyers and sellers driven by changes in the tightness of housing markets (as is predicted by competitive search theory) may be a significant source of fluctuations in house prices.

We develop here a framework that introduces frictions of these types into a housing market where both the entry of new buyers and the construction of new houses evolve endogenously. The value of living in a particular city is determined by an exogenous housing dividend which we think of as relative income. New buyers enter the market as renters and search for a house whenever the expected value of doing so exceeds their next best alternative. New houses are constructed and offered for sale or for rent by profit maximizing firms. Resident home-owners may also put their houses up for sale or rent them out and exit the market when they experience changes in their life situation, which we model as driven by the the realization of an exogenous shock. Exchange in the housing market is characterized by directed search as proposed by Moen (1997). In this environemt we establish the existence of a unique stationary growth path characterized by constant rates of both population growth and construction.

We study the implications of shocks to relative income in a version of the model calibrated to data on U.S. cities. The model generates short-term price momentum in equilibrium even in the absence of persistent income growth (*i.e.* even if income follows a first-order autoregressive process). While in equilibrium an increase in the value of living in a city generates an immediate increase in search activity as households enter the market, it takes time for these buyers to find a house through the matching process. Initially, in fact, the rate at which individual buyers find matches actually declines. Similarly, although both sales and the *rate* at which houses sell rise immediately, construction of new housing takes time to respond. Thus, although the value of searching declines after just one period (due

[3] Case and Shiller (1989) argue that serial correlation in rents does not explain momentum in price changes.

to mean reversion in income), the tightness of the housing market (*i.e.* the ratio of buyers to sellers) continues to rise for several periods, driving up further the both value of vacant houses and the transaction prices for house. Eventually, as income reverts to its long-run relative level, the stock of buyers declines as entry slows and residents become home-owners. As this happens the decline in vacancies slows and eventually reverses due to construction. Tightness declines and the house price eventually reverts back to its steady state value.

Our analysis is related to that in two other recent papers on housing markets. Diaz and Jerez (2010) also develop and calibrate a competitive search model of the housing market. They consider a version of Wheaton's (1990) model with neither entry of buyers nor house construction. Our approach is motivated in part by their insight that competitive search may magnify the effects of exogenous changes on house prices due to movements in the shares of surplus going to buyers and sellers. In our experiments, however, we find the opposite, a version of the model with constant shares (owing to a Cobb-Douglas matching function) generates *greater* variance in house prices rather than less.

Glaeser et al. (2010) develop a dynamic, rational expectations model with no search frictions. House prices are determined by relative income movements, which induce entry in the short run, and housing supply conditions which pin down prices in the long run. They calibrate a version of their model and study its dynamics driven by an estimated process on incomes. The possibility of short–term price momentum and mean reversion in prices and construction arises because of the observed "hump-shaped" pattern of relative incomes.[4] Their model is reasonably successful in accounting for longer term movements in prices and construction and for overall volatility in the median market. Their calibrated model, however, cannot generate any short term momentum in prices and similarly, the persistence of fluctuations in construction rates is too low.

Although a number of other papers have studied the role of search and matching frictions in housing markets (e.g. Wheaton, 1990; Krainer, 2001; Albrecht at al., 2007; and Head and Lloyd-Ellis, 2010), these generally focus treat the aggregate housing stock as fixed and consider steady-state implications. While Caplin and Leahy (2008) do consider the non-steady state implications of their model, they also assume a fixed housing stock. In contrast, we focus on the role of frictions for the transitional dynamics of prices and construction of new homes. Although we do allow for turnover of existing homes, this is not crucial for the qualitative nature of price and investment dynamics (although it does matter quantitatively).

[4] They also assume utility is decreasing in local population size which has a dampening effect on prices. However, in their calibration this effect is tiny so, in fact, the shock process drives everything.

Models of housing investment and construction (e.g. Davis and Heathcote, 2007 and Glaeser et al. 2010), on the other hand, generally abstract from trading and matching frictions in the market for houses in order to focus on supply side factors. In this paper we bring together aspects of both literatures.

In Section 2 we document some of the key empirical features of housing market dynamics. Section 3 develops the basic strucure of our model. The equilibrium is characterized in Section 4 and the steady state is analyzed in Section 5. Section 6 presents the calibration. Section 7 considers the qualitative dynamic implications of the model and Section 8 contains the quantitative analysis. Section 9 concludes. All proofs and extended derivations are in contained the appendix.

2 Empirical properties of relative house prices, income, construction, and city populations

In this section, we characterize the dynamics of relative income, house prices, construction rates and population growth across US cities. Our data consist of annual observations between 1980 and 2008 for 98 metropolitan statistical areas (MSAs). Details of sources and data construction are provided in Appendix A. Our measure of income at the city level is total income less labor earnings from construction, and we approximate the stock of housing by accumulating permits. Since we are interested in *relative dynamics* across cities, we first transform the data by removing common time effects. That is, we first run a panel regression for the logarithm of each series on time dummies and study only the residual components from those regressions.

Relative movements in house prices, construction and city populations could arise in response to all kinds of shocks at both the city and aggregate levels. Here, we want to isolate the dynamics that result from changes in relative city incomes. To do this, we estimate a panel vector autoregression model. We restrict our econometric model with assumptions based on the theory that we develop below. In particular, our theory implies that a persistent, positive shock to local income induces households to enter a city more rapidly. This, in turn, drives up the demand for housing relative to trend and to some extent spurs construction, although the latter takes time. Changes in the ratio of buyers to sellers in the housing market drive movements in both house prices and rents. These increase in the short-run, slowing entry to some extent. Eventually, as incomes revert to their mean, entry slows relative to construction, prices decline, and the city economy returns to its long-run trend.

This broad description motivates us to consider a panel VAR of the following form:

$$\mathbf{\Gamma X}_{ct} = \sum_{i=1}^{T} \mathbf{A}_i \mathbf{X}_{ct-i} + \mathbf{F}_c + \boldsymbol{\varepsilon}_{ct} \tag{1}$$

where $\mathbf{X}_{ct} = [Y_{ct}, P_{ct}, g^H_{ct}, g^N_{ct}]'$ denotes the vector of the log of income less construction earnings *per capita*, the log of house prices, the growth in the stock of houses (cumulative permits) and population growth in each city at each date. Here $\mathbf{\Gamma}$, $\mathbf{A}_i$ are matrices of parameters, $\mathbf{F}_c$ is a vector of city fixed–effects and $\boldsymbol{\varepsilon}_{ct} = [\varepsilon^Y_{ct}, \varepsilon^P_{ct}, \varepsilon^H_{ct}, \varepsilon^N_{ct}]'$ are the structural shocks. To estimate the structural parameters of this model, we must impose a set of identifying restrictions. Specifically, we assume that the shocks are orthogonal and that there are restrictions on $\mathbf{\Gamma}$ motivated by our theory. Specifically, we effectively assume that income does not depend contempraneously on any of the other variables, prices depend contemporaneously only on income, construction growth depends contemporaneously on income and prices and population growth can depend on all the other variables. These restrictions are consistent with the model we present below, and seem reasonable more generally. Note that g^H_{ct} and g^N_{ct} growth rates going forward (i.e. $g^H_{ct} = \ln H_{t+1} - \ln H_t$), so it seems reasonable that these variables to be able to respond to time t shocks to in income and prices.[5] Moreover, although there could be agglomeration effects of population on income, this effect is unlikely to occur contemporaneously. Although there are 4 structural shocks in the estimated model, our focus is on the affects of shocks to local income only.

In our baseline estimation we use the system GMM estimator proposed by Arellano and Bover (1995) and Blundell and Bond (1998). This estimator is asymptotically consistent when the number of panels becomes large for a given time-dimension, thereby avoiding the so-called incidental parameters problem associated with fixed-effects estimators (Nickell, 1981). There are several reasons that we focus on results using this estimator over other possibilities. Firstly, it is generally found to outperform other standard GMM estimators such as that of Arellano and Bond (1991) when the endogenous variables are persistent.[6] Secondly, its asymptotic properties are fairly well understood and it has been extended to the context of panel VARs by Binder, Hsiao and Pesaren (2005). Finally, the standard

[5] The ordering of g^H_{ct} and g^N_{ct} in the system makes very little difference to our results.

[6] Essentially, the system GMM estimator instruments the endogenous variables in levels using lagged differences. An alternative, which is asymptotically equivalent but has been found to perform better in finite samples is to instrument with lagged deviations from the forward mean of the remaining sample.

Table 1: **Moments from Structural PVAR—Income Shock, System GMM**

	Relative Std. Dev.	Corr. with Income	Autocorrelation year 1	year 2	year 3	year 4
Per capita Income growth	1.0000	1.0000	0.2480	0.0221	-0.0546	-0.0759
House Price Appreciation	1.3483	0.8049	0.7066	0.2988	0.0274	-0.1144
Construction Rate	0.1144	0.4600	0.8946	0.6589	0.4116	0.2117
Population Growth	0.1791	0.7115	0.7149	0.4533	0.2526	0.1219

fixed–effects estimator has been found to exhibit a significant finite-sample bias for samples with similar dimensions to ours (i.e. moderately large time and panel dimension. See Judson and Owen (1999)). However, there are also some potential pitfalls in using the system GMM estimator in finite samples. We discuss these in more detail in Appendix B and compare the implied estimates with two alternatives.[7]

Figure 1 depicts the implied impulse response functions for a relative income shock together with the associated 95% confidence intervals.[8] In response to the shock, relative income exhibits positively auto-correlated growth, peaking after one year, and is quite persistent. The resulting movement in the relative house price exhibits considerably more momentum, continuing to rise for 4 years before mean-reverting. Mean reversion in house prices is more rapid than in incomes. Population growth responds immediately to the shock then slows, whereas the construction rate responds more sluggishly, peaking after two periods. One consequence of this is that the ratio of city population to the stock of housing rises and remains persistently high following a shock to income.

These observations are quantified in Table 1. Notice that, in response to income shocks alone, the unconditional volatility of house prices relative to income is about half as large as in the data overall. The persistence of house price movements, however, is somewhat larger. Similar implications hold for construction and population growth rates.

Table 2 provides key moments for local earnings, house prices, construction rates, and ratios of housing stocks to city population based on shocks to local income in the Panel VAR for each of the three sub-samples. Several, key observations are apparent. The

[7]We have alse estimated the system with more than 2 lags, but this make little difference to our results.

[8]Confidence intervals are computed using a Monte Carlo simulation for panel VAR provided by Love and Zicchino (2006).

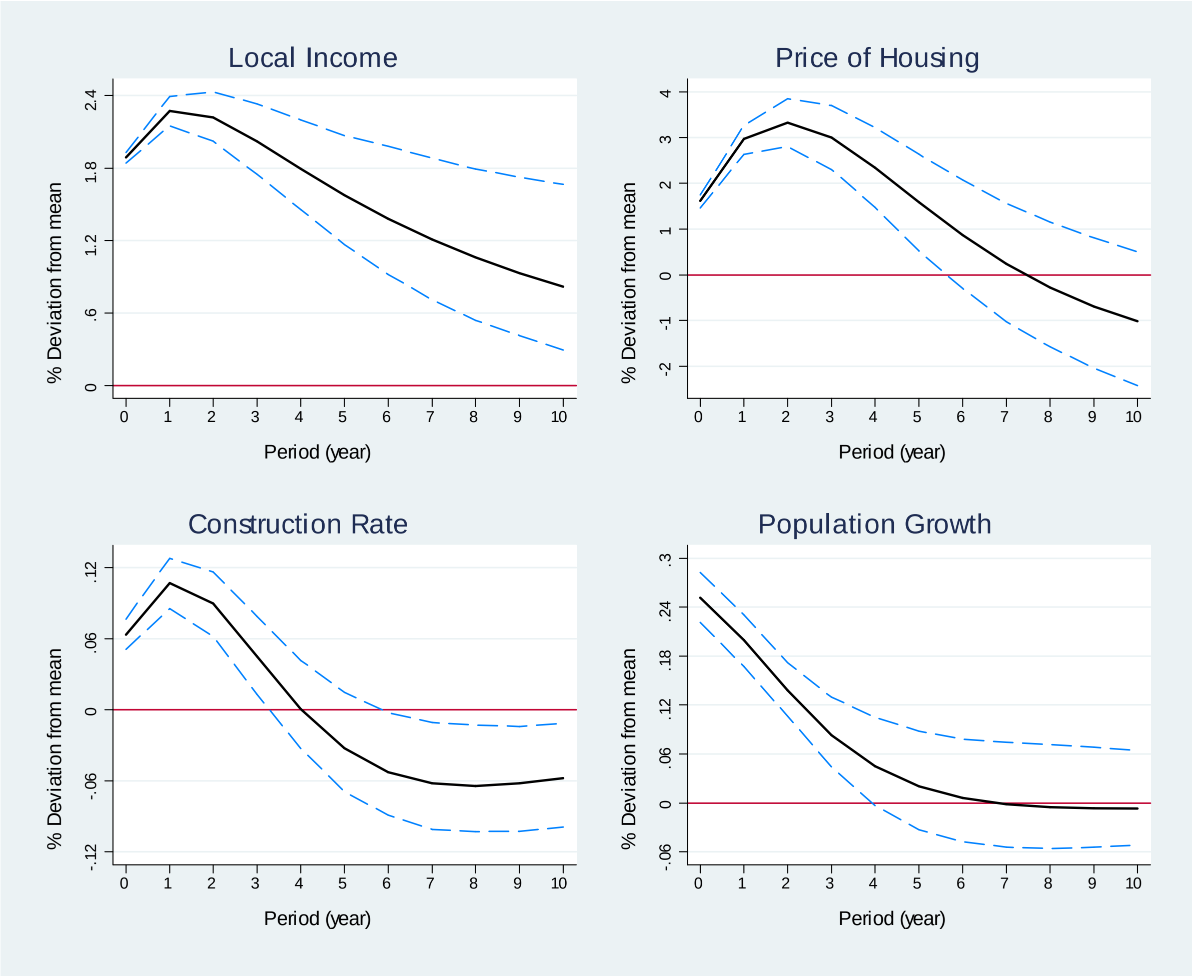

Figure 1: Impulse response functions from panel VAR: income shock

standard deviation of house prices is roughly equal to that of local earnings in the full sample. Both construction rates and housing stock-population ratios are much less volatile than local earnings. House prices, construction rates and housing stock-population ratios are all strongly positively correlated with local earnings, although for inland cities these correlations are somewhat weaker. The higher and more persistent autocorrelation in both house price appreciation and population growth relative to earnings growth can also be observed in all the sub-samples.

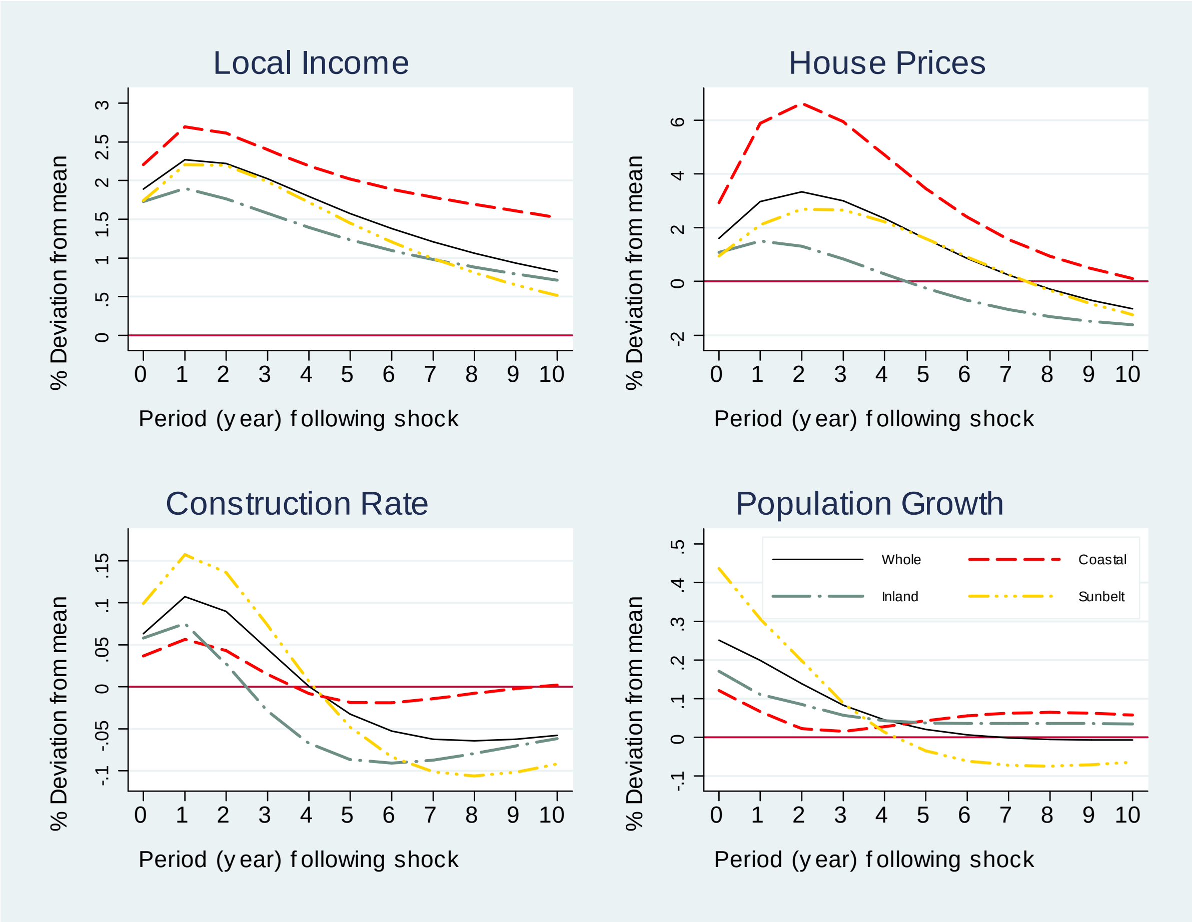

Figure 2: Impulse responses from panel VAR for sub-regions: income shock

Table 2: **Moments from Strucural PVAR by Sub-sample—Income Shock**

		Relative Std. Dev.	Corr. with Inc. Growth	Autocorrelation year 1	year 2	year 3	year 4
Income Growth	Coastal	1	1	0.2433	-0.0147	-0.0813	-0.0843
	Interior	1	1	0.1405	-0.0305	-0.0660	-0.0683
	Sunbelt	1	1	0.3263	0.0664	-0.0491	-0.0883
Price Appreciation	Coastal	2.1073	0.7783	0.6939	0.2314	-0.0361	-0.1413
	Interior	0.9218	0.8014	0.6152	0.2526	0.0420	-0.0679
	Sunbelt	1.1937	0.7067	0.8168	0.4756	0.1820	-0.0144
Construction Rate	Coastal	0.0429	0.5512	0.7945	0.3724	-0.0192	-0.2572
	Interior	0.1453	0.4382	0.9141	0.7353	0.5629	0.4320
	Sunbelt	0.1885	0.5229	0.8904	0.6431	0.3676	0.1287
Population Growth	Coastal	0.0987	0.4268	0.8082	0.6706	0.6278	0.6289
	Interior	0.1453	0.5258	0.7480	0.6036	0.4918	0.4334
	Sunbelt	0.3194	0.8402	0.6704	0.3897	0.1535	-0.0031

3 The Environment

Time is discrete and indexed by t. We consider economic activity in a single housing market (which we refer to as a *city*), and treat activity outside this market as exogenous. The *aggregate* economy is populated by measure Q_t of *ex ante* identical households, which grows exogenously at net rate μ. Each period, new households enter the city through a process described below. All households resident within the city require housing, and they may either own their own home or rent. On entry to the city, households are permanently differentiated randomly into those that *ceteris paribus* would prefer to own the house in which they live, and those who have no interest in doing so. All households discount the future at rate $\beta \in (0, 1)$. We assume that capital markets are perfect and that the gross interest rate is $1/\beta$.

In period t, there is within the city a stock of identical housing units, H_t, which can be occupied by a resident owner, rented to a resident, or offered for sale. The measure of resident home-owners is denoted N_t, and that of resident renters by $B_t + F_t$, where B_t is the measure of renters who would like to own a house (and so are currently searching) and F_t that of renters that are not interested in owning. Measure S_t houses are for sale, where $S_t = H_t - N_t - B_t - F_t$. Houses for sale include both newly built houses that are currently owned by developers, and houses put up for sale by resident owners who either don't want them anymore or are moving elsewhere.

All households are infinitely-lived and endowed with two types of labour: general labour and construction labour. At each date t, a household supplies one unit of general labour inelastically and l_t units of construction labour endogenously, taking the construction wage, w_t, as given. General labour earns y_t per unit supplied, where y_t is exogenous and follows a stationary stochastic process in levels.[9] We parameterize specific processes for y_t in our analysis of equilibrium dynamics contained in Sections 7 and 8.

At date t, preferences over consumption, c_t, construction labour and housing are given by:

$$U_t(c_t, l_t, z_t) = c_t - v(l_t) + z_t, \tag{2}$$

where

$$v(l_t) = \frac{l_t^{1+\frac{1}{\eta}}}{\zeta^{\frac{1}{\eta}}\left(1+\frac{1}{\eta}\right)} \tag{3}$$

and η and ζ are constants. The variable z_t denotes a utility premium derived from owning the house in which the household lives. For a household that has no interest in owning, $z_t = 0$, for all t. For other households, $z_t = z^H$ in periods when they own a house that they like and $z_t = 0$ in periods when they either rent or live in a house that they don't like. We assume that z^H reflects the quality of the house. This value is constant over time because any depreciation resulting from occupancy is assumed to be offset by maintenance. We let m denote the cost of this maintenance incurred by the owner.

At the beginning of each period, a house that is not currently owner-occupied can either be rented or listed for sale. Let H_t^R denote the stock of houses available for rent in period t. A rented house earns rent, r_t, less the constant maintenance cost, m. The rental market is competitive. Houses that are designated for sale must remain vacant while on the market and their value at time t is denoted V_t. It follows that the value of a house that is not currently owner–occupied is given by

$$\tilde{V}_t = \max\left[r_t - m + \beta E_t \tilde{V}_{t+1}, \quad V_t\right]. \tag{4}$$

There are also in the economy a large number of *developers* who behave competitively and operate a technology for the construction of new housing units. Each new house requires one unit of land, which can be purchased in a competitive market at unit price q_t, and $1/\phi$

[9]It is straightforward to generalize preferences so that general labour is supplied endogenously too. Provided the disutility of supplying each type of labour is separable, however, this would makes no difference to our analysis (see below).

units of construction labour. The stock of houses thus evolves over time according to:

$$H_{t+1} - H_t = \phi L_t \tag{5}$$

where L_t is the total amount of construction labour supplied city-wide. Houses constructed at time t become available either for sale or for rent at time $t+1$ and do not depreciate over time. There is free entry into construction and we assume that the price of land depends on the stock of houses relative to its trend value, H_t^*:

$$q_t = \bar{q}\left(\frac{H_t}{H_t^*}\right)^{\frac{1}{\xi}}. \tag{6}$$

Here ξ represents the elasticity of land supply with respect to its price. This elasticity could depend on many factors including topology, land regulations and local politics (see Saiz, 2010).

Newly built houses are identical to pre-existing ones. Developers can either rent them or designate them for sale, in which case they remain vacant for at least one period and have exactly the same value, V_t, as existing vacant houses. Only houses that are occupied require maintenance to offset depreciation.

At the beginning of period t, a measure μQ_{t-1} of new households enters the economy and receives an alternative value, ε, to entering the city. Here ε is distributed across the new households according to a stationary distribution function, $G(\varepsilon)$, with support $[0, \bar{\varepsilon}]$. There exists a critical alternative value, ε_t^c, at which a new household is just indifferent to entering the city:

$$\varepsilon_t^c = \bar{W}_t. \tag{7}$$

where $\bar{W}_t$ is the value of being a new entrant to the city. All non-resident households with $\varepsilon \leq \varepsilon_t^c$ enter the city and are immediately separated into two types. A fraction ψ derive utility from owning their own home *per se* and becomes potential buyers. A fraction $1-\psi$ do not and become perpetual renters. Letting W_t denote the value of being a potential buyer and W_t^f the value of being a perpetual renter, it follows that

$$\bar{W}_t = \psi W_t + (1-\psi) W_t^f \tag{8}$$

Searching for a house to own takes at least one period, and during this time potential buyers also rent. At the end of each period, perpetual renters may, with probability $\pi_f \in (0, 1)$, experience an exogenous shock that induces them to leave the city. On receiving this shock they move out immediately and receive a continuation value, Z. Otherwise they remain as renters in the next period.

Home-owners are subject to two exogenous shocks. With probablity $\pi_h \in (0,1)$ owners receive a shock that causes them to want to leave the city. Like renters, upon receiving this shock they move out immediately, receive a continuation value, Z, and put their house up either for sale or for rent. With probability $\theta \in (0,1)$ the remaining $(1-\pi_h)N_t$ of owners at date t will find that they no longer derive the utility premium z^H from owning their current house. In our baseline model, we assume that such "mis-matched" owners immediately move out of their current house and rent while searching for a new one.[10]

The housing market is decentralized and characterized by competitive search as proposed by Moen (1997). We may think of it as consisting of a variety of sub-markets, each of which is characterized by a house price, P_t and a pair of matching probabilities, one for buyers and one for sellers. We imagine that there exist a large number of *market makers*, who decide which sub-market(s) to open (if any). Search is directed in the sense that buyers and sellers observe the price and matching probabilities of all sub-markets, and then decide which *single* market to enter. There is no cost to entering any sub-market. Matching within each sub-market is random, because buyers and sellers cannot coordinate. Once matched, buyers and sellers exchange at the price specified *ex ante* for their sub-market. In a competitive search equilibrium, the matching technology and entry decisions together imply matching probabilities that are consistent with those specified for the corresponding sub-markets. Moreover, the set of open sub-markets is *complete* in the sense that the opening of no additional sub-market can make some buyers and/or sellers better off.

Let B_t and S_t be the respective number of buyers and sellers present in any sub-market. The number of matches per period is determined by a matching function, $\mathcal{M}(B_t, S_t)$, where $\mathcal{M}$ is increasing in both arguments and exhibits constant returns to scale. Let $\omega_t = B_t/S_t$, which we refer to as the *tightness* of the market. It follows that a buyer who enters the housing market will find a vacant house with probability

$$\lambda_t = \frac{\mathcal{M}(B_t, S_t)}{B_t} = \lambda(\omega_t). \tag{9}$$

Similarly, a seller in a sub-market will find a buyer with probability

$$\gamma_t = \frac{\mathcal{M}(B_t, S_t)}{S_t} = \gamma(\omega_t) = \omega_t \; \lambda(\omega_t). \tag{10}$$

[10] As we shall see below, in equilibrium, mis-matched owners are indifferent between this and remaining in their own house while searching. Assuming some or all mis-matched owners remain in their current home while searching yields almost identical results.

Let $\epsilon(\omega_t)$ denote the elasticity of the number of matches with respect to the number of buyers, *i.e.*,

$$\epsilon(\omega_t) = \frac{B}{\mathcal{M}} \cdot \frac{\partial \mathcal{M}}{\partial B} = \frac{1}{1 - \frac{\gamma(\omega_t)/\gamma'(\omega_t)}{\lambda(\omega_t)/\lambda'(\omega_t)}}. \tag{11}$$

We impose the following assumption on the matching function:

Assumption 1. *There exists an interval* $(\underline{\omega}, \bar{\omega})$ *such that for all* $\omega \in [\underline{\omega}, \bar{\omega}]$,*: (i)* $\lambda(\omega) \in [0,1]$, $\gamma(\omega) \in [0,1]$, *and* $\lim_{\omega \to \underline{\omega}} \lambda(\omega) = \lim_{\omega \to \bar{\omega}} \gamma(\omega) = 1$; *(ii)* $\lambda'(\omega) < 0$, $\gamma'(\omega) > 0$*; (iii)* $\epsilon'(\omega) \leq 0$.

Part (i) is a regularity condition. Part (ii) requires intuitively that the matching probability for a buyer decreaes with the market tightness whereas that for a seller increases with market tightness. Part (iii) requires that as the market tightness increases, the increase in the number of matches in response to the increase in the number of buyers decreases. This requirement generates a positive relationship between the market tightness and the value of houses. As the tightness increases, houses are sold at a higher rate, and for a given selling price this drives up the value of a vacant house. We associate the rate at which houses sell with their *liquidity*; when this rate increases (decreases) houses become more (less) liquid. We parameterize a specific matching function as part of our calibration in Section 6.

4 Equilibrium

Linearity of preferences in consumption together with the assumption that capital markets are perfect implies that households are indifferent with regard to the timing of their consumption.

Lemma 1: *Each household makes its housing decisions so as to solve*

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left[y_t + x(w_t) + z_t - \Omega_t \right] \tag{12}$$

where Ω_t *represents the net value of all housing-related transactions that take place in period* t *and*

$$x(w_t) = \frac{\zeta w_t^{1+\eta}}{1+\eta}. \tag{13}$$

Perpetual renters never choose to search for a house and remain as renters until they exogenously move to another location. It follows that we can express the value of being such a renter as

$$W_t^f = u_t^R + \pi_f \beta Z + (1 - \pi_f) \beta E_t W_{t+1}^f. \tag{14}$$

where

$$u_t^R = y_t + x(w_t) - r_t. \tag{15}$$

The stock of perpetual renters evolves according to

$$F_{t+1} = (1 - \pi_f) F_t + (1 - \psi) G(\varepsilon_t^c) \mu Q_t. \tag{16}$$

The value of being a home-owner, J_t, is given by

$$\begin{aligned} J_t \;=\; & u_t^H + \beta \pi_n \left(Z + E_t \tilde{V}_{t+1} \right) + (1 - \pi_n) \theta \left(E_t W_{t+1} + E_t \tilde{V}_{t+1} \right) \\ & + (1 - \pi_n)(1 - \theta) E_t J_{t+1}] \end{aligned} \tag{17}$$

where

$$u_t^H = y_t + x(w_t) + z^H - m. \tag{18}$$

Sellers are free to enter any sub-market without cost and choose the one which maximizes their return. By entering sub-market i, the seller sells a house at P_t^i with probability $\gamma(\omega_t^i)$. If the house fails to sell at the posted price, the seller holds on to it until the next period and receives the value of a house that is not currently owner-occupied (and thus can be either rented or offered again for sale at that time). It follows that the value of a vacant house for sale satisfies

$$V_t = \max_i \left\{ \gamma\left(\omega_t^i\right) P_t^i + \left[1 - \gamma\left(\omega_t^i\right)\right] \beta E_t \tilde{V}_{t+1} \right\}. \tag{19}$$

From (19), a seller is willing to enter only sub-markets that offer $P_t^i \geq \beta E_t \tilde{V}_{t+1}$. At the beginning of each period, a house that is not currently owner-occupied can either be rented or listed for sale.

Buyers must decide in each period which sub-market to enter, and like sellers they will choose that which maximizes their expected return. A buyer who successfully matches in sub-market i pays the posted price, P_t^i, and becomes a home-owner in the next period, receiving value J_{t+1}. One who remains unmatched continues to search in the next period. Buyers, who are by definition searching for a house, are subject to neither separation nor preference shocks. Recall that they are currently renting, and so receive the renter utility u_t^R for the current period. The value of being a buyer, W_t, is therefore given by

$$W_t = u_t^R + \max_i \left\{ \lambda\left(\omega_t^i\right) \left(\beta E_t J_{t+1} - P_t^i\right) + \left[1 - \lambda\left(\omega_t^i\right)\right] \beta E_t W_{t+1} \right\}. \tag{20}$$

It is clear that a buyer is willing to enter a sub-market if and only if $P_t^i \leq \beta[E_t J_{t+1} - E_t W_{t+1}]$.

When choosing a sub-market to open, market makers choose the submarket's characteristics, (P_t, ω_t), to maximize the value of a buyer W_t, subject to the value of a vacant house given by (19). In equilibrium, the set of sub-markets is *complete* in the sense that there is no unopened sub-market that could improve the welfare of any buyer or seller. We focus on equilibria in which the total trade surplus in the housing market is strictly positive, *i.e.*,

$$E_t[J_{t+1} - W_{t+1} - \tilde{V}_{t+1}] > 0. \tag{21}$$

The stock of potential buyers at date t is given by:

$$B_t = \theta(1 - \pi_h)N_{t-1} + \psi G(\varepsilon_t^c)\mu Q_{t-1} + \sum_i (1 - \lambda\left(\omega_{t-1}^i\right))B_{t-1}^i \tag{22}$$

The stock of owners evolves via

$$N_t = (1 - \pi_n)(1 - \theta)N_{t-1} + \sum_i \lambda\left(\omega_{t-1}^i\right) B_{t-1}^i. \tag{23}$$

4.1 Equilibrium Definition

Definition: A *competitive search equilibrium* is a sequence

$$\left\{J_t, V_t, W_t, \bar{W}_t, W_t^f, P_t, \varepsilon_t^c, B_t, N_t, H_t, \omega_t, L_t, l_t, w_t, r_t, H_t^R, s_t\right\}_{t=0}^{\infty}$$

such that the following hold for any given evolution of $\{y_t, Q_t\}_{t=0}^{\infty}$:

i. New households enter the market optimally so that (7) and (22) are satisfied;

ii. The trade surplus in the housing market is strictly positive, *i.e.* (21) holds;

iii. The value of being a home-owner satisfies (17); the value of a vacant house satisfies (19) and the value of a buyer satisfies (20);

iv. The owner of a vacant house is indifferent between putting the unit up for rent and for sale:

$$\tilde{V}_t = r_t - m + \beta E_t \tilde{V}_{t+1} = V_t; \tag{24}$$

v. The market for rental housing clears:

$$H_t^R = B_t + F_t; \tag{25}$$

vi. Given house prices and construction wages, there is free entry into construction:

$$\beta E_t \tilde{V}_{t+1} \leq \frac{w_t}{\phi} + q_t, \qquad H_{t+1} \geq H_t, \tag{26}$$

where the two inequalities hold with complementary slackness;

vii. The market for construction labour clears:

$$L_t = (N_t + B_t + F_t)\ l_t; \tag{27}$$

viii. Boundary conditions on value functions rule out bubbles: $\lim_{T\to\infty} \beta^T E_t J_{t+T} = 0$;

ix. Market makers design sub-markets to solve (20) subject to (19).

x. All active sub-markets must have $\gamma(\omega_t)$, $\lambda(\omega_t) \in (0,1)$ to guarantee entry of both buyers and sellers.

4.2 Characterizing the equilibrium

Free entry of sellers implies that all sub-markets must offer them the same payoff. It follows that (19) determines a relationship between the listed price and the market tightness that must be satisfied by all active sub-markets:

$$\gamma(\omega_t(P)) = \frac{V_t - \beta E_t \tilde{V}_{t+1}}{P - \beta E_t \tilde{V}_{t+1}}. \tag{28}$$

Thus, it is sufficient to index sub-markets by the price listed in them. >From (28) it is clear that the probability of a vacant house being sold in a given period is lower in sub-markets with higher prices. This in turn implies that an individual house's time-on-the-market is positively related to its price, as is extensively documented by empirical studies.[11] Thus, the market-maker's optimization problem can be expressed as

$$\max_P \quad \{\lambda(\omega_t(P))\,(\beta E_t J_{t+1} - P - \beta E_t W_{t+1})\}, \tag{29}$$

where,

$$\omega_t(P_t) = \gamma^{-1}\left[\frac{V_t - \beta E_t V_{t+1}}{P_t - \beta E_t V_{t+1}}\right] \tag{30}$$

[11] The positive correlation between time-on-the-market and transaction price is found in the work of Forgey et al (1996), Kang and Gardner (1989), Leung, Leong and Chan (2002), Anglin et al (2003), and Merlo and Ortalo-Magné (2004), among others.

as implied by (28). We have the following proposition:

Proposition 1. *In a competitive search equilibrium, only one sub-market with price P_t and tightness ω_t is active. In that market the share of the surplus from house transactions that accrues to the buyer is equal to the elasticity of the number of matches with respect to the number of buyers*[12]*:*

$$s(\omega_t) = \epsilon(\omega_t) \tag{31}$$

With competitive search, the shares of the total match surplus accruing to buyers and sellers in a transaction depends on the tightness of the market. In particular, house prices satisfy:

$$P_t = (1 - s(\omega_t))\beta \left(E_t J_{t+1} - E_t W_{t+1}\right) + s(\omega_t)\beta E_t V_{t+1}, \tag{32}$$

where using (31) and (11), $s'(\omega) \leq 0$.

Combining (19) and (24), it is apparent that, in equilibrium, the stocks of rental and ownable housing must be such that the return to renting a house for a period equals the expected gain from holding it vacant for sale:

$$r_t - m = \gamma_t \left(P_t - \beta E_t V_{t+1}\right). \tag{33}$$

We focus on equilibria in which construction of houses is always positive, that is, $H_{t+1} > H_t$.[13] It follows, then, from (5), (82) and (27) that the quantity of new housing constructed in period t is given by

$$H_{t+1} - H_t = \phi \left(N_t + B_t + F_t\right) \zeta w_t^{\eta} \tag{34}$$

Similarly, with $H_{t+1} > H_t$ it follows from (26) and (24) that

$$H_{t+1} - H_t = \phi^{1+\eta} \left(N_t + B_t + F_t\right) \zeta \left(\beta E_t V_{t+1} - \bar{q}\right)^{\eta}. \tag{35}$$

To obtain a stationary representation of the economy, we normalize the state variables by the total population, Q_t. We use lower case letters to represent *per capita* (*i.e.* per household) values. It follows that the dynamic equations for *per capita* renters, buyers, owners and houses, respectively, can be written as

$$(1+\mu)f_t = (1-\psi)\,\mu G\left(\bar{W}_t\right) + (1-\pi_f)f_{t-1} \tag{36}$$

$$(1+\mu)b_t = \psi\mu G\left(\bar{W}_t\right) + \left[1 - \lambda(\omega_{t-1})\right] b_{t-1} + \theta(1-\pi_n)n_{t-1} \tag{37}$$

$$(1+\mu)n_t = (1-\theta)(1-\pi_n)n_{t-1} + \lambda(\omega_t)b_{t-1} \tag{38}$$

$$(1+\mu)h_{t+1} = h_t + \zeta\phi^{1+\eta}\left(n_t + b_t + f_t\right)\left(\beta E_t V_{t+1} - q_t\right)^{\eta}. \tag{39}$$

[12]This result is a special case of that derived by Moen (1997).

[13]It is straightforward to show that this will be the case in any competitive search equilibrium for an economy with sufficient population growth.

It also follows that the tightness of the housing market is given by

$$\omega_t = \frac{b_t}{h_t - b_t - f_t - n_t}, \tag{40}$$

and that market-clearing in the rental market implies

$$h_t^R = b_t + f_t \tag{41}$$

5 Deterministic steady-state

We now consider a steady-state in which general non-construction income *per capita* is constant and normalized to unity: $y_t = 1$. In this setting $\bar{W}$ and ω are constant, and (36) implies that the normalized measure of renters is

$$f^* = \frac{(1-\psi)\mu G(\bar{W}^*)}{\mu + \pi_f}. \tag{42}$$

Similarly, from (37) the measure of buyers each period satisfies,

$$b^* = \frac{\psi\mu G(\bar{W}^*)}{\mu + \lambda(\omega^*) - \frac{\theta(1-\pi_n)\lambda(\omega^*)}{\mu+\pi_n+\theta(1-\pi_n)}}, \tag{43}$$

(38) implies that the steady-state fraction of the total population located in the city is

$$n^* = \frac{\lambda(\omega^*)}{\mu + \pi_n + \theta(1-\pi_n)} b^*, \tag{44}$$

and (39) yields that the housing stock per capita satisfies

$$h^* = \frac{\zeta\phi^{1+\eta}\left(n^* + b^* + f^*\right)}{\mu}\left(\beta V^* - \bar{q}\right)^{\eta}. \tag{45}$$

Lemma 2: *In the steady-state, there exists a negative "supply-side" relationship between the value of a house for sale and market tightness:*

$$V^* = V^S(\omega^*) = \frac{1}{\beta}\left[\frac{\mu}{\zeta\phi^{1+\eta}}\left(1 + \frac{\psi\left(\mu + \pi_f\right)}{A\gamma\left(\omega^*\right) + B\omega^*}\right)\right]^{\frac{1}{\eta}} + \frac{\bar{q}}{\beta}, \tag{46}$$

where $A = \frac{\mu+\psi\pi_f+(1-\psi)\pi_n}{\mu+\pi_n+\theta(1-\pi_n)}$ *and* $B = \mu + \psi\pi_f$.

The relationship described in (46) can be interpreted as follows. As the value of vacant housing rises, new construction is stimulated and more houses become available for sale. This drives down the ratio of buyers to houses for sale, ω.

In the steady-state, the values of owners, buyers and vacant houses, the house price and rent, must satisfy the the following set of equations:

$$
\begin{aligned}
J^* &= \bar{u}^H + \pi_n \beta Z + \pi_n \beta V^* + (1-\pi_n)\theta\beta\,(W^* + V^*) + (1-\pi_n)\,(1-\theta)\,\beta J^* &(47)\\
W^* &= \bar{u}^R + \lambda(\omega^*)(\beta J^* - P^*) + (1-\lambda(\omega^*))\beta W^* &(48)\\
V^* &= \gamma\,(\omega^*)\,P^* + (1-\gamma\,(\omega^*))\beta V^* &(49)\\
P^* &= (1-s(\omega^*))\beta\,(J^* - W^*) + s(\omega^*)\beta V^* &(50)\\
r^* &= m + \gamma\,(\omega^*)\,(P^* - \beta V^*) &(51)\\
w^* &= \phi\,(\beta V^* - \bar{q}) &(52)\\
W_f^* &= \bar{u}^R + \pi_f \beta Z + (1-\pi_f)\beta W_f^* &(53)\\
\bar{W}^* &= \psi W^* + (1-\psi) W_f^*, &(54)
\end{aligned}
$$

where $s(\omega^*)$ in (50) is the share of surplus to a buyer expressed in terms of steady-state market tightness, ω^*, $\bar{u}^H = \bar{y} + x(w^*) + z^H - m$ and $\bar{u}^R = \bar{y} + x(w^*) - r^*$. One can solve the first five equations of this system for J^*, W^*, V^*, P^* and r^*. Then the last two equations can be used to determine $\bar{W}^*$ and W_f^*. The above yields another relationship between the value of houses for sale and market tightness:

Lemma 3: *In a steady-state, there exists a positive "demand–side" relationship between the value of a house for sale and market tightness:*

$$
V^* = V^D(\omega^*) = \frac{\gamma\,(\omega^*)\,(1-s(\omega^*))\beta z^H}{(1-\beta)\,[1-\beta(1-\theta)(1-\pi_n)] + (1-\beta+\pi_n\beta)\,\beta\lambda\,(\omega^*)\,s(\omega^*)}. \tag{55}
$$

Intuitively, a higher ratio of buyers to sellers, (*i.e.*, a tighter market), has two effects. Firstly, it increases the rate at which houses sell, γ. For a given selling price, this drives up the value of a vacant house. Secondly, it *lowers* the rate at which buyers find houses, which increases the gain from becoming an owner. This raises the selling price of houses, which also drives up the value of a house for sale. Lemmas 2 and 3 then yield the following proposition:

Proposition 2. *There exists a unique steady-state equilibrium if the following condition holds:*

$$
\frac{1}{\beta}\left[\frac{\mu}{\zeta\phi^{1+\eta}}\left(1+\frac{\psi\,(\mu+\pi_f)}{A+B\bar{\omega}}\right)\right]^{\frac{1}{\eta}} + \frac{\bar{q}}{\beta} < \tag{56}
$$

$$
\frac{(1-s(\bar{\omega}))\beta z^H}{(1-\beta)\,[1-\beta(1-\theta)(1-\pi_n)] + (1-\beta+\pi_n\beta)\,\beta\lambda\,(\bar{\omega})\,s(\bar{\omega})}
$$

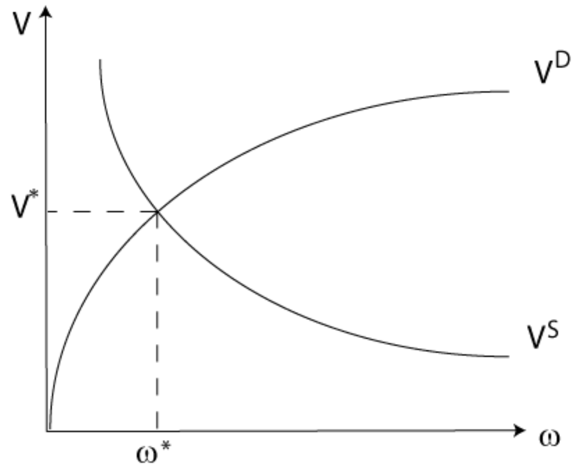

Figure 3: Steady State Equilibrium

where $\bar{\omega} = \gamma^{-1}(1)$.

Figure 3 depicts, in (V, ω) space, the existence of a steady-state at the intersection of (46) and (55). The existence of the steady-state depends on the values of a variety of parameters which together ensure that the surplus from housing transactions is positive given that new houses are always being built in the city. Intuitively, a land price, $\bar{q}$, that is too high, for example, or an ownership premium z^H that is too low may cause the surplus from housing transactions to become negative. In our calibration below, the parameters are such that that this condition is never violated.

6 Calibration

6.1 The baseline parameterization

In this section we discuss the calibration scheme. In our baseline model, we suppose the matching function takes a simple Cobb-Douglas form[14] given by

$$M = \kappa B_t^{\delta} S_t^{1-\delta}, \tag{57}$$

[14] This specification violates Assumption 1 (part i) as $\lambda(\omega)$ is not bounded above by 1. We deal with this in our calibration by choosing κ sufficiently small that this constraint is never binding in any of our experiments.

where $\kappa > 0$ and $\delta \in (0, 1)$. In this case, $\epsilon(\omega) = \delta$ so the share of the surplus accruing to each party in a housing transaction is constant, and in particular, is not sensitive to the tightness of the market.[15]

Table 3 gives the parameter values we set for our baseline calibration. Numbers above the line are set to match the indicated targets directly. Values below the line are set jointly so that the specified steady state values generated by the model match the given targets. For illustrative purposes, however, in the table we associate these parameters with a specific target for which it is particularly relevant. We define a period to equal one quarter. We set β to reflect an annual interest rate of 4% and μ is chosen to match annual population growth during the 1990s.[16] We normalize $\bar{y} = 1$. Thus, present values and prices are all measured relative to the steady-state *per capita* income.

The parameter ϕ represents the labor productivity of the construction sector. The rato of permits issued in the US each quarter to the numbers of employees in residential construction is approximately 0.1 on average. If the average working week is 35 hours this amounts to about 400 hours per quarter, which yields a number of permits produced per hour worked equal to about 0.00025 (this amounts to 4000 man hours per house).

We set π_f to match the annual fraction of renters that move between counties; about 12% on average according to the Census Bureau. Similarly, π_n is set to match the annual fraction of home-owners that move between counties (12%) and θ the fraction of owners that move but do not change counties (60%). Note that Dieleman, Clark and Deurloo (2000) estimate an overall housing turnover rate of 8% annually (see also Caplin and Leahy, 2008), which is consistent with our quarterly value of $\pi_n + (1 - \pi_n)\theta \simeq 0.02$.

An important parameter is the elasticity of the labour supply in construction, η. There is much debate over aggregate labour supply elasticities in general which indicates that it is important to distinguish between variation in labour supply on the intensive and extensive margins. In the case of construction labour supply most evidence suggests that it is much more elastic than other types of labour. In our model, this elasticity determines the elasticity of new housing construction with respect to the price of housing. Most recent estimates of this using annual data, both at the national level and at the city levels suggest that it is quite large. Early estimates range from about 1.6 to 5 at the national level (see Topel and Rosen (1988), Poterba (1991) and Blackley (1999)) and up to 25 in some cities. For our

[15] This implies that the competitive search model is effectively equivalent to a random search model with house prices determined by Nash Bargaining in which the Hosios condition (buyers' bargaining weight equal to δ) is imposed.

[16] Population growth has slowed somewhat in recent years.

baseline calibration, we set the value equal to the median elastcity for the 45 cities estimated by Green, Malpezzi and Mayo (2005) which is $\eta = 5$. However we consider the sensitivity of our results to variations in its value.

A related parameter is the price elasticity of land supply, ξ. In our model, this parameter effectively relates movements in the price of housing to the total stock of housing (as opposed to new construction). Saiz (2010) studies the relationship between house prices and the stock of housing based on a long difference estimation between 1970 and 2000 for 95 US cities.[17] In particular, by instrumenting using new measures of regulatory restrictions and geographical constraints, he is able to infer city level price elasticities that vary due to natural and man-made land constraints. His supply elasticity estimates vary from 0.60 to 5.45 with a population–weighted average of 1.75 (2.5 unweighted). We therefore set $\xi = 1.75$ in our our baseline, but again we will consider the sensitivity of our results to variations in its value.

The steady-state unit price of land, $\bar{q}$, is set so that the relative share of land in the price of housing is 30% (see Davis and Palumbo, 2008 and Saiz, 2010). The average price of a house is approximately three times annual income or 12 times quarterly income. This implies a ratio of the land price to income of $0.3 \times 12 = 3.6$.

We choose the remaining parameters so that several key steady state statistics match their average counterparts in U.S. data. In particular, the value of ψ is calibrated so that the average fraction of households that rent in the steady-state is 32%:

$$\chi = \frac{b + f}{n + b + f} = 0.32. \tag{58}$$

The maintenance cost m is chosen so that the rent is just under 14% of median income. Note that the income of the average renter in the US is less than half of that of the average owner, reflecting the fact that the characteristics of owners and renters differ systematically. On average, a renter in the U.S. allocates 24% of his after-tax income to rent (see Davis and Ortalo-Magne, 2009). Since in our model all agents are homogeneous, we target the ratio of rent to the median income of owners and renters, which is somewhat lower (see Head and Lloyd-Ellis, 2010 for details). As described earlier, we assumed that the maintenance cost is just enough to offset depreciation. If d denotes the rate of depreciation, it follows that the implicit flow utility derived from owning a house is given by

$$z^h = \frac{(1 - \beta)\, m}{d}. \tag{59}$$

[17] In this sense, the estimated relationship picks up long term dynamics associated with ξ. In contrast, the estimates of Green, Malpezzi and Mayo (2005) relate to short run dynamics assoitade with η.

Table 3: **Baseline Calibration Parameters: Steady State**

Parameter	Value	Target
β	0.99	Annual real interest rate = 4%
μ	0.003	Annual population growth rate = 1.2%
ϕ	0.00025	Quarterly permits/construction employment (hours)
π_f	0.03	Annual mobility of renters = 12%
π_n	0.008	Annual mobility of owners = 3.2%
θ	0.012	Fraction of moving owners that stay local = 60%
η	5	Median price elasticity of new construction = 5
ξ	1.75	Median price elasticity of land supply = 1.75
$\overline{q}$	3.6	Average land price-income ratio
ψ	0.43	Fraction of households that rent = 32%
m	0.02	Average rent to average income ratio, $r^* = 0.137$
z^h	0.025	Zero net-of-maintenance depreciation
κ	0.76	Vacancy rate = 2%
δ	0.09	Months to sell = months to buy
$\zeta^{\frac{1}{\eta}}$	800	$P^* = 12$

Harding et al. (2007, p. 212) estimate the gross-of maintenance rate of depreciation for a house of median age in the US to be about 3% annually ($d = 0.008$).[18] It follows from (59) that $z^h = 0.025$.

Given the other parameters of the model, those of the matching function, κ and δ, jointly determine the steady-state values of the vacancy rate and market tightness. Average vacancy rates for the US economy and by MSA are available from the Census Bureau's Housing Vacancy Survey (HVS). In our model, houses that are vacant in equilibrium are designated to be for sale. The HVS distinguishes the category "vacant units which are for sale only". In 2000, for example, this category constituted 1% of the overall housing stock. Since owned homes constituted approximated two-thirds of the housing stock, this corresponds to a home-owner vacancy rate of about 1.5%.[19] Housing units that are in the category "vacant units for rent" actually consist, however, of vacant units offered for rent only and those offered *both* for rent and sale. In 2000, for example. houses both for rent and sale constituted a further 2.6% of the overall housing stock.[20] In addition, only about half

[18] The resulting actual depreciation rate is rather less than 1% precisely because maintenance is undertaken.

[19] This number is close to the average over the period 1980-2008. However, more recently homeowner vacancy rates have exceeded 2.5%

[20] Again, since rental units constitute about a third of the housing stock, this corresponds to a rental vacancy rate of about 8%.

of all vacant units are included in either of the two categories. The remainder include units that are held off the market for various other reasons. For example, this category includes vacant units located in a multi-unit structure which is for sale.

In our model, vacant units are technically available for rent in the subsequent period, so it makes sense to include those vacant units offered for both rent and sale in our measure of vacancies. For this reason, we assume an additional 1% of the housing stock is vacant and for sale, so that $v = 0.02$. Again, we will consider the sensitivity of our results to alternative values of v.

We assume that, in steady-state, the time taken to sell a house is equal to the time taken to buy, so that $\omega^* = 1$. It follows that the overall vacancy rate is

$$v = \frac{h - b - f - n}{h} = \frac{b}{b + b + f + n}. \tag{60}$$

Using (44), this implies that

$$\begin{aligned} \gamma^* &= \lambda^* = (\mu + \pi_n + \theta(1 - \pi_n)) \frac{n^*}{b^*} && (61) \\ &= (\mu + \pi_n + \theta(1 - \pi_n)) (1 - \chi) \frac{1 - v}{v}. && (62) \end{aligned}$$

Given the values of μ, θ and π_n from Table 3 and our targets for χ and v, the implied value is $\gamma = 0.76$, leading to an average time for a house to be on the market of just under 4 months. This may seem somewhat high given that according to the National Association of Realtors, the time taken to sell a typical house is about 2 months.[21] Their estimate of "time on the market", is, however, somewhat misleading because houses are sometimes strategically de-listed and quickly re-listed in order to reset the "days on market" field in the MLS listing. In their detailed analysis of the housing market in 34 Cook county (Illinois) suburbs over the period 1992-2002, Levitt and Syverson (2008) compute time-to-sale by "summing across all of a house's listing periods that are separated by fewer than 180 days." They estimate that the average time on the market for a house that eventually sells is 94 days (3.07 months). Moreover, in their sample of 127,000 houses, 22% of houses put up for sale never sell. In less active markets it is likely that the time on the market is even longer.

Given the other parameters, ζ, is chosen so that the price of a house is three times annual income or 12 times quarterly income. Note that the value of δ required to hit these targets,

[21] There are varying estimates of the time to buy and the time to sell. Diaz and Jerez (2008) use 2 months based on a report from the National Association of Realtors. Piazzesi and Schneider (2009) suggest using 6 months. Anglin and Arnott (1999) report estimates of up to 4 months.

given the other parameters, implies that over 90% of the surplus from housing transactions goes to the seller. With competitive search, this also implies that

The dynamics of the model depend crucially on the shape of $G(\cdot)$ in the vicinity of ε^c since this determines the responsiveness of new entrants to changes in the value of search. In the steady-state, however, the only variable that depends on $G(\cdot)$ is the measure of searching households per capita, b^*. This is not something that is likely to be directly observable and so the parameters determining the relevant characteristics of $G(\cdot)$ are not possible to identify in this way. Our approach is to use our estimate of the relative standard deviation of population growth in response to income shocks from Table 1 to calibrate the elasticity of $G(\cdot)$, evaluated at ε^c:

$$\alpha = \frac{\varepsilon^c G'(\varepsilon^c)}{G(\varepsilon^c)}. \tag{63}$$

6.2 An alternative economy with no housing market frictions

It is useful to compare our baseline results to those from an economy with no frictions in the housing market. In this economy new entrants can either rent or purchase a house immediately and move in. Since households derive more utility from owning and construction costs are the same, only pure renters will choose to rent in equilibrium. With no frictions, the dynamic system can be written as

$$\begin{aligned}
(1+\mu)f_t &= (1-\pi_f)f_{t-1} + (1-\psi)\,\mu G\left(\bar{W}_t\right) && (64)\\
(1+\mu)n_t &= (1-\pi_n)n_{t-1} + \psi\mu G\left(\bar{W}_t\right) && (65)\\
(1+\mu)h_{t+1} &= h_t + \zeta\phi^{1+\eta}\,(n_t+f_t)\,(\beta E_t P_{t+1} - q_t(h_t/h_t^*))^\eta && (66)\\
h_t &= n_t + f_t && (67)\\
J_t &= u_t^H + \beta\pi_n\,(Z + E_t P_{t+1}) + \beta(1-\pi_n)E_t J_{t+1} && (68)\\
W_t^f &= u_t^R + \pi_f\beta Z + (1-\pi_f)\,\beta E_t W_{t+1}^f && (69)\\
\bar{W}_t &= \psi\,(J_t - P_t) + (1-\psi)W_t^f && (70)\\
r_t &= m + P_t - \beta E_t P_{t+1} && (71)
\end{aligned}$$

The economy with no frictions is comparable in several aspects to the model discussed by Glaeser et al. (2010). An important difference, however, is that they assume the outside alternative to living in a the city yields a homogeneous payoff. This effectively implies immediate entry of buyers until the price of housing adjusts to keep the value of entering constant. This tends to generate high variance in both prices and construction in response to income shocks. In our model there is a distribution of alternatives, so that as households enter the critical outside value rises, helping to stem the flow of additional households into the city.

For the model with no frictions, the parameters are chosen as in Table 6 except that we re-set ψ, m, z^h and ζ so that the steady state of the frictionless model matches the relevant targets.[22] In the stationary equilibrium with no search frictions, the price is given by

$$P^* = \frac{1}{\beta}\left[\frac{\mu}{\zeta\phi^{1+\eta}}\right]^{\frac{1}{\eta}} + \frac{\bar{q}}{\beta}. \qquad (72)$$

We use (72) to derive the value of ζ such that $P^* = 12$.

7 Qualitative Dynamics

In order to study the dynamics of the model, we linearize the dynamic systems for both the baseline and economy with no frictions in neighborhoods of their respective deterministic steady–states. For our calibrated parameters, the resulting systems of first–order linear difference equations satisify the conditions for saddle-path stability. We solve numerically for the implied local dynamics driven by stochastic movements in y_t using a Generalized Schur decomposition due to Klein (2007).

For now we assume that the process followed by the log of non-construction income, $\ln y_t$, is a simple AR(1) process with persistence parameter $\rho = 0.98$ and innovation standard deviation $\sigma_\varepsilon = 0.01$. We use this example to illustrate that the model's qualitative dynamics are not driven by the hump-shaped dynamics of incomes observed in the data. Note that in a framework with no frictions, persistence of this form cannot translate into momentum (*i.e.* postively auto-correlated growth) in asset (*i.e.* house) prices. As we show below, without frictions the impulse response of house prices simply inherits the shape of that for local general labour earnings, which may be seen as reflecting local housing dividends.[23] In the baseline economy, due to the frictions present, this is not generally the case. Rather, the theory will produce momentum in house prices even in cases when housing dividends exhibit none.

[22] Obviously, the parameters of the matching function κ and δ are not relevant in this case.

[23] Glaeser et al. (2010) consider an ARMA(1,1) process for u_t, but house prices still mean revert very quickly.

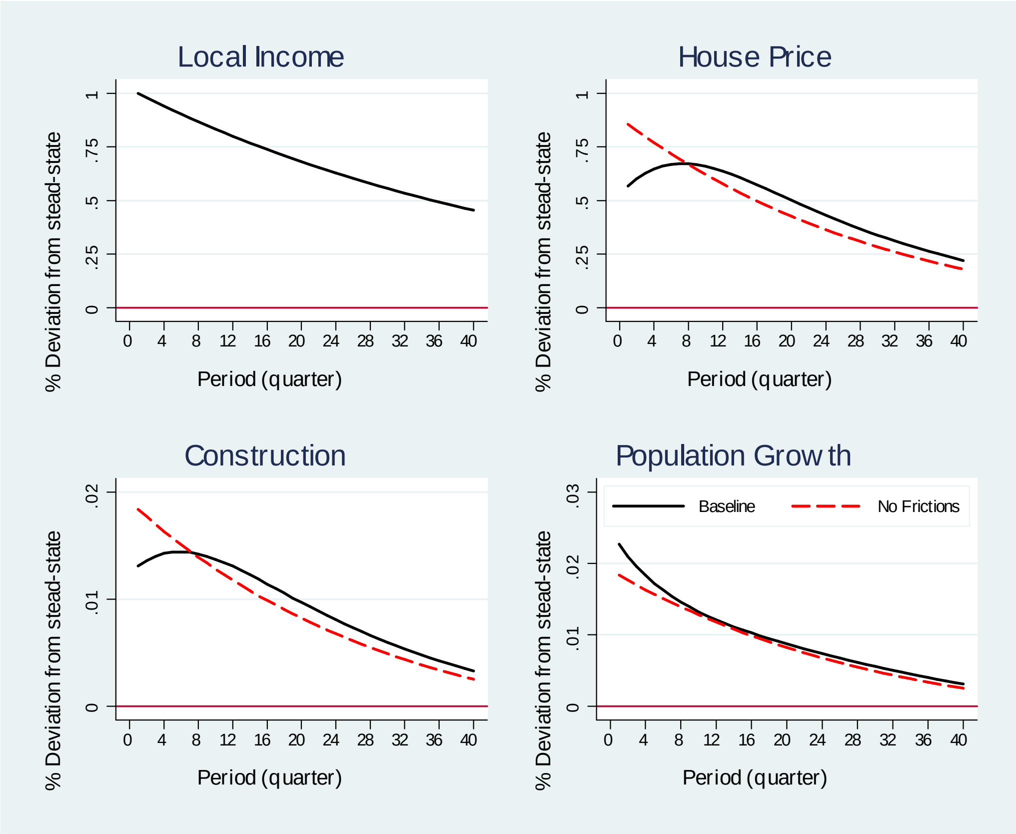

Figure 4: Impulse response functions for AR(1) shock process

The implied impulse response functions (IRF's) following a shock to income are depicted in Figures 4 and 5. Each panel in Figure 4 depicts the IRFs for incomes, house prices, construction rate and population growth relative to trend, for the economies with and without search frictions. As may be seen, the model with frictions can generate impulse responses that are qualitatively very similar to those illustrated in Figure 1. Clearly, in both scenarios the housing market friction acts so as to generate momentum in house prices (*i.e.* a "hump–shaped" IRF). In contrast, the economy with no frictions generates no momentum in prices, in spite of the fact that there is substantial momentum in the housing stock.[24]

There are two forces at play generating serial correlation in house price appreciation in the economy with search frictions. The initial rise in the value of living in a city generates an immediate increase in search activity as households enter. It takes time, however, for buyers to find a house through the matching process. Also, although sales and the probability

[24]In this economy, of course, "tightness" is always equal to one.

of selling rise immediately, construction of new housing takes time to respond. Thus, as can be seen in the lower left panel of Figure 5, the ratio of buyers to sellers ("tightness") rises. Even if the value of searching subsequently declines (due to subsequent reductions in entry and increased construction), the number of searchers continues to rise and the stock of vacancies to decline for some time. Consequently, the ratio of buyers to sellers continues to rise in the short term, further driving up the rate at which houses sell and hence the value of a vacant house. Since, in equilibrium, the house price partly reflects this value, it rises, too. Eventually, the stock of buyers begins to fall as they find and purchase houses, and the decline in vacancies slows (and eventually reverses) as construction rates catch up. This causes the ratio of buyers to sellers to decline and, in anticipation of this, the house price eventually reverts back towards its steady-state value.

A second factor relates to the interaction of the markets to rent and own. Given that prices are expected not only to rise initially in response to the shock, but also to rise in the future due to continued increases in market tightness, there is an increase in the measure of unoccupied houses which are rented rather than put up for sale immediately (see the bottom right panel of Figure 5). This increased relative supply of rental housing keeps the rental rate from rising too rapidly and reinforces the continued entry of buyers which drives the subsequent price appreciation. This self-fulfilling effect tends to magnify the underlying momentum in house prices.

8 Quantitative Analysis

8.1 Baseline Calibration

In the previous section we used our calibrated model to illustrate the role played by search frictions in generating momentum in house prices in our model. We now consider the quantitative implications of our theory by comparing the model's output under an empirically relevant process for local earnings with the stylized facts reported in Section 2.

Mechanically, this entails replacing the arbitrary AR(1) process for $\ln y_t$ considered in the previous section with one based on our empirical findings in Section 2. This is complicated, however, by the mismatch between the frequency of available city-level income data and the period length assumed in our calibrated model. The former, which was used in our data analysis above, is available annually, whereas the baseline calibration assumes that each period is a quarter. While the period length could, of course, be increased to one year in

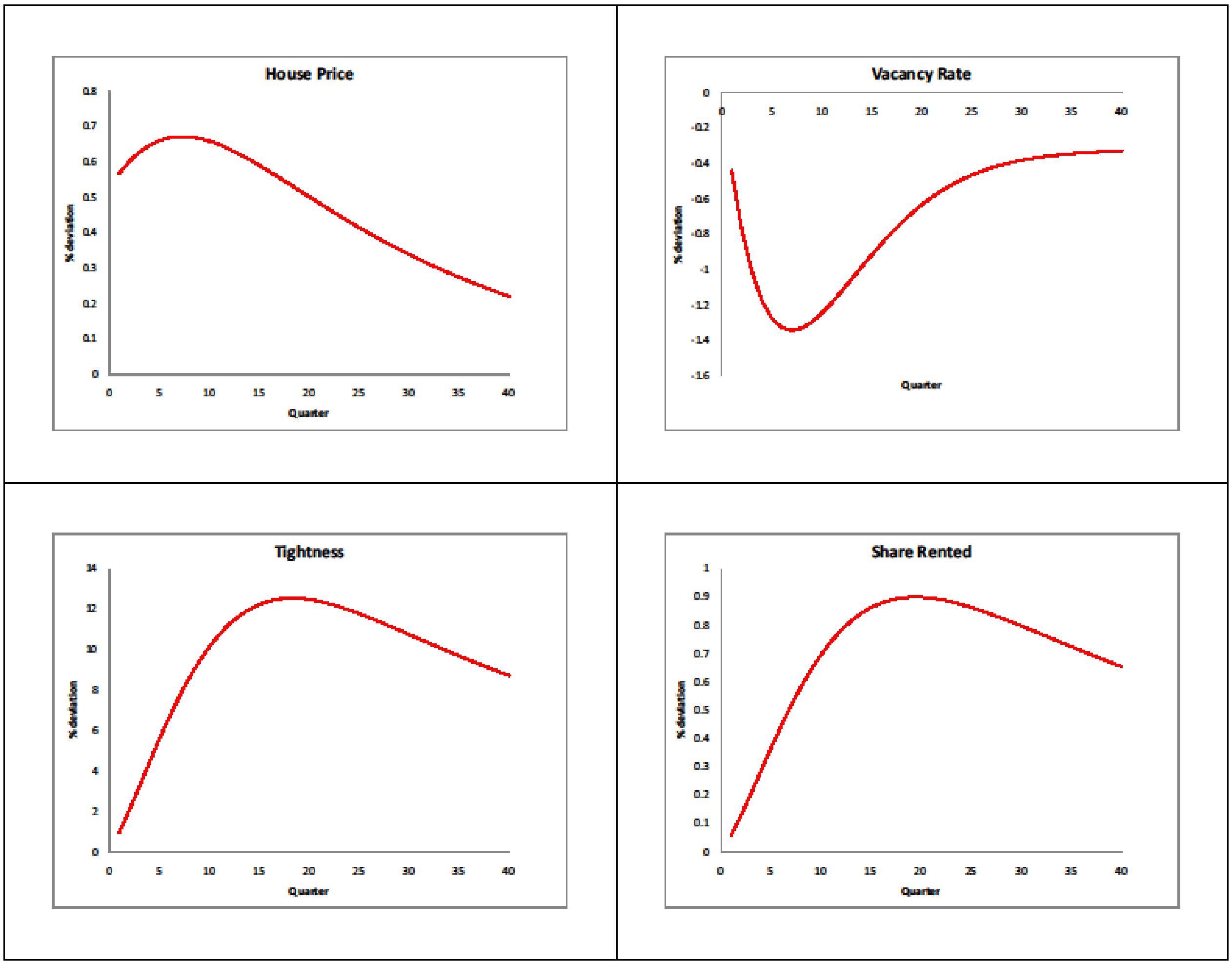

Figure 5: Impulse responses for AR(1) shock process

the model, this would be restrictive as it would require that houses for sale remain vacant for at least a year, and this is clearly counterfactual.

The approach taken here is instead to derive a quarterly process for income that shares certain key properties at annual frequencies as the process estimated in our data panel VAR in Section 2. For a full description of procedure by which we constructed this process, see the appendix. Here, in Table 4, we report selected moments for both the estimated and constructed processes; specifically, the variance and autocorrelation at annual frequencies for the first four years:

Table 4: Calibration of Earnings Process

	σ_y	ρ_1	ρ_2	ρ_3	ρ_4
Data	0.02	0.25	0.02	-0.05	-0.08
Model	0.02	0.21	-0.03	-0.03	-0.03

Using the constructed process and the linearized model calibrated as above, we generate sample paths for the key variables of the model and use these to construct "annual" time series for the economy. Moments for these series, along with the corresponding moments for the U.S. economy, are presented in Tables 5 and 6.

Consider first the moments in Table 5, which contains both the standard deviations of house prices, the growth of housing stock, and the city population growth with local income, and the correlations of those variables with local income. The first column of the table report the numbers from our analysis of the data in Section 2. The second column reports the results for our baseline calibration and the third column reports the results for the model without frictions. The remaining columns contain results for the model with frictions, but considers the implications of alternative choices of the specified parameter. In each case the other targets listed in Table 3 remain fixed. This implies that some parameters (i.e. ψ, m, z^h, δ, κ, ζ and α) may be adjusted to match these targets. Table 6 contains the first four autocorrelation coefficients for price appreciation, housing growth and population growth for each of these cases.

Table 5: Volatilities and Co-movements: Calibrated Earnings Shocks

Moment	US Cities	Interior	Coastal	Sunbelt	Baseline	No frictions
σ_p/σ_y	1.35	0.92	2.11	1.19	0.67	0.90
σ_h/σ_y	0.11	0.15	0.04	0.19	0.17	0.17
σ_n/σ_y	0.18	0.15	0.10	0.32	0.18	0.18
σ_{py}	0.80	0.80	0.78	0.71	0.97	0.91
σ_{hy}	0.46	0.44	0.55	0.52	0.35	0.39
σ_{ny}	0.71	0.53	0.43	0.84	0.37	0.39

Table 6: Autocorrelations: Calibrated Earnings Shocks

Moment	US Cities	Interior	Coastal	Sunbelt	Baseline	No frictions
ρ_1^p	0.71	0.62	0.69	0.82	0.23	-0.01
ρ_2^p	0.30	0.25	0.23	0.48	0.08	-0.04
ρ_3^p	0.03	0.04	-0.04	0.18	0.0	-0.04
ρ_4^p	-0.11	-0.07	-0.14	-0.01	-0.05	-0.04
ρ_1^h	0.89	0.91	0.79	0.89	0.94	0.89
ρ_2^h	0.66	0.74	0.37	0.64	0.85	0.79
ρ_3^h	0.41	0.56	-0.02	0.37	0.76	0.70
ρ_4^h	0.21	0.43	-0.26	0.13	0.67	0.61
ρ_1^n	0.71	0.75	0.81	0.67	0.88	0.89
ρ_2^n	0.45	0.60	0.67	0.39	0.78	0.79
ρ_3^n	0.25	0.49	0.63	0.15	0.70	0.70
ρ_4^n	0.21	0.43	0.63	0.00	0.62	0.61

8.2 Sensitivity Analysis

Here we consider the sensitivity of our results to changes in the values of key parameters

Table 7: Volatilities and Co-movements: Sensitivity Results

		Labour supply elasticity		Land supply elasticity		Vacancy Rate		Entry (demand) elasticity	
Moment	Baseline	$\eta = 2$	$\eta = 20$	$\xi = .5$	$\xi = 5$	$v = .01$	$v = .03$	$\alpha = 5$	$\alpha = 20$
σ_p/σ_y	0.67	1.71	0.16	0.74	0.65	0.83	0.53	0.51	1.52
σ_h/σ_y	0.17	0.17	0.17	0.17	0.17	0.175	0.166	0.14	0.34
σ_n/σ_y	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.15	0.35
σ_{py}	0.97	0.94	0.73	0.97	0.97	0.96	0.96	0.97	0.94
σ_{hy}	0.35	0.29	0.42	0.38	0.34	0.39	0.30	0.35	0.34
σ_{ny}	0.37	0.32	0.39	0.39	0.36	0.37	0.37	0.38	0.34
α	6.5	12	5	6.8	6.4	7.0	6.1		

Table 8: Autocorrelations: Sensitivity Results

Moment	Baseline	Labour supply elasticity $\eta = 2$	$\eta = 20$	Land supply elasticity $\xi = .5$	$\xi = 5$	Vacancy Rate $v = .01$	$v = .03$	Entry (demand) elasticity $\alpha = 5$	$\alpha = 20$
ρ_1^p	0.23	0.07	0.48	0.19	0.24	0.11	0.33	0.33	0.05
ρ_2^p	0.08	0.02	0.04	0.07	0.07	-0.02	0.18	0.12	0.00
ρ_3^p	0.0	-0.01	-0.10	-0.01	-0.01	-0.06	0.06	-0.01	-0.03
ρ_4^p	-0.05	-0.03	-0.11	-0.04	-0.06	-0.06	-0.01	-0.06	-0.04
ρ_1^h	0.94	0.94	0.94	0.93	0.94	0.91	0.95	0.94	0.92
ρ_2^h	0.85	0.87	0.83	0.84	0.85	0.81	0.88	0.86	0.84
ρ_3^h	0.76	0.80	0.71	0.74	0.76	0.71	0.80	0.76	0.75
ρ_4^h	0.67	0.74	0.61	0.65	0.67	0.63	0.72	0.67	0.68
ρ_1^n	0.88	0.88	0.89	0.87	0.88	0.88	0.88	0.88	0.88
ρ_2^n	0.78	0.79	0.78	0.77	0.78	0.78	0.78	0.78	0.79
ρ_3^n	0.70	0.72	0.69	0.68	0.70	0.69	0.69	0.69	0.72
ρ_4^n	0.62	0.67	0.60	0.60	0.62	0.61	0.62	0.61	0.65

From these tables it is clear that the baseline calibration understates both the relative volatility of house prices and its first-order serial correlation.

Momentum in house prices requires search frictions, and even this may not be sufficient. The theory has a difficult time generating momentum in house prices to the degree observed, and this hinges on the responsiveness of entry to changes in local earnings. The baseline calibration generates to little momentum in house prices. Momentum approaching that observed can be obtained for lower elasticities of entry, but this comes at the expense of reduced volatility of all three variables in response to local earnings shocks.

8.3 Robustness

We now consider two alternative environments to assess the robustness of our findings. Qualitatively, none of these affect our results significantly, and so in that respect we find our results to be be very robust. Quantitatively, we find that our findings with respect to both volatility and the co-movements among the variables that we have considered are very robust. With regard to the degree of price momentum, however, our results are to some extent sensitive to fluctuations in the share of the surplus accruing to buyers and sellers in housing transactions.

8.3.1 Generalized urn-ball matching

We first consider an alternative matching function, for which the equilibrium shares of the surplus received by the buyers and sellers are not constant. Specifically, consider

$$\mathcal{M}(B,S)=S\varphi(1-e^{-\tau\frac{B}{S}}). \tag{73}$$

If $\tau=1$, the matching probabilities are equivalent to the "urn-ball" matching process assumed by Diaz and Jerez (2010). Here, we consider a somewhat more general form in order to calibrate the model to the same targets as for our baseline calibration above. This generalization could be motivated along the lines of Albrecht, Gauthier, and Vroman (2003), where τ denotes the average number of applications to purchase made per period and φ indexes the effort required to process each application. Given the other parameters of our baseline calibration, the matching function parameter values needed to achieve the same targets as above are $\varphi=0.792$ and $\tau=3.295$.

The surplus accruing to the buyer for this matching function is

$$s(\omega)=\epsilon(\omega)=\frac{\theta\omega}{e^{\theta\omega}-1}, \tag{74}$$

which is decreasing in market tightness, ω. That is, as the ratio of buyers to sellers increases, the share received by buyers falls.

Figure 6 illustrates the effect of a shock to general earnings on house prices and market tightness in both the baseline economy and in that with generalized urn-ball matching. Clearly, the form of the matching function has a significant effect on price momentum, and this can be traced to the effect of an increase in local earnings on the initial response of prices and the extent of entry. In the urn-ball matching case, the share of the surplus received by the buyer falls as tightness rises. Thus, the initial price increase in prices is greater, and this discourages entry as can be seen in the response of tightness. Since tightness responds by less, prices peak earlier and return to their steady-state level faster than with Cobb-Douglas matching.

Note that while the form of the matching function has a significant effect on momentum, it has little effect on either volatility or the co-movements among house prices, local earnings, tightness, and construction rates.

8.3.2 Mismatched owners remain in their houses

In our basic model, we assumed that mis-matched owners put their houses up for sale immediately and become renters. In fact

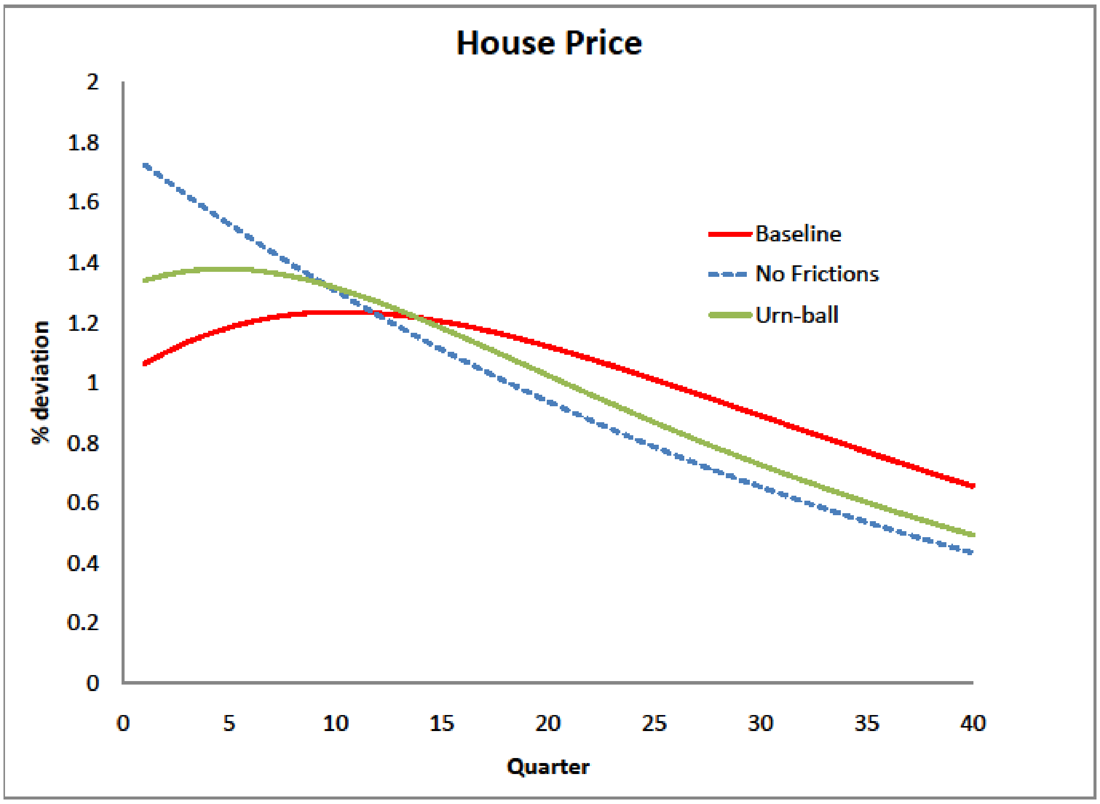

Figure 6: Alternative matching functions

Proposition 3: *In equilibrium, mismatched owners are indifferent between*

(1) putting up their house for sale or rent immediately and renting while searching and

(2) remaining in their current house while searching, then putting their vacant house up for sale once they match with a new one.

Suppose instead that they remain in their houses until they find a new house, then put their vacant house up for sale. Since owners who become mis-matched are indifferent between the two alternatives, the value functions remain unchanged. Let $\tilde{n}_t$ denote these mis-matched owners. Then the flows of households between states is now described by (36) and

$$(1+\mu)\tilde{n}_t = \theta(1-\pi_n)n_{t-1} + [1-\lambda(\omega_{t-1})]\,\tilde{n}_{t-1} \tag{75}$$

$$(1+\mu)b_t = \psi\mu G\left(\bar{W}_t\right) + [1-\lambda(\omega_{t-1})]\,b_{t-1} \tag{76}$$

$$(1+\mu)n_t = (1-\theta)(1-\pi_n)n_{t-1} + \lambda(\omega_{t-1})\left(b_{t-1}+\tilde{n}_{t-1}\right) \tag{77}$$

Market tightness is given by

$$\omega_t = \frac{b_t+\tilde{n}_t}{h_t-b_t-\tilde{n}_t-f_t-n_t} \tag{78}$$

and the housing stock evolves according to

$$(1+\mu)h_{t+1} = h_t + \zeta\phi^{1+\eta}\left(n_t+\tilde{n}_t+b_t+f_t\right)\left(\beta E_t V_{t+1}-\bar{q}\right)^{\eta} \tag{79}$$

When we change the model in this way and retain the same calibration targets as before we find that our results hardly change. The steady state probabilities of buying and selling increase somewhat to $\lambda = \gamma = 0.83$ (which implies a time to sell of 3.6 months). The effects on the model's dynamics are, however, negligible.

9 Concluding Remarks

This paper makes two main contributions. First, we provide a parsimonious characterization of the impact of relative income shocks across US cities on the short run dynamics of average house prices, construction and population growth. Specifically we estimate a panel VAR with city level fixed effects and use it to isolate the impact of relative income shocks by making structural assumptions consistent with our theory. In particular, our estimates are consistent with previous findings (e.g. Glaeser et al. 2010) that house price appreciation exhibits substantial serial correlation in the short term and long-run mean reversion. Moreoever, we find that the volatility of house price movements that occur in response to income shocks is high relative to the volatility of local incomes. City level population growth responds quite quickly in response to income shocks, whereas construction rates tend to be quite sluggish initially. Our interpretation of this is that although construction is quite elastic, net entry of the population into cities seems to be even more so.

Our second main contribution is to build a model that helps to rationalize these movements in house prices, construction and population. To do this we introduce competitive search into a dynamic model of housing markets with endogenous entry and construction. An important feature of the model is that households rent while searching for a house to own. This implies that that they can obtain the relative gains from living in the city without buying a house. Although this increases the demand for rental housing, owners of unnoccupied housing have an incentive to rent out their houses and delay selling them if they expect prices to rise in the future. Consequently house prices don't rise to their maximum immediately and then fall, as they would in a frictionless market, but rather appreciate for several periods as the ratio of buyers to houses for sale grows.

We calibrate the model so that its steady state matches key long run averages in US data and assess whether it can quantitatively account for the estimated moments in the data. We find that a calibrated version of our model captures the qualitative movements in the data quite well, but generally understates them quantitatively.

10 Appendix A: Data appendix

This appendix provides details on data sources, definitions and calculatios. Our unit of observation is a core-based statistical area (metropolitan statistical area or MSA). We use the 2006 MSA definitions. Our sample consists of 98 MSAs.

Populations: City populations are taken from the REIS. Throughout we assume that city populations are proportional to the number of households.

Local incomes: We define local incomes as the total income for all sources less construction earnings. Our MSA level data are from the Regional Economic Information System (REIS) compiled by the Bureau of Economic Analysis (BEA, Table CA34). We subtract construction earnings because they are endogenous in our model. However, whether or not we include them make little difference to the empirical results. We could have defined local income as non-construction earnings. We chose not to because we would expect the incentives to move to a given location to depend on total income. However, when we estimate the panel VAR using this definition instead, the results were qualitatitively similar.

House prices: Following Van Niewenburgh and Weil (2010), we form a time series of home prices for each city by combining level information from the 2000 Census with time series information from the FHFA. From the 2000 Census, we use nominal home values for the median single-family home. From the FHFA we use the Home Price Index (HPI) from 1980 to 2008. The HPI is a repeat-sales index for single family properties purchased or refinanced with a mortgage below the conforming loan limit. As a repeat-sale index, it is a constant quality house price index. In contrast to Van Niewenburgh and Weil (2010), we combine prices for MSA divisions into those for MSAs by using population–weighted averages of the division level prices. We need to do this because the housing stock data (described below) can only be constructed using permits at the MSA level.

Housing Stocks: In a similar fashion to Glaeser et al. (2010), we form a time series for housing stocks for each city by combining information form the 2000 Census with times series information from the US Departmenet of Housinga and Urban Development (HUD). From the 2000 Census, we use the estimated number of single-family homes. This data was only available at the county level, so we summed across the counties within the relevant MSAs. From HUD we used annual permits issued for each city from 1980 to 2008. According to the US Census Bureau, approximately 97.5% of permits issued each year translate into housing starts and, of these, 96% are completed. We therefore constructed housing stocks H_t according to

$$H_{t+1} = H_t + 0.936 \times \text{Permits}_t.$$

11 Appendix B: Empirical Results

11.1 Full Panel VAR Results

Table B.1: System GMM (2SLS) estimates

	Y		P		g^H		g^N	
$Y(-1)$	1.22	(0.05)	0.45	(0.10)	0.01	(0.01)	0.07	(0.03)
$P(-1)$	-0.01	(0.01)	1.25	(0.05)	0.01	(0.00)	-0.02	(0.00)
$g^H(-1)$	0.57	(0.14)	1.19	(0.27)	0.74	(0.05)	0.40	(0.11)
$g^N(-1)$	-0.19	(0.18)	0.11	(0.26)	0.11	(0.05)	0.26	(0.19)
$Y(-2)$	-0.30	(0.05)	-0.60	(0.08)	-0.03	(0.01)	-0.07	(0.03)
$P(-2)$	0.01	(0.01)	-0.31	(0.06)	-0.01	(0.00)	0.02	(0.00)
$g^H(-2)$	-0.53	(0.11)	-0.88	(0.20)	-0.14	(0.03)	-0.22	(0.05)
$g^N(-2)$	0.13	(0.07)	0.59	(0.13)	0.05	(0.02)	0.17	(0.17)
	0.92		0.93		0.60		0.44	

Table 4 provides overall summary statistics from our baseline panel VAR. The first column shows the average standard deviation of each series relative to that of the growth in *per capita* income. The second column shows the correlation with *per capita* income growth. The remaining columns show the first four coefficients of autocorrelation. Several observations can be made:

1. House prices are much more (by a factor exceeding three) volatile than incomes.

2. Price changes much more persistent that income growth, with a first-order autocorrelation of 0.4 as compared with 0.2.

3. Population growth rates are more volatile on average than rates of construction

4. Rates of construction are more persistent than population growth rates

Table 4: **Summary Statistics**

	Standard Deviation	Corr. with Inc. growth	Autocorrelation in Growth Rates year 1	year 2	year 3	year 4
Income growth	1.0000	1.0000	0.2425	0.0194	-0.0560	-0.0758
Price appreciation	2.6601	0.4086	0.4138	0.1164	-0.0252	-0.0826
Construction Rate	0.3547	0.1839	0.7643	0.5249	0.3372	0.2035
Population Growth	0.5271	0.2485	0.4335	0.3339	0.1905	0.1181

11.2 Alternative estimators

Estimating a panel VAR raises a number of econometric issues. A basic problem in dynamic panel data models with fixed effects is that the lagged dependent variables are, by construction, correlated with the individual effects. This renders the least squares estimator biased and inconsistent. Consistent estimation requires some transformation to eliminate fixed effects. A within transformation wipes out the individual effects by taking deviations from sample means, but the resulting within-group estimator is inconsistent when the number of panels becomes large for a given time-dimension (Nickell, 1981).

Given this inconsistency, the literature focuses mainly on a first-difference transformation to eliminate the individual effect while handling the remaining correlation with the (transformed) error term using instrumental variables and GMM estimators (e.g. Arellano and Bond, 1991). However, the Arellano-Bond estimator is known to suffer from a weak instruments problem when the relevant time series are highly persistent, as they are in our case. As Blundell and Bond (1998) demonstrate this can result in large finite-sample biases. In our baseline estimation we use the system GMM estimator proposed by Arellano and Bover (1995) and Blundell and Bond (1998). This estimator is consistent when the number of panels becomes large for a given time-dimension and is less likely to suffer from the weak instruments problem. Another reason for focussing on this estimator is that its properties are fairly well understood and it has been studied in the context of panel VARs by Binder, Hsiao and Pesaren (2005).

There are however several potential problems with using the system GMM estimator for a sample with the dimensions considered here. While it is usually thought to be suitable for typical microeconometric panels, with only a few waves but a large number of individuals, here we have moderately large number of cities and a moderately long time series. Moreover, GMM estimators tend to have a larger standard error compared to the within-group estimator and may suffer from a finite sample bias due to weak instruments. Here we address these issues by comparing our estimates with those of two alternative estimators: a standard within-groups estimator and an "orthogonal to backward mean" within-group estimator, inspired by Everaert (2011). Although the former is inconsistent as the number of panels becomes large, this should be less of a problem given the dimensions of our sample. While the latter is also inconsistent, this inconsistency is expected to be negligibly small for moderately long time periods.

Table B.3: OLS estimates

	Y		P		g^H		g^N	
$Y(-1)$	1.23	(0.05)	0.40	(0.07)	0.04	(0.01)	0.07	(0.04)
$P(-1)$	0.03	(0.01)	1.52	(0.05)	-0.00	(0.00)	-0.03	(0.01)
$g^H(-1)$	0.46	(0.13)	0.92	(0.26)	0.81	(0.05)	0.49	(0.14)
$g^N(-1)$	-0.34	(0.16)	0.53	(0.29)	0.11	(0.05)	0.28	(0.25)
$Y(-2)$	-0.24	(0.05)	-0.37	(0.07)	-0.04	(0.01)	-0.08	(0.04)
$P(-2)$	-0.03	(0.01)	-0.54	(0.05)	0.00	(0.00)	0.04	(0.01)
$g^H(-2)$	-0.36	(0.09)	-1.89	(0.19)	-0.11	(0.03)	-0.14	(0.05)
$g^N(-2)$	0.12	(0.07)	0.55	(0.11)	0.07	(0.02)	0.22	(0.12)
	0.99		0.98		0.70		0.50	

Table B.4: Within-group estimates

	Y		P		g^H		g^N	
$Y(-1)$	1.16	(0.05)	0.39	(0.06)	0.04	(0.01)	0.08	(0.03)
$P(-1)$	0.03	(0.01)	1.46	(0.05)	0.00	(0.00)	-0.03	(0.00)
$g^H(-1)$	0.49	(0.11)	1.17	(0.22)	0.74	(0.05)	0.46	(0.11)
$g^N(-1)$	-0.42	(0.16)	0.26	(0.21)	0.10	(0.05)	0.08	(0.22)
$Y(-2)$	-0.26	(0.04)	-0.30	(0.07)	-0.04	(0.01)	-0.05	(0.02)
$P(-2)$	-0.04	(0.01)	-0.58	(0.05)	-0.00	(0.00)	0.01	(0.00)
$g^H(-2)$	-0.24	(0.08)	-1.07	(0.17)	-0.19	(0.04)	-0.11	(0.05)
$g^N(-2)$	0.02	(0.04)	0.32	(0.10)	0.06	(0.01)	0.02	(0.07)
	0.90		0.87		0.55		0.10	

11.2.1 Within-Group Estimator

Here we report empirical results from our panel VAR based on a standard within–group estimator.

Table 5: **Moments from Structural PVAR—Income Shock, Within-Group**

	Relative Std. Dev.	Corr. with Income	Autocorrelation in Growth Rates			
			year 1	year 2	year 3	year 4
Per capita Income	1.0000	1.0000	0.1980	0.0051	-0.0568	-0.0858
House Prices	1.5970	0.8871	0.8359	0.4780	0.1114	-0.1698
Construction Rate	0.0580	0.7912	0.8601	0.5475	0.2312	-0.0087
Population Growth	0.0806	0.6279	0.5929	0.2835	0.0595	-0.0887

Table 6: **Moments from Structural PVAR—Income Shock, Orthogonal to Backward Mean**

	Relative Std. Dev.	Corr. with Income	Autocorrelation in Growth Rates			
			year 1	year 2	year 3	year 4
Per capita Income	1.0000	1.0000	0.2340	0.0735	0.0200	-0.0082
House Prices	0.8275	0.6772	0.8359	0.4803	0.1286	-0.1325
Construction Rate	0.0336	0.3843	0.8991	0.6729	0.4332	0.2341
Population Growth	0.0428	0.2922	0.6604	0.3642	0.1713	0.0651

11.2.2 Orthogonal-to-Backward Mean Estimator

12 Appendix C: Math appendix

12.1 Proofs and Derivations

Proof of Lemma 1: The household's optimization problem can be expressed as

$$\max_{c_t,l_t} E_t \sum_{t=0}^{\infty} \beta^t U_t(c_t, l_t, z_t) \qquad \text{s.t.} \qquad E_t \sum_{t=0}^{\infty} \beta^t c_t \leq E_t \sum_{t=0}^{\infty} \beta^t \left[y_t + w_t l_t - \Omega_t\right] \tag{80}$$

It follows their dynamic optimization problem is equivalent to

$$\max_{l_t} E_t \sum_{t=0}^{\infty} \beta^t \left[y_t + w_t l_t - v(l_t) + z_t - \Omega_t\right] \tag{81}$$

The solution to the (static) household construction labour supply problem yields

$$l_t = l^s(w_t) = \zeta w_t^{\eta}. \tag{82}$$

Hence,

$$w_t l(w_t) - v\left(l(w_t)\right) = \zeta w_t^{1+\eta} - \frac{\zeta^{\frac{1+\eta}{\eta}} w_t^{1+\eta}}{\zeta^{\frac{1}{\eta}}\left(1+\frac{1}{\eta}\right)} = \frac{\zeta w_t^{1+\eta}}{1+\eta} \tag{83}$$

Proof of Proposition 1. The first-order condition to the market-maker's optimization problem (29) yields

$$\lambda'(\omega_t)\omega_t'(P_t)\left(\beta E_t J_{t+1} - P_t - \beta E_t W_{t+1}\right) - \lambda\left(\omega_t(P_t)\right) = 0, \tag{84}$$

where $\omega_t(P_t)$ and $\omega_t'(P_t)$ are determined by (30). This implies

$$\frac{\beta E_t J_{t+1} - P_t - \beta E_t W_{t+1}}{P_t - \beta E_t \tilde{V}_{t+1}} = -\frac{\lambda(\omega_t(P_t))/\lambda'(\omega_t(P_t))}{\gamma\left(\omega_t(P_t)\right)/\gamma'(\omega_t(P_t))}, \tag{85}$$

which can be used together with (30) to solve for P_t. Then one can solve for ω_t from (30). Note that (30) implies that $\omega_t'(P_t) < 0$ given the assumption that $\gamma'(\omega) > 0$.

Recall from equilibrium condition (iii) that trade surplus in the housing market is strictly positive. Given the boundary condition that $\lim_{T\to\infty} \beta^T E_t J_{t+T} = 0$, it is clear from that the household's equilibrium values are bounded, which implies that the trade surplus is also bounded. Together we have $\beta E_t J_{t+1} - \beta E_t W_{t+1} - \beta E_t V_{t+1} \in (0, \infty)$, where we have incorporated that $V = \tilde{V}$ in the equilibrium. Recall equilibrium condition (iii) that $\gamma\left(\omega_t\right),\ \lambda\left(\omega_t\right) \in (0,1)$ for all active sub-markets. Also recall from part (ii) of Assumption 1 that $\lambda'(\omega) < 0$,

$\gamma'(\omega) > 0$. These conditions imply that $\epsilon(\omega) \in (0,1)$ by (11). Define $LHS(P_t)$ as the left-hand side of (85) and $RHS(P_t)$ the right-hand side. Given (11), it is clear that

$$RHS(P_t) = \frac{\epsilon(\omega_t(P_t))}{1-\epsilon(\omega_t(P_t))}. \tag{86}$$

Because $\epsilon(\omega) \in (0,1)$, we have $RHS(P_t) \in (0,\infty)$ for all P_t. Moreover, recall $\omega_t'(P_t) < 0$ from (30) and $\epsilon'(\omega) \leq 0$ from Assumption 1. Thus $RHS'(P_t) \geq 0$.

Given (19) and (20), free entry of both buyers and sellers imply that the values V_t and W_t are constant across all active sub-markets. Then (4), (17) and (24) imply that $E_t J_{t+1}$, $E_t W_{t+1}$ and $E_t V_{t+1}$ do not vary across sub-markets, either. Thus for any given V_t, J_t, W_t, one can verify that $LHS'(P_t) < 0$ because $\beta E_t J_{t+1} - \beta E_t W_{t+1} - \beta E_t V_{t+1} > 0$. Recall from (19) and (20) that the price in an active sub-market satisfies

$$\beta E_t V_{t+1} \leq P_t \leq \beta E_t J_{t+1} - \beta E_t W_{t+1}. \tag{87}$$

It follows that

$$LHS(P_t = \beta E_t V_{t+1}) = \infty > RHS(P_t = \beta E_t V_{t+1}) \tag{88}$$

$$LHS(P_t = \beta E_t J_{t+1} - \beta E_t W_{t+1}) = 0 < RHS(P_t = \beta E_t J_{t+1} - \beta E_t W_{t+1}), \tag{89}$$

where the two inequalities are because $RHS(P_t) \in (0,\infty)$ for all P_t. The above results imply a unique $P_t^* \in (\beta E_t V_{t+1},\ \beta E_t J_{t+1} - \beta E_t W_{t+1})$ that satisfies

$$\frac{\beta E_t J_{t+1} - P_t^* - \beta E_t W_{t+1}}{P_t^* - \beta E_t V_{t+1}} = -\frac{\lambda(\omega_t^*(P_t^*))/\lambda'(\omega_t^*(P_t^*))}{\gamma(\omega_t^*(P_t^*))/\gamma'(\omega_t^*(P_t^*))}, \tag{90}$$

and a unique $\omega_t^*(P_t^*)$ that satisfies

$$\omega_t^*(P_t^*) = \gamma^{-1}\left(\frac{V_t - \beta E_t V_{t+1}}{P_t^* - \beta E_t V_{t+1}}\right). \tag{91}$$

Thus, there is a single active sub-market in the directed search equilibrium.

Equation (90) may be written as

$$\frac{s(\omega)}{1-s(\omega)} = \frac{\epsilon(\omega)}{1-\epsilon(\omega)}, \tag{92}$$

where $s(\omega)$ denotes the buyer's share of the surplus in a sub-market with tightness ω. The right-hand side of the above is the ratio of the elasticities of the number of matches with respect to the numbers of buyers and sellers. It follows that $s(\omega) = \epsilon(\omega)$. **QED**

Proof of Lemma 2. First use (40) and (41) to derive

$$h^* = \frac{b^*}{\omega_t} + b^* + n^* + f^*. \tag{93}$$

Then use the above and (45) to eliminate h^*:

$$\frac{b^*}{\omega_t} + b^* + n^* + f^* = \frac{\zeta \phi^{1+\eta} (n^* + b^* + f)}{\mu} (\beta V^* - \bar{q})^\eta . \tag{94}$$

It follows that (46) can be obtained by substituting (**??**), (43) and (44) into the above. It follows that $V^S(\omega^*)$ is strictly decreasing in ω^* because $\gamma'(\omega) > 0$ from Assumption 1. **QED**

Proof of Lemma 3: From (50) we have

$$P^* = (1 - s(\omega^*))\beta (J^* - W^* - V^*) + \beta V^* \tag{95}$$

$$r^* = m + \gamma(\omega^*)(1 - s(\omega^*))\beta (J^* - W^* - V^*) \tag{96}$$

It follows that

$$V^* = \gamma(\omega^*)(1 - s(\omega^*))\beta (J^* - W^* - V^*) + \beta V^* \tag{97}$$

$$W^* = Z + \frac{\lambda(\omega^*) s(\omega^*)}{1-\beta} \beta (J^* - W^* - V^*) \tag{98}$$

and

$$J - W - V = \bar{u}^H + \pi_n \beta Z + \pi_n \beta V^* + (1-\pi_n)\theta\beta (W^* + V^*) + (1-\pi_n)(1-\theta)\beta J^* \tag{99}$$

$$-\bar{u}^R - \lambda(\omega^*) s(\omega^*) \beta (J^* - W^* - V^*) - \beta W \tag{100}$$

$$-\gamma(\omega^*)(1 - s(\omega^*))\beta (J^* - W^* - V^*) - \beta V^* \tag{101}$$

Observe that

$$\bar{u}^H - \bar{u}^R = z^H + \gamma(\omega^*)(1 - s(\omega^*))\beta (J^* - W^* - V^*) \tag{102}$$

It follows that

$$J - W - V = z^H - \pi_n \beta (W - Z) + [(1-\pi_n)(1-\theta) - \lambda(\omega^*) s(\omega^*)] \beta (J^* - W - V) \tag{103}$$

$$J - W - V = z^H + \left[(1-\pi_n)(1-\theta) - \left(\frac{1 - \beta + \pi_n \beta}{1-\beta} \right) \lambda(\omega^*) s(\omega^*) \right] \beta (J^* - W - V) \tag{104}$$

and so

$$J - W - V = \frac{z^H}{1 - \beta(1-\pi_n)(1-\theta) + \left(\frac{1-\beta+\pi_n\beta}{1-\beta} \right) \beta \lambda(\omega^*) s(\omega^*)} \tag{105}$$

Substitution yields (55). It is clear from (92) that $s(\omega) = \epsilon(\omega)$, where we implicitly express P as a function of ω. Recall from Assumption 1 that $\lambda'(\omega) < 0$, $\gamma'(\omega) > 0$ and $\epsilon'(\omega) \leq 0$. It follows that the right-hand side of (55) is increasing in ω^*. **QED**

Proof of Proposition 2. Because $V^S(\omega^*)$ is decreasing in ω^* and $V^D(\omega^*)$ is increasing in ω^*, a steady-state equilibrium must be unique if it exists. Existence basically requires that the curves intersect at a value of $\omega \in (0, \bar{\omega})$. That is, an (interior) equilibrium exists if $V^D(0) < V^S(0)$ and $V^D(\bar{\omega}) > V^S(\bar{\omega})$. Recall the definition of $\epsilon(\omega)$ from (11). Also recall from Assumption 1 that $\lambda(\bar{\omega}) = \gamma(0) = 0$, $\lambda(0) = \gamma(\bar{\omega}) = 1$, $\lambda'(\omega) < 0$ and $\gamma'(\omega) > 0$. It follows that $V^S(0) = \infty$, $V^D(0) = 0$ and

$$V^S(\infty) = \frac{1}{\beta}\left[\frac{\mu}{\zeta\phi^{1+\eta}}\left(1 + \frac{\psi(\mu + \pi_f)}{A + B\bar{\omega}}\right)\right]^{\frac{1}{\eta}} + \frac{\bar{q}}{\beta} \tag{106}$$

$$V^D(\infty) = \frac{(1 - s(\bar{\omega}))\beta z^H}{(1-\beta)\left[1 - \beta(1-\theta)(1-\pi_n)\right] + (1 - \beta + \pi_n\beta)\beta\lambda(\bar{\omega})s(\bar{\omega})}. \tag{107}$$

It follows that the condition for the existence and uniqueness of a steady state is given by (56). **QED**

Solving the dynamic system: The dynamic system is given by

$$\ln y_t = (1-\rho)\ln \bar{y} + \sum_{i=1}^{T} \rho_i \ln y_{t-i} + s_t \tag{108}$$

$$(1+\mu)f_t = (1-\psi)\mu G(\bar{W}_t) + (1-\pi_f)f_t \tag{109}$$

$$(1+\mu)b_t = \psi\mu G(\bar{W}_t) + [1 - \lambda(\omega_{t-1})]b_{t-1} + \theta(1-\pi_n)n_{t-1} \tag{110}$$

$$(1+\mu)n_t = (1-\theta)(1-\pi_n)n_{t-1} + \lambda(\omega_{t-1})b_{t-1} \tag{111}$$

$$(1+\mu)h_{t+1} = h_t + \zeta\phi^{1+\eta}(n_t + b_t + f_t)(\beta E_t V_{t+1} - \bar{q})^\eta. \tag{112}$$

$$\omega_t = \frac{b_t}{h_t - b_t - f - n_t} \tag{113}$$

$$J_t = u_t^H + \beta\pi_f Z + \beta\left[(\pi_f + (1-\pi_f)\theta)E_t\tilde{V}_{t+1} + (1-\pi_f)\theta E_t W_{t+1} + (1-\pi_f)(1-\theta)E_t J_{t+1}\right] \tag{114}$$

$$W_t = u_t^R + \lambda(\omega_t)(\beta E_t J_{t+1} - P_t) + (1-\lambda(\omega_t))\beta E_t W_{t+1} \tag{115}$$

$$V_t = \gamma(\omega_t)P_t + (1-\gamma(\omega_t))\beta E_t V_{t+1} \tag{116}$$

$$P_t = (1-s(\omega_t))\beta E_t(J_{t+1} - W_{t+1}) + s(\omega_t)\beta E_t V_{t+1} \tag{117}$$

$$r_t = m + \gamma(\omega_t)(P_t - \beta E_t V_{t+1}). \tag{118}$$

$$W_t^f = u_t^R + \pi_f\beta Z + (1-\pi_f)\beta E_t W_{t+1}^f \tag{119}$$

$$\bar{W}_t = \psi W_t + (1-\psi)W_t^f \tag{120}$$

Proof of Proposition 3: The value of being a mis-matched owner who remains in their house while they search for a new one is given by

$$\tilde{J}_t = y_t + x_t - m + \lambda(\omega_t)\left(\beta E_t J_{t+1} - P_t + \beta E_t V_{t+1}\right) + (1-\lambda(\omega_t))\beta E_t \tilde{J}_{t+1} \tag{121}$$

The value of becoming a renter immediately and putting the vacant house up for sale is given by

$$\begin{aligned} W_t + V_t &= u_t^R + \lambda(\omega_t)\left(\beta E_t J_{t+1} - P_t\right) + (1-\lambda(\omega_t))\beta E_t W_{t+1} + \gamma(\omega_t) P_t + (1-\gamma(\omega_t))\beta E_t V_{t+1} \quad (122) \\ &= u_t^R + \lambda(\omega_t)\left(\beta E_t J_{t+1} - P_t\right) + (1-\lambda(\omega_t))\beta E_t W_{t+1} + \gamma(\omega_t)(1-s(\omega_t))\beta E_t\left(J_{t+1} - W_{t+1}\right) \\ &\quad + \gamma(\omega_t)s(\omega_t)\beta E_t V_{t+1} + (1-\gamma(\omega_t))\beta E_t V_{t+1} \quad (123) \\ &= y_t + x_t - r_t + \lambda(\omega_t)\left(\beta E_t J_{t+1} - P_t + \beta E_t V_{t+1}\right) + \gamma(\omega_t)(1-s(\omega_t))\beta E_t\left(J_{t+1} - W_{t+1} - V_{t+1}\right) \\ &\quad + (1-\lambda(\omega_t))\beta\left(E_t W_{t+1} + E_t V_{t+1}\right) \quad (124) \end{aligned}$$

$$W_t + V_t = y_t + x_t - m + \lambda(\omega_t)\left(\beta E_t J_{t+1} - P_t + \beta E_t V_{t+1}\right) + (1-\lambda(\omega_t))\beta E_t\left[W_{t+1} + V_{t+1}\right] \tag{125}$$

Since $\lim_{T\to\infty} \beta^T E_t \tilde{J}_{t+T} = \lim_{T\to\infty} \beta^T E_t\left[W_{T+1} + V_{T+1}\right] = 0$, solving forwards implies that

$$\tilde{J}_t = W_t + V_t. \tag{126}$$

12.2 Stationary Equilibrium with no search

In a stationary equilibrium there are no shocks so that $u_t^H = \bar{u}^H$. Housing market clearing implies

$$h^* = n^* + b^* \tag{127}$$

and it follows directly that the stationary equilibrium price is

$$P^* = \frac{1}{\beta}\left[\frac{\mu}{\zeta\phi^{1+\eta}}\right]^{\frac{1}{\eta}} + \frac{\bar{q}}{\beta} \tag{128}$$

The value of being a home-owner is then

$$J^* = \frac{\bar{u}^H + \beta\pi Z + \beta\pi P^*}{1-\beta(1-\pi)} \tag{129}$$

Given stationary values for J and P, the new entrants per period is

$$b^* = \frac{\mu}{1+\mu} G\left(J^* - P^*\right) \tag{130}$$

and the steady-state fraction of the total population located in the city is

$$n^* = \frac{1}{\mu+\pi} b^*. \tag{131}$$

Finally, the housing stock per capita is

$$h^* = \frac{\zeta \phi^{1+\eta}\left(n^* + b^*\right)}{\mu} \left(\beta P^* - \bar{q}\right)^{\eta} \tag{132}$$

12.3 Calibration of quarterly income shock process

If we now think of a period as a quarter, we can write an annual AR(2) process as

$$x_t = b_1 x_{t-4} + b_2 x_{t-8} + \varepsilon_t. \tag{133}$$

Let $y_t = x_{t-4}$. Then we can write this as a stacked system given by

$$X_t = \mathbf{B} X_{t-4} + \mathbf{e}_t \tag{134}$$

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} b_1 & b_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_{t-4} \\ y_{t-4} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \end{bmatrix} \tag{135}$$

Now consider a VAR(1) given by

$$X_t = \mathbf{A} X_{t-1} + \mathbf{v}_t \tag{136}$$

where $\mathbf{v}_t = [v_t \quad 0]'$. Iterating on this yields

$$X_t = \mathbf{A}^4 X_{t-4} + \mathbf{A}^3 \mathbf{v}_{t-3} + \mathbf{A}^2 \mathbf{v}_{t-2} + \mathbf{A} \mathbf{v}_{t-1} + \mathbf{v}_t \tag{137}$$

It follows that $\mathbf{A} = \mathbf{B}^{\frac{1}{4}}$ and $\mathbf{e}_t = \mathbf{A}^3 \mathbf{v}_{t-3} + \mathbf{A}^2 \mathbf{v}_{t-2} + \mathbf{A} \mathbf{v}_{t-1} + \mathbf{v}_t$. We can decompose the VAR(1) as

$$x_t = a_{11} x_{t-1} + a_{12} y_{t-1} + v_t \tag{138}$$

$$y_t = a_{21} x_{t-1} + a_{22} y_{t-1} \tag{139}$$

But since $y_t = x_{t-4}$ this is

$$x_t = a_{11}x_{t-1} + a_{12}x_{t-5} + v_t \tag{140}$$

$$x_{t-4} = a_{21}x_{t-1} + a_{22}x_{t-5} \tag{141}$$

Substituting out x_{t-5} yields

$$x_t = a_{11}x_{t-1} + \frac{a_{12}}{a_{22}}\left(x_{t-4} - a_{21}x_{t-1} - v_{2t}\right) + v_t \tag{142}$$

$$x_t = \left(a_{11} - \frac{a_{12}a_{21}}{a_{22}}\right)x_{t-1} + \frac{a_{12}}{a_{22}}x_{t-4} + v_t \tag{143}$$

Thus the AR(2) process at the annual frequency translates into a particular AR(4) process at the quarterly frequency. There is of course a loss of information.

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