Investment Cycles

Patrick Francois
Department of Economics
University of British Columbia
and
CentER, Tilburg University
p.francois@uvt.nl

Huw Lloyd–Ellis
Department of Economics
Queen's University
and
CIRPÉE
lloydell@qed.econ.queensu.ca

November, 2003

Abstract

Keynes viewed aggregate investment as an exogenous cause of business cycles driven by "animal spirits". More recently, it has become common to model investment fluctuations as a rational response to fluctuating incentives, driven by exogenous aggregate technology changes. However, this approach raises a number of questions. Why treat investments in physical capital as endogenous, while treating those in intangible capital as exogenous? Relatedly, why would technology changes exhibit such strong co–movement across diverse sectors of the economy, and why are the short–run, empirical relationships between aggregate investment and standard measures of investment incentives so weak? We address these and other related issues by extending the existing literature on endogenous business cycles to incorporate physical capital. In doing so, we demonstrate the crucial role played by endogenous innovation and incomplete contracting, inherent to the process of creative destruction, in driving investment cycles.

Key Words: Long–term contracting, investment irreversibility, putty–clay technology, asset–specificity, Endogenous cycles and growth
JEL: E0, E3, O3, O4

Funding from Social Sciences and Humanities Research Council of Canada and the Netherlands Royal Academy of Sciences is gratefully acknowledged. This paper has benefitted from the comments of Paul Beaudry and David R. F. Love and seminar participants at Dalhousie, McMaster, Queen's, UWO, the 2003 Midwest Macroeconomics Conference and the 2003 meetings of the Canadian Economics Association meetings. The usual disclaimer applies.

1 Introduction

Fluctuations in the aggregate investment rate are a central feature of the business cycle. As Figures 1 and 2 illustrates, the rate of U.S. investment in fixed, non–residential assets displays regular, and recurring patterns of activity over time. The investment rate fell during all post–war, NBER–dated recessions (the shaded regions in Figure 1) and typically rose during expansions. The only exceptions were around 1967 and 1987, which saw large declines in investment but output slowdowns that were not quite large enough to be dated as NBER recessions.

According to Keynes (1936), investment fluctuations played a central *causal* role in driving business cycles. He argued that the co–movement of investment across diverse sectors of the economy was exogenously driven by a kind of mass psychology or "animal spirits".[1] More recently, economists have attempted to understand movements in aggregate investment as an optimal response to measurable incentives. In the canonical Real Business Cycle (RBC) model, for example, fluctuations in aggregate investment are driven by fluctuations in aggregate technology that change the incentives to produce investment goods relative to consumption goods. However, this increasingly standard approach raises a number of conceptual and quantitative questions:

• First, why treat investment in tangible, physical capital as an endogenous response to incentives, while implicitly treating the investments in intangible, knowledge capital, which shift the production function, as exogenous shocks? It is not clear that this is conceptually any better than treating investment in physical assets as exogenous.[2] Over the past 15 years there have been considerable advances in understanding the potential and actual role of endogenous innovation on long run growth, but this has had relatively little influence on business cycle research. One clue to the potential importance of viewing some technology shifts as, at least partially, endogenous comes from the work of Hall (1988) who finds that the Solow residual is significantly correlated with factors that do not seem likely to have a direct impact on technology.

• Second, why do these apparent shifts in technology take place in such a clustered fashion across diverse sectors of the economy? Assuming from the outset that technological change affects all sectors symmetrically again seems no better on a conceptual level from directly assuming that investment co–moves across sectors because of animal spirits. One possibility is that these shifts

[1] One modern incarnation of this idea is to model animal spirits as purely exogenous, but self–fulfilling changes in expectations (see e.g. Farmer and Guo 1990). In this case, investment is optimal but the aggregate incentives are stochastic.

[2] The RBC literature generally takes this clustering of productivity improvements as given, and focusses on the propagation mechanism.

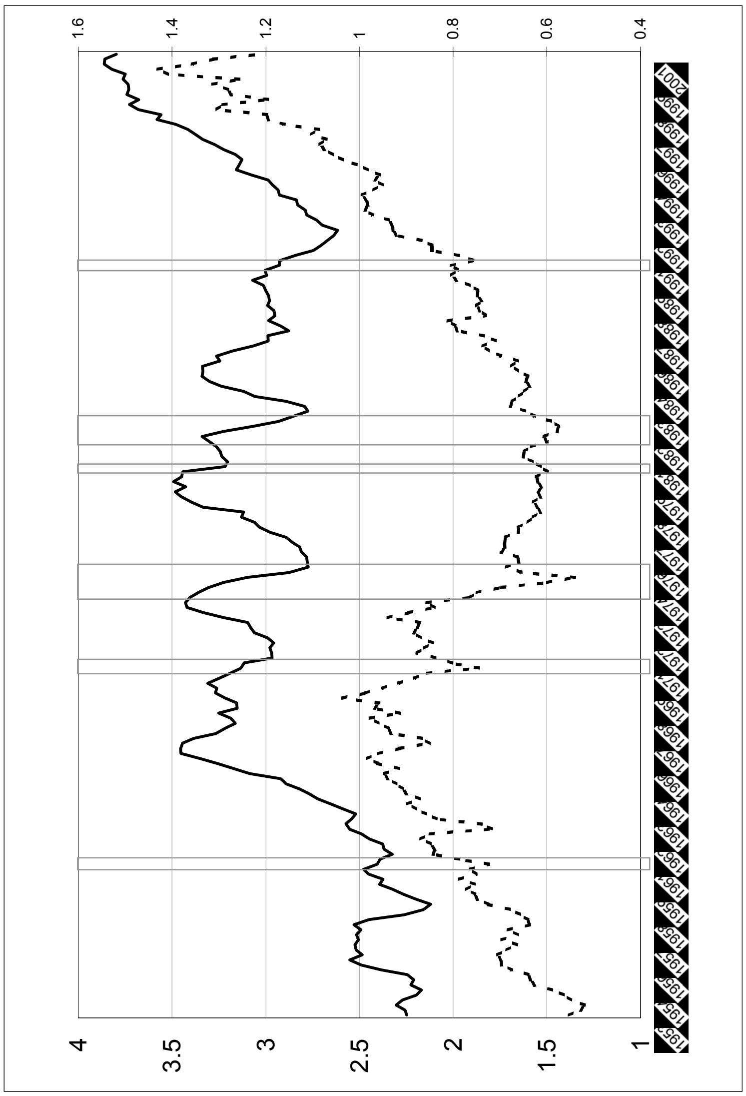

Figure 1: Q and I/K

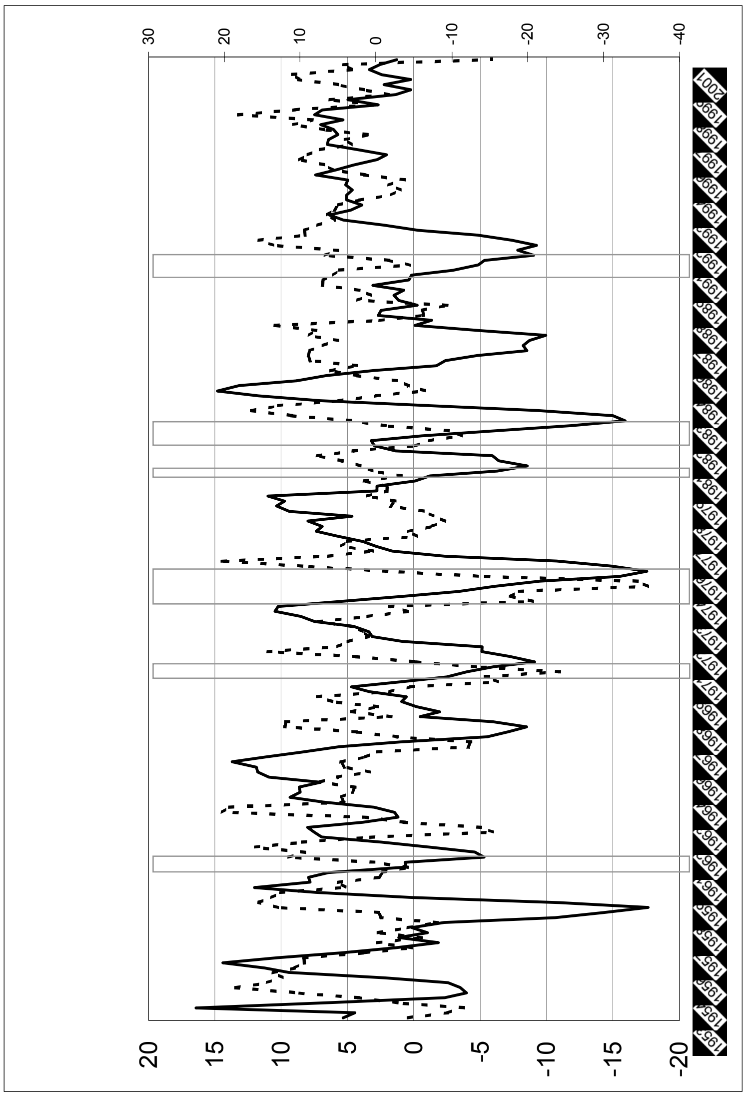

Figure 2: Q and I/K: Year over year % change

are the result of general purpose technologies (GPTs) which affect all sectors. However, there is little evidence supporting this idea at business cycle frequencies.[3] As Lucas (1981) reasons, while technology shocks may be important at the firm level, it is not immediately obvious why they would be important for economy–wide aggregate output fluctuations.

• Finally, if investment really is optimally determined, why is the short–term empirical relationship between aggregate investment and contemporaneous measures of investment incentives apparently so weak? In particular, while there is some evidence of a long run relationship, neither micro nor macro level empirical work has generally found a significant short–run relationship between investment and Tobin's Q — the ratio of the equity value of firms, to the book value of the capital stock.[4] As is well known, one cannot infer from this that investment is sub–optimal because Tobin's Q need not reflect the *marginal* incentives to invest,[5] and because equity values are likely to include the values of intangible, as well as tangible, capital.[6] But then the question arises as to what kind of relationship should we expect to observe between investment and *measurable* proxies of financial incentives. Figures 1 and 2 suggest that Tobin's Q appears to lead investment especially during the latter part of expansions and recessions, with Q falling several quarters before investment declines and rising several quarters prior to expansions.[7]

In this article, we take the view that in order to understand the relationships between aggregate investment, productivity growth and the stockmarket over the business cycle, one must ultimately deal head-on with the source of productivity fluctuations, the reasons for sectoral co–movement, and the apparent delay in investment in response to incentives. One possible starting point for thinking about these issues is Shleifer's (1986) model of "implementation cycles". He shows that in the presence of imperfect competition, the implementation of a productivity improvement by one firm may increase the demand for anothers' products by raising aggregate demand. This may induce innovators, who anticipate short-lived profits due to imitation, to

[3]There have been seven major technological innovations identified in the last 200 years; ?? et. al. in Laitner Stolyarov. These correspond to the Schumpeterian 30 year cycles, or the long cycle.

[4]First suggested by Tobin (1968) and Brainard and Tobin (1969). See Cabellero, 1999 for a recent survey.

[5]As shown by Abel (1979) and Hayahsi (1982), when there are adjustment costs, marginal and average Q need not be equal.

[6]See Hall (2002).

[7]One rationalization of the lead–lag behaviour observed towards the end of recessions is that the stock market anticipates a subsequent boom driven by a GPT. The stock market (and hence Q) moves immediately with the arrival of the information, but investment is delayed, for some reason, until productivity actually rises. This is the kind of explanation pursued by Hobijn and Jovanovic (199?) and Laitner and Stolyarov (2003), for the long term relationship between investment and the stockmarket between 1973 and 2003. However, as noted earlier there is little evidence supporting the arrival of GPTs at business cycle frequencies.

delay the implementation of their own innovation until others implement, thereby generating self–enforcing booms in aggregate activity. Unfortunately, though capable of generating both co–movement and delay in implementation, Shleifer's model cannot serve as a framework for understanding investment cycles. This is because the sectoral co–movement that he establishes is not robust to the introduction of capital or, in fact, *any* storable commodity. Anticipating a boom, producers would produce early, store the output, and sell it in the boom, thus undermining the cycle.[8]

Recently, Francois and Lloyd-Ellis (2003) show how a similar process of endogenous clustering can arise due to the process of "creative destruction" familiar from Schumpeterian endogenous growth models. Like imitation, potential obsolescence limits incumbency and provides incentives to cluster implementation. However, in their framework, where productive resources are needed to generate new innovations, allowing for the possibility of storage does not rule out clustering, and in fact yields a *unique* cyclical equilibrium.[9] Moreover, because this costly innovation tends to be bunched just before a boom, it causes a downturn in aggregate output (even if the measure of GDP includes this investment). Because of its ability to accommodate storage, this framework is more promising as a vehicle for understanding investment. However, the addition of fully–reversible physical capital would still undermine their cyclical equilibrium because, in anticipation of a boom, households would "eat" some of their capital in order to smooth consumption.

Full reversibility and ex–post flexibility of installed physical capital is, however, at odds with much recent evidence on investment. In particular, there is considerable direct evidence that many types of physical investment are not reversible and feature inflexible characteristics once installed (see Ramey and Shapiro, 2001, Kasahara, 2002). Doms and Dunne (1993) have also documented the considerable "lumpiness" of plant level investments, while Cabellero and Engel (1998) have demonstrated the high skewness and kurtosis observed in aggregate investment data.[10] Moreover, the variation in "shiftwork" over the business cycle (see Bresnahan and Ramey 1994, Hamermesh 1989 and Mayshar and Solon, 1993) is consistent with some degree of inflexibility

[8] His model is also subject to a number of other criticisms including the fact that there are no downturns in his model and that there is a continuum of multiple cyclical equilibria, making the predictions of the model rather imprecise. Moreover, while the timing of implementation booms is endogenous, the innovations themselves arise exogenously.

[9] Shleifer (1986) assumes that innovations arise exogenously. When innovation is endogenous, growth is intimately related to the business cycle.

[10] A related literature emphasizes the role of plant level investment irreversibilities. However, recent work (see Veracierto, 2002 and Thomas 2002,) in the RBC tradition has found that the effects of such irreversibilities at the aggregate level are virtually non–existent.

in factor propotions, since it implies that capital is being used less intensively during recessions than is optimal ex ante.

Here we introduce physical capital into the framework developed by Francois and Lloyd-Ellis (2003) in order to understand the business–cycle relationships between investment, productivity and the stockmarket.[11] We model production in a way which captures, as simply as possible, the inflexibility of installed capital relative to uninstalled capital. Specifically, we assume that, once installed, capital becomes irreversible, lumpy and sector–specific. Moreover, we assume that while the utilized capital to labor ratio can be increased as output expands and more capital is added, it cannot be adjusted as output contracts and no new capital is being added. To fix ideas, consider the example of a car manufacturer. As the demand for cars expands, it can add new equipment to a given workforce working at maximum capacity, thereby raising the capital–labour ratio and increasing labour productivity. However, as output contracts the manufacturer retains the installed capital (due to irreversibility), but uses it less intensively and reduces the number of shifts in proportion, so that utilized capital–labour hours ratio remains fixed. The lumpiness assumption implies that the manufacturer cannot rent out the capital to another car manfacturer during breaks between shifts.

In our model we assume that the owners of physical capital and the owners of intangible capital are distinct entities (e.g. banks and entrepreneurs). Threat of entry from replacement capitalists during an expansion induces the incumbent capitalist to offer a user cost sequence whose present value is just sufficent to cover the cost of the capital. However, during a downturn, the competition faced by incumbent capitalists is diminished because the cost saving from waiting until the subsequent boom to produce replacement capital exceeds the return from renting it to the producer during the downturn. If they could, incumbent capitalists would therefore raise the price above the competitive level that they had originally offered. To circumvent this problem, capitalists offer binding price commitments to entrepreneurs in their sector. These prices commitments are offered before the downturn occurs, and are therefore competitively determined by the threat of entry by competing capital owners.

A consequence of modelling capital this way is that it need not be fully utilized throughout the cycle, nor smoothly accumulated even in the absence of uncertainty. Specifically, capital ac-

[11] Using the simpler model of Shleifer (1986) as a vehicle for this analysis will not work, even with inflexible capital. Storage of *any* kind undermines the clustering of activity there. The endogenous innovation, present in Francois and Lloyd-Ellis (2003), is a necessary part of the equilibrium.

cumulation becomes highly procyclical, and goes through two distinct phases; one expansionary and one stagnant. The expansionary phase occurs in the output boom of the cycle, which is triggered by the implementation of stored productivity improvements. Since these improvements arrive stochastically across sectors, and thus increase firm value, the value of Tobin's Q starts to rise through the recession. However since firms optimally choose to delay the implementation of improvements until the boom, investment lags behind the increase in Q. During the expansion, capital is accumulated continuously and smoothly. At its end, the economy enters into a recessionary phase where capital ceases to be accumulated. Since demand falls then, and since installed capital has a fixed utilized capital–labor intensity, producers continuously reduce capital utilization throughout the recession. The anticipated decline in demand leads to a fall in firm values, and hence Tobin's Q, even though it remains optimal for firms to invest up until the contraction commences; thus Q leads investment into the recession too.

Although our focus here is on the nature of investment cycles, this is delivered in a framework where the economy's aggregate fluctuations arise endogenously. Thus, there are no simple causal relationships between the variables of interest studied here, instead all of these are general equilibrium implications arising from the growth process of an economy which is formally identical to that studied in the Schumpeterian endogenous growth literature, as in Aghion and Howitt (1992) and Grossman and Helpman (1991). Expansions are "neoclassical", supply–side phenomena which directly raise both *potential* output, through the delayed implementation of productivity improvements, and *actual* output through increased production labor and subsequent capital accumulation. Recessions are "Keynesian" demand–side contractions during which actual output falls below its potential, investment stops, and some capital resources are left under–utilized. These reductions in aggregate demand are an equilibrium response to the anticipated future expansion, as effort shifts into long–run growth promoting activities, and out of current production.[12]

The paper proceeds as follows: Section 2 sets out the framework, Section 3 characterizes the cycling steady state, Section 4 presents the necessary conditions for existence of the steady state and Section 5 provides numerical examples of cycling economies, and comparative statics. Section 6 deals with the model's dynamics and Section 7 concludes with a discussion of the main implications. All proofs are in the Appendix.

[12] This has the flavor of the so–called "paradox of thrift": current savings are chanelled into investments whose return will not be realized until the long run (when, according to Keynes (1936), "we are all dead").

2 Previous Literature

Our treatment of capital has many similarities to putty-clay treatments, as first developed by Johansen (1959). A standard neoclassical treatment of capital allows for smooth substitution between factors irrespective of whether capital is installed or not, as well as full convertability of capital back to output. A putty-clay treatment instead emphasizes a sharp distinction between substitutability of factors prior to and after capital installation. Putty-clay approaches usually assume that investment embodies both the capital labor ratio and the level of technology at the time the capital is installed. Our approach is very similar to these, for example Gilchrist and Williams (2000), in that the ex ante choice of capital comes from a standard Cobb-Douglas production function, but, when installed, Leontief adjustment thereafter; with the factor ratio dictated by that chosen at the time of installation. The main difference between our treatment and these approaches, however, arises from their emphasis on the technological specificity of the capital. In our framework, irreversibility is the critical factor that we wish to explore, and to highlight this, we shut down issues of capital obsolescence: in contrast to putty-clay models, here capital is NOT vintage specific and is infinitely lived. As in putty-clay treatments however, the irreversibility we use implies that short-run capital labor substitutability induces a tight connection between changes in demand and changes in employment and capacity. Fuss (1977) presents evidence supporting a putty-clay view of capital, Gilchrist and Williams (2000) argue that a putty-clay depiction is confirmed by minimum distance estimation of two sector models that formally nest these characteristics and the fully flexible neoclassical treatments of capital. As in their model, our model's microfoundations are also consistent with the importance of plant shut-down as a short run adjustment margin, see for example Bresnahan and Ramey (1994). We also assume that investments are lumpy, in the sense that they cannot be partly dismantled and used elsewhere. Once again, this is consistent with micro evidence; Cooper, Haltiwanger and Power (1999).

Our treatment is also related to models which explicitly introduce costs of capital adjustment; see Caballero (1997) for a simplified discussion of these, and Dixit (1993) for a more thorough survey. An important distinction between our approach and most others however, is that these costs arise asymmetrically. When expanding, we assume that capital adjustment is unimpeded; when firms invest, they raise the capital-labor ratio by augmenting the amount (in efficiency units) of capital that each worker works with. However, once in place, capital is infinitely costly to adjust; it cannot be converted into a consumption good, nor into another capital good of different capital-labor intensity.

Though irreversibility models of investment are appealing at the micro level, recent work by Veracierto (2003) suggests that they will be less helpful in explaining aggregate investment's behavior. This is in sharp contrast to what we will find here, and also contrasts to an earlier literature, Coleman (1997), Faig (1997) and Ramey and Shapiro (1998). The reason for the difference with that literature is that the magnitude of the shocks considered in his framework are smaller, as implied from callibrated Solow residuals. With the smaller shocks, the non-negativity constraint on investment rarely binds and aggregate level investment is quantitatively similar to that yielded by a model without constraints. In contrast, it differs from our approach because our model features recessions - periods of protracted decline in demand, where capacity exceeds output requirements. Recessions are anticipated aggregate shocks in our framework, (though shock is a misnomer). These endogenous reductions in demand and output combine with the ex post fixity of capital to lead to underutilization and reduced incentives for investment. In his framework, in contrast, irreversibility constraints bind as a consequence of idiosyncratic, not aggregate, shocks, and hence wash out at the economy wide level.

The current paper is also related to aggregate level studies highlighting the anticipatory effect of economy wide changes on firm values. Hall (2001), Hobijn and Jovanovic(2001) and Laitner and Stolyarov (2003) all emphasize the long run implications of the IT revolution, the anticipation of which is dated to the early 70's. Laitner and Stolyarov (2003) are particulary concerned with Tobin's Q, and with its fall below unity for the decade starting in the mid 70's. They emphasize the capital and knowledge obsolescence sparked by the arrival of the GPT. Though similar in its emphasis on a destructive effect of new innovation, this analysis cannot reasonably be applied to business cycle frequencies without a theory of sectoral comovement. Our work is thus complementary to these in its business cycle length focus.

A final relationship is to the literature on endogenous cycles and growth. Though there has been a large literature on cycles and growth, it is mostly restricted to single sector settings. These imply a long-cycle focus because they can only be consistent with a GPT type explanation for fluctuations. Previous work favouring this major innovation interpretation includes: Jovanovic and Rob (1990), Cheng and Dinopolous (1992), Helpman and Trajtenberg (1998), Li (2000), Aghion and Howitt (1998), Evans, Honkephoja and Romer (1998) and Walde (2001). These works cannot be translated readily into a multi-sector setting because they include no force generating co-movement, so that aggregate fluctuations disappear in multi-sector extensions. There also seems to be little empirical support for the idea that the factors driving long cycles (estimated to be at least 30 years) are operating at business cycle frequencies, see Jovanovic and Lach (1998) and Andolfatto and Macdonald (1998), and the discussion of estimated lengths of GPTs in Laitner and Stolyarov (2003).

Apart from Shleifer (1986), exceptions to the single sector focus are provided by Francois and Lloyd-Ellis (2003) which, as discussed above, we follow closely here, and two contributions of Matsuyama (1999 and 2001). Matsuyama constructs a framework of endogenous growth and cycles featuring aggregate endogenous upturns in a multiple sector setting. Like Shleifer's (1986) original model of implementation cycles, limited monopoly power plays a key role, and also like Shleifer he allows the source of limited monopoly power to be exogenous immitation. A significant contrast with Shleifer is that his model does not depend on delay to generate cyclical behavior, and is thus not sensitive to allowing for storage. This enables him to explore a meaningful role for capital accumulation through the cycle. However, Matsuyama's model emphasizes an innovative process which is capital intensive, suggesting R&D plays a central role, and again suggesting a long-cycle focus.[13] Through his cycle, there is no decline in production, no reduction in capacity utilization and no reduction in consumption. Furthermore, Matsuyama's interpretation of his model suggests a long-cycle aim; examples to which his model are addressed are the productivity slowdown and the post-war Asian economy growth experiences.

3 The Model

3.1 Assumptions

There is no aggregate uncertainty. Time is continuous and indexed by $t \geq 0$. We consider a closed economy with no government sector. The representative household has isoelastic preferences

$$U(t) = \int_t^\infty e^{-\rho(\tau - t)} \frac{C(\tau)^{1-\sigma} - 1}{1 - \sigma} d\tau \tag{1}$$

where ρ denotes the rate of time preference and σ represents the inverse of the elasticity of intertemporal substitution. The household maximizes (1) subject to the intertemporal budget

[13] In fact R&D is really modelled as a fixed entry cost.

constraint

$$\int_t^{\infty} e^{-[R(\tau)-R(t)]}C(\tau)d\tau \leq S(t) + \int_t^{\infty} e^{-[R(\tau)-R(t)]}w(\tau)d\tau \tag{2}$$

where $w(t)$ denotes wage income, $S(t)$ denotes the household's stock of assets (firm shares and capital) at time t and $R(t)$ denotes the discount factor from time zero to t. The population is normalized to unity and each household is endowed with one unit of labor hours, which it supplies inelastically.

Final output can be used for the production of consumption, $C(t)$, investment, $\dot{K}(t)$, or can be stored at an arbitrarily small flow cost of $\nu > 0$ per unit time. It is produced by competitive firms according to a Cobb–Douglas production function utilizing a continuum of intermediates, x_i, indexed by $i \in [0,1]$:

$$C(t) + \dot{K}(t) \leq Y(t) = \exp\left(\int_0^1 \ln x_i(t)di\right). \tag{3}$$

For simplicity we also assume that there is no physical depreciation.

Output of intermediate i depends upon the state of technology in sector i, $A_i(t)$, utilized capital, $K_i^u(t)$, which cannot exceed the stock of installed capital, $K_i(t)$, and labor hours, $L_i(t)$, according to the following production technology:

$$x_i^s(t) = \begin{cases} [K_i^u(t)]^{\alpha}[A_i(t)L_i(t)]^{1-\alpha} & \text{where } K_i^u(t) = K_i(t) \\ \kappa_i(z)^{\alpha}A_i(t)^{1-\alpha}L_i(t) & \text{where } K_i^u(t) = \kappa_i(z)L_i(t) < K_i(t) \end{cases} \tag{4}$$

where $\kappa_i(z)$ is the capital–labor ratio chosen at $z < t$, the time at which the last increment to capital was installed. The unit measure of labor hours is perfectly mobile across sectors and inelastically supplied by households in aggregate. However, the amount of this supply that is used in production of intermediates potentially varies due to its opportunity cost in an alternative activity, specified shortly. Installed capital, $K_i(t)$, is sector–specific and is owned by "capitalists" who rent it to entrepreneurs at the rate $q_i(t)$. Installed capital is non-divisible so that any part of it that is not utilized cannot be used elsewhere.[14] Once installed, sector–specific capital cannot be converted back into output, i.e., $\dot{K}_i \geq 0$. We assume that intermediates are completely used up in production, but can be produced and stored for use at a later date. Incumbent intermediate producers must therefore decide whether to sell now, or store and sell later at the flow storage cost ν.

[14] For similar specifications of putty-clay capital, see Johansen (1959), Gilchrist and Williams (2000) and Kasahara (2003). Evidence of underutilization is widespread: for example, Bils and Cho (1994) and Basu (1996).

An implication of this structure is that during an expansion, when new capital is being built, firms can choose (K, L) combinations along the Cobb–Douglas production isoquants (curved in Figure 1) and choose an optimal capital–labor ratio that reflects relative factor prices. However during a contraction, if labor is removed and the firm produces below capacity, production must utilize factors along a ray from the chosen point on the full-capacity isoquant, which reflects the same factor ratios. In such a situation, the installed capital is used less intensively in proportion to the labor hours allocated to production. One interpretation of this is that there are fewer shifts.

This production set up implies that if firms were to reduce output below capacity, one of the following three outcomes must occur:

(1) the entire stock of capital is rented to another (presumably more productive) entrepreneur, if one exists;

(2) capital is fully utilized, but some output is stored to be sold later, or

(3) the installed capital remains in place but is under–utilized, $K_i^u(t) < K_i(t)$.

Along the cyclical growth path that we will study here, only outcome (3) turns out to be consistent with equilibrium. However, we will discuss in some detail the conditions that rule out (1) and (2).

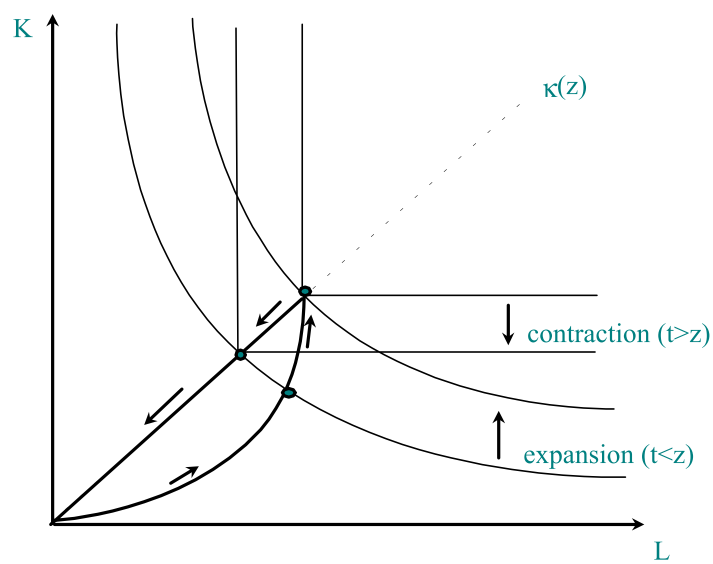

Figure 3: Implications of Irreversibility and Putty–Clay Technology

3.1.1 Innovation

Competitive entrepreneurs in each sector attempt to find ongoing marginal improvements in productivity by allocating labor effort to innovation rather than production.[15] They finance their activities by selling equity shares to households. The probability of an entrepreneurial success in instant t is $\delta H_i(t)$, where δ is a parameter, and H_i is the labor effort allocated to innovation in sector i. At any point in time, entrepreneurs decide whether or not to allocate labor effort to innovation, and if they do so, how much. The aggregate labor hours allocated to innovation is given by $H(t) = \int_0^1 H_i(t)dt$.

New ideas and innovations dominate old ones by a factor e^{γ}. Successful entrepreneurs must choose whether or not to implement their innovation immediately or delay implementation until a later date.[16] Once they implement, the knowledge associated with their improvement becomes publicly available, and can be built upon by rival entrepreneurs. However, prior to implementation, the knowledge is privately held by the entrepreneur.[17] We let the indicator function $Z_i(t)$ take on the value 1 if there exists a successful innovation in sector i which has not yet been implemented, and 0 otherwise. The set of instants in which entrepreneurial successes are implemented in sector i is denoted by Ω_i. We let $V_i^I(t)$ denote the expected present value of profits from implementing a success at time t, and $V_i^D(t)$ denote that of delaying implementation from time t until the most profitable time in future.

3.1.2 Contracts

The nature of innovation is such that entrepreneurs cannot simply "sell" their ideas to capitalists, but must be involved in its implementation themselves. We assume that entrepreneurs do not have the wealth required to purchase the capital stock needed to implement, and hence must borrow from the capitalists. In effect, there is a separation of ownership and control with respect to the capital stock of the firm, which may necessitate the writing of a long–term capital supply contract. The effective user cost of capital is the outcome of such a contractual relationship between the

[15] All of the labor considered here is skilled and capable of substituting between the two activities. We discuss the implications of allowing unskilled labor in the concluding section.

[16] As in Francois and Lloyd–Ellis (2003) we adopt a broad interpretation of innovation. Recently, Comin (2002) has estimated that the contribution of measured R&D to productivity growth in the US is less that 1/2 of 1%. As he notes, a larger contribution is likely to come from unpatented managerial and organizational innovations.

[17] Even for the case of intellectual property, Cohen, Nelson and Walsh (2000) show that firms make extensive use of secrecy in protecting productivity improvements. Secrecy likely plays a more prominent role for entrepreneurial innovations, which are the key here.

entrepreneur and the capitalists in each sector. Incumbent capital owners are limited in the extent of their monopoly pricing by the threat of "replacement" capital being built in their sector.[18]

Capital Supply Contracts: Intermediate producers and capital owners in every sector i can contract over a future binding utilized capital level, $K_i^u(\tau)$, and a price for each unit of utilized capital, $q_i(\tau)$, for all τ up to a chosen contract termination date, T_i^K. Thus a contract signed at time t is a tuple $\left\{\{K_i^u(\tau), q_i(\tau)\}_{\tau\in[t,T_i^K]}, T_i^K\right\}$.[19] Since the productive advantage of an intermediate producer lasts only until a superior technology is implemented in that sector, contracts allow the termination of agreements before T_i^K if shutting down production. Otherwise, the parties can break contracts only by mutual agreement.

Although supply contracts with particular entrepreneurs only last until their ideas become obsolete, capitalist owners can retain their incumbency permanently (subject to competition from other capitalists). The present value of the capitalist's net income in sector i under the utilization–price sequence $\{K_i^u(\tau), q_i(\tau)\}_{\tau=t}^{\infty}$ is therefore:

$$V_i^K(t) = \int_t^{\infty} e^{-[R(\tau)-R(t)]}\left[q_i(\tau)K_i^u(\tau) - \dot{K}_i(\tau)\right]d\tau. \tag{5}$$

Intermediate Supply Contracts: Final goods producers are also able to contract intermediate good deliveries from each of the intermediate producing sectors, i. Such contracts written at t involve a similar tuple: $\left\{\{x_i(\tau), p_i(\tau)\}_{\tau\in[t,T_i^X]}, T_i^X\right\}$ where the unit price is $p_i(t)$ and the contract termination date is T_i^X. Contracts can be altered under the same conditions as in capital contracts.[20] The value of final goods producers is denoted $V^Y(t)$.

3.2 Definition of Equilibrium

Given initial state variables[21] $\{A_i(0), Z_i(0), K_i(0)\}_{i=0}^1$, an equilibrium for this economy is:

(1) a sequence of capital supply contracts $\left\{\widehat{T}_{iv}^K, \left\{\widehat{K}_i^u(t), \widehat{q}_i(t)\right\}_{t\in[\widehat{T}_{iv-1}^K, \widehat{T}_{iv}^K]}\right\}_{v\in I}$,

(2) a sequence of intermediate supply contracts $\left\{\widehat{T}_{iv}^X, \{\widehat{x}_i(t), \widehat{p}_i(t)\}_{t\in[\widehat{T}_{iv-1}^X, \widehat{T}_{iv}^X]}\right\}_{v\in I}$,

[18] In order to maintain competition in capital supply it will be assumed that, in the event of a competing capital stock being built, ties in tended prices are always broken in favour of the entrant. Due to storage costs, entry of replacement capital will imply scrapping of the pre-existing stock.

[19] Identical results obtain if instead of specifying the time-varying utilization rate of capital, $K_i^u(\tau)$, contracts can only be written over $K(\tau)$.

[20] Though conceptually feasible, contracts written over the supply of labor and final output are redundant in the equilibria we study and will not be considered further.

[21] Without loss of generality, we assume no stored output at time 0.

(3) sequences $\left\{\widehat{K}_i(t),\ \widehat{L}_i(t),\ \widehat{H}_i(t),\ \widehat{A}_i(t),\ \widehat{Z}_i(t),\ \widehat{V}_i^I(t),\ \widehat{V}_i^D(t),\ \widehat{V}_i^K(t)\right\}_{t\in[0,\infty)}$ for each intermediate sector i, and

(4) economy wide sequences $\left\{\widehat{Y}(t),\ \widehat{R}(t),\ \widehat{w}(t),\ \widehat{V}^Y(t),\ \widehat{C}(t),\ \widehat{S}(t)\right\}_{t\in[0,\infty)}$

which satisfy the following conditions:

• Households allocate consumption over time to maximize (1) subject (2). The first–order conditions of the household's optimization require that

$$\widehat{C}(t)^{\sigma} = \widehat{C}(\tau)^{\sigma} e^{\widehat{R}(t)-\widehat{R}(\tau)-\rho(t-\tau)} \qquad \forall\ t,\tau, \tag{6}$$

and that the transversality condition holds

$$\lim_{\tau\to\infty} e^{-\widehat{R}(\tau)}\widehat{S}(\tau) = 0 \tag{7}$$

• Labor markets clear:

$$\int_0^1 \widehat{L}_i(t)di + \widehat{H}(t) = 1 \tag{8}$$

• Arbitrage trading in financial markets implies that for all assets that are held in strictly positive amounts by households, the rate of return between time t and time s must equal $\frac{\widehat{R}(s)-\widehat{R}(t)}{s-t}$.

• Free entry into innovation — entrepreneurs select the sector in which they innovate so as to maximize the expected present value of the innovation, and

$$\delta \max[\widehat{V}_i^D(t), \widehat{V}_i^I(t)] \leq \widehat{w}(t), \quad \widehat{H}_i(t) \geq 0 \qquad \text{with at least one equality.} \tag{9}$$

• At instants where there is implementation, entrepreneurs with innovations must prefer to implement rather than delay until a later date

$$\widehat{V}_i^I(t) \geq \widehat{V}_i^D(t) \quad \forall\ t \in \widehat{\Omega}_i. \tag{10}$$

• At instants where there is no implementation, either there must be no innovations available to implement, or entrepreneurs with innovations must prefer to delay rather than implement:

$$\begin{aligned} \text{Either } \widehat{Z}_i(t) &= 0, \\ \text{or if } \widehat{Z}_i(t) &= 1,\ \widehat{V}_i^I(t) \leq \widehat{V}_i^D(t) \quad \forall\ t \notin \widehat{\Omega}_i. \end{aligned} \tag{11}$$

• For all capital supply contracts written at date t, the equilibrium contract is such that no other contract dominates for the capitalist and all existing entrepreneurs indexed by technologies

$A_i(\tau) \leq A_i(t)$ in sector i:

$$\max\left[\widehat{V}_i^I(t), \widehat{V}_i^D(t)\right] + \widehat{V}_i^K(t) \quad \geq \quad \max\left[V_i^I(t), V_i^D(t)\right] + V_i^K(t) \tag{12}$$

$$\forall \text{ mutually determined } V_i^J \quad \neq \quad \widehat{V}_i^J, \quad J = I, D, K. \tag{13}$$

• For all intermediate supply contracts written at date t, the equilibrium contract is such that no other contract dominates for the final goods producer and all existing entrepreneurs in sector i:

$$\max\left[\widehat{V}_i^I(t), \widehat{V}_i^D(t)\right] + \widehat{V}^Y(t) \quad \geq \quad \max\left[V_i^I(t), V_i^D(t)\right] + V^Y(t), \tag{14}$$

$$\forall \text{ mutually determined } V_i^J \quad \neq \quad \widehat{V}_i^J, \quad J = I, D, K, \tag{15}$$

where $V^Y(t)$ holds contracts with other intermediate suppliers fixed.

• Free entry into final output production: $\widehat{V}^Y(t) \leq 0$

• Free entry of replacement capital: $\widehat{V}_i^K(t) \leq \widehat{K}_i(t)$

4 The Acyclical Balanced Growth Path

In this section, we briefly consider the existence of an equilibrium growth path along which the utilized capital of firms grows monotonically, entrepreneurship is continuous and implementation is never delayed. We derive the equilibrium without utilizing long–term contracts, so that all transactions occur in spot markets, since it will be seen that allowing for them does not affect the equilibrium. While the acyclical growth path is not our main focus, it is useful for understanding our later results.

Consumption satisfies the familiar differential equation:

$$\frac{\dot{C}(t)}{C(t)} = \frac{r(t) - \rho}{\sigma}, \tag{16}$$

where $r(t) = \dot{R}(t)$, denotes the continuous time interest rate. In the absence of uncertainty or adjustment costs, and as long as utilized capital is anticipated to grow, capitalists never acquire more capital than is needed for production, so that

$$K_i^u(t) = K_i(t). \tag{17}$$

Within each sector, i, the existence of potential capital entrants implies that capital owners cannot earn excess returns on marginal units. Hence:

Lemma 1 *: As long as new capital is being built, free–entry into capital markets implies that*

$$q_i\,(t) = q\,(t) = r\,(t) \qquad \forall\ i. \tag{18}$$

Final goods producers choose intermediates to maximize profits, taking their prices as given. The derived demand for intermediate i is

$$x_i^d(t) = \frac{Y(t)}{p_i(t)}. \tag{19}$$

The unit elasticity of demand for intermediates implies that limit pricing, which drives out the previous incumbent, is optimal:

Lemma 2 *: The limit price is given by*

$$p_i(t) = \frac{q(t)^{\alpha} w(t)^{1-\alpha}}{\mu e^{-(1-\alpha)\gamma} A_i^{1-\alpha}(t)}. \tag{20}$$

where $\mu = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$.

The resulting instantaneous profit earned in each sector is given by

$$\pi(t) = (1 - e^{-(1-\alpha)\gamma})Y(t). \tag{21}$$

Aggregate final output can be expressed as

$$Y(t) = [K(t)]^{\alpha} \left[\overline{A}(t)L(t)\right]^{1-\alpha}, \tag{22}$$

where $\overline{A}\,(t) = \exp\left(\int_0^1 \ln A_i(t)di\right)$. Along the acyclical steady–state growth path, a constant fraction of the labor force is allocated to entrepreneurship. The standard solution method yields the following steady state implication:

Proposition 1 *: If*

$$\frac{(1 - e^{-(1-\alpha)\gamma})\gamma(1-\sigma)}{1 - \alpha e^{-(1-\alpha)\gamma}} < \frac{\rho}{\delta} \tag{23}$$

then there exists an acyclical equilibrium with a constant growth rate given by

$$g^a = \max\left[\frac{[\delta(1 - e^{-(1-\alpha)\gamma}) - \rho(1-\alpha)e^{-(1-\alpha)\gamma}]\gamma}{1 - \alpha e^{-(1-\alpha)\gamma} - \gamma\,(1-\sigma)\,(1-\alpha)\,e^{-(1-\alpha)\gamma}}, 0\right]. \tag{24}$$

Along this equilibrium growth path, the inequality in (23) implies that $r(t) > g^a(t)$ at every moment. Along a balanced growth path, this condition must hold for the transversality condition to be satisfied and hence for utility to be bounded. However, this condition also ensures both that no output is stored, and that the implementation of any innovation is never delayed (see Francois and Lloyd–Ellis, 2003, for further elaboration). Allowing long–term supply contracts would only undermine the existence of this equilibrium growth path if contracting for non-spot market prices could make both parties to a contract better off. However, since all quantities are chosen optimally in the spot market in this equilibrium, such contracts would necessarily involve one side being made worse off.

5 The Posited Cyclical Growth Path

In this section, we begin by informally positing a cyclical equilibrium growth path in which, due to the rigid nature of capital, under–utilization may occur during downturns. We then posit the equilibrium behavior for capitalists and entrepreneurs over the cycle and detail the implications for contracting, consumption and aggregate entrepreneurship. In Section 5, we derive more formally the implications of this behavior over each phase of the cycle, and Section 6 then demonstrates the consistency of the posited behavior of entrepreneurs and capitalists in an equilibrium steady state and derives the conditions for existence.

Figures 4 and 5 depict the movement of key variables during the cycle. Cycles are indexed by the subscript v, and feature a consistently recurring pattern through their phases. The vth cycle features three distinct phases:

• The **expansion** is triggered by a productivity boom at time T_{v-1} and continues through subsequent capital accumulation, leading to continued growth in output, consumption and wages. Over this expansion phase the interest rate falls and investment, though positive, declines as the capital stock rises. Also, since labor's productivity in manufacturing intermediates is relatively high, no labor is allocated to entrepreneurship. Through time, continued capital accumulation lowers returns to further investment, rendering entrepreneurship relatively more attractive. Eventually

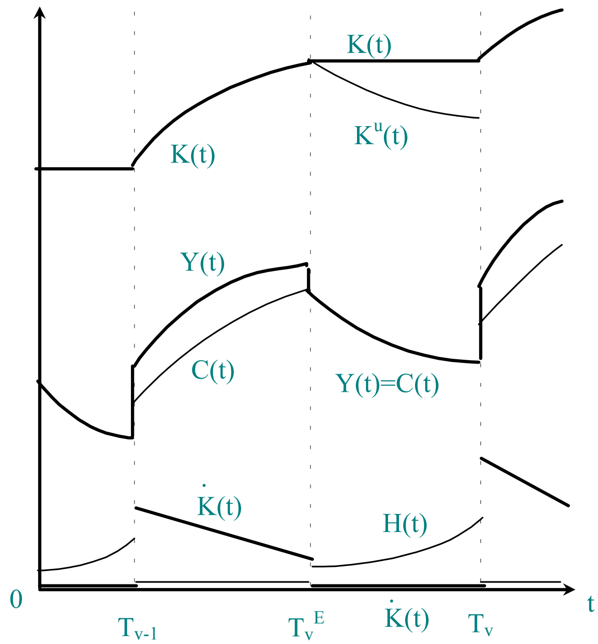

Figure 4: Evolution of Key Aggregates over the Cycle

innovation and reorganization re-commence, drawing labor hours from production. At this point, the return on investment in physical assets drops to zero, and investment ceases temporarily.

• The **contraction** starts with a collapse in fixed capital formation at time T_v^E as investment shifts towards longer-term focused activities. Intermediate producers experience a reduction in aggregate demand and then optimally re-allocate resources to relatively labor intensive entrepreneurial reorganization in order to raise productivity for the forthcoming boom. Due to irreversibility and the putty-clay nature of installed capital, labor's departure from production implies that capital cannot be fully utilized. Through the downturn, capital utilization falls and is traded at a constant price. Innovation and reorganization continue to increase throughout this phase so that the economy continues to contract through declining consumption expenditure.

• The **boom** occurs at an endogenously determined date, T_v, when the value of implementing stored innovations first exceeds the value of delaying their implementation. At that point, successful entrepreneurs implement their innovation, starting the upswing once again. During the boom the returns to production rise above those of innovation and re–organization, drawing skilled labor out of entrepreneurship and into production. Returns to capital also rise with the new more productive technologies, so that capital accumulation recommences and the cycle begins again.

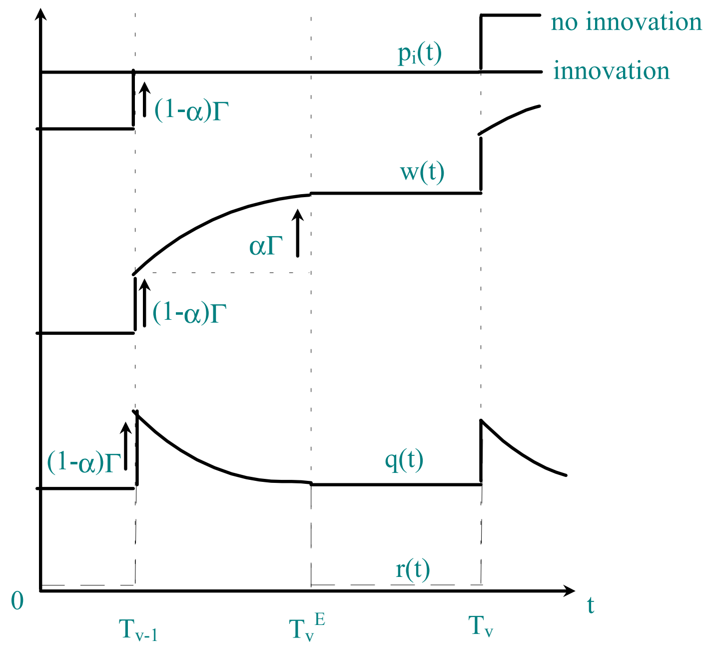

Figure 5: Evolution of Prices over Cycle

5.1 Contracts

Because competition from replacement capitalists weakens during a recession, contracts arise endogenously as capitalists compete to offer guaranteed prices to capital users in advance of the downturn. At this point, we anticipate the form these contracts will take and derive their implications. In Section 6 we shall verify that these contracts are constrained optimal given these implications. Contracts are written during the expansion of each cycle and terminate just before the boom of the next. Contracts between intermediate and final goods producers specify a sequence of quantities and prices for intermediates through the cycle. These contracted prices reflect the marginal costs of the main competitor: the previous incumbent holding the next best technology. The prices and quantities agreed to in the intermediate goods contract take the same form as the spot market values along the acyclical growth path:

$$p_i^c(t) = \frac{q(t)^{\alpha} w(t)^{1-\alpha}}{\mu e^{-(1-\alpha)\gamma} A_i^{1-\alpha}(T_{v-1})} \quad \forall t \in [T_{v-1}, T_v). \tag{25}$$

$$x_i^c(t) = \frac{Y(t)}{p_i^c(t)} \quad \forall t \in [T_{v-1}, T_v). \tag{26}$$

Through the upturn, contracts between capitalists and entrepreneurs specify steady expansion in each sector's capital stock, with capital being traded at a declining market–clearing price.

During a contraction, the contracts specify a fixed rental price for capital and a declining utilization rate. The equilibrium contracts are written before T_{v-1}^{E} over rental rate and utilized capital $\{q_i^c(t), K_i^c(t), T_v\}$ and take the following form:

$$q_i^c(t) = \begin{cases} \alpha e^{-(1-\alpha)\gamma}\overline{A}^{1-\alpha}(T_{v-1})K(t)^{\alpha-1} & \forall \ t \in [T_{v-1}, T_v^E] \\ \alpha e^{-(1-\alpha)\gamma}\overline{A}^{1-\alpha}(T_{v-1})K(T_v^E)^{\alpha-1} & \forall \ t \in (T_v^E, T_v) \end{cases} \tag{27}$$

$$K_i^c(t) = \begin{cases} K(t) & \forall \ t \in [T_{v-1}, T_v^E] \\ \lambda(t)K(T_v^E) & \forall \ t \in (T_v^E, T_v), \end{cases} \tag{28}$$

where $\lambda(t) < 1$ denotes the utilization rate. Note that the posited contracts are symmetric across sectors. In fact, as we will see, a contract specifying a constant price through the downturn is not necessary. What will be required is that the price sequence is such that its average equals a unique value. However, a constant price with this property is an equilibrium, and if there is any arbitrarily small cost to price adjustment it is unique.

5.2 Consumption

Over intervals during which the discount factor does not jump, consumption is allocated as described by (16). However along the cyclical growth path, the discount rate jumps at the boom, so that consumption exhibits a discontinuity during implementation periods. We therefore characterize the optimal evolution of consumption from the beginning of one cycle to the beginning of the next by the difference equation

$$\sigma \ln \frac{C_0(T_v)}{C_0(T_{v-1})} = R(T_v) - R(T_{v-1}) - \rho (T_v - T_{v-1}). \tag{29}$$

where the 0 subscript is used to denote values of variables the instant after the implementation boom. Note that a sufficient condition for the boundedness of the consumer's optimization problem is that $\ln \frac{C_0(T_v)}{C_0(T_{v-1})} < R(T_v) - R(T_{v-1})$ for all v, or that

$$\frac{(1-\sigma)}{T_v - T_{v-1}} \ln \frac{C_0(T_v)}{C_0(T_{v-1})} < \rho \quad \forall \ v. \tag{30}$$

In our analysis below, it is convenient to define the discount factor that will be used to discount from some time t during the cycle to the beginning of the next cycle. This discount factor is given by

$$\beta(t) = R(T_v) - R(t) = R(T_v) - R(T_{v-1}) - \int_{T_{v-1}}^{t} r(s)ds. \tag{31}$$

5.3 Innovation

Let $P_i(s)$ denote the probability that, since time T_v, no entrepreneurial success has been made in sector i by time s. It follows that the probability of there being no entrepreneurial success by time T_{v+1} conditional on there having been none by time t, is given by $P_i(T_{v+1})/P_i(t)$. Hence, the value of an incumbent firm in a sector where no entrepreneurial success has occurred by time t during the vth cycle can be expressed as

$$V_i^I(t) = \int_t^{T_{v+1}} e^{-\int_t^\tau r(s)ds} \pi_i(\tau) d\tau + \frac{P_i(T_{v+1})}{P_i(t)} e^{-\beta(t)} V_{0,i}^I(T_{v+1}). \tag{32}$$

The first term here represents the discounted profit stream that accrues to the entrepreneur with certainty during the current cycle, and the second term is the expected discounted value of being an incumbent thereafter.

Lemma 3 *In a cyclical equilibrium, successful entrepreneurs can credibly signal a success immediately and all entrepreneurship in their sector will stop until the next round of implementation.*

In the cyclical equilibrium, entrepreneurs' conjectures ensure no more entrepreneurship in a sector once a signal of success has been received, until after the next implementation. The expected value of an entrepreneurial success occurring at some time $t \in \left(T_v^E, T_v\right)$ but whose implementation is delayed until time T_v is thus:

$$V_i^D(t) = e^{-\beta(t)} V_{0,i}^I(T_v). \tag{33}$$

Since no implementation occurs during the cycle, the entrepreneur is assured of incumbency until at least T_{v+1}. Incumbency beyond that time depends on the probability that there has not been another entrepreneurial success in that sector up until then.[22] The symmetry of sectors implies that entrepreneurial effort is allocated evenly over all sectors that have not yet experienced a success within the cycle. This clearly depends on some sectors not having already received an entrepreneurial innovation, an equilibrium condition that will be imposed subsequently (see Section 6). Thus the probability of not being displaced at the next implementation is

$$P_i(T_v) = \exp\left(-\int_{T_v^E}^{T_v} \widetilde{H}_i(\tau) d\tau\right), \tag{34}$$

[22] A signal of further entrepreneurial success submitted by an incumbent is not credible in equilibrium because incumbents have incentive to lie to protect their profit stream. No such incentive exists for entrants since, without a success, profits are zero.

where $\widetilde{H}_i(\tau)$ denotes the quantity of labor that would be allocated to entrepreneurship if no entrepreneurial success had occurred prior to time τ in sector i. The amount of entrepreneurship varies over the cycle, but at the beginning of each cycle all industries are symmetric with respect to this probability: $P_i(T_v) = P(T_v)\ \forall i$.

6 The Three Phases of the Cycle

6.1 The (Neoclassical) Expansion

We denote the improvement in aggregate productivity during implementation period T_v (and, hence, the growth in the average unit cost) by $e^{(1-\alpha)\Gamma_v}$, where

$$\Gamma_v = \ln\left[\overline{A}_v/\overline{A}_{v-1}\right], \tag{35}$$

and $\overline{A}_v = \exp\left(\int_0^1 \ln A_i(T_v)di\right)$. Following an implementation boom, the state of technology in use remains unchanged for the rest of the cycle. An implication of the Cobb–Douglas structure is that, through competition, the unit factor price index simply reflects this level of technology.

Lemma 4 *: The input price index for $t \in [T_{v-1}, T_v^E]$ is constant and uniquely determined by the level of technology*

$$q(t)^{\alpha}w(t)^{1-\alpha} = \mu e^{-(1-\alpha)\gamma}\overline{A}_{v-1}^{1-\alpha}. \tag{36}$$

As a result of the boom, wages rise rapidly. Since the next implementation boom is some time away, the present value of engaging in entrepreneurship falls below the wage, $\delta V^D(t) < w(t)$. During this phase, no labor is allocated to entrepreneurship and no new innovations come on line. However, final output grows in response to capital accumulation financed from household savings. In equilibrium the Euler equation and aggregate resource constraint imply dynamics that are almost identical to those of the Ramsey model:[23]

Proposition 2 *During the expansion, capital and consumption evolve according to:*

$$\frac{\dot{C}(t)}{C(t)} = \frac{\alpha e^{-(1-\alpha)\gamma}\overline{A}_{v-1}^{1-\alpha}K(t)^{\alpha-1} - \rho}{\sigma} \tag{37}$$

$$\dot{K}(t) = \overline{A}_{v-1}^{1-\alpha}K(t)^{\alpha} - C(t). \tag{38}$$

[23] Note that, unlike the Ramsey model, the rate of return on savings is not equal to the marginal product of capital, but rather is a fraction $e^{-(1-\alpha)\gamma}$ of it. This reflects the entrepreneurial share of this marginal product accruing as a monopoly rent.

Since all capital is utilized, Lemma 1 applies so that

$$r(t) = q(t) = \alpha e^{-(1-\alpha)\gamma} \overline{A}_{v-1}^{1-\alpha} K(t)^{\alpha-1}. \tag{39}$$

Thus, as capital accumulates, the interest rate declines. Since technology is unchanging, Lemma 4 implies the wage must be rising

$$\frac{\dot{w}(t)}{w(t)} = -\left(\frac{\alpha}{1-\alpha}\right)\frac{\dot{q}(t)}{q(t)} = \alpha\frac{\dot{K}(t)}{K(t)} > 0 \tag{40}$$

During the expansion, the expected value of entrepreneurship, $\delta V^D(t)$, is necessarily growing at the rate of interest, but continues to be dominated by the wage in production. After enough capital has been accumulated, however, $\delta V^D(t)$ eventually equals $w(t)$. At this point, if all workers were to remain in production, returns to entrepreneurship would strictly dominate those in production. As a result labor hours are re–allocated from production and into innovation, which triggers the contractionary phase.

6.2 The (Keynesian) Contraction

Because of the putty–clay nature of capital, as labor starts to be withdrawn from production the capital–labor ratio cannot be adjusted from $\kappa(T_v^E)$ and output must contract. Since technology is also fixed during this phase, the wage must be constant:

Lemma 5 *: The wage for $t \in [T_v^E, T_v]$ is constant and determined by the level of technology and the capital–labor ratio chosen at the last peak, $\kappa(T_v^E)$:*

$$w(t) = \bar{w}_v = (1-\alpha)e^{-(1-\alpha)\gamma} \overline{A}_{v-1}^{1-\alpha} \kappa(T_v^E)^{\alpha}. \tag{41}$$

Since there is free entry into entrepreneurship, $w(t) = \delta V^D(t)$, so that the value of entrepreneurship, $\delta V^D(t)$, is also constant. Since the time until implementation for a successful entrepreneur is falling and there is no stream of profits, because implementation is delayed, the instantaneous interest rate necessarily equals zero. If it were not, entrepreneurial activity would be delayed to the instant before the boom. Therefore:

$$r(t) = \frac{\dot{V}^D(t)}{V^D(t)} = \frac{\dot{w}(t)}{w(t)} = 0. \tag{42}$$

Note that this zero interest rate is consistent with the fact there is now excess (under–utilized) capital in the economy. Since marginal returns to capital in this phase are zero, physical investment ceases and the only investment is that in innovation, undertaken by entrepreneurs.

Lemma 6 *: At T_v^E, investment in physical capital falls discretely to zero and entrepreneurship jumps discretely to $H_0(T_v^E) > 0$.*

A switch like this across types of investment is also a feature of the models of Matsuyama (1999, 2001) and Walde (2002). However, here factor intensity differences between entrepreneurship and investment lead to a crash in output followed by continued decline through the recession. Although investment falls discretely at $t = T_v^E$, consumption must be constant across the transition between phases because the discount factor does not change discretely. With putty–clay technology, the decline in output due to the fall in investment demand is proportional to the fraction of labor hours withdrawn from production. It follows that the fraction of labor hours allocated to entrepreneurship at the start of the downturn, $H_0(T_v^E)$, which we denote as H_v from now on, equals the rate of investment at the peak of the expansion:

$$H_v = \frac{\dot{K}\left(T_v^E\right)}{Y\left(T_v^E\right)} = 1 - \frac{C\left(T_v^E\right)}{\overline{A}_{v-1}^{1-\alpha} K(T_v^E)^\alpha}. \tag{43}$$

Although consumption cannot fall discretely at T_v^E, the zero interest rate implies that consumption must be declining after T_v^E,[24]

$$\frac{\dot{C}(t)}{C(t)} = \frac{r(t) - \rho}{\sigma} = -\frac{\rho}{\sigma}, \tag{44}$$

as resources flow out of production and into entrepreneurship.

Since $Y(t) = C(t)$, the growth rate in the hours allocated to production is also given by (44) and so aggregate entrepreneurship at time t is given by

$$H(t) = 1 - (1 - H_v)\, e^{-\frac{\rho}{\sigma}[t - T_v^E]}. \tag{45}$$

Note that the putty–clay nature of capital implies that as labor leaves current production, capital utilization falls in the same proportion. It follows that the capital utilization rate specified in the equilibrium contract (28) is given by

$$\lambda(t) = (1 - H_0(T_v^E))e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}. \tag{46}$$

During the downturn, in the absence of a capital contract, entrepreneurs would be vulnerable to an increasing rental price through the downturn. To see why, observe that, in order to forestall

[24] Although $r = 0$, strict preference for zero storage results from the arbitrarily small storage costs.

entry by a competing capitalist, the incumbent capitalist is constrained to offer a price–quantity sequence which satisfies

$$V^K\left(K(t),t\right) = \int_t^{T_v} e^{-[R(\tau)-R(t)]}\left[q\left(\tau\right)K^u(\tau) - \dot{K}(\tau)\right]d\tau + e^{-\beta(T_v)}V^K\left(K(t),T_v\right) \leq K(t), \quad (47)$$

where $V^K\left(K(t),\tau\right)$ denotes the value of the installed capital at time τ. During the downturn $r\left(t\right)=0$ and $\dot{K}(\tau)=0$, so that for $t \in \left[T_v^E, T_v\right]$, the condition becomes:

$$\int_t^{T_v} q\left(\tau\right)\lambda(\tau)K(T_v^E)dt + e^{-\beta(T_v)}V^K\left(K(T_v^E),T_v\right) \leq K(T_v^E). \quad (48)$$

However competition from potential replacement capitalists at the beginning of the next cycle ensures that $V^K\left(K(T_v^E),T_v\right) = K(T_v^E)$. Dividing by $K(T_v^E)$ and re–arranging, using (46), yields a necessary restriction to forestall entry during the downturn:

$$\int_t^{T_v} q\left(\tau\right)(1-H_v)e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}d\tau \leq 1 - e^{-\beta(T_v)}. \quad (49)$$

The right hand side of this expression is constant throughout the downturn, but the left–hand side would be decreasing through the downturn if $q\left(t\right)$ were constant. It follows that, in the absence of a contracted price, the capitalist could raise q through the downturn and still satisfy (49).

The main implication of this is that, without a contract written before T_v^E delineating the price charged by the capitalist for the remainder of the cycle, entrepreneurs will face an increasing rental rate for capital through the downturn. Given the potential for such price gouging, entrepreneurs will demand the writing of such contracts before T_v^E, when the cost of replacement capital is low.

The first thing to note about any such contract is that it must satisfy the capital feasibility constraint above, which will bind at $t = T_v^E$:

Lemma 7 *Any capital supply contract $\{q_i^c(\tau), K_i^u(\tau)\}$ signed at some date $t \in [T_{v-1}, T_v^E)$ must satisfy:*

$$\int_{T_v^E}^{T_v} q^c\left(\tau\right)(1-H_v)e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}d\tau = 1 - e^{-\beta(T_v)} \quad (50)$$

There are a number of price sequences $q^c(t)$ that could satisfy this condition, however the average level of prices through $t \in [T_{v-1}, T_v^E)$ is unique. Let this average in the vth cycle be

$$\overline{q}_v \equiv \frac{\int_{T_v^E}^{T_v} q^c\left(\tau\right)(1-H_v)e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}d\tau}{\int_{T_v^E}^{T_v}(1-H_v)e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}d\tau}. \quad (51)$$

Using 50, and integrating the denominator through the downturn, Δ_v^E, this implies:

$$\overline{q}_v = \frac{1 - e^{-\beta(T_v)}}{(1 - H_v)\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma}\right)}. \tag{52}$$

A further feature of such contracts is that they must induce a cost minimizing capital/labor ratio, in order again to forestall entry by competing capital providers. The standard marginal condition applies at every instant through the upturn. With putty/clay capital and zero discounting through the downturn, it is possible to treat the whole of the contractionary phase as if it were a single production period. Consequently, a condition analogous to the standard marginal condition applies to the optimal capital/labor ratio through the downturn,

$$\frac{(1-\alpha)\,K\left(T_v^E\right)}{\alpha L\left(T_v^E\right)} = \frac{\overline{w}_v}{\overline{q}_v}.$$

Since, $L\left(T_v^E\right) = 1$, it follows that:

Proposition 3 *For a capital–supply contract to be efficient through the downturn it is necessary that capital is installed only up to the point at which the marginal return to capital is equal to its average rental price:*

$$q(T_v^E) = \alpha e^{-(1-\alpha)\gamma}\overline{A}_{v-1}^{1-\alpha}K(T_v^E)^{\alpha-1} = \overline{q}. \tag{53}$$

Equating (52) and (53), substituting for $1 - H_v$ using (43), it follows that the capital–consumption ratio at the height of the expansion can be expressed as:

$$\frac{K\left(T_v^E\right)}{C\left(T_v^E\right)} = \frac{\alpha e^{-(1-\alpha)\gamma}\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma}\right)}{1 - e^{-(1-\alpha)\Gamma_v}}. \tag{54}$$

Note that $K\left(T_v^E\right)$ is also the effective capital stock at the beginning of the next boom since there is no depreciation and no capital is accumulated through the recession.

6.3 The Boom

Productivity growth at the boom is given by $\Gamma_v = (1 - P(T_v))\gamma$, where $P(T_v)$ is defined by (34). Substituting in the allocation of labor to entrepreneurship through the downturn given by (45) and letting

$$\Delta_v^E = T_v - T_v^E, \tag{55}$$

yields the following implication.

Proposition 4 *: In an equilibrium where there is positive entrepreneurship only over the interval* $(T_v^E, T_v]$*, the growth in productivity during the succeeding boom is given by*

$$\Gamma_v = \delta\gamma\Delta_v^E - \delta\gamma(1 - H_v)\left(\frac{1 - e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma}\right). \tag{56}$$

For an entrepreneur who is holding an innovation, $V^I(t)$ is the value of implementing immediately. During the boom, for entrepreneurs to prefer to implement immediately, it must be the case that

$$V_0^I(T_v) > V_0^D(T_v), \tag{57}$$

recalling that 0 subscripts denote values immediately after implementation. Just prior to the boom, when the probability of displacement is negligible, the value of implementing immediately must equal that of delaying until the boom:

$$\delta V^I(T_v) = \delta V^D(T_v) = w\left(T_v\right). \tag{58}$$

From (57), the return to entrepreneurship at the boom is the value of immediate (rather than delayed) incumbency. It follows that free entry into entrepreneurship at the boom requires that

$$\delta V_0^I(T_v) \leq w_0\left(T_v\right). \tag{59}$$

The opportunity cost of financing entrepreneurship is the rate of return on shares in incumbent firms in sectors where no innovation has occurred. Just prior to the boom, this is given by the capital gains in sectors where no entrepreneurial successes have occurred;

$$\beta(T_v) = \log\left(\frac{V_0^I(T_v)}{V^I(T_v)}\right). \tag{60}$$

Note that since the short–term interest rate is zero over this phase, $\beta(t) = \beta(T_v)$, $\forall\ t \in (T_v^E, T_v)$. Combined with (58) and (59) it follows that asset market clearing at the boom requires

$$\beta(T_v) \leq \log\left(\frac{w_0\left(T_v\right)}{w(T_v)}\right) = (1-\alpha)\Gamma_v. \tag{61}$$

Free entry into entrepreneurship ensures that $\beta(T_v) > (1-\alpha)\Gamma_v$ cannot obtain in equilibrium.

Provided that $\beta(t) > 0$, households will never choose to store final output from within a cycle to the beginning of the next either because it is dominated by the long–run rate of return on claims to future profits. However, unlike final output, the return on stored intermediate output in sectors with no entrepreneurial successes is strictly positive, because of the increase in its price that occurs as a result of the boom. Even though there is a risk that the intermediate becomes obsolete at the boom, if the anticipated price increase is sufficiently large, households may choose to purchase claims to intermediate output rather than claims to firm profits.

If innovative activities are to be financed at time t, it cannot be the case that households are strictly better off buying claims to stored intermediate goods. In sectors with no entrepreneurial success, incumbent firms could sell such claims, use them to finance greater current production and then store the good to sell at the beginning of the next boom when the price is higher. In this case, since the cost of production is the same whether the good is stored or not, the rate of return on claims to stored intermediates in sector i is $\log p_{i,v+1} / p_{i,v} = (1-\alpha)\Gamma_v$.

It follows that the long run rate of return on claims to firm profits an instant prior to the boom must satisfy

$$\beta(T_v) \geq (1-\alpha)\Gamma_v. \tag{62}$$

Free–entry into arbitrage ensures that $\beta(T_v) < (1-\alpha)\Gamma_v$ cannot obtain in equilibrium. Because there is a risk of obsolescence, this condition implies that at any time prior to the boom the expected rate of return on claims to stored intermediates is strictly less than $\beta(t)$.

Combining (61) and (62) yields the following implication of market clearing during the boom for the long–run growth path:

Proposition 5 *Asset market clearing at the boom requires that*

$$\beta(T_v) = (1-\alpha)\Gamma_v. \tag{63}$$

Asset market–clearing thus yields a unique relationship between the discount applied over the boom, and productivity growth.[25]

The growth in output at the boom exceeds the growth in productivity for two reasons: first labor is re–allocated back into production, and second the previously unutilized capital is now

[25] Shleifer's (1986) model featured multiple expectations–driven steady state cycles. Such multiplicity cannot occur here because, unlike Shleifer, the possibility of storage that we allow forces a tight relationship between Γ_v and Δ_v^E as depicted in Proposition 4. Since Γ_v, Δ_v^E pairs must satisfy this restriction as well, in general, multiple solutions cannot be found. This however does not rule out cycles of a qualitatively different nature to those analyzed here.

being used productively. Since just before the boom, both inputs are a fraction $(1-H_v)e^{-\frac{\rho}{\sigma}\Delta_v^E}$ of their peak levels, output growth through the boom is given by

$$\begin{aligned}\Delta \ln Y(T_v) &= (1-\alpha)\Gamma_v + (1-\alpha)\Delta \ln L + \alpha \Delta \ln K^u \\ &= (1-\alpha)\Gamma_v + \frac{\rho}{\sigma}\Delta_v^E - \ln(1-H_v) \end{aligned} \tag{64}$$

It follows directly from Proposition 5 that growth in output exceeds the discount factor across the boom. Since profits are proportional to output, this explains why firms are willing to delay implementation during the downturn.

The boom in output can be decomposed into a boom in consumption and investment. From the Euler equation, we can compute consumption growth across the boom:

$$\Delta \ln C(T_v) = \frac{(1-\alpha)}{\sigma}\Gamma_v. \tag{65}$$

Notice that whether the growth in consumption exceeds the growth in productivity at the boom, depends on the value of σ. In particular, if $\sigma < 1$, consumption growth must exceed aggregate productivity growth. Finally, since in the instant prior to the boom $C(T_v) = Y(T_v)$, it follows that the investment rate at the boom jumps to

$$\frac{\dot{K}_0(T_v)}{Y_0(T_v)} = 1 - (1-H_v)e^{\left(\frac{1-\sigma}{\sigma}\right)(1-\alpha)\Gamma_v - \frac{\rho}{\sigma}\Delta_v^E} \tag{66}$$

7 Optimal Behavior During the Cycle

Given the dynamics implied above, in this section we derive conditions which must be satisfied in order for the posited behavior of capitalists and entrepreneurs to be optimal.

7.1 Optimal Entrepreneurship and Implementation

Equilibrium entrepreneurial behavior imposes the following requirements on our hypothesized cycle:

• Successful entrepreneurs at time $t = T_v$ must prefer to implement immediately, rather than delay implementation until later in the cycle or the beginning of the next cycle:

$$V_0^I(T_v) > V_0^D(T_v). \tag{E1}$$

• Entrepreneurs who successfully innovate during the downturn must prefer to wait until the beginning of the next cycle rather than implement earlier and sell at the limit price:

$$V^I(t) < V^D(t) \qquad \forall\, t \in (T_v^E, T_v) \tag{E2}$$

• No entrepreneur wants to innovate during the slowdown of the cycle. Since in this phase of the cycle $\delta V^D(t) < w(t)$, this condition requires that

$$\delta V^I(t) < w(t) \qquad \forall\, t \in (0, T_v^E) \tag{E3}$$

The conditions on the value functions above take as given that entrepreneurs do not produce in excess of current demand and store their output until the boom. Provided that the incumbent entrepreneur does not terminate the capital supply contract, (63) ensures that storage across the boom is not optimal. However, since in the posited equilibrium the capital stock is being under–utilized, it is possible that just before the boom a rival entrepreneur who has successfully innovated may be able to "buy out" the contract and utilize all the capital, meeting the current demand for output and storing the remainder until the boom.

This rival would not benefit from taking over the capital contract of the incumbent under identical terms. From (63), producing output and storing it until the boom is not optimal if he must pay a constant amount $\bar{q}$ for capital. Moreover, under (E2) implementation and sale before the boom is not optimal. However, the rival may be willing to take-over the use rights if able to pay $\overline{q}$ for the amount $K^c(t)$ as in the incumbent's contract, utilize extra units of idle capital at some price $\widetilde{q} < \overline{q}$, and store. Clearly any $\widetilde{q} > 0$ for the excess units would be amenable to the capitalist. The most the rival will be willing to pay per period for the current capital is $\overline{q}K(T_v^E)$, since $e^{-\beta(T_v)}q(T_v) = \overline{q}$. To buy out the contract, the rival must compensate the incumbent for the loss of profits sustained for the remainder of the cycle and must offer the capitalist at least the payment he is currently receiving, $\overline{q}(1 - H_v)e^{-\frac{\rho}{\sigma}(t - T_v^E)}K(T_v^E)$ per period. It follows that such a contract buy–out will not a be mutually acceptable at time t if

$$\int_t^{T_v} \pi(\tau)d\tau + \int_t^{T_v} \overline{q}(1 - H_v)e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}K(T_v^E)d\tau \geq \int_t^{T_v} \overline{q}K(T_v^E)d\tau. \tag{67}$$

The following proposition provides a sufficient condition for this to hold throughout the downturn:

Proposition 6 *: If*

$$(1-(1-\alpha)e^{-(1-\alpha)\gamma})(1-H_v)e^{-\frac{\rho}{\sigma}\Delta_v^E} > \alpha e^{-(1-\alpha)\gamma} \tag{E4}$$

then entrepreneurs who successfully innovate during the downturn prefer to wait until the beginning of the next cycle rather than displace the incumbent, produce now and store until the boom.

In effect, condition (E4) explains how it is possible for there to be under–utilized capital during a recession even though there exist rivals who could potentially use the capital stock more profitably. The reason is that the capital stock is "lumpy", so that the rival cannot use a part of it while the incumbent continues to produce. For this reason the rival must compensate the incumbent for his profit loss and this "endogenous" fixed cost is too large for entry to be profitable under recessionary demand conditions. Entry does not become profitable until the boom. There, demand is high and entry costs low because the previous incumbent's profits do not need to be compensated as they have already been destroyed by the implementation of a superior production process.

Note finally that in constructing the equilibrium above we have implicitly imposed the requirement that the downturn is not long enough that all sectors innovate. Thus the following condition must be satisfied with strict inequality:

$$P(T_v) > 0. \tag{E5}$$

Taken together conditions (E1) through (E5) are restrictions on entrepreneurial behavior that must be satisfied for the cyclical growth path we have posited to be an equilibrium. However, we must first check that under these conditions, the contracts we have specified are indeed undominated.

7.2 The Optimality of Contracts

7.2.1 The Intermediate Good Supply Contract

The need for an intermediate contract arises because the lumpiness and sector specificity of installed capital implies that only one intermediate producer can use the sector's capital. Though it is possible to commission the building of a new capital stock, if guaranteed a sufficiently high rental rate, the rental rate so required increases through the cycle, so that the threat of entry provides progressively weaker restraint through time. By negotiating a contract before the downturn, bids from the previous intermediate producer force limit pricing by the current incumbent at a relatively low marginal cost, since at this time the cost of building replacement capital is relatively low. Thus the intermediate goods contract guards the final goods producers against the increasing monopoly power of the incumbent through the downturn by pinning the producer down to a price/quantity pair while the previous incumbent's threat of entry is greatest.

Lemma 8 *The contracted price sequence for intermediate i, $p_i^c(t)$, and quantity $x_i^c(t)$, for $t \in [T_{v-1}, T_v)$, satisfying (25) and (26) is optimal, given a sequence of input prices $w(t), q(t)$ for $t \in [T_{v-1}, T_v)$ faced by the previous incumbent.*

The input price for labor, $w(t)$ is determined in the per period spot market so it remains now to determine the sequence of capital prices.

7.2.2 The Capital Supply Contract

The aim of capital supply contracts is to forestall hold-up by the capitalist, but the contract's "reach" is limited on the entrepreneur's side. Unlike capital which is infinitely lived, entrepreneurs lose their productive advantage when displaced by superior producers, so that they cannot make unconditional promises to purchase capital into the indefinite future. All contracts are thus contingent upon the entrepreneur's continuing production. We show now that the earlier posited contract comprising (27) and (28) is an optimal response to the posited behavior of other agents over the cycle:

Proposition 7 *Provided (E1)–(E5) hold then, in each sector i, at the boom of every cycle (T_v, $v = 1...\infty$), an equilibrium contract for the capitalist and leading entrepreneur is a sequence of prices $q^c(t)$ and capital $K^c(t)$ for all $t \in [T_v, T_{v+1})$ that satisfies (27) and (28).*

Note that the cyclical equilibrium is supported by the limitations on contracting that we have imposed. The critical, and we think realistic, assumption is that only future prices and quantities can be contracted ex ante. Allowing for a richer set of contracting possibilities would overturn this result. The sort of environments required would need to allow that, in addition to a time varying price q and quantity K for capital, it would be possible to condition transfers between the parties on other actions that they or other parties take. For example if the new incumbent entrepreneur (who arrives probabilistically in the downturn) could somehow be party to the contract at time T_v, then full utilization of the capital through the downturn could also be contracted ex ante. Such a rich contracting environment, however, seems to require unrealistically complex and difficult to observe details to be enforceable between the parties. Thus endogenous underutilization, which corresponds to that observed in actual business cycles, arises here due to seemingly natural limitations in contracting.

8 The Stationary Cyclical Growth Path

Here we characterize the stationary cyclical growth path implied by Propositions 2 to 5. To allow a stationary representation, we normalize all aggregate by dividing by $\bar{A}_{v-1}$ and denote the result with lower case variables.

First recall from Proposition 2, that the dynamics of the economy during the expansion are analagous to those in the Ramsey model without technological change. Let $c_v = c(T_v^E)$ and $k_v = k(T_v^E)$ denote the normalized values of consumption and capital at the peak of the vth expansion. Given initial values $c_0(T_{v-1})$ and $k_0(T_{v-1})$, and an expansion length Δ_v^X, it is possible to summarize the expansion as follows:

$$c_v = f(c_0(T_{v-1}), k_0(T_{v-1}), \Delta_v^X) \tag{68}$$

$$k_v = g(c_0(T_{v-1}), k_0(T_{v-1}), \Delta_v^X), \tag{69}$$

where $f(\cdot)$ and $g(\cdot)$ are well–defined functions. Since capital accumulation stops in the recession, and $\overline{A}$ rises by $e^{\Gamma_{v-1}}$, it follows that $k_0 = e^{-\Gamma_{v-1}} k_{v-1}$. From (44), consumption declines by a factor $e^{-\frac{\rho}{\sigma}\Delta_{v-1}^E}$ in the recession. When combined with its increase at the boom, from (65), this yields $c_0 = e^{\left(\frac{1-\alpha}{\sigma}-1\right)\Gamma_{v-1}-\frac{\rho}{\sigma}\Delta_{v-1}^E} c_{v-1}$. Substituting for c_0 and k_0 then yields

$$c_v = f(e^{\left(\frac{1-\alpha}{\sigma}-1\right)\Gamma_{v-1}-\frac{\rho}{\sigma}\Delta_{v-1}^E} c_{v-1}, e^{-\Gamma_{v-1}} k_{v-1}, \Delta_v^X) \tag{70}$$

$$k_v = g(e^{\left(\frac{1-\alpha}{\sigma}-1\right)\Gamma_{v-1}-\frac{\rho}{\sigma}\Delta_{v-1}^E} c_{v-1}, e^{-\Gamma_{v-1}} k_{v-1}, \Delta_v^X). \tag{71}$$

Substituting for $1 - H_v$ in Proposition 4 using (43), we can express the size of the boom as

$$\Gamma_v = \delta\gamma\Delta_v^E - \delta\gamma \frac{c_v}{k_v^\alpha}\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma}\right), \tag{72}$$

Propositions 5 and 3 yield

$$\alpha e^{-(1-\alpha)\gamma}\frac{c_v}{k_v} = \frac{1-e^{-(1-\alpha)\Gamma_v}}{\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma}\right)}. \tag{73}$$

Finally, asset market clearing over the boom (conditions (58) to (61)) imply:

$$\delta v_0^I(c_v, c_{v-1}, k_v, k_{v-1}, \Gamma_v, \Gamma_{v-1}, \Delta_v^E, \Delta_v^X) = \frac{w_0\left(T_{v-1}\right)}{\overline{A}_{v-1}} = (1-\alpha)e^{-(1-\alpha)\gamma}e^{-\alpha\Gamma_{v-1}}k_{v-1}^{\alpha}, \tag{74}$$

where $v_0^I = V_0^I(T_{v-1})/\bar{A}_{v-1}$ is explicitly derived in the appendix.

In the stationary cycle $\Gamma_v = \Gamma$, $k_v = k$, $c_v = c$, $\Delta_v^E = \Delta^E$ and $\Delta_v^X = \Delta^X$ for all v. Imposing these on the equations above, yields a system of five equations in the five unknowns:

$$\begin{aligned}
\Gamma &= \delta\gamma\Delta^E - \delta\gamma\frac{c}{k^\alpha}\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta^E}}{\rho/\sigma}\right) && (75)\\
\alpha e^{-(1-\alpha)\gamma}k^{\alpha-1}\frac{c}{k^\alpha} &= \frac{1-e^{-(1-\alpha)\Gamma}}{\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta^E}}{\rho/\sigma}\right)} && (76)\\
c &= f(e^{\left(\frac{1-\alpha}{\sigma}-1\right)\Gamma-\frac{\rho}{\sigma}\Delta^E}c, e^{-\Gamma}k, \Delta^X) && (77)\\
k &= g(e^{\left(\frac{1-\alpha}{\sigma}-1\right)\Gamma-\frac{\rho}{\sigma}\Delta^E}c, e^{-\Gamma}k, \Delta^X) && (78)\\
\delta v_0^I(c, k, \Gamma, \Delta^E, \Delta^X) &= (1-\alpha)e^{-(1-\alpha)\gamma}e^{-\alpha\Gamma}k^\alpha. && (79)
\end{aligned}$$

We demonstrate existence of the stationary cycle, and analyze this system numerically in the next section. However, the dynamics of the model can be understood heuristically from the phase diagram in Figure 6. Here the process of capital accumulation in the expansionary phase, $t \in \left(T_{v-1}, T_v^E\right)$, within a cycle, when in steady state, is depicted.

The economy does not evolve along the standard stable trajectory of the Ramsey model terminating at the steady state, S. Instead, the evolution of the cycling economy during the expansion is depicted by the path between A and B in the figure. Capital is accumulated starting at the point k_0 corresponding to point A in the diagram, according to (37) and (38). The point k_0 denotes the inherited capital stock at the boom. Accumulation ends at $k\left(T^E\right)$, at which point investment stops until the next cycle. Note that if allowed to continue along such a path the economy would eventually violate transversality, but capital accumulation stops and consumption declines so that the economy evolves from B to C through the downturn. During this phase, the dynamics of the economy are no longer dictated by the Ramsey phase diagram. When this phase ends, implementation of stored productivity improvements occurs at the next boom, and

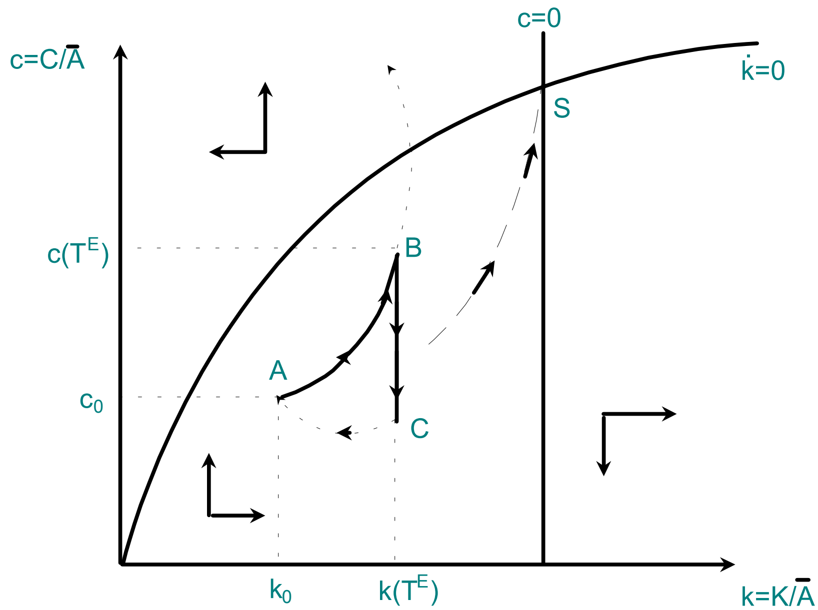

Figure 6: Phase Diagram

$\bar{A}$ increases, so that k fall discretely. If $\sigma < 1$, consumption grows by more than productivity at the boom, so that c rises discretely. The boom is therefore depicted by the dotted arrow back to point A. At this point, investment in the expansionary phase recommences for the next cycle. The connection between the two phases of the cycle arises due to the allocation of resources to entrepreneurship. This allocation of resources will be reflected in the size of the increment to $\bar{A}$, Γ.

8.1 Existence of the Stationary Cycle

To demonstrate the existence of the stationary cycle, we numerically solve the model for various combinations of parameters and check the existence conditions (E1)–(E5). We choose parameters to fall within reasonable bounds of known values, and present a baseline case given in Table 1:

Table 1: Baseline Parameters

Parameter	Value
α	0.22
γ	0.13546
σ	0.79
ρ	0.02
δ	1.39

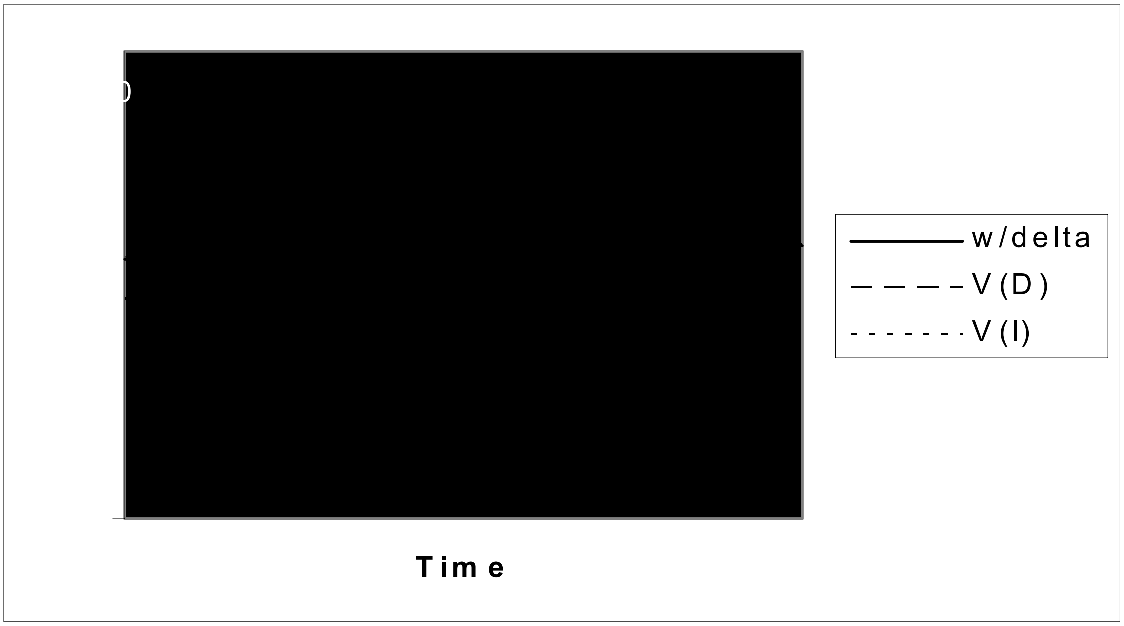

Figure 7: Existence

The parameters α and γ were chosen so as to obtain a labor share of 0.7, a capital share of 0.2 and a profit share of 0.1. These values correspond approximately to those estimated by Atkeson and Kehoe (2002). The value of γ corresponds to a markup rate of around 15%. The intertemporal elasticity of substitution $\frac{1}{\sigma}$ is slightly high, but we solve for various values below, including $\sigma = 1$. Given $\sigma = 0.79$, we calibrated δ and ρ so as to match a long–run annual growth rate of 2.2% and an average risk–free real interest rate of 3.8%, values which correspond to annual data for the post–war US. The baseline case above yields a cycle length of a little less than 4 years, $H_v = .2044$, and $k_v = 7.668$. In this, and all simulations we have computed, steady state values are unique.[26]

Numerically simulated value functions are plotted for the baseline case in Figure 5. The figure allows direct verification of conditions E1, E2, and E3.[27]

The plot starts with implementation at a boom, when $V^I > V^D$ and w/δ. For a discrete interval, V^I remains above V^D implying that, in the event of entrepreneurial success, implementation would dominate delay. However, over this part of the phase, the relative value of labor in production, w/δ, exceeds returns to entrepreneurship, so that no entrepreneurial successes are

[26] Francois and Lloyd-Ellis (2003) explicitly establish uniqueness of the stationary cycle when capital accumulation is not allowed. It seems likely that the introduction of capital would not lead to an additional stationary cycle here, but we have not been able to establish this analytically.

[27] Condition E4 and E5 are not depicted but can be directly checked.

available to implement. Throughout this expansionary phase, investment occurs so that the wage continues to rise. At the same time, V^D is also rising because the time until implementation of entrepreneurial successes falls. Note that this increase in V^D is simply a function of discounting, i.e., the ensuing boom at which stored successes will be implemented is drawing closer. Throughout this phase V^I declines as the duration of guaranteed positive profits falls.

The end of the expansion corresponds to the commencement of entrepreneurship, i.e., when the increasing value of a delayed entrepreneurial success eventually meets the opportunity cost of labor in production. This corresponds to the point at which $w/\delta = V^D$. Since V^D rose due to discounting during the expansion, during the contraction, when capital is no longer accumulated and interest rates are zero, V^D remains constant. This implies that, by arbitrage, the wage must also be constant, until the contraction ends. Through the downturn V^I continues to fall, but must eventually rise again as the probability of remaining the incumbent at the boom, given that an entrepreneurial success has not arrived in one's sector, increases. This increase in V^I is the force that will eventually trigger the next boom that ends the recession. It occurs when V^I just exceeds V^D and entrepreneurs implement stored entrepreneurial successes, leading to an increase in productivity, a jump in demand, movement of labor back to production, and full capacity utilization.

Table 2 lists the numerical implications for growth, cycle length and terminal values of capital stocks for various combinations of parameter values, including the baseline case. A first thing to note is the extreme sensitivity of cycle length, $\Delta = \Delta^X + \Delta^E$, to changes in parameters. In contrast, the long–run growth rate is much less sensitive to changes in parameters than along the acyclical growth path. Generally, increases in parameters that directly raise the impact of entrepreneurship, δ and γ, increase the growth rate, as in the acyclical steady state. Changes in σ and ρ also have effects similar to those present in the acyclical steady state. Additionally, however, changes in these parameters alter cycle length in ways which counterveil, and sometimes overshadow, the direct effects. For example, increasing σ, lowering inter-temporal substitutability, generally induces lower growth in the acyclical steady state because consumers are less willing to delay consumption to the future. A similar effect is present here. However, as the table shows, this increase also raises cycle length and amplitude, inducing more entrepreneurship and a larger boom. The net effect, as the table shows, is an increase in growth rate for this configuration. A similar sequence of effects is present for increases in ρ. Increasing the capital share, α, increases capital accumulation and lowers interest rates, but because this also induces a shorter cycle length, the net effect is a fall in growth rate.

Values of σ closer to 1 do not satisfy our existence conditions given the values of other parameters assumed in the baseline case. However, if we allow δ to rise somewhat, higher values of σ are consistent with the cycle (see the last two rows of Table 2). Intuitively, with higher entrepreneurial productivity, both the size of booms and the average growth rate tend to be higher in equilibrium. As a result, households are willing to delay consumption enough even for low elasticities of intertemporal substitution. As can be seen, the long–run growth rate in such cases tends to be higher and the cycles shorter.

Table 2: Comparative Stationary Cycles

Parameters					$H\left(T_v^E\right)$	$k(T_v^E)$	g	Δ
σ	δ	ρ	α	γ				
.79	1.39	.02	.22	.13546	0.2044	7.668	2.2	3.92
.78					0.2141	7.577	2.185	1.69
.80					0.1963	7.743	2.213	5.87
	1.38				0.1995	7.778	2.191	5.06
	1.40				0.2099	7.553	2.207	2.65
		.0197			0.2154	7.565	2.183	1.46
		.0203			0.1949	7.74	2.213	6.05
			.213		0.1942	7.332	2.213	5.61
			.227		0.2151	8.008	2.185	2.08
				.13446	0.1977	7.843	2.177	5.29
				.13646	0.2115	7.491	2.221	2.47
.9	1.593				0.2008	6.574	2.541	5.29
1	1.861				0.2065	5.407	2.967	4.12

8.2 Qualitative Behavior of Key Variables

8.2.1 Investment and Consumption

Investment is strongly pro-cyclical here. Contemporaneous with the productivity boom, investment jumps discretely to its highest point in the cycle. It remains positive throughout the expansionary phase and then declines sharply as the economy enters recession. Note importantly that the driving force for investment here is the marginal product of capital in production: investment is zero in the recession when interest rates are zero, and jumps to its highest point at the start of the boom, when interest rates are at their highest level.[28] Consumption is also strongly pro-

[28] Output and sales' growth closely follow investment, whereas investment has almost no relationship, at business cycle frequencies, with the user cost of capital; see Hassett and Hubbard (1996).

cyclical, but evolves more smoothly than investment At the boom, consumption jumps discretely, and continues to increase throughout the expansionary phase. Consumption falls smoothly in the recession and continues to decline throughout the downturn until the next boom.

The model also generates pro-cyclical allocation of labor to production of consumables and investment goods, as has been reported (e.g. Christiano and Fisher 1995). Although the investment and consumption good sectors are not distinguished, per se, in the model, the allocation of labor to consumption good production can be inferred from equation (65). As long as $\sigma < 1$ consumption growth exceeds productivity growth so that the allocation of labor and capital to consumption must have risen at the boom. The reason labor in both consumption and investment good production can rise is because of the endogenous shutting down of entrepreneurship at the boom. This mechanism is similar to that generated by introducing "homework" in Benhabib, Rogerson and Wright (1991).

8.2.2 Productivity

Even though underlying productivity improvements arise in a decentralized and time-varying manner across sectors, the incentives for strategic delay identified here lead to simultaneous implementation at the boom. Consequently total factor productivity (TFP) rises uniformly there. Output jumps discretely at the boom also, in part, due to the reallocation of additional resources to production then. The effects of the productivity boom persist in further output increases, as capital is then accumulated through the expansion. Note that it is not optimal to accumulate capital before the boom because its short term returns there are zero. It is this delayed accumulation of capital in response to the productivity boom which leads to a prolonged expansion in output. This is consistent with the evidence that expansions tend to start with relatively aggressive growth, and are then followed by milder increases thereafter (see Dahl and Gonzalez–Rivera, 2003).

In the expansion, all labor is used in production and capital is fully utilized. In the contraction, labor is reallocated to entrepreneurial activity, capital utilization falls, and output declines.[29] If utilized capital and labor were correctly measured this would imply that measured productivity should remain constant through the recession. As already discussed, capacity utilization is well

[29] The fall in output does not result from failure to measure the productive contribution of entrepreneurs alone. Even if this were fully measured, which is unlikely to be the case, output would still fall since during the contraction. The reallocation of labor from production, where workers are paid less than their marginal value (due to profits), to entrepreneurship, which if measured correctly implies workers being paid marginal value, will lead to a decline in measured output.

known to fall in recessions. However, even if capital utilization is correctly measured in US data, but labor allocated to innovation is not fully measured, then it will appear that labor is being hoarded (see Fay and Medoff 1985).[30] If this occurs, measured productivity would fall, which is consistent with the evidence (see for example Fernald and Basu 1999). Even if entrepreneurship is being correctly measured, but the fall in capital utilization is not, measured productivity falls. Through the expansion, total factor productivity is constant, and labor productivity rises.

8.2.3 Returns to Factors

In US data, corporate profits increase mildly through upturns but show clear and marked falls in contractions. This is again consistent with the model. As productivity is unchanged through the cycle, equation (21) shows profit to be proportional to demand, so that the pattern of profits will be consistent with this pro–cyclical pattern. Wages rise rapidly in the boom due to the increase in labor productivity. The wage then continues to increase throughout the expansionary phase as capital is accumulated, but is constant through the contraction. This is because the marginal product of labor in production is constant. Competition does not put downward pressure on wages because labor in this phase enters into its alternative activity, entrepreneurship or reorganization. The wage then here is best interpreted as the skilled wage, since it is assumed that all labor is equally able to work in entrepreneurship. The introduction of unskilled labor, which has no role in these tasks, would see a similar increase in wages (and full employment) through the economy's upturn and then a decline (in either wages, and/or employment if labor market frictions are modeled) during the recession.

8.2.4 The Term Spread

In US data the spread between interest rates on a ten year treasury note and a three month treasury bill tends to be large in recessions (i.e. the long term interest rate exceeds the short term). In expansions smaller and seems to be a good predictor of recessions. In particular, relatively low values of the term spread, high short term interest rates relative to long, suggest a higher probability of recession. The cycle analyzed here exhibits a low value of the yield curve through the expansion, and a high value in the recession. The highest value of the yield curve is at

[30] Entrepreneurship is, at best, likely to be only partially measured in the data, since much of it involves activities that will raise long-term firm profits but have little directly recorded output value contemporaneously.

the start of the recession. Towards the end of the recession it tracks down as the three month rate starts to include the increased discount over the boom. This implies, particular for short cycles in the model, a good fit with the data. Estrella and Mishkin (1996) argue that the yield curve is a superior predictor over other leading indicators at leads from 2 to 4 quarters. Similarly, at the start of the expansion the value of the yield curve is at its lowest point, thus again providing a leading indication of the imminent contraction to follow the expansionary phase.

8.2.5 Tobin's Q

The aggregate behavior of Tobin's Q, defined as the ratio of the value of firms to the book value of their capital stock, is illustrated in Figure 8. In our model Tobin's Q is given by

$$Q(t) = \frac{V^K(t) + \Pi(t)}{K(t)}, \tag{80}$$

where $\Pi(t)$ denotes the stock market value of the intangible capital tied up in firms, and recall that $V^K(t)$ is the market value of their physical capital.

During an expansion $V^K(t) = K(t)$ and, the value of intangible capital with the value of incumbent firms: $\Pi(t) = V^I(t)$. It follows that

$$Q(t) = 1 + \frac{V^I(t)}{K(t)} \qquad \forall t \in (T_{v-1}, T_v^E). \tag{81}$$

Since $V^I(t)$ declines and $K(t)$ grows during the expansion, $Q(t)$ must decline.

In the downturn, the value of the physical capital stock declines below the capital stock, so that

$$V^K(t) = \left[\bar{q}\int_t^{T_v} \lambda(\tau)d\tau + e^{-\beta(T_v)}\right] K(T_v^E) < K(T_v^E). \tag{82}$$

Also some sectors experience innovations, so there exist terminal firms who are certain to be made obsolete at the next round of innovation. At any point in time the measure of sectors in which no innovation has occurred is $P(t)$, therefore the total value of firms on the stockmarket is given by

$$\Pi(t) = (1 - P(t))[V^T(t) + V^D(t)] + P(t)V^I(t), \tag{83}$$

where $V^T(t)$ denotes the value of "terminal" firms who are certain to be made obsolete during the next wave of implementation. The value of these firms can be written as

$$V^T(t) = V^I(t) - \frac{P(T_v)}{P(t)}V^D(t). \tag{84}$$

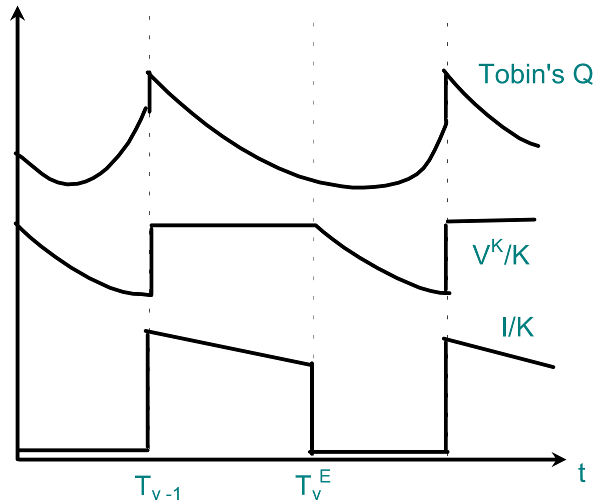

Figure 8: Tobin's Q, average Q and the Investment Rate

Substituting into (83) yields

$$\hat{\Pi}(t) = V^I(t) + (1 - P(t)) \left[1 - \frac{P(T_v)}{P(t)} V^D(t)\right]. \tag{85}$$

Through the downturn, the value of intangible capital initially falls and then rises again as the economy approaches the next boom.[31] Immediately prior to the boom $P(t) = P(T_v)$, so that again $\Pi(T_v) = V^I(T_v)$. The value of Q during the downturn is thus given by

$$Q(t) = \bar{q} \int_t^{T_v} \lambda(\tau) d\tau + e^{-\beta(T_v)} + \frac{\hat{\Pi}(t)}{K(T_v^E)} \qquad \forall\ t \in [T_v^E, T_v). \tag{86}$$

During the contraction, then, $Q(t)$ initially declines as $K(t)$ remains unchanged and the decline in $V^k(t)$ dominates. However, eventually the growth in the value of intangible capital, $\hat{\Pi}(t)$, starts to dominate as we approach the boom, so that $Q(t)$ rises in anticipation. At the boom, since the book value of capital remains unchanged, but the market value of both physical and capital growth by a factor $e^{(1-\alpha)\Gamma_v}$, Tobin's Q rises rapidly.

The qualitative behavior of Tobin's Q in our model thus accords quite well with its aggregate counterpart in US data. As illustrated by Figures 1 and 2, Tobin's Q tends to reach a peak prior to the peak of expansions and then reaches a minimum midway through NBER–dated recessions. The most rapid periods of growth in Tobin's Q therefore start to occur before the end of recessions and continue through the subsequent boom just as they do in our stationary cycle.

[31] This cyclical anticipation of future profits implicit in aggregate stock prices accords well with the findings of Hall (2001).

9 Concluding Remarks

In addition to an endogenous treatment of growth, our model *endogenously* generates the following qualitative behavior of key aggregates and prices over the business cycle:

• Implementation of innovations is strongly pro–cyclical, so that total factor productivity rises during booms, but remains constant during downturns.

• Labour productivity is strongly pro–cyclical.

• Wages rise during booms and expansions, but do not fall during contractions.

• Investment and consumption are strongly pro–cyclical, but investment is more volatile. Investment is strongly correlated with output and sales growth.

• Labor and capital inputs into consumption and investment sectors are both pro–cyclical.

• Capacity utilization is strongly pro–cyclical.

• Profits are strongly pro–cyclical.

• Term spread is flat during expansions and steep midway through contractions.

The basic mechanism underlying the model discussed in this paper captures a simple and compelling reason for cross-sectoral co–movement: entrepreneurs delay innovative activity (or reorganization) until demand conditions are slack, and delay implementing productivity improvements until the point at which demand conditions are favorable and the costs of acquiring the necessary capital are sufficiently low — this is when other entrepreneurs are doing the same. Since ensuing prices must also satisfy consumers' optimization and asset market clearing when there is the possibility of storage and arbitrage trading, this does not lead to the possibility of multiple self-fulfilling cycles. Moreover, these conditions do not rule out cycles because the re–allocation of resources inherent to the process of endogenous growth ensures that profits grow more than the discount rate across the boom.

The model generates movements in aggregates over the cycle which are qualitatively similar in many respects to those observed in US data. It should be reiterated that these results arise in a framework where both the economy's cyclical behavior and its growth path are fully endogenized. Moreover, the framework we explore has remarkably few degrees of freedom; the model is fully specified by five exogenous parameters: two summarizing household preferences, two underlying the productivity of entrepreneurship, and one pinning down factor shares in production. We do not claim that the current framework is capable of providing a quantitative account of the business cycle. However, in future work we will build on this parsimonious structure to explore a number of key extensions:

• Aggregate uncertainty and stochastic cycle lengths — The length and other characteristics of actual business cycles, vary from cycle to cycle and look rather different from the deterministic stationary equilibrium cycle described here. Introducing some degree of aggregate uncertainty would help to address this. However, in order to develop such an extension we need to develop a deeper understanding of the local transitional dynamics of the model. It turns out that these dynamics are not as complex as one might expect at first blush. The reason is that the path back to the stationary cycle (at least locally) involves the accumulation of only one factor: either physical capital or intangible. Although a full analysis of these local dynamics is beyond the scope of the current paper, we believe it is feasible.

• Unemployment — A natural way to introduce unemployment into the model is to allow for unskilled labour which cannot be used in entrepreneurship and is not directly substitutable with skilled labor in production. With putty–clay production, the marginal value of this unskilled labor falls to zero during the downturn and some fraction of unskilled workers would become unemployed (just like physical capital). In a competitive labor market, this would drive unskilled wages down to their reservation level. However, in the presence of labor market imperfections, such as efficiency wages and search frictions, the dynamics of unemployment and wages interact with the process of creative destruction in a more complex manner. In further work we explore these dynamics more fully.

• Government policy — The framework developed here (as well as its extensions) provide a natural framework for thinking about counter–cyclical policy. First, the question arises as to whether removing or reducing cycles is a valid policy objective at all. In Francois and Lloyd–Ellis (2003) we showed that switching from the cyclical equilibrium to a corresponding acyclical one would raise long–run growth but lower welfare. Similar results are likely to carry over the stationary cycle in the current model. A second issue is that of how to implement a counter–cyclical policy. The recession here is Keynesian in that it is associated with deficient demand, and the government could intervene, for example, by raising demand for goods and services and taxing savings. However, such a policy would effectively channel resources away from innovative activities and may dampen growth. On the other hand the anticipation of higher demand during a downturn might stimulate innovation, so the overall effect is unclear.

10 Appendix

Proof of Lemma 1: Differentiating (5) with respect to time yields

$$\dot{V}_i^K(t) = r(t)\, V_i^K(t) - q_i(t)\, K_i(t) + \dot{K}_i(t) = \dot{K}_i(t)\,. \tag{87}$$

Since $V_i^k(t) = K_i(t)$, (18) follows.■

Proof of Lemma 2: Given factor prices $q(t)$ and $w(t)$, entrepreneurs choose the combination of capital and labor that minimizes the cost of producing $x_i(t)$:

$$K_i(t) = \frac{x_i(t)}{A_i^{1-\alpha}(t)}\left[\left(\frac{\alpha}{1-\alpha}\right)\frac{w(t)}{q(t)}\right]^{1-\alpha} \quad \text{and} \quad L_i(t) = \frac{x_i(t)}{A_i(t)^{1-\alpha}}\left[\left(\frac{1-\alpha}{\alpha}\right)\frac{q(t)}{w(t)}\right]^{\alpha} \tag{88}$$

The resulting unit cost is:

$$\frac{w(t)}{A_i(t)^{1-\alpha}}\left[\left(\frac{1-\alpha}{\alpha}\right)\frac{q(t)}{w(t)}\right]^{\alpha} + \frac{q(t)}{A_i^{1-\alpha}(t)}\left[\left(\frac{\alpha}{1-\alpha}\right)\frac{w(t)}{q(t)}\right]^{1-\alpha} = \frac{q(t)^{\alpha}w(t)^{1-\alpha}}{\mu A_i^{1-\alpha}(t)}. \tag{89}$$

Since the productivity of the most productive rival is $e^{-\gamma}A_i(t)$, the limit price is given by (20).■

Proof of Proposition 1: Using (19) and (20) to substitute for x_i and p_i into (88) yields $K_i = K$, and $L_i = L = 1$ for all i with q and w given by:

$$q(t) = \frac{\alpha e^{-(1-\alpha)\gamma}Y(t)}{K(t)} \tag{90}$$

$$w(t) = (1-\alpha)e^{-(1-\alpha)\gamma}Y(t). \tag{91}$$

Since $q(t) = r(t) > 0$, accumulating capital dominates storage, so that:

$$\dot{K}(t) = Y(t) - C(t), \tag{92}$$

Since all successes are implemented immediately, the aggregate rate of productivity growth is

$$g(t) = \delta\gamma H(t) \tag{93}$$

No–arbitrage implies that

$$r(t) + \delta H(t) = \frac{\pi(t)}{V^I(t)} + \frac{\dot{V}^I(t)}{V^I(t)} \tag{94}$$

Since, innovation occurs in every period, free entry into entrepreneurship implies that

$$\delta V^I(t) = w(t). \tag{95}$$

Along the balanced growth path, all aggregates grow at the rate g. From the Euler equation it follows that

$$r(t) = \rho + \sigma g. \tag{96}$$

Differenting (91) and (95) w.r.t. to time, using these to substitute for $\frac{\dot{V}^I(t)}{V^I(t)}$ in (94), and using (96) to substitute for $r(t)$ and (21) to sustitute for $\pi(t)$, we get

$$\rho + \sigma g + \frac{g}{\gamma} = \frac{\delta(1 - e^{-(1-\alpha)\gamma})}{(1-\alpha)e^{-(1-\alpha)\gamma}} + g. \tag{97}$$

Solving for g yields (24).■

Proof of Lemma 3 We show: (1) that if a signal of success from a potential entrepreneur is credible, other entrepreneurs stop innovation in that sector; (2) given (1) entrepreneurs have no incentive to falsely claim success.

Part (1): If entrepreneur i's signal of success is credible then all other entrepreneurs believe that i has a productivity advantage which is e^{γ} times better than the existing incumbent. If continuing to innovate in that sector, another entrepreneur will, with positive probability, also develop a productive advantage of e^{γ}. Such an innovation yields expected profit of 0, since, in developing their improvement, they do not observe the non-implemented improvements of others, so that both firms Bertrand compete with the same technology. Returns to attempting innovation in another sector where there has been no signal of success, or from simply working in production, $w(t) > 0$, are thus strictly higher.

Part (2): If success signals are credible, entrepreneurs know that upon success, further innovation in their sector will cease from Part (1) by their sending of a costless signal. They are thus indifferent between falsely signalling success when it has not arrived, and sending no signal. Thus, there exists a signalling equilibrium in which only successful entrepreneurs send a signal of success.■

Proof of Lemma 4: From the production function we have

$$\ln Y(t) = \int_0^1 \ln \frac{Y(t)}{p_i(t)} di \tag{98}$$

Substituting for $p_i(t)$ using (20) yields

$$0 = \int_0^1 \ln \frac{q(t)^{\alpha} w(t)^{1-\alpha}}{\mu e^{-(1-\alpha)\gamma} A_i^{1-\alpha}(T_{v-1})} di \tag{99}$$

which re-arranges to (36).■

Proof of Lemma 6: Suppose instead that there exists an intermediate phase in which neither capital is accumulated nor entrepreneurship occurs. Consider the first instant of that phase. Since in the instant prior to that capital was being accumulated, the marginal return to investment in physical capital must exceed ρ. Since the marginal product of capital cannot jump downwards discretely at full capital utilization, there are only two possibilities: either (1) $r(T_v^E) = \rho$ at the start of the intermediate phase or (2) $r(T_v^E) > \rho$ at the start of the intermediate phase. Situation (2) can be ruled out directly since, by assumption, in the intermediate phase there is no entrepreneurship, and so it must be the case that $r > \rho$ and investment will occur. Situation (1) occurs if the marginal return to capital converges continuously to $r = \rho$ along the neoclassical accumulation phase. But this corresponds exactly with the path of accumulation along the stable trajectory of the Ramsey model which does not converge in finite time — this would then imply an infinite length to the capital accumulation phase.■

Proof of Proposition 3: Given the constant wage w_v and the average rental rate on capital through the downturn, $\bar{q}$, the efficient contract will be constructed so as to solve the following cost–minimization problem through the downturn:

$$\min_{\tilde{K}_i,\tilde{L}_i} \; w_v\tilde{L}_i + \bar{q}\tilde{K}_i \quad \text{s.t.} \quad \tilde{x}_i \leq \tilde{K}_i^{\alpha}(A_i\tilde{L}_i)^{1-\alpha} \tag{100}$$

where $\tilde{L}_i = \int_{T_v^E}^{T_v} L_i(t)dt$, $\tilde{K}_i = \int_{T_v^E}^{T_v} K_i^u(t)dt$ and $\tilde{x}_i = \int_{T_v^E}^{T_v} x_i(t)dt$. This temporal aggregation is possible for two reasons: (1) the interest rate is zero through the downturn and intermediate prices are constant, so that the value of a unit of output is time–independent, and (2) as labour is withdrawn from production, the capital labour ratio is constant. Thus the entire downturn can be treated as if it were a single production period. The necessary condition from the problem is simply that

$$\frac{\tilde{K}_i}{\tilde{L}_i} = \left(\frac{\alpha}{1-\alpha}\right)\frac{w_v}{\bar{q}}. \tag{101}$$

At the peak of the expansion, the optimal capital labour ratio is

$$\kappa_i(T_v^E) = \left(\frac{\alpha}{1-\alpha}\right)\frac{w_v}{q(T_v^E)}. \tag{102}$$

Since the capital labour ratio through the downturn must equal that at the peak and the wage is constant through the downturn, it follows that the efficient contract must satisfy $\bar{q} = q(T_v^E)$.■

Proof of Proposition 4: From (35), long–run productivity growth is given by

$$\Gamma_v = (1 - P(T_v))\gamma \tag{103}$$

Integrating (45) over the downturn and substituting for $H(\cdot)$ using (45) yields

$$1 - P(T_v) = \delta \int_{T_v^E}^{T_v} \left(1 - (1 - H_v)e^{-\frac{\rho}{\sigma}[t - T_v^E]}\right) d\tau. \tag{104}$$

Substitution into (103) and integrating gives (56).■

Proof of Lemma 8: $p_i(t) = \frac{q(t)^{\alpha} w(t) 1-\alpha}{\mu e^{-(1-\alpha)\gamma} A_i^{1-\alpha}(t)}$ is the marginal cost of the previous incumbent given the sequence $w(t), q(t)$. Due to the unit elasticity of final producer demand, intermediate producing entrepreneurs wish to set price as high as possible. Thus, contracting a lower price at any instant is not optimal for the leader in i. Offering a $p_i^c(t) > p_i(t)$ in any instant would lead to a bid by the previous incumbent that would be both feasible and preferred by the final good producer. Thus $p_i^c(t)$ is the profit maximizing price and $x_i^c(t) = \frac{Y(t)}{p_i^c(t)}$for all $t \in [T_v, T_{v+1})$.■

Proof of Lemma 7: If $V_i^k(t) > K_i(t)$ it is feasible for the leading producer to write an alternative $\{q_i(t), K_i(\tau)\}$ with the builder of a new capital stock in sector i which would lead to new capital being constructed and which would be preferred by the producer. A preferred sequence for the leading producer would be one in which prices were no higher than the contracted sequence above, but which had a strictly lower price in at least one instant. This is feasible if $V_i^k(t) > K_i(t)$. Finally, no new capitalist would enter offering a sequence $V_i^k(t) < K_i(t)$, so that any equilibrium price sequence must at least satisfy (50).■

Proof of Proposition 6: Condition (67) can be expressed as

$$\begin{aligned}
\int_t^{T_v} \pi(\tau) d\tau &\geq \int_t^{T_v} \overline{q} K(T_v^E) \left(1 - (1 - H_v)e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}\right) d\tau \\
(1 - e^{-(1-\alpha)\gamma}) Y(T_v^E) \int_t^{T_v} (1 - H_v)e^{-\frac{\rho}{\sigma}(\tau - T_v^E)} d\tau &\geq \overline{q} K(T_v^E) \int_t^{T_v} \left(1 - (1 - H_v)e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}\right) d\tau
\end{aligned}$$

Since $\overline{q} K(T_v^E) = \alpha e^{-(1-\alpha)\gamma} Y(T_v^E)$, this can be expressed as

$$\begin{aligned}
(1 - e^{-(1-\alpha)\gamma}) \int_t^{T_v} (1 - H_v)e^{-\frac{\rho}{\sigma}(\tau - T_v^E)} d\tau &\geq \alpha e^{-(1-\alpha)\gamma} \int_t^{T_v} \left(1 - (1 - H_v)e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}\right) d\tau \\
(1 - (1 - \alpha)e^{-(1-\alpha)\gamma})(1 - H_v) \int_t^{T_v} e^{-\frac{\rho}{\sigma}(\tau - T_v^E)} d\tau &\geq \alpha e^{-(1-\alpha)\gamma}(T_v - t)
\end{aligned}$$

Since this holds with equality at $t = T_v$, a sufficient condition is that the left hand side declines more rapidly with t than the right hand side. That is

$$(1-(1-\alpha)e^{-(1-\alpha)\gamma})(1-H_v)e^{-\frac{\rho}{\sigma}(t-T_v^E)} > \alpha e^{-(1-\alpha)\gamma} \tag{105}$$

This will hold for all t if holds for $t = T_v$. Hence, condition (68) follows.■

Proof of Proposition 7: The proof proceeds by demonstrating that there does not exist another mutually improving price/quantity sequence that could be written at time T_v. Note first that in considering alternative contracts we can restrict attention to those that also satisfy Lemma 7. Since contracts not satisfying this will always be dominated by alternatives that do so for at least one of the parties. Since $q^c(t) = r(t)$ for $t \leq T_v^E$ and $K^c(t)$ satisfies (37), there is no potential for mutually beneficial changes in the terms of the contract through the upturn, as capital is efficiently utilized then. At the peak of boom $K^c\left(T_v^E\right) = K\left(T_v^E\right)$ and thereafter, through the downturn, the level of installed capital remains at $K\left(T_v^E\right)$, but since proportion $1-(1-H_0(T_v^E))e^{-\frac{\rho}{\sigma}(\tau-T_v^E)}$ remains idle every instant, utilization is not efficient, which indicates a possibly improving contract. We show that no such contract exists.

If the entrepreneur does not store intermediate, there is no demand for the idle capital because $K^c(t)$ meets production needs. Even free use of excess capital to produce and sell more intermediate output at any instant τ during the downturn would be rejected due to the unit elasticity of demand. That is, since total revenue remains fixed at $p_i(t)x_i^d(t) = Y(t)$, producing more simply incurs extra labor costs with unchanged revenues.

Now consider alternative sequences under which the entrepreneur may store some output, and which increase capital utilization. Entrepreneurs will never choose to store output beyond the boom when a new incumbent has arrived. If competing with a new incumbent, pricing at the new incumbent's marginal cost requires - $p_i(t) = \frac{r(T_v)^\alpha w(T_v)1-\alpha}{\mu e^{-(1-\alpha)\gamma}A_i^{1-\alpha}(t+1)}$ which is less than the price if selling before the boom - $p_i(T_v) = \frac{r\left(T_{v+1}^E\right)^\alpha w\left(T_{v+1}^E\right)1-\alpha}{\mu e^{-(1-\alpha)\gamma}A_i^{1-\alpha}(t-1)}$ since $r\left(T_{v+1}^E\right)^\alpha w\left(T_{v+1}^E\right)e^\gamma = r\left(T_v\right)^\alpha w\left(T_v\right)$, but $A_i^{1-\alpha}(t+1) = e^{2\gamma}A_i^{1-\alpha}(t-1)$. Thus, terminal entrepreneurs strictly prefer to sell stored output before the boom. Proposition 5 ensures the same holds for an entrepreneur who expects to remain the incumbent at the boom.

Capitalists may be willing to modify the contract to allow storage if they receive a positive rent on the proportion $1-(1-H_0(T_v^E))e^{-\frac{\rho}{\sigma}(\tau-T_v^E)}$ which lies idle in the equilibrium contract. We show now that capitalists would only accept a rent of $\overline{q}$, on the extra units. To see why, suppose that the capitalist were to accept a modified contract which maintained the price $\overline{q}$ for units $K^c(t)$ but allowed a lower price $\widetilde{q}$ for utilization of the idle units. Given such a contract the entrepreneur would optimally choose to meet contracted production at each t by utilizing $K^c(t)$ and utilize the remaining $\widetilde{K} = 1 - (1 - H_0(T_v^E))e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}$ to produce and store output. The entrepreneur will then optimally choose to shut down at time τ given by

$$\int_{\tau}^{T_{v+1}} K^c(t)^{\alpha} A_i(t) L_i^*(t)^{1-\alpha} = \int_{T_v^E}^{\tau} \widetilde{K}^{\alpha} A_i(t) L_i^*(t)^{1-\alpha}, \tag{106}$$

where $L_i^*(t)$ is the optimal amount of labor to use at time t. Recall that shutting down production is always an available option for the entrepreneur. The RHS of (106) is the stored output produced using the extra capital up to τ and the LHS is the demand for output under the contract with the final goods producer from τ on. By utilizing the extra units of capital at $\widetilde{q} < \overline{q}$ the entrepreneur simply replaces future demand for capital at price $\overline{q}$ with stored output produced using capital priced at $\widetilde{q}$ and thus lowers costs. Note also that a similar path of storage and shut down would be chosen for a non-stationary price on the extra units, $\widetilde{q}(t)$, provided that, for some instants at least, $\widetilde{q}(t) < \overline{q}$.

Now consider returns to the capitalist from such a deviation. There are two cases to consider depending on whether there has been an entrepreneurial success in sector i or not. If there has not been a success up to time $t > \tau$, which occurs with probability $P_i(t)$, then demand for capital at $t = 0$. As already established, the entrepreneur shuts down at τ in (106) under this deviation, so that his demand is zero. Moreover the previous incumbent cannot sell output to the final goods sector since the current incumbent has contracted sale to the final goods sector, under (25) (26). Thus the previous incumbent (and all earlier) have zero demand for capital for $t < T_v$. However, with probability $(1 - P_i(t))$ there has been a success in sector i at time t. In that case a single entrepreneur has some use for the capital before the boom. But note that it is not essential for him to access capital then, since he does not have an intermediate delivery contract yet, and along the equilibrium path will wait for implementation until the boom. Since such an individual is the only producer with demand for the capital, she is able to drive the capitalist down to marginal cost, which implies a price $q(t) \to 0$ over the interval $t \in [\tau, T_{v+1}]$, where τ solves (106), under this deviation. Since this yields strictly lower returns for the capitalist than (27) (28) a deviation allowing intermediate storage will not be accepted by the capitalist.

It thus follows that any alternative contract which allows utilization of capital in the downturn above $K^c(t)$ cannot be mutually improving. Since $K^c(t)$ is the maximal contracted level of capital possible, and any alternative rental rate necessarily makes one party worse off, the contract $(q^c(t), K^c(t))$ is not dominated by an alternative contract that could be written at T_v.

There is finally the possibility of remaining uncontracted, which would be preferred by the capitalist. But, since this would allow for a strictly increasing unit price for capital through the downturn, in accordance with (49), this would be rejected by the entrepreneur. If the sector's capital owner refused to contract, the entrepreneur would contract capital to be built with another producer at terms $(q^c(t), K^c(t))$. ■

Definition of v_0^I :

$$\begin{aligned}
v_0^I &= V_0^I(T_{v-1})/\bar{A}_{v-1} && (107)\\
&= \int_{T_{v-1}}^{T_v^E} e^{-\int_{T_{v-1}}^{\tau} r(s)ds} \frac{\pi(\tau)}{\bar{A}_{v-1}} d\tau + e^{-\int_{T_{v-1}}^{T_v^E} r(s)ds} \int_{T_v^E}^{T_v} \frac{\pi(\tau)}{\bar{A}_{v-1}} d\tau + P(T_v)\frac{V_0^I(T_v)}{\bar{A}_v}. && (108)\\
&= (1-e^{-(1-\alpha)\gamma})\left[\int_{T_{v-1}}^{T_v^E} e^{-\int_{T_{v-1}}^{\tau} r(s)ds} y(\tau)d\tau + e^{-\int_{T_{v-1}}^{T_v^E} r(s)ds}\int_{T_v^E}^{T_v} y(\tau)d\tau\right] + \left(1-\frac{\Gamma_v}{\gamma}\right)\frac{w_0(T_v)}{\delta \bar{A}_v} && (109)\\
&= (1-e^{-(1-\alpha)\gamma})e^{-\alpha\Gamma_{v-1}}k_{v-1}^{\alpha}\int_{T_{v-1}}^{T_v^E} e^{-\int_{T_{v-1}}^{\tau} r(s)ds}\left(\frac{k(\tau)}{k_0(T_{v-1})}\right)^{\alpha} d\tau\\
&\quad + e^{-\int_{T_{v-1}}^{T_v^E} r(s)ds}\left\{(1-e^{-(1-\alpha)\gamma})c_v\delta\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma}\right)\right.\\
&\quad\quad \left. + \left[1-\delta\Delta_v^E + \delta\frac{c_v}{k_v^{\alpha}}\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma}\right)\right](1-\alpha)e^{-(1-\alpha)\gamma}k_v^{\alpha}\right\}. && (110)
\end{aligned}$$

11 References

Aghion, Philippe and Howitt, Peter, "A Model of Growth Through Creative Destruction," *Econometrica*, vol. 60, 1992, pp. 323–351.

Aghion, Philippe and Howitt, Peter (1998) *Endogenous Growth Theory*, Cambridge, MIT Press.

Acemoglu, Daron, Philippe Aghion and Fabrizio Zilibotti (2002a) "Distance to Frontier, Selection and Economic Growth." *NBER Working Paper* no. w9066, and *CEPR Discussion Paper* no. 3467.

Acemoglu, Daron, Philippe Aghion and Fabrizio Zilibotti (2003) "Vertical Integration and Distance to Frontier," NBER Working Paper no. w9191. Forthcoming in the *Journal of the European Economic Association*, 2003.

Andolfatto, David and Glenn M. MacDonald, (1998) "Technology Diffusion and Aggregate Dynamics," *Review of Economic Dynamics*, vol. 1 (2), April, pp. 338-70.

Atkeson, Andrew and Patrick Kehoe (2002), "Measuring Organization Capital," NBER Working Paper #W8722.

Basu, Susanto, "Procyclical Productivity: Increasing Returns or Cyclical Utilization?" *Quarterly Journal of Economics,* vol. 111 (3), 1996, pp. 719-51.

Benhabib, Jess, Richard Rogerson and Randall Wright (1991) "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," *Journal of Political Economy*, 99, 1166-1187.

Bils, Mark and Jang Ok Cho (1994) "Cyclical Factor Utilization" *Journal of Monetary Economics*, 33(2), pp. 319-54.

Boldrin, Michele, Lawrence J. Christiano and Jonas Fisher, "Habit Persistence, Asset Returns and the Business Cycle," *Amerian Economic Review*, vol. 91 (1), March 2001, pp. 149–166.

Bresnahan, T. and V. Ramey (1994) "Output Fluctuations at the Plant Size," *Quarterly Journal of Economics*, Vol. 109, 593-624.

Burnside, Craig, Martin Eichenbaum and Sergio Rebelo, "Capital Utilization and Returns to Scale" Centre for Economic Policy Research, Discussion Paper No. 1221, 1995.

Burns, Arthur and W.C. Mitchell, (1946) *Measuring Business Cycles, Studies in Business Cycles, No. 2*, New York: National Bureau of Economic Research.

Caballero, Ricardo J. (1999) "Aggregate Investment", in *Handbook of Macroeconomics,* Volume 1B, edited by John B. Taylor and Michael Woodford, Amsterdam; Elsevier Science, North-

Holland, pp. 813-62

Caballero, Ricardo J. and Eduardo Engel (1998) “Nonlinear aggregate Investment Dynamics: Theory and Evidence”, *NBER Working Paper* No. 6420.

Christiano, Laurence J. and Terry J. Fitzgerald. (1998) “The Business Cycle: It’s still a puzzle”, *Economic Perpectives*, p. 56-84.

Cohen, W., R. Nelson and J. Walsh (2000) “Protecting Their Intellectual Assets: Appropriability Conditions and Why US firms Patent or Not” *National Bureau of Economic Research Working Paper*, # 7552.

Comin, Diego (2002), “R&D? A Small Contribution to Productivity Growth,” mimeo, New York University.

Dahl, Christian, M. and Gloria Gonzalez-Rivera (2003) “Testing for Neglected Nonlinearity in Regression Models. A Collection of New Tests based on the Theory of Random Fields” *Journal of Econometrics*, 114 (1) pp. 141-164.

Deneckere, Raymond-J and Kenneth L Judd (1992) “Cyclical and Chaotic Behavior in a Dynamic Equilibrium Model, with Implications for Fiscal Policy” in Benhabib,-Jess, ed. *Cycles and chaos in economic equilibrium.* Princeton, Princeton University Press, pp. 308-29.

Doms, Mark and Timothy Dunne, (1998) “Capital Adjustment Patterns in Manufacturing Plants,” *Review of Economic Dynamics,* 1(2) pp. 409-29.

Estrella, Arturo and Frederic Mishkin (1996) “The Yield Curve as a Predictor of Recessions in the United States and Europe.” In *The Determination of Long-Term Interest Rates and Exchange Rates and the Role of Expectations,* Bank for International Settlements.

Fay, Jon A. and James L. Medoff, “Labor and Output over the Business Cycle: Some Direct Evidence” *American Economic Review,* vol. 75 (4), 1995, pp. 638-55.

Fernald, John G; and Susanto Basu (1999) Why is Productivity Procyclical? Why Do We Care? *Board of Governors of the Federal Reserve System, International Finance Discussion Paper* No. 638.

Francis, Neville and Valerie A. Ramey (2002) “Is the Technology-Driven Real Business Cycle Hypothesis Dead?” *NBER Working Paper* No. w8726.

Francois, Patrick and Shouyong Shi, “Welfare Improving Cycles” *Journal of Economic Theory*, 86, 1999, pp. 226-257.

Francois, Patrick and Huw Lloyd–Ellis, “Animal Spirits Through Creative Destruction,” forthcoming *American Economic Review*, June 2003.

Francois, Patrick and Joanne Roberts (2003) "Contracting Productivity Growth" *Review of Economic Studies* 70, 1, 59-86.

Freeman, Scott, Dong Pyo Hong and Dan Peled, "Endogenous Cycles and Growth with Indivisible Technological Developments" *Review of Economic Dynamics*, vol. 2, 1999, 403–432.

Gale, Douglas (1996) "Delay and Cycles," *Review of Economic Studies*, 63(2) pp. 169-98.

Geroski, P.A. and C.F. Walters, "Innovative Activity over the Business Cycle" (1995) *Economic Journal*, 105, pp. 916-28.

Gilchrist, Simon; and John C. Williams (2000) "Putty-Clay and Investment: A Business Cycle Analysis", *Journal of Political Economy,* 108(5) pp. 928-60.

Goodfriend, Marvin and Robert King (1997), "The New NeoClassical Synthesis and the Role of Monetary Policy," in Ben Bernanke and Julio Rotemberg, eds. *NBER Macroeconomics Annual 1997.* Cambridge, Mass. MIT Press, pp. 231–83.

Grilliches, Z. (1984) "Comment on R&D and Innovation: Some Empirical Findings," in *R&D, Patents, and Productivity*, edited by Zvi Grilliches, Chicago, University of Chicago Press, 148-149.

Grossman, Gene and Elhanan Helpman, *Innovation and Growth in the Global Economy*, Cambridge, MA.: MIT press, 1991.

Hall, Robert E. (2000) "Reorganization" *Carnegie Rochester Conference Series on Public Policy*; 52(0), pp. 1-22.

Hall, Robert E. (2001) "The Stock Market and Capital Accumulation", *American Economic Review,* December 91(5) pp. 1185-1202.

Hamermesh, Dan. (1989) "Labor Demand and the Structure of Adjustment Costs," *American Economic Review*, 79, 674-689.

Hassett, Kevin A, and R. Glenn Hubbard, (1996) "Tax Policy and Investment", *National Bureau of Economic Research Working Paper,* No. 5683.

Johansen, Leif (1959) "Substitution versus Fixed Production Coefficients in the Theory of Growth: A synthesis", *Econometrica*, 27, pp.157-76.

Jovanovic, Boyan and Lach, Saul, (1998) "Product Innovation and the Business Cycle," *International Economic Review*; 38(1) pp. 3-22.

Kasahara, Hiroyuki (2003), "Technology Adjustment under Uncertainty: The Putty–Clay Investment Model," *mimeo*, Queen's University.

Keynes, John M., (1936) *The General Theory of Employment, Interest and Money*, London, Macmillan.

King, Robert G. and Sergio Rebelo, "Resuscitating Real Business Cycles" Rochester Center for Economic Reserach Working Paper No. 467, 2000.

Lucas, Robert E., Jr., (1981) *Studies in Business-Cycle Theory,* MIT Press, Cambridge, Massachusetts.

Martimort, David and Thierry Verdier (2000) "The Internal Organization of the Firm, Transaction Costs and Macroeconomic Growth," *Journal of Economic Growth*, 5, (4) 315-340.

Matsuyama, Kiminori, "Growing Through Cycles" *Econometrica,* 67 (2), 1999, pp. 335–47.

Matsuyama, Kiminori, (2001) "Growing through Cycles in an Infinitely Lived Agent Economy" *Journal of Economic Theory* 100(2): 220-34.

Mayshar, J. and G. Solon (1993) "Shift work and the Business Cycle," *American Economic Review Papers and Proceedings*, 83, 229-39.

Nickell, Stephen, Daphne Nicolitsas, Malcolm Patterson, "Does Doing Badly Encourage Management Innovation?" *Oxford Bulletin of Economics and Statistics*, 63 (1), 2001, pp. 5-28.

Ramey, Valerie A and Matthew D. Shapiro (2001) "Displaced Capital: A Study of Aerospace Plant Closings", *Journal of Political Economy,* 109(5) pp. 958-92.

Romer, Paul (1986) "Increasing Returns and Long-run Growth", *Journal of Political Economy* 94(5), pp. 1002-37.

Schumpeter, Joseph (1927) "The Explanation of the Business Cycle" *Economica,* pp. 286–311.

Schumpeter, Joseph, (1950) *Capitalism, Socialism and Democracy*, 3rd edition, New York.

Segerstrom, Paul S., T. C. A. Anant and Elias Dinopoulos, "A Schumpeterian Model of the Product Life Cycle" *American Economic Review*, Vol. 80, No. 5., 1990, pp. 1077-1091.

Shleifer, Andrei, "Implementation Cycles" *Journal of Political Economy*, vol. 94 (6), 1986, pp. 1163–1190.

Siu, Henry (1999) "Comovement of labor and heterogeneous skill," *mimeo*, University of British Columbia.

Thomas, Julia (2002) "Is Lumpy Investment Relevant for the Business Cycle?" Journal of Political Economy 110 (3) 508-534.

Veraciarto, Marcelo (2002), "Plant–Level Irreversible Investment and Equilibrium Business Cycles," *American Economic Review*, vol. 92 (1), pp. 181–197.

Wälde, Klaus (2002), "The Economic Determinants of technology Shocks in a Real Business Cycle Model," *Journal of Economic Dynamics and Control*, vol. 27, pp. 1–28.