Housing and the Business Cycle

Morris Davis and Jonathan Heathcote

Winter 2009

Motivation

- Need to distinguish between housing and non-housing investment
- \hookrightarrow produced using different technologies
- \hookrightarrow different rates of depreciation
- \hookrightarrow housing yields "home production" services not in National Accounts
 - "Stylized facts" for models with heterogeneous capital goods:
- (1) comovement between consumption and investment in different assets
- (2) residential investment is 2x as volatile as business investment
- (3) residential investment leads cycle, business investment lags it

Model Overview

Households - consume goods and housing services, supply land and labour Real estate developers — combine land and structures to build houses Two final goods sector — one produces structures; the other C and K Three intermediate sectors — construction, manufacturing and services Labour, capital and productivity shocks

Main Findings

- Purely neoclassical model can account for "puzzles" (1) and (2), but not (3)
- Also matches facts on relative volatility of sub-sectors
- Implies pro-cyclical house prices, but not volatile enough
- Why positive comovement and high volatility?
- $\,\hookrightarrow\,$ not due to correlated shocks
- \hookrightarrow final goods sectors use all intermediates
- \hookrightarrow housing requires land, which acts like an adjustment cost
- \hookrightarrow residential investment is relatively labour intensive
- $\,\hookrightarrow\,$ low depreciation of housing

Population

- Gross population growth: $\eta>1$
- All variables in per capita terms

Intermediate sectors

• Intermediate firms' output:

$$x_{it} = k_{it}^{\theta_i} \left(z_{it} n_{it} \right)^{1-\theta_i} \qquad i \in \{b, m, s\}$$

- \hookrightarrow rent capital at rate r_t and hire labour at w_t
- \hookrightarrow output prices p_{it}
- \hookrightarrow productivity shocks:

$$\mathbf{\hat{z}}_{t+1} = \mathbf{B}\mathbf{\hat{z}}_t + \mathbf{\varepsilon}_{t+1} \qquad \mathbf{\varepsilon}_{t+1} \sim \mathbf{N}(\mathbf{0}, \mathbf{V})$$

where

$$\hat{\mathbf{z}}_t = [\ln \tilde{z}_{bt}, \ln \tilde{z}_{mt}, \ln \tilde{z}_{st}]' \ln \tilde{z}_{it} = \ln z_{it} - t \ln g_{zi} - \ln z_{i0}$$

Final goods sectors

• Final goods' output:

$$y_{jt} = b_{jt}^{B_j} m_{jt}^{M_j} s_{jt}^{S_j}$$
 $j \in \{c, d\}$, $S_j = 1 - B_j - M_j$

 \hookrightarrow output prices, $p_{ct} = 1$ (numeraire) and $p_{dt} =$ relative price of residential investment

Land and real estate

- Each household sells one unit of land each period
- Developers combine new residential structures, x_d, and new land, x_l, to build houses:

$$y_{ht} = x_{lt}^{\phi} x_{dt}^{1-\phi}$$

- Structures depreciate at rate δ_s
- Total stock of "effective" housing:

$$\eta h_{t+1} = x_{dt}^{1-\phi} x_{lt}^{\phi} + (1-\delta_s)^{1-\phi} h_t$$

Let

$$1-\delta_h=(1-\delta_s)^{1-\phi}$$

Households

• Optimization problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \eta^t U(c_t, h_t, n_t) \quad \text{s.t.}$$

$$c_{t} + \eta k_{t+1} + \eta p_{ht} h_{t+1} = (1 - \tau_{n}) w_{t} n_{t} + [1 - (1 - \tau_{k}) (r_{t} - \delta_{k})] k_{t} + (1 - \delta_{h}) p_{ht} h_{t} + p_{lt} x_{lt} + \xi_{t}$$

where

$$U(c_t, h_t, n_t) = \frac{1}{1-\sigma} \left(c_t^{\mu_c} h_t^{\mu_h} (1-n_t)^{1-\mu_c-\mu_h} \right)^{1-\sigma}$$

• First-order conditions:

$$U_{c}(t) = -U_{n}(t)(1-\tau_{n})w_{t}$$

$$U_{c}(t) = \beta E_{t} \left[U_{c}(t+1) \left(1 - (1-\tau_{k}) \left(r_{t+1} - \delta_{k} \right) \right) \right]$$

$$U_{c}(t)p_{ht} = \beta E_{t} \left[U_{c}(t+1) \left(1 - \delta_{h} \right) p_{ht+1} + U_{h}(t+1) \right]$$

where

$$\begin{aligned} U_c(t) &= \mu_c c_t^{-1} \left(c_t^{\mu_c} h_t^{\mu_h} (1-n_t)^{1-\mu_c-\mu_h} \right)^{1-\sigma} \\ U_n(t) &= -(1-\mu_c-\mu_h) (1-n_t)^{-1} \left(c_t^{\mu_c} h_t^{\mu_h} (1-n_t)^{1-\mu_c-\mu_h} \right)^{1-\sigma} \\ U_h(t) &= \mu_h h_t^{-1} \left(c_t^{\mu_c} h_t^{\mu_h} (1-n_t)^{1-\mu_c-\mu_h} \right)^{1-\sigma} \end{aligned}$$

Market Clearing Conditions

• Final goods and real estate

$$c_{t} + \eta k_{t+1} + g_{t} = y_{ct} + (1 - \delta_{k}) k_{t}$$

$$\eta h_{t+1} = y_{ht} + (1 - \delta_{h}) h_{t}$$

$$x_{dt} = y_{dt}$$

$$x_{lt} = 1$$

Intermediate goods

$$b_{ct} + b_{dt} = x_{bt}$$
$$m_{ct} + m_{dt} = x_{mt}$$
$$s_{ct} + s_{dt} = x_{st}$$

• Factor markets

$$k_{bt} + k_{mt} + k_{st} = k_t$$
$$n_{bt} + n_{mt} + n_{st} = n_t$$

Government budget constraint:

$$\xi_t + g_t = \tau_n w_t n_t + \tau_k \left(r_t - \delta_k \right) k_t$$

Equilibrium prices

• Factor prices

$$r_t = p_{it}\theta_i k_{it}^{\theta_i - 1} (z_{it}n_{it})^{1 - \theta_i} \qquad i \in \{b, m, s\}$$
$$w_t = z_{it}p_{it}(1 - \theta_i) k_{it}^{\theta_i} (z_{it}n_{it})^{-\theta_i}$$

• Prices of intermediates

$$p_{bt} = B_c \frac{y_{ct}}{b_{ct}} = B_d \frac{p_{dt} y_{dt}}{b_{dt}}$$

$$p_{mt} = M_c \frac{y_{ct}}{m_{ct}} = M_d \frac{p_{dt} y_{dt}}{m_{dt}}$$

$$p_{st} = S_c \frac{y_{ct}}{s_{ct}} = S_d \frac{p_{dt} y_{dt}}{s_{dt}}$$

• Prices of structures and land

$$p_{dt} = (1-\phi)\frac{p_{ht}y_{ht}}{x_{dt}}$$
$$p_{lt} = \phi\frac{p_{ht}y_{ht}}{x_{lt}}$$

Implication for House prices

Relative price of residential investment can be written as

$$\begin{array}{ll} \mathsf{n}\, p_{dt} & = & \left(B_c - B_d\right) \left(1 - \theta_b\right) \mathsf{ln}\, z_{bt} + \left(M_c - M_d\right) \left(1 - \theta_m\right) \mathsf{ln}\, z_{mt} \\ & \quad + \left(S_c - S_d\right) \left(1 - \theta_s\right) \mathsf{ln}\, z_{st} + \mathsf{other \ terms} \end{array}$$

- \hookrightarrow a positive shock in sector *i* will reduce p_{dt} if residential investment is relatively intensive in input *i*
- \Rightarrow implications for comovement
 - Price of new housing

$$\ln p_{ht} = -\ln(1-\phi) + \phi \ln y_{dt} + \ln p_{dt}$$

Mapping between model and NIPA

 In NIPA private consumption includes imputed value for rents from owner–occupied housing:

$$PCE_t = c_t + q_t h_t$$

where

$$q_t = \frac{U_h(c_t, h_t, n_t)}{U_c(c_t, h_t, n_t)}$$

 In NIPA, raw land is not part of GDP ⇒ should only include value of residential investment, not of new houses:

$$GDP_t = y_{ct} + p_{dt}y_{dt} + q_th_t$$

 Real private consumption and GDP defined using balanced growth prices ⇒ does not capture short-run price movements

Balanced Growth Path

- Although each sector has different growth rates, a BGP exists due to Cobb-Douglas assumptions
- All variables are made stationary by dividing by gross growth rate

$$\hat{x}_t = \frac{x_t}{g_x^t}$$

• Model is solved using Klein (2000)

Table 1: Growth Rates on Balanced Growth	Path (growth rates gross, variables per-capita)
n _b , n _m , n _s , n, r	1
$k_{b}, k_{m}, k_{s}, k, c, i_{k}, g, y_{c}, w$	$g_{k} = \left[g_{zb}^{B_{c}(1-\theta_{b})}g_{zm}^{M_{c}(1-\theta_{m})}g_{zs}^{S_{c}(1-\theta_{s})}\right]^{\frac{1}{1-B_{c}\theta_{b}-M_{c}\theta_{m}-S_{c}\theta_{s}}}$
b_c, b_h, x_b	${m g}_b={m g}_k^{ heta_b}{m g}_{zb}^{1- heta_b}$
m_c, m_h, x_m	${m g}_m={m g}_k^{ heta_m}{m g}_{zm}^{1- heta_m}$
s _c , s _h , x _s	$\boldsymbol{g}_s = \boldsymbol{g}_k^{\theta_s} \boldsymbol{g}_{zs}^{1-\theta_s}$
X _d	${m g}_d={m g}_b^{B_h}{m g}_m^{M_h}{m g}_s^{S_h}$
X ₁	$g_l = \eta^{-1}$
y _h , h	$g_h = g_l^{\phi} g_d^{1-\phi}$
$p_h y_h, p_d x_d, p_l x_l, p_b x_b, p_m x_m, p_s x_s$	g_k

Table 1: Growth Rates on Balanced Growth Path (growth rates gross, variables per-capita)

Table 2: Tax Rates, Depreciation Rates, Adjustment Costs, Preference Parameters

	Davis Heathcote	Grenwood Hercowitz (GH)
Tax rate on capital income: τ_k	0.3788	0.50
Tax rate on labor income: τ_n	0.2892	0.25
Govt. cons. to GDP	0.179*4	0.0
Transfers to GDP	0.076*	
Depreciation rate for capital: δ_k	0.0557*	0.078
Depreciation rate for res. structures: δ_s	0.0157*	0.078
Land's share in new housing: ϕ	0.106	
Population growth rate: η	1.0167*	0.0
Discount factor: β	0.9512	0.96
Risk aversion: σ	2.00*	1.00
Consumption's share in utility: μ_c	0.3139	0.2600
Housing's share in utility: μ_h	0.0444	0.0962
Leisure's share in utility: $1-\mu_c-\mu_h$	0.6417	0.6438

⁴ Starred parameter vales are chosen independently of the model.

Calibration

- Period is one year
- μ_c and μ_h chosen so that $\hat{n} = 0.3$ and value of stock of residential structures = GDP
- τ_k and τ_n chosen so that non-residential capital stock = 1.5 x annual output and ξ/GDP = 0.076
- Shock processes estimated as VAR
- \hookrightarrow little evidence of spillovers weak correlation of shocks
- \hookrightarrow shocks to construction and manufacturing much more volatile
 - input shares based on input-output tables

Table 3: Production Technologies				
	Con.	Man.	Ser.	GH
Input shares in cons/inv production $B_{c_{\text{s}}}M_{c_{\text{s}}}S_{c}$	0.0307	0.2696	0.6997	
Input shares in res. structures $B_{d_{,}}M_{d_{,}}S_{d}$	0.4697	0.2382	0.2921	
Capital's share by sector $\theta_{b}, \theta_{m_{s}} \theta_{s}$	0.132	0.309	0.237	0.30
Trend productivity growth (%) g_{zb} , g_{zm} , g_{zs}	-0.27	2.85	1.65	1.00
Autocorrelation coefficient		see table 4		$\rho = 1.0$
Std. dev. innovations to logged productivity		see table 4		0.022

Table 4: Estimation of Exogenous Shock Process

System estimated: $\widetilde{z}_{t+1} = B\widetilde{z}_t + \varepsilon_{t+1}$ where $\widetilde{z}_t = \begin{pmatrix} \log \widetilde{z}_{bt} \\ \log \widetilde{z}_{mt} \\ \log \widetilde{z}_{st} \end{pmatrix}$, $\varepsilon_t = \begin{pmatrix} \varepsilon_{bt} \\ \varepsilon_{mt} \\ \varepsilon_{st} \end{pmatrix}$ and $\varepsilon_t \sim N(0, V)$.⁵

Autoregressive coefficients in matrix B

(Seemingly unrelated regression estimation method: standard errors in parentheses)

	$\log \widetilde{z}_{b,t+1}$	$\log \widetilde{z}_{m,t+1}$	$\log \widetilde{z}_{s,t+l}$		
$\log \widetilde{z}_{bt}$	0.707 (0.089)	-0.006 (0.078)	0.003 (0.038)		
$\log \widetilde{z}_{mt}$	0.010 (0.083)	0.871 (0.073)	0.028 (0.036)		
$\log \widetilde{z}_{st}$	-0.093 (0.098)	-0.150 (0.087)	0.919 (0.042)		
R^2	0.551	0.729	0.903		
	Correlations	of innovation.	\$	Sta	andard deviation of innovations
	ε _b	ε _m	ε _s		
ε _b	1	0.089	0.306	ε _b	0.041
ε _m		1	0.578	ε _m	0.036
ε _s			1	ε _s	0.018

⁵ All variables are linearly detrended prior to estimating this system.

Main Results

- Steady state implications look "reasonable"
- Second moments:
- \hookrightarrow accounts well for high relative volatility of residential investment
- \hookrightarrow yields comovement between investment sectors
- \hookrightarrow relative volatilities of sub-sectors is correct
- $\,\hookrightarrow\,$ house price volatility is too low
- \hookrightarrow housing investment does not lead cycle

Table 5: Decomposition of Final Expenditure into Final Sales From Industries (%)(based on 1992 IO-Use Table)

	PCE	BFI + RESI	RESI ⁶	BFI	G^7
Construction	0.0	43.9	100.0	22.6	33.6
Manufacturing	23.3	41.3	0.0	56.9	44.2
Services	76.7	14.8	0.0	20.5	22.2

	PCE	BFI + RESI	RESI	BFI	PCE + BFI + GOVI ⁸
Construction	1.4	21.3	47.0	11.6	3.1
Manufacturing	23.0	40.6	23.8	46.9	27.0
Services	75.7	38.1	29.2	41.5	70.0

Table 7:	Properties	of Steady	State:	Ratios to	GDP %

- •		
	Data (1948-2001)	Model
Capital stock (K)	152	152
Residential structures stock (P _d x S)	100	100
Private consumption (PCE)	63.8	63.9
Government consumption (G)	17.9	17.9
Non-residential inv (non-RESI)	13.5	13.9
Residential inv (P _d x RESI)	4.7	4.4
Construction $(P_b \times Y_b)$	5.2 ⁹	4.8
Manufacturing (P _m x Y _m)	32.8	24.7
Services $(P_s \times Y_s)$	61.5	70.6
Real after tax interest rate (%)		6.0

⁶ We attribute all \$225.5 billion of residential investment in 1992 to sales from the construction industry, since the first I/O use table does not have a 'residential investment' column. We defend this choice in the data appendix.

⁷ G is government expenditure, which includes government consumption and government investment expenditures.

⁸ GOVI is government investment. We assume that the value-added composition of government investment by intermediate industry is the same as business fixed investment.

⁹ The shares of construction, manufacturing and services do not add to exactly one, since the product approach to computing GDP does not give exactly the same answer as the expenditure approach. In both model and data, imputed rental income from owner-occupied housing is attributed to the service sector.

able 8: Business Cycle P	roperties ¹⁰						
		Data	ı (1948-2	2001)		Model	
% Standard Deviations	GDP		2.26			1.73	
(rel. to GDP)	PCE		0.78			0.48	
	Labor (N)		1.01			0.41	
	Non-RESI		2.30			3.21	
	RESI		5.04			6.12	
	House prices (P _h)	1.37	(1970-2	.001)		0.40	
Constr	uction output (Y _b)		2.74			4.02	
Manufac	turing output (Y _m)		1.85			1.58	
Se	ervices output (Y _s)		0.85			0.99	
Const	ruction hours (N _b)		2.32			2.15	
Manufao	cturing hours (N _m)		1.53			0.39	
Se	ervices hours (N _s)		0.66			0.37	
Correlations	PCE, GDP		0.80			0.95	
	P _h , GDP	0.65	(1970-2	001)		0.65	
	PCE, non-RESI		0.61			0.91	
	PCE, RESI		0.66			0.26	
	non-RESI, RESI		0.25			0.15	
	P _h , RESI	0.34	(1970-2	.001)		-0.20	
	N _b , N _m		0.75			0.48	
	N _b , N _s		0.86			0.23	
	N _m , N _s		0.79			0.96	
Lead-lag correlations		i = 1	i = 0	i = -1	i = 1	i = 0	i = -
1	non-RESI _{t-i} , GDP _t	0.25	0.75	0.48	0.45	0.94	0.33
	RESI _{t-i} , GDP _t	0.52	0.47	-0.22	0.19	0.44	0.14
n	on-RESI t-i, RESI t	-0.37	0.25	0.53	0.07	0.15	0.08

¹⁰ Statistics are averages over 500 simulations, each of length 54 periods, the length of our data sample. Prior to computing statistics all variables are (i) transformed from the stationary representation used in the solution procedure back into non-stationary representation incorporating trend growth, (ii) logged, and (iii) Hodrick-Prescott filtered with the smoothing parameter, λ , set to 100.

Understanding the results

- Model GH one sector RBC model with some capital entering (log) utility function
- Model A $\sigma = 2$ and non-permanent shocks
- Model B adding land
- Model C sector specific shocks
- Model D a distinct technology for residential structures
- Model E sector specific capital shares
- Model F asset-specific depreciation rates (benchmark)

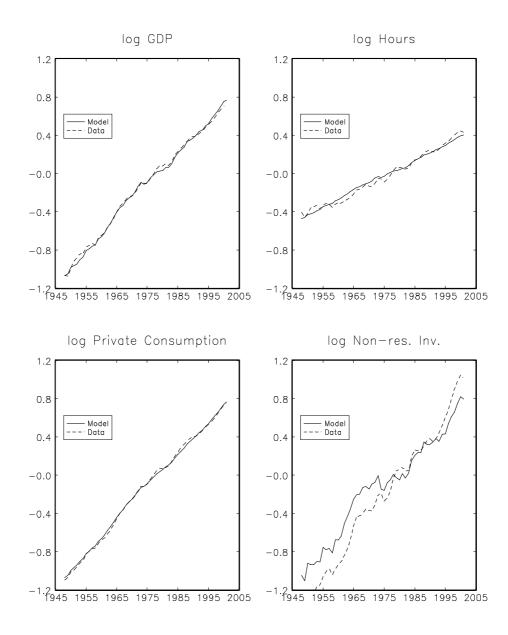
Model	Description	Selected parameter values
GH	Greenwood and Hercowitz	see tables 2 and 3
A	One sector model, housing in utility (re-parameterized <i>GH</i>)	$\begin{split} \sigma &= 2, \ \rho = 0.85, \ \sigma(\epsilon) = 0.022\\ \delta_k &= \delta_s = 0.0557\\ \theta_b &= \theta_m = \theta_s = 0.25\\ B_d &= B_c, \ M_d &= M_c, \ S_d &= S_c \end{split}$
В	A + land	$\phi = 0.106$
С	B + sector-specific shocks	see table 4
D	C + two final goods technologies	see table 3
E	D + sector-specific capital shares	$\theta_{\rm b} = 0.132, \theta_{\rm m} = 0.309, \theta_{\rm s} = 0.2$
F (Benchmark)	E + different depreciation rates	$\delta_{\rm s} = 0.0157$

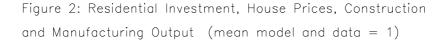
Table 10: Alternat	tive Parar	neterizati	ons: Busi	ness Cycle	e Properti	es		
	Data	GH	Α	В	С	D	Ε	F
GDP (% std dev)	2.26	1.37	1.93	1.88	1.69	1.69	1.67	1.73
Std. dev. relative to	GDP							
PCE	0.78	0.60	0.39	0.39	0.42	0.44	0.45	0.48
Ν	1.01	0.36	0.47	0.48	0.45	0.44	0.45	0.41
Non-RESI	2.30	2.74	3.92	3.55	3.41	3.46	3.30	3.21
RESI	5.04	2.08	2.86	1.22	1.22	4.25	5.10	6.12
Y _b	2.74	1.25	1.15	1.16	1.82	3.66	4.36	4.02
Y _m	1.85	1.25	1.15	1.16	1.80	1.79	1.65	1.58
Y _s	0.85	1.25	1.15	1.16	1.06	1.05	1.05	0.99
P_{h}	1.37	0.00	0.00	0.13	0.13	0.41	0.45	0.40
Correlations								
Non-RESI, RESI	0.25	0.88	-0.10	0.73	0.75	-0.07	-0.07	0.15
P _h , RESI	0.34	-	-	1.00	1.00	-0.44	-0.48	-0.20
Lead-lag pattern: c	$orr(x_{t-1}, G$	$(DP_t) - con$	$rr(x_{t+1}, Gl$	OP_t				
$\mathbf{x} = \mathbf{RESI}.$	0.74	-0.11	-0.93	-0.48	-0.46	0.04	0.11	0.12
x = Non-RESI	-0.23	0.37	0.46	0.21	0.20	0.11	0.07	0.05

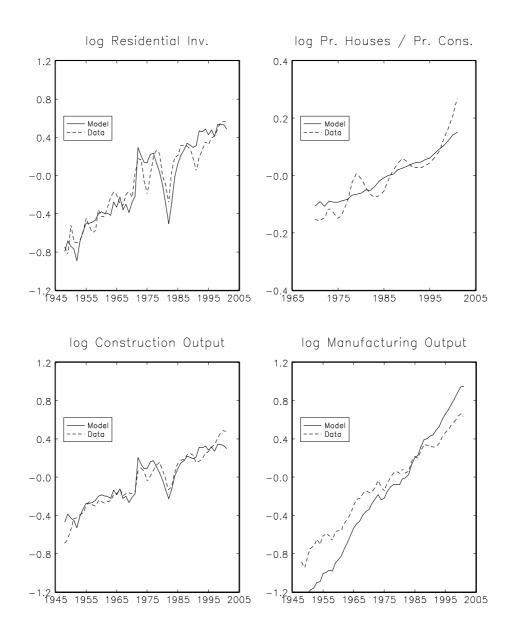
Comparison to US Historical Time Series

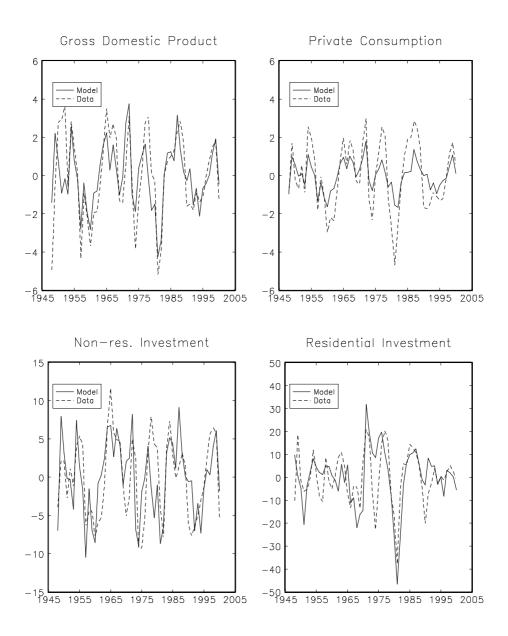
- Model matches trend growth in hours, consumption and output
- Residential investment exhibits relatively slow growth (due to low prod. growth in construction)
- Does not match growth of non-residential investment or manufacturing
- Model does reasonably well at business cycle frequencies
- Does not account well for house price dynamics











Supply vs. Demand Shocks

- Traditional view based on regressions of residential investment on house prices (controlling for other factors)
- \hookrightarrow positive coefficient \Rightarrow demand shocks more important than supply shocks
 - Regression using simulated data from model yields positive coefficient, despite no demand shocks
- \hookrightarrow due to omitted variable bias