ECON 835 — LECTURE NOTES

Accounting For Economic Growth: Theory and Emprics

Recently, relatively high quality (though far from perfect) data on broad macroeconomic aggregates that are comparable across different countries have become available for researchers to use. The main data set that researchers use is called the Penn World Tables, developed by Robert Summers and Alan Heston (this databank is available at Queens). This data has allowed economists and policy makers to make meaningful (PPP adjusted) cross–country comparisons, and to try to evaluate alternative theories regarding the determinants of both the level of per capita income and its growth rate. The kinds of questions that researchers have been interested in include the following:

1. What accounts for cross–country differences in income per capita and labour productivity ? — For example, GNP per capita in the US in 1990 was 20 times that of India and over 75 times that of Rwanda. More generally, the richest 5% of countries have income per capita that is more than 30 times that of the poorest 5%.

2. To what extent have the per capita incomes of various countries diverged or converged over time, and what determines this ? — For example, some countries such as Ireland, S. Korea and Turkey experienced growth rates far in excess of the OECD average in the 1990s, while many countries, especially in S. America and Africa had negative growth rates.

3. What determines long–run growth rates and persistent differences in them across countries ?

4. How do economic institutions and policies impact upon growth ? — For example, what is the role of social infrastructure, government regulations and labour market institutions in accounting for different levels of productivity and per capita income.

1 The Solow Growth Model

Solow's (1956) version of the neoclassical growth model has become a benchmark framework for thinking about the long–run evolution of the macroeconomy and for organizing aggregate data. Although it is extremely simple, it provides a useful starting point for thinking about first–order cross–country differences. Indeed many economists believe it provides a surprisingly accurate account of the cross–country data, at least once it is augmented appropriately. Others argue, however, that it is not a useful theory of development because not enough of the cross–country variation in per capita income can be explained by capital (physical and human) accumulation alone.

1.1 Assumptions

The basic Solow model describes a closed economy and consists of just two equations. The first describes the relationship between aggregate output, Y_t, and aggregate inputs:

$$Y_t = F(K_t, A_t L_t) \tag{1}$$

Here K_t denotes the aggregate productive capital stock, L_t denotes the aggregate labour force and A_t represents the "effectiveness of labour". The aggregate capital stock evolves according to

$$\Delta K_t = sY_t - \delta K_t, \tag{2}$$

where s denotes the fraction of output that is saved and invested, and δ denotes the physical depreciation rate.

The model imposes several crucial assumptions on the aggregate production function, $F(\cdot)$ and the evolution of economic variables:

• Technical progress represented by increments in A_t and is assumed to be labour augmenting (Harrod neutral). As we'll see this assumption guarantees that the capital–output ratio is constant in the long run, which is roughly consistent with the data for most OECD economies.

• The production function exhibits constant returns to scale in K_t and $A_t L_t$. This assumption implies that for any scalar $\lambda > 0$ we can write

$$F(\lambda K_t, \lambda A_t L_t) = \lambda F(K_t, A_t L_t) = \lambda Y_t. \tag{3}$$

Constant returns to scale is consistent with a balanced growth path along which output, capital and effective labour grow at the same rate. In particular, it allows us to express the model in **intensive form**, which permits a stationary representation in the long run.

• To obtain the intensive form, let $\lambda = \frac{1}{A_t L_t}$, so that the above equations become

$$F\left(\frac{K_t}{A_t L_t}, 1\right) = \frac{F(K_t, A_t L_t)}{A_t L_t} = \frac{Y_t}{A_t L_t}. \tag{4}$$

If $k_t = \frac{K_t}{A_t L_t}$ and $y_t = \frac{Y_t}{A_t L_t}$, then we can define the intensive form production function $f(k_t) = F\left(\frac{K_t}{A_t L_t}, 1\right)$, so that

$$y_t = f(k_t). \tag{5}$$

For example, if the aggregate production function is Cobb–Douglas, $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$, then the intensive form is

$$y_t = k_t^{\alpha}. \tag{6}$$

• The intensive form production function satisfies the Inada conditions:

$$f(0) = 0, \;\; f'(k) > 0, \;\; f''(k) < 0, \;\; \lim_{k \to 0} f'(k) = \infty, \;\; \lim_{k \to \infty} f'(k) = 0. \tag{7}$$

These conditions will be sufficient to ensure a unique balanced growth path (see below).

• The population growth is given by

$$n = \frac{\Delta L_t}{L_t} \tag{8}$$

and the rate of technical progress is given by

$$g = \frac{\Delta A_t}{A_t}. \tag{9}$$

Both are constant and exogenously determined.

1.2 The Dynamics of the Model

The capital stock per unit of effective labour is given by

$$k_t = \frac{K_t}{A_t L_t}. \tag{10}$$

It follows that its growth rate is given by

$$\frac{\Delta k_t}{k_t} = \frac{\Delta K_t}{K_t} - \frac{\Delta A_t}{A_t} - \frac{\Delta L_t}{L_t} = \frac{\Delta K_t}{K_t} - g - n \tag{11}$$

Multiplying through by k_t yields

$$\begin{aligned} \Delta k_t &= \frac{\Delta K_t}{A_t L_t} - (n+g)k_t && (12) \\ &= \frac{sY_t - \delta K_t}{A_t L_t} - (n+g)k_t && (13) \\ &= sy_t - (n+g+\delta)k_t && (14) \end{aligned}$$

Thus, we can express the dynamics of the capital stock per unit of effective labour by

$$\Delta k_t = sf(k_t) - (n+g+\delta)k_t. \tag{15}$$

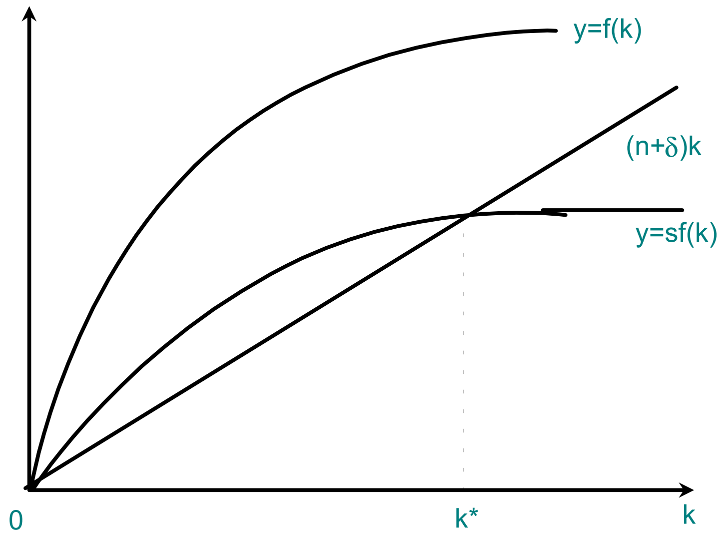

Figure 1: The Solow Growth Model

The first term on the right hand side is the actual rate of investment per unit of effective labour. The second term is the "break–even" level of investment — the investment needed to keep the capital stock per unit of effective labour constant in the face of technical change, population growth and depreciation. Observe that

$$\text{if } sf(k_t) \gtreqless (n+g+\delta)k_t \text{ then } \Delta k_t \gtreqless 0. \tag{16}$$

In particular there is a value k^* such that $\Delta k_t = 0$, which is implicitly given by

$$sf(k^*) - (n+g+\delta)k^*. \tag{17}$$

This is the balanced growth path (BGP) value of the capital stock per unit of effective labour. This is illustrated in Figure 1. Observe that if $k < k^*$, $\Delta k > 0$ and if $k > k^*$, $\Delta k < \dot{0}$. This implies that that the BGP is **stable**, since the economy always converges to it.

1.3 Properties of the BGP

- The characteristics of the long–run growth path are independent of initial conditions.
- Along the BGP, the capital stock grows at the same rate as income, so that K/Y is constant:

$$\frac{K}{Y} = \frac{k}{y} = \frac{k^*}{f(k^*)} = \frac{s}{n+g+\delta}. \tag{18}$$

• Along the BGP, the level of income per worker depends positively on the savings rate and negatively on the population growth rate. That is

$$\frac{Y_t}{L_t} = A_t y = A_t f(k^*). \tag{19}$$

From Figure 1, an increase in s leads to an increase in k^* and hence an increase in $\frac{Y_t}{L_t}$.

• Along the BGP, the rate of growth of income per worker depends *only* on the rate of technological progress — not on the rates of saving and population growth.

• Along the BGP, the marginal product of capital is constant, whereas the marginal product of labour grows at the rate g.

1.4 Competitive Markets in the Solow Model

The description of the Solow model above does not explicitly describe product or factor markets, except to the extent that it assumes that markets clear. The simplest way to motivate this assumption is to assume that product and factor markets are competitive, and that firms choose combinations of capital and labour so as to minimize the costs of producing a given level of output, taking prices as given. The level of output is then determined by price adjustment in the goods market, so that firms make zero profits.[1]

Suppose the firms take the prices of capital and labour as given. Then the total real costs faced by a particular firm i are given by

$$TC_i = w_t L_i + q_t K_i \tag{20}$$

where w_t denotes the real wage rate and q_t denotes the real user–cost of capital, both of which are assumed to be the same for all firms. Suppose further that each firm minimizes the costs of producing output X_i given a Cobb Douglas production function:

$$X_i = A_t^{1-\alpha} K_i^{\alpha} L_i^{1-\alpha}. \tag{21}$$

We can express (20) as an **isocost line:**

$$K_i = \frac{TC_i}{q_t} - \frac{w_t}{q_t} L_i. \tag{22}$$

Clearly the slope of this isocost curve is given by $-w_t/q_t$. The cost minimizing combination of capital and labour is then depicted as the point of tangency between the isocost line and the

[1] Note, however, the Solow model does not necessarily require perfect competition

isoquant in Figure 2. In other words, where the relative marginal products equal relative prices

$$\frac{F_L}{F_K} = \frac{w_t}{q_t}. \tag{23}$$

In the Cobb–Douglas case, this implies that

$$\frac{K_t}{L_t} = \left(\frac{\alpha}{1-\alpha}\right)\frac{w_t}{q_t} \tag{24}$$

or, since $k_t = K_t/A_tL_t$,

$$k_t = \left(\frac{\alpha}{1-\alpha}\right)\frac{w_t}{A_tq_t} \tag{25}$$

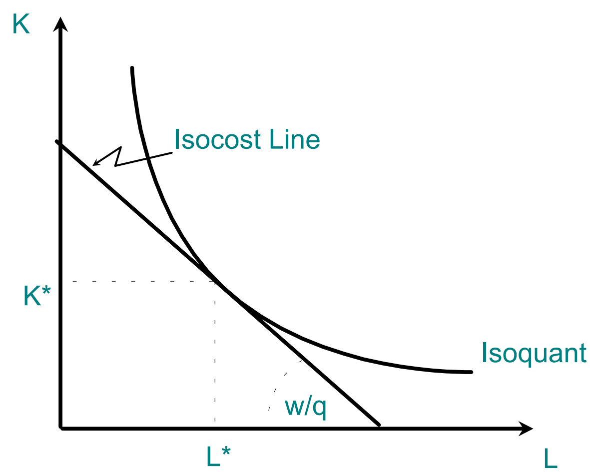

Figure 2: Cost Minimization

1.4.1 Perfectly Competitive Goods Market

When there is perfect competition in the goods market then firms are price takers and earn zero profits. If firm i produces Y_i it follows that the real price must satisfy

$$\frac{P_i}{P} = \frac{wL_i + qK_i}{X_i}. \tag{26}$$

Here, P_i is the price of good i and P is the aggregate price index, so the ratio is the real price of the good. The right hand side of this equation is the real **unit cost of production**: it represents the cost of producing one unit of output. But, from equation (24),

$$K_i = \left(\frac{\alpha}{1-\alpha}\right)\frac{w_t}{q_t}L_i \tag{27}$$

and using this to substitute in (21) we get

$$X_i = \left[\left(\frac{\alpha}{1-\alpha} \right) \frac{w}{q} \right]^{\alpha} L_i \tag{28}$$

Using (27) and (28) to substitute for K and X in (26) give us

$$\frac{P_i}{P} = \frac{w^{1-\alpha} q^{\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} A_t^{1-\alpha}} \tag{29}$$

Since they face the same factor prices, all firms charge the same price and so $P_i = P$. Then the above equation implies that

$$w_t^{1-\alpha} q_t^{\alpha} = \alpha^{\alpha} (1-\alpha)^{1-\alpha} A_t^{1-\alpha} \tag{30}$$

Using (25) to substitute out q_t yields the implied real wage

$$w_t = (1-\alpha) A_t k_t^{\alpha}. \tag{31}$$

The right hand side of this expression is simply the marginal product of labour (in intensive form). To see this, simply let $k = K/AL$. Notice that along a BGP, where k is constant, the real wage grows at the same rate as technology. Using (31) to substituting for w_t in (30) yields the user cost of capital

$$q_t = \alpha k_t^{\alpha-1}. \tag{32}$$

As before, the right hand side of this expression is simply the marginal product of capital (in intensive form). The user cost of capital is equal to the real interest, r_t, rate plus depreciation. Thus, the model provides a prediction regarding the real interest rate: $r_t = q_t - \delta$. Notice that if we are on a BGP the real interest rate is therefore constant.

1.5 Evaluation of the Basic Solow Model

1. Convergence

According to a strict interpretation of the Solow growth model, in which all economies have similar parameters (s, n, g, δ), and access to a common pool of technological knowledge, poor (low capital) countries should eventually converge with rich countries. Baumol (1986) addresses this issue for 16 industrialized economies and finds that indeed the poorest countries in 1870 grew the fastest between 1870 and 1980, whereas the reverse was true for rich countries in 1870 (see Figure 3.8, p. 77 in Ray). This appears to be consistent with convergence to a BGP in the Solow model.

However, De Long (1988) shows that this observation is largely a result of sample "selection bias". Those countries that were rich in 1980 are also the ones with the longest data sets. Thus, countries that were relatively poor in 1870 are only in the data set if the grew rapidly. Similarly, those countries that were rich in 1870 are included even if they grew slowly. When De Long considers an expanded data set of the 23 richest countries as of 1870, he shows that the evidence for convergence breaks down (see Figure 3.9, p. 78 in Ray). Moreover, when one undertakes a similar exercise to examine convergence during the post–war period for all the countries in the Penn World Tables, there is very little evidence of convergence (see Figure 10, p. 81 in Ray).

2. Cross–country rates of return

An alternative way to evaluate the Solow model was suggested by Lucas (1990). Because of diminishing returns to capital per unit of effective labour, the Solow model predicts that rich countries should have lower rates of return on investment than poor countries, and as a result we should see capital flowing from rich to poor countries. Of course, there are various barriers and friction that restrict the free flow of capital across borders. However, it is difficult to see how large differences in rates of return could persist for long periods.

To quantify the differences in rates of return that are implied by the model consider the Cobb–Douglas example, where $y = k^\alpha$. The marginal product of capital is given by $\alpha k^{\alpha-1}$. In a competitive economy, this should reflect the rate of return on capital (ignoring depreciation). Thus we can compare in the rates of return in, say, India and the US as follows:

$$\frac{r_I}{r_{US}} = \left(\frac{k_I}{k_{US}}\right)^{\alpha-1} = \left(\frac{y_{US}}{y_I}\right)^{\frac{1-\alpha}{\alpha}} \tag{33}$$

If we take the typical estimate of $\alpha = 0.3$, then

$$\frac{r_I}{r_{US}} = \left(\frac{y_{US}}{y_I}\right)^2 = \left(\frac{Y_{US}/L_{US}}{Y_I/L_I} \times \frac{A_I}{A_{US}}\right)^2 \tag{34}$$

Output per worker in the US is on the order of 20 times that of India. If both economies have access to the same technology/skills ($A_{US} = A_I$), then the relative rate of return would be

$$\frac{r_I}{r_{US}} = 20^2 = 400 \tag{35}$$

Observed differences in real rates of return are nowhere near this big (more like a ratio of 3 or 4 at most). For the model to be consistent with actual rates of return, the technology/skill differences would have to satisfy

$$4 = \left(20 \times \frac{A_I}{A_{US}}\right)^2 \tag{36}$$

$$\frac{A_I}{A_{US}} = \frac{1}{10} \tag{37}$$

Thus, the effectiveness of labour in the US would have to be 10 times that of India (and probably more). While this may be so, the basic Solow model does not provide an *explanation* for these differences (A is exogenous), and hence is not very useful as a theory of differences in the level of development.

3. Conditional Analysis

Mankiw, Romer and Weil (1992) adopt a less strict interpretation of the basic Solow model, by allowing for variation in the underlying parameters of the model (in particular, savings rates and population growth rates). Thus, they *condition* their evaluation of the model on observable cross–country differences. Note that allowing for these differences (but not differences in g) implies that countries converge to parallel long–run paths (see Figure 3.11, p. 83 in Ray).

Consider the Cobb–Douglas version of the model as a first approximation. Recall that the BGP value of k is given by

$$s_i k_i^{\alpha} = (n_i + g + \delta) k_i \tag{38}$$

where i indexes the country. Solving for k, yields

$$k_i = \left(\frac{s_i}{n_i + g + \delta} \right)^{\frac{1}{1-\alpha}} \tag{39}$$

Since $y = k^{\alpha}$, this implies that

$$\frac{Y_i}{A_i L_i} = y_i = \left(\frac{s_i}{n_i + g + \delta} \right)^{\frac{\alpha}{1-\alpha}} \tag{40}$$

Multiplying through by A_i and taking logs yields the predicted log of output per worker:

$$\ln \frac{Y_i}{L_i} = \ln A_i + \frac{\alpha}{1-\alpha} \left[\ln s_i - \ln(n_i + g + \delta) \right]. \tag{41}$$

Using data for 98 countries from the Penn World Tables, mankiw, Romer and Weil (1992) regress the log of output per worker in 1985 on the average savings rate and population growth rates for each country between 1965 and 1985, according to the following specification

$$\ln \frac{Y_i}{L_i} = a + b \ln s_i + c \ln(n_i + 0.5) + \varepsilon_i \tag{42}$$

where a, b, and c are parameters and they set $g + \delta = 0.5$ for all countries. Note that in running this OLS regression, they are implicitly treating variations in A_i across countries as a random residual that is uncorrelated with either s_i or n_i.

They obtain the following results:

• More than half the worldwide variation in per capita GDP in 1985 can be accounted for by the variations in savings rates and population growth rates. The $R^2 = 0.59$.

• As predicted by the Solow model $\hat{b} > 0$ and $\hat{c} < 0$ and both parameter estimates are significant.

• However, the parameter estimates are too large to be anywhere close to 0.5 (which is what the should be if $\alpha = 0.3$). In fact $\hat{b} = 1.42$ and $\hat{c} = -1.97$. Moreover, the coefficients are far from being of similar magnitude (i.e. the restriction that $\hat{b} = -\hat{c}$ is rejected).

2 The Augmented Solow Model

Although, once we allow for variations in savings rates and population growth rates, the basic Solow model begins to make some sense of the data, it is far from satisfactory. In particular, the implied capital share is over 0.6, which is inconsistent with most findings and relatedly, the extent of diminishing returns to capital per effective worker implied by a factor share of 0.3 seems to be inconsistent with small differences in observed rates of return. Although there are other more general problems that we will discuss below, Mankiw, Romer and Weil (1992) argue that one way to address these two issues is to broaden the definition of capital to include human capital (and perhaps other forms of intangible capital). This seems like a reasonable approach since these other forms of "capital" require significant investment and, hence, foregone consumption, so there is no reason to exclude them.

Mankiw, Romer and Weil (1992) formulate an augmented Solow model, by hypothesizing an aggregate production function given by

$$Y_t = K_t^{\alpha} H_t^{\beta} (A_t L_t)^{1-\alpha-\beta} \tag{43}$$

They model the accumulation of human and physical capital in a symmetric fashion:

$$\Delta K_t = s_K Y_t \tag{44}$$

$$\Delta H_t = s_H Y_t, \tag{45}$$

where I have ignored depreciation for simplicity. This framework implicitly assumes that the production function producing human capital is the same as that for physical capital. Letting $k = K/AL$, $h = H/AL$ and $y = Y/AL$ as before yields the intensive form production function

$$y_t = k_t^{\alpha} h_t^{\beta}. \tag{46}$$

As in the basic model, we can derive the dynamics physical and human capital in intensive form also:

$$\Delta k_t = s_K k_t^{\alpha} h_t^{\beta} - (n+g)k_t \tag{47}$$

$$\Delta h_t = s_H k_t^{\alpha} h_t^{\beta} - (n+g)h_t \tag{48}$$

It can be shown that the economy converges to a stable BGP where $\Delta k_t = \Delta h_t = 0$. Along this BGP, h and k must satisfy

$$s_K k_t^{\alpha} h_t^{\beta} = (n+g)k_t \tag{49}$$

$$s_H k_t^{\alpha} h_t^{\beta} = (n+g)h_t \tag{50}$$

Solving yields

$$k = \left(\frac{s_K^{1-\beta} s_H^{\beta}}{n+g} \right)^{\frac{1}{1-\alpha-\beta}} \quad \text{and} \quad h = \left(\frac{s_K^{\alpha} s_H^{1-\alpha}}{n+g} \right)^{\frac{1}{1-\alpha-\beta}}. \tag{51}$$

Substituting these values into the production function yields

$$y = \left[\frac{s_K^{\alpha} s_H^{\beta}}{(n+g)^{\alpha+\beta}} \right]^{\frac{1}{1-\alpha-\beta}} \tag{52}$$

Taking logs and noting that $y = Y/AL$ we get

$$\ln \frac{Y}{L} = \ln A + \frac{\alpha}{1-\alpha-\beta} \ln s_K + \frac{\beta}{1-\alpha-\beta} \ln s_H + \frac{\alpha+\beta}{1-\alpha-\beta} \ln(n+g). \tag{53}$$

2.1 Empirical Evaluation

Using the same data set as before, but now including a proxy for s_H for each country: the fraction of the working age population in secondary school, Mankiw, Romer and Weil estimate the regression

$$\ln \frac{Y_i}{L_i} = a + b \ln s_{Ki} + c \ln s_{Hi} + d \ln(n_i + 0.5) + \varepsilon_i. \tag{54}$$

They obtain the following results:

• The augmented model can now account for almost 80% of the world wide variation in output per worker with just three explanatory variables.

• The parameter estimates are significant and have the right sign $b > 0$, $c > 0$ and $d < 0$.

• The implied values of the factor shares are $\alpha = 0.31$ and $\beta = 0.28$. Thus, the estimate of α is consistent with observed capital shares and they argue that the estimate of β is also "reasonable".

• The restriction that $b + c = -d$, cannot be rejected at the 5% level.

2.2 Problems

On the face of it it seems like the augmented Solow model does a pretty good job in accounting for cross–country income differences. However, the approach taken by Mankiw, Romer and Weil (1992) has come under significant criticism. These include:

• It is not clear that it is reasonable to treat the explanatory variables as exogenous. Even though they are predetermined, it is likely that household incentives to save, invest and reproduce may depend on their expectations about future per capita income. This reverse causality will cause the coefficient estimates to be biased upwards. Moreover, it is then not clear how to interpret the regression results.

• The implicit assumption that the effectiveness of labour A_i is uncorrelated with s_{Hi} or s_{Ki} across countries is unlikely to be true. One might expect people to invest more in education when they are in an economy that is anticipated to have a high level of TFP growth. This "omitted variable bias" will result in an overestimate of the impact of the savings rates on per capita income. Howitt (2000) shows that when one introduces a proxy of TFP growth such as investment in R&D as a percentage of GDP, the coefficients on s_{Hi} and s_{Ki} are reduced.

• The proxy for s_H seems arbitrary. If one chose the fraction of the population in primary and secondary education, the explanatory power of the model drops down to 60%.

• The Solow model assumes that rates of technical change are largely exogenous. However, Bernanke and Gurkaynak (2001) find that TFP growth rates across countries are significantly correlated with behavioural variables such as the savings rate. These findings suggest that growth is at least partially endogenous.

• Although the model simultaneously explains how rates of return to physical as well as the wage rate for unskilled labour might be low for developing nations, there is a problem with this argument. Due to diminishing returns to human capital, the model also predicts that the rate of payment to human capital must be higher in developing countries. This seems inconsistent with our observations: it does not seem that workers from OECD countries are "knocking down the door" to earn high wages in LDCs!

Example: To illustrate the last point consider our Cobb Douglas economy again. The production function in intensive form is

$$y = k^{\alpha} h^{\beta}$$

The return to physical capital is

$$r = \alpha k^{\alpha-1} h^{\beta}$$

This equation can be re–written as

$$h = \left(\frac{r}{\alpha}\right)^{\frac{1}{\beta}} k^{\frac{1-\alpha}{\beta}} \tag{*}$$

The return to human capital is

$$w_h = \beta k^{\alpha} h^{\beta-1}$$

Substituting for h using (*) yields

$$\begin{aligned} w_h &= \beta k^{\alpha} \left(\frac{r}{\alpha}\right)^{\frac{\beta-1}{\beta}} k^{\frac{(1-\alpha)(\beta-1)}{\beta}} \\ &= \beta \left(\frac{\alpha}{r}\right)^{\frac{1-\beta}{\beta}} k^{\frac{\alpha\beta-(1-\alpha)(1-\beta)}{\beta}} \\ &= \beta \left(\frac{\alpha}{r}\right)^{\frac{1-\beta}{\beta}} k^{\frac{\alpha+\beta-1}{\beta}} \end{aligned}$$

Since $\alpha + \beta < 1$ the return to human capital is *lower* in countries with a high return to capital and a high capital–labour ratio (holding technology constant). This seems inconsistent with our observations: it does not seem that skilled workers from OECD countries are "knocking down the door" to earn high wages in LDCs!

Basically, the only way to generate situations in which the return to every input is lower in developing countries is to allow total factor productivities, A, to vary across countries. Other possibilities like raising α and β, may raise the return to physical and human capital, but will lower the return to raw labour, and anyway, in a sense, varying these parameters also constitutes varying technologies.