

## ECON435/835: Topics in Development Economics

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### Assignment #3

Due Date: 4pm Wednesday March 2, 2011

1. Consider an economy, like that considered by Murphy, Shleifer and Vishny (1989), consisting of three (representative) households. Each household supplies 1 unit of labour which could be used either in manufacturing or agriculture. Household 1 owns no shares, household 2 owns a fraction  $\hat{\gamma} < \frac{1}{2}$  of the claims to both manufacturing firm profits and agricultural rents, and household 3 owns the remaining  $1 - \hat{\gamma} > \frac{1}{2}$ .

(a) The agricultural sector produces output using only labour according to

$$Y_A = \theta L_A^{\frac{1}{2}}.$$

Derive the agricultural profit function,  $\pi_A(L_A)$ , and wage function,  $w(L_A)$  implied by profit maximization.

(b) Let  $z = 1$  be the maximum food requirement of all households. Assuming that  $w < z$ , derive an expression for the equilibrium amount of labour effort in agriculture as a function of  $\theta$ . For what range of values of  $\theta$  is it true that  $w < z$ .

(c) Manufacturing production is the same as in Murphy, Shleifer and Vishny. The fixed labour requirement for the modern technology is  $C = 0.5$  and the traditional technology is 50% less productive than the modern technology:  $\alpha = 1.5$ . Show that if all sectors up to  $Q$  industrialize, only household 3 will demand output from the  $Q$ th sector.

(d) Derive the equilibrium level of profits in the manufacturing sector as a function of  $\theta$ ,  $\hat{\gamma}$  and  $L_A$ . How do the equilibrium profits depend on these parameters?

(e) Using your answers to (b) and (d) characterize the impact of an increase in  $\theta$  on the equilibrium value of  $L_A$ .

(f) Ceteris paribus the increase in  $\theta$  raises the marginal product of, and hence the demand for, labour in agriculture. Explain intuitively how your answer in part (e) comes about despite this.

2. Consider the following credit market. The output of a risk-neutral farmer depends stochastically on her effort level,  $x$ . Specifically, suppose her output is  $Q$  with probability  $p(x) = 1 - e^{-ax}$  and is 0 otherwise. The utility cost to the farmer of providing this effort is  $x$ . In order to undertake any production at all, the farmer requires fixed capital  $K$ . The farmer has wealth  $W$  which she could either invest in her own production or place in a development bank and earn interest  $i$ .

- (a) Suppose  $W > K$ , so that the farmer can self-finance. What would the farmer's optimal choice of effort,  $x^*$ , be?
- (b) Now suppose the farmer only has wealth  $W < K$  and must borrow the remainder  $K - W$ . The repayment to the lender is  $R = (1+r)(K - W)$ , where  $r \geq i$  is the lending rate. What would the farmer's utility-maximizing level of effort be? How does it depend on  $r$ ?
- (c) If lenders are competitive and face the marginal cost of funds  $i$ , what must be the relationship between  $r$  and  $x$ ? Explain.
- (d) Characterize (but don't solve for) the interest rate  $\hat{r}$  and the effort level  $\hat{x}$  that obtains in an information-constrained Pareto efficient equilibrium. (Hint: use a diagram)
- (e) Suppose  $a = \frac{1}{2}$ ,  $Q = 30$ ,  $K = 20$ ,  $W = 10$  and  $i = 0$ . What are the constrained-efficient values of  $\hat{r}$  and  $\hat{x}$ ? How does the latter compare with  $x^*$  computed for these parameters?

3. A risk-neutral farmer produces output,  $Y$ , using capital,  $K$ , according to

$$Y = K^{\frac{1}{2}}$$

The capital is completely used up in production each period and the opportunity cost of each dollar invested is  $1 + r$ , where  $r = 0.1$ .

(a) If the farmer were to finance the investment out of her own savings, what would the optimal level of investment be in each period? What is the farmer's income?

Now suppose that the farmer has no savings and must finance the investment in each period by borrowing from a risk-neutral lender. The loan contract specifies that in return for the initial loan of  $K$ , the farmer should pay the lender  $R$ . The marginal cost of funds to the lender is  $1 + r$ . There is no uncertainty, but the lender has no means to enforce repayment directly. Assume that if the farmer defaults in any period she cannot access credit in the future. In this "autarky" case the farmer moves to an urban area where she can earn a wage  $v$  per period. It is prohibitively costly for the lender to track her down. The farmer discounts the future at rate  $\delta = 0.6$ .

- (b) Write down the farmer's incentive constraint and the lender's participation constraint.
- (c) Assuming that  $v = 0$ , illustrate these constraints on a diagram with  $R$  on the vertical and  $K$  on the horizontal axis. Derive the value of  $K$  at all intersections of the two constraints, if any.
- (d) If the debt contract is designed so as to maximize the farmer's income, is the incentive constraint binding? Explain your answer using the diagram in part (c). What is the implied level of investment?
- (e) Now assume that opportunities in the urban sector improve (e.g. due to globalization), so that  $v = 0.1$ . Does the incentive constraint bind now? What is the implied level of investment?