Department of Economics

Queen's University

ECON435/835: Development Economics

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Assignment #1

Due Date: 4 p.m., Wednesday, January 26, 2011

1. Suppose the economy consists of a large number of households with identical preferences represented by the indirect utility function

$$u(y_h) = Ay_h^{\alpha},$$

where y_h is the income of household h and α and A are constants.

(a) For any general distribution of household income, is the growth in average household utility proportional to the growth in average household income ? Explain why.

Now suppose also that income is **log–normally** distributed across households in the economy. This means that the natural log of y, $\ln y$, is distributed normally with mean μ and variance, $\text{Var}(\ln y) = \sigma^2$. This kind of distribution is actually a pretty good first approximation to the actual distribution of income in a typical OECD economy — skewed to the left with a long, thin upper tail.[1] A key property is that the mean value of income is given by

$$E[y] = e^{\mu + \frac{1}{2}\sigma^2}.$$

You should also know that since $\ln y$ is normally distributed with mean μ and variance σ^2, then any linear transformation of $\ln y$, $a + b \ln y$, is also normally distributed with mean $a + b\mu$ and variance $b^2\sigma^2$.

(b) What is the average utility of households in terms of α, μ and σ.

(c) Suppose we (incorrectly) assume that the economy consists of a hypothetical single representative household whose income is equal to the average income in the actual economy. Write down the utility of that hypothetical household in terms of α, μ and σ.

(d) Income inequality changes very slowly over time in Canada. What does this imply for your answer to part (a) under the log-normal distribution case ? What about the comparison of per capita income across countries that have very different degrees of inequality ?

[1] You can find out more about this distribution at http://en.wikipedia.org/wiki/Log-normal_distribution.

2. Consider an economy in which the representative household has utility over 2 goods given by

$$u(x_1, x_2) = \beta \ln x_1 + (1 - \beta) \ln x_2.$$

with price p_1 and p_2. The household spends all its income, m, on these two goods. The two goods are produced by separate industries which have production functions given by

$$x_1 = \theta_1 k_1^{\gamma} l_1^{1-\gamma} \qquad \text{and} \qquad x_2 = \theta_2 k_2^{\delta} l_2^{1-\delta},$$

where $\theta_1 \neq \theta_2$ are measures of TFP for each industry and $\gamma \neq \delta$.

(a) Derive an index of real GDP growth in terms of the growth in the final outputs of the two goods.

(b) Derive an index of real GDP growth for this economy in terms of the growth of factor inputs and TFP, assuming competitive markets.

Now consider an economy in which households have preferences over a single composite consumption good index, C, given by

$$u(C) = \ln C$$

and that good is produced by a single production sector according to

$$Y = AK^{\alpha} L^{1-\alpha},$$

where A, K and L represent composite indices of TFP, real capital and labour services, respectively.

(c) Derive an index of real GDP growth for this economy in terms of the growth in A, K and L.

(d) Show that it is possible for the single–sector economy to represent the two–sector economy above. What would the composite indices C, A, K and L and the parameter α have to be? (Hint: In growth accounting, the weights used to compute the growth of an index from the weighted average of its sub-components must add up to 1).

3. An economy produces final output using capital, K, and labour, L, according to the technology

$$Y = K^{\alpha}(AL)^{1-\alpha},$$

where A denotes the effectiveness of labour. Total output is growing at the rate of 5% per year. The rental rate per unit of capital is equal to 0.1 units of final output. The physical capital–output ratio is 3:1. The stocks of capital and population are growing at the rate of 3 and 2% respectively. Assume that everybody works.

(a) Under the assumption that all output is paid in wages and rent, calculate the implied shares of capital and labour in national income.

(b) Using standard growth accounting techniques, estimate the implied rate of growth in the effectiveness of labour in this economy.

Suppose that the effectiveness of labour is given by $A = TH$, where T is TFP and H is human capital. The effect of an increase in schooling on an individual i's wage within the economy at a given level of technology is estimated to be given by

$$\Delta \ln w_i = 0.1 \Delta s_i.$$

(c) Assuming that the economy is approximately competitive, how could this information be used to construct an index of aggregate human capital growth (see Hall and Jones, 1999)?

(d) If average years of schooling increases by 0.1 years per year, decompose the growth in A into that component arising from TFP growth and that arising from human capital accumulation.