Money-Wage Dynamics and Labor-Market Equilibrum*

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If the economy were always in macroeconomic equilibrium then perhaps the full-employment money-and-growth models of recent vintage would suffice to explain the time paths of the money wage and the price level. But since any actual economy is almost continuously out of equilibrium we need also to study wage and price dynamics under arbitrary conditions.

The numerous Phillips-curve studies of the past ten years have done this with a vengeance in offering countless independent variables in numerous combinations to explain wage movements. But it is difficult to choose among these econometric models, and rarely is there a clear rationale for the model used. This paper presents a modest start toward a unified and empirically applicable theory of money-wage dynamics. At the same time it tries to capture the role of expectations and thus to work into the theory the notion of labor-market equilibrium.

I. Evolution of the Phillips Curve and its Opposition

Keynes' *General Theory* (1936) and virtually all formal macroeconomic models of the postwar era postulated a minimum unemployment level—a full-employment level of unemployment—which could be maintained with either stable prices or rising prices. In this happy state, additional aggregate demand would produce rising prices and wages but no reduction of unemployment. The full-employment quantity of unemployment was identified as "frictional" and "voluntary"; and frictional unemployment was (mistakenly) assumed to be unresponsive to demand.[1] Hence there was no need to choose between low unemployment and price stability.

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[1] A monetary economy can choose among different levels of frictional unemployment that correspond to different levels of aggregate demand and job vacancies. In fact, therefore, there is no unique full-employment quantity of frictional unemployment.

This doctrine depended on Keynes' notions of money-wage behavior. At more than minimum unemployment, a rise (fall) of demand and employment would produce a once-for-all rise (fall) of the money wage, prices constant; a rise (fall) of the price level would cause a rise (fall) of the money wage in smaller proportion. Hence, in a stationary economy at least, his theory did not predict the possibility of a secular rise of money-wage rates at normal unemployment rates—let alone wage rises exceeding productivity growth—only the one-time "semi-inflation" (Keynes, 1936, p. 301) of prices and wages during the transition to minimum unemployment.

This doctrine was quickly disputed by Robinson (1937, pp. 30–31), who wrote of a conflict between moderately high employment and price stability. Dunlop (1938) suggested that the *rate of change* of the money wage depends more on the *level* of unemployment than upon the rate of change of unemployment, as Keynes had it. After the war, Singer (1947), Bronfenbrenner (1948), Haberler (1948), Brown (1955), Lerner (1958), and many others wrote that at low albeit above-minimum unemployment levels there occurs a process of "cost inflation," "wage-push inflation," "income inflation," "creeping inflation," "sellers' inflation," "dilemma inflation," or the "new inflation"—a phenomenon which was attributed to the discretionary power of unions or oligopolies or both to raise wages or prices or both without "excess demand."[2]

I believe this customary attribution of cost inflation to the existence of such large economic units to be unnecessary and insufficient. Like the theory of unemployment, the theory of cost inflation requires a non-Walrasian model in which there is no auctioneer continuously clearing commodity and labor markets. Beyond that, it is not clear to me what monopoly power contributes. An increase of monopoly power—due, say, to increased concentration—will raise prices relative to wages at any given unemployment rate and productivity level; but once, at the prevailing unemployment rate, the real wage has fallen (relative to productivity) enough to accommodate the higher markup, this process will stop and any continuation of inflation will depend on other sources.[3]

[2] Some wage-push theorists like Weintraub (1959) appear to treat inflation as almost spontaneous, virtually independent of the unemployment rate over any relevant range, and hence not induced by aggregate demand. I once tested the hypothesis that the 1955–57 inflation was more of this character than were the two earlier postwar inflations, making the assumption that autonomous "wage push" or "profit push" would be uneven in its sectoral incidence, so that the coefficient of correlation between sector price changes and sector output changes would (if the hypothesis were true) be algebraically smaller in the 1955–57 period than it was earlier (1961). It was algebraically smaller, but the statistical significance of the decline was impossible to determine. Incidentally, Selden's correlation test (1959) wrongly attributes significance to the positivity of the coefficient in 1955–57 instead of to the magnitude of the decline.

[3] The answer of Ackley (1966) and Lerner (1967) that corresponding to every unemployment rate and productivity level there is a natural real wage that is irreducible

Similarly, I doubt that the existence of labor unions is remotely sufficient to explain the cost inflation phenomenon. Whether the unions significantly exacerbate the problem—whether they increase that unemployment rate which is consistent with price stability—is, however, a difficult question. The affirmative answer frequently starts from the theory, set forth by Dunlop (1950), that a union, to maximize its utility, seeks to "trade off" the real wage rate against the unemployment of its members, raising the former (relative to productivity) until the gain from a further real wage increase is offset by the utility loss from the increase in unemployment expected to result from it. At an unemployment level below the unions' optimum, the unions then push up wage rates faster than productivity. But firms pass these higher costs on to consumers, so the real wage gains are frustrated, and as long as the government maintains the low unemployment level the rounds of inflation will continue.

I have trouble applying such a model to the American economy. Almost three-quarters of the civilian labor force do not belong to unions. This fact casts doubt on the quantitative importance of the model. And perhaps the fact goes much deeper. If the union members whom the unions make unemployed have no good prospect of future union employment, they will be inclined to seek employment elsewhere. If, at the other extreme, the union unemployment is shared in the form of a short workweek, this unemployment—while real enough to the extent that members do not "moonlight"—does not add to the official unemployment rate as it is measured. Certainly the unions *participate* in the cost inflation process, and they may even increase a little the volume of unemployment consistent with price stability. But I should think that a union must offer its membership a frequency of employment opportunities that is roughly comparable to that elsewhere in order to thrive and that appreciably reduced employment opportunities require a greater wage differential between union and other employment than is commonly observed.[4]

Phillips' successful fitting of what we now call the Phillips curve (1958) to a scatter diagram of historical British data deprived the discussions of some of their institutional color, but epitomized the new concept of cost inflation—if by that term we mean (as I think most of the aforementioned writers intended) *that kind of inflation which can be stopped only by a reduction of the employment rate* through lower aggregate demand and which

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despite structural changes, so that money wages will keep pace with prices until unemployment is allowed to increase, seems to me to be terribly implausible. In any case, if this paper is right, cost inflation theory does not require any such "double monopoly" argument.

[4] It is certainly likely, however, that an *increase* of union power, even if localized, will raise the average money-wage level at any constant unemployment rate (see Hines, 1964).

thus raises a cruel dilemma for fiscal and monetary policy.[5] The Phillips curve portrayed the rate of wage change as a continuous and decreasing function of the unemployment rate, with wage increases exceeding typical productivity growth at sufficiently low albeit above-minimum unemployment rates. Hence, if prices are tied to marginal or average costs, the smaller the level at which aggregate demand sets the unemployment rate the greater is the *continuing* rate of inflation.

Strikingly, Phillips found that the nineteenth-century data pointed to a trade-off between wage increases and unemployment in the same way as contemporary data. Lipsey's sequel (1960) showed a statistically significant Phillips-curve relation for the subperiod 1861–1913. In fact, this early Phillips curve was *higher* (by about one percentage point) than the Phillips curve he fitted to the period 1929–57.[6] Apparently the cost inflation tendency, if real, is not "new" in history; in Britain anyway it may be no worse than it used to be.

But is the Phillips trade-off real, serious, and not misleading? I shall discuss briefly two challenges to the Phillips curve to which this paper is relevant. The first is the question of whether the slope of the wage increase-unemployment relation is great enough to pose a serious dilemma for aggregate demand policy. Though proponents of an American Phillips curve had tough sledding at first—numerous other variables were held to be important (Bowen, 1960; Bhatia, 1962; Eckstein and Wilson, 1962)—Perry's synthesis (1964) of much of this early work left a quantitatively important role for the unemployment rate (as well as for the profit rate and the rate of change of prices) in explaining money-wage movements in U.S. manufacturing. But in 1963 Bowen and Berry (1963) found that the *decrease* of the unemployment rate was far more important than the level of the unemployment rate in contributing to wage increases. The recent study of annual long-term wage data by Rees and Hamilton (1967) also showed a negligible (and statistically insignificant) relation between the steady-state unemployment rate and the rate of wage increase (though

[5] By contrast, in the pure "demand inflation" of Keynes and the classics, a reduction of the price trend could be achieved without cost to output and employment, since aggregate demand is necessarily superfluous to begin with. "Demand inflation" may be worth preserving, since a regime of "mixed inflation" is conceivable.

My earlier paper (1961) contains a fairly complete taxonomy of inflations (see also Fellner, 1959). Incidentally, the occasional definition of cost inflation as an autonomous upward shift of the Phillips curve is very awkward and does not imply the "policy dilemma" with which inflation analysts were concerned in the fifties.

[6] At a constant price level and an unemployment rate of 2 per cent, Lipsey's (1960) 1862–1913 regression (his equation [10]) predicts a 2.58 per cent wage increase annually, while the 1929–57 regression (his equation [13]) predicts a 1.65 per cent annual increase. At the same 3 per cent productivity growth in both periods, for example, price stability would have permitted smaller unemployment in the latter period. But Lipsey's Table 2 (p. 30) is evidence of the early Phillips curve's underestimation of the wage increases after World War II.

wage-change effects on prices feed back strongly on wages in their equation). This evidence strongly supports the neo-Keynesian revival led by Sargan (1964) and Kuh (1967) who make the level of the unemployment rate, together with productivity and the price level, determine the *level* of the money wage.[7] The underlying theory is apparently that a rise of aggregate demand creates "bottlenecks" and hence a rise of wage rates in certain areas and skills at the same time that it increases employment; once these bottlenecks have melted away and employment has reached its new and higher level there is no longer upward wage pressure. On this theory, money-wage increases go hand in hand with employment growth and not intrinsically with a high level of the employment rate.

Less frontal in a way but having equally profound policy implications is the second issue of the so-called stability of the Phillips curve. Continental economists like von Mises (1953, pp. 418–20) always emphasized the role of expectations in the inflationary process. In our own day, William Fellner and Henry Wallich are most closely associated with the proposition that the maintenance of too low an unemployment rate and the resulting continued revision of disappointed expectations will cause a runaway inflation. These ideas are reflected in the modern-day models of steady, "anticipated" inflation, begun by Lerner (1949), which imply (or assume) that high inflation confers no benefits in the form of higher employment if (or as soon as) the inflation rate is fully anticipated by firms and workers.[8] Recently, Friedman (1966) and I (1967) have sought to reconcile the Phillips hypothesis with the aforementioned axiom of anticipated inflation theory. I postulated that the Phillips curve, in terms of percentage price increase (or wage increase), shifts uniformly upward by one point with every one point increase of the expected percentage price increase (or expected wage increase). Then the *equilibrium* unemployment rate—the rate at which the actual and expected price increases (or wage increases) are equal—is independent of the rate of inflation. If one further postulates, as Friedman and I did, an "adaptive" or "error-correcting" theory of expectations, then the persistent underestimation of price or wage increases which would result from an unemployment level consistently below the equilibrium rate would cause expectations continually to be revised upward so that the rate of inflation would gradually increase without limit; and, similarly, a very high, *constant* rate of inflation, while "buying" a very low unemployment rate at first, would require a gradual rise of the unemployment rate toward the equilibrium rate as expectations

[7] If the *real* wage rate were made a rapidly increasing function of the employment rate, the Kuh-Sargan model could then produce (cost) inflation at low, yet above-minimum, unemployment rates.

[8] Lerner (1967) now recants. A paper of mine (1965) on anticipated inflation contains many of the references. Two recent money-and-growth models which study the consequences of alternative anticipated price trends are those by Tobin (1965) and Sidrauski (1967).

of that inflation developed. Therefore, society cannot trade between steady unemployment and steady inflation, on this theory. Society must eventually drive (or allow) the unemployment rate toward the equilibrium level or force it to oscillate around that equilibrium level.[9]

This paper is addressed primarily to these two issues. The next section offers a theory of why, given expectations, both the level of unemployment *and* the rate of change of employment should be expected to explain money-wage movements. The following section presents a theory of the influence of expected wage changes upon the Phillips curve. Some econometric tests of the predictions of these theories are reported in a statistical appendix.

II. "Turnover" and "Generalized Excess Demand"

For most of this section, until I try to accommodate other factors, I shall deal only with a more or less "atomistic" labor market in which there is no collective bargaining between unions and firms. But I exclude any Walrasian auctioneer to clear the labor market—the labor market is never properly cleared in this model—and I do not require that commodity markets be cleared. Firms may be said to have some dynamic monopsony power in that they need to pay a higher wage the faster they wish to attract labor, other recruitment activities held constant.

The model postulates considerable variety in the kinds of jobs and workers and postulates imperfect information about their availabilities.[10] Firms must incur "search costs" to find round pegs to fill round holes, and unemployed workers must also expend money and energy to find suitable employment. As a consequence, positive unemployment and positive job vacancies tend to persist in a growing labor market and even under stationary labor supply because of the turnover or attrition of firms' employment rolls. Total vacancies can be positive for every kind of job and total unemployment can be positive for every type of worker because

[9] On certain assumptions regarding preferences and other matters, I showed that society (or the world) would choose between an "overemployment" route *down* to the equilibrium employment rate (thus leaving a heritage of a high Phillips curve corresponding to inflationary expectations) and an "underemployment" route *up* to the equilibrium employment rate on the basis of "time preference." The role of time preference is illuminated by Friedman's (1966) characterization of "the true trade-off" (p. 59) as one between "unemployment today and unemployment at a later date"; there is such an intertemporal trade-off in the model under discussion if one holds eventual inflation rates constant, in the same way that the Fisherian trade-off between consumption today and consumption tomorrow holds subsequent wealth or capital constant. But there remains at any moment of time a statical trade-off between unemployment and inflation (with the expected inflation rate a parameter), analogous to the statical trade-off between consumption and capital formation (with initial capital stock a parameter) which lies at the roots of the intertemporal trade-off.

[10] Works by Stigler (1962), by Alchian and Allen (1964, xxxi), and by Holt and David (1966) contain some economics of such labor markets.

of spatial mismatching among jobs and people. In the formal model I shall exclude serious bottlenecks in one or more kinds of labor in order to speak aggregatively of "the" wage rate, "the" unemployment rate, and "the" vacancy rate as if they were pretty much uniform over the spectrum of workers and jobs.

As defined here, "aggregate unemployment," denoted U, consists of both those individuals without employment who are actively seeking a job (at going real wage rates) and the more passive without work who would accept a job opportunity (at the going rate) were it known to them. "Aggregate job vacancies," denoted V, consist both of those jobs which employers are actively seeking at a cost to fill and of the quantity of unfilled jobs that would be filled if and only if workers presented themselves without recruitment cost to the firm. Though it is doubtful that "active" unemployment and vacancies are equivalent, respectively, to "passive" unemployment and vacancies in their consequences for wage rates, I merge these active and passive components for simplicity.[11]

Letting N denote the number of persons employed, we have as a definition of labor supply, L, the relation

$$L = N + U. \tag{1}$$

Labor demand, N_D, is defined by

$$N_D = N + V. \tag{2}$$

L may depend upon the usual factors like the real wage rate, income, wealth, and demographic factors; N_D may depend on the technology, the product wage (net of interest and "depreciation" on the investment outlays to process and train a new employee), the degree of monopoly power, and, if prices do not clear the commodity markets, upon aggregate demand as well.

The concept of "excess demand" for labor, denoted X, is usually defined as

$$X = N_D - L, \tag{3}$$

when

$$X = V - U. \tag{4}$$

The usual excess-demand theory of money-wage dynamics states that the proportionate rate of change of the money wage is proportional to the excess demand *rate*, denoted x. The latter is excess demand per unit of labor supply, and hence equal to the excess of the vacancy rate, v, over the unemployment rate, u:

$$x = v - u, \quad x = X/L, \quad v = V/L, \quad u = U/L. \tag{5}$$

[11] Econometric analysis by Simler and Tella (1967) shows total unemployment to explain wage movements better than active or "measured" unemployment alone.

The modal rationale for the simple Phillips-curve relation between wage change and the unemployment rate is that, at least in sectors or economies with little or no unionization, the unemployment rate is a good proxy for the excess-demand rate and that the latter largely explains wage movements (apart from aggregation phenomena like changes in the employment mix).[12] Even if excess demand were the sole determinant of wage changes—this paper seeks to generalize that theory and to make it accommodate the influence of expectations—it is not obvious that the unemployment rate is a good proxy for it. What if, at times, the vacancy rate in (5) enjoys a life of its own, moving independently of the unemployment rate? (I shall later discuss the evidence on this.) Lipsey's paper (1960) brilliantly deduces from a model of employment dynamics a well-behaved relationship between the vacancy rate (hence the excess-demand rate) and the *steady* unemployment rate. I shall show, however, using a similar model, that in the non-steady-state case the unemployment rate is an inadequate indicator of the excess-demand rate and that the rate of change of employment constitutes an essential additional indicator for inferring the excess-demand rate.[13]

The excess-demand explanation of wage movements is unlike the law of gravity in that this explanation itself calls for an underlying explanation. When we try to rationalize it, however, its restrictiveness becomes clear. It implies that a one-unit increase of the vacancy rate always has the same

[12] The most extensive exposition is Lipsey's (1960). In criticizing the reliance solely on the unemployment rate which this rationale promotes, Perry (1966) wrote, "If the rate of wage change is proportional to the amount of excess demand which in turn is measured by unemployment, there is no room for other variables" (p. 22). I believe his abandonment of the excess-demand theory *on this ground* was mistaken. This paper adduces three explanatory variables from what is essentially an excess-demand theory.

[13] These two points can perhaps be understood simply from the following exercise: Draw a non-negatively sloped labor supply curve and a non-positively sloped labor demand curve in the customary real wage-employment plane. Consider now the locus of points corresponding to a given unemployment rate; this iso-unemployment-rate curve will lie to the left of the supply curve and will also be non-negatively sloped. It is immediately obvious that if the demand curve is negatively sloped, or the supply curve positively sloped, then not all points on the locus represent equal algebraic excess demand; in particular, as we move down this locus from its intersection with the demand curve, vacancies and excess demand increase despite constancy of the unemployment rate. Thus the latter is not necessarily a sufficient proxy for excess demand. (This demonstration in no way contradicts the proposition that, *vacancy rate constant*, excess demand is decreasing in unemployment. The zero-vacancy, on-the-demand-curve case is a familiar example. This paper tries to get away from the supposition that we are always "on the demand curve," even the Keynesian demand curve arising from excess supply in commodity markets.)

However, as we consider situations of higher vacancies, the unemployment rate unchanged, we should expect the rate of increase of employment likewise to be higher as employers seek to reduce vacancies through greater recruitment. The *two* pieces of information—the unemployment rate, and the rate of increase of employment—may together constitute a satisfactory proxy, or a better proxy, for excess demand.

wage effect as a one-unit decrease of the unemployment rate. Second, the excess-demand theory implies that most of the time, in the neighborhood of "equilibrium" (see Part III), vacancies will equal unemployment and that a *disequilibrium* rise of wage rates requires vacancies to exceed unemployment. That vacancies almost never exceed unemployment[14] may be due in part to the behavior of unions, as conceded earlier, and in part to the existence of "unemployables" and the resistence to money-wage cuts in sectors and trades where the market calls for them. But I suspect that a part of the reason is the inaccuracy of the excess-demand theory on its own terms.

I shall now describe and try to rationalize a *generalized* excess-demand theory of money-wage movements, one which is less restrictive than the simple excess-demand theory but which admits it as a special case. Elements of this approach have previously been discussed by James Duesenberry[15] (1958, pp. 300–9). Until Part III, where expectations are introduced, I hold constant the rate at which each firm expects *other* firms to change over time the wage they pay their labor. For ease of exposition, it is assumed simply that each firm expects the wage paid elsewhere to be constant for the near future.

An important element of this theory is the cost to the firm of its "turnover rate." Given a constant differential between the firm's wage rate and the wage rates paid by other firms, a fall of the unemployment rate will tend to increase the quit rate experienced by the firm. Unless the firm's employment was excessive to begin with, the increase of its quit rate will impose costs: The firm must either allow its output to decrease, thus losing profits, or incur the recruitment, processing, and training costs of replacing the departing workers (or choose some combination of these two losses). At a sufficiently high quit rate corresponding to a low unemployment rate, the firm will want to increase the differential between the wage it pays and the average wage paid elsewhere, on the ground that the savings from lower turnover costs will more than pay for the extra wage bill. As all firms attempt to raise this differential, the general wage index rises.[16] (The theory will work in reverse as well: There presumably exists a sufficiently high unemployment rate such that the quit rate is low enough to induce the firm to want to pay a wage below that paid by others on the ground that the wage savings will more than pay for the extra turnover costs.) Thus one role of unemployment in this theory stems from its effect upon quit rates rather than from any supposed underbidding for jobs by unemployed workers.

Undoubtedly job vacancies also play a part. First of all, the quit rate may depend upon both the unemployment rate and the vacancy rate since

[14] Ross (1966, p. 98) reports American evidence that only at an unemployment rate as low as 2.5 per cent does the vacancy rate equal the unemployment rate.

[15] I have also benefited from a conversation on this subject with Professor Duesenberry, but he is not responsible for deviations and errors on my part.

[16] For impressive empirical support of this part of the theory, see Eagly (1965).

these two variables together can be supposed to affect accession rates and hence the expected duration of unemployment by anyone contemplating quitting. Second, when a firm finds it has unfilled jobs it will respond with some combination of additional recruitment expenditures and an attempted increase of the differential between the wage it pays and the wage paid elsewhere, in order to facilitate recruitment and encourage workers to seek employment at the firm as they learn of the higher differential.[17] The magnitude of the desired differential *on this account*, for the ith firm, depends presumably upon the number of vacancies in the firm, V_i, the size of the unemployment pool, U, the number of workers employed elsewhere, $N - N_i$, and the size of the labor force, L.

Let Δ_i^* denote the ith firm's desired wage differential as defined by

$$\Delta_i^* = \frac{w_i^* - w}{w}, \tag{6}$$

where w is the average wage paid by all firms and w_i^* is the wage rate which the ith firm wishes to pay. Then the above theory states that

$$\Delta_i^* = j^i(u, v, U, V_i, N - N_i, L). \tag{7}$$

Suppose now that j^i is homogeneous of degree zero in the last four variables. Then we may write

$$\Delta_i^* = k^i(u, v, v_i), \quad v_i = V_i/L, \tag{8}$$

if we neglect the small discrepancy (in the atomistic case) between N/L and $(N - N_i)/L$. Now if all firms are much alike, we can express the *average* desired wage differential, denoted Δ^*, as a function of both the unemployment rate and the aggregate vacancy rate, $v = \Sigma v_i$ (as given in [5]):

$$\Delta^* = m(u, v), \quad u, v > 0, \tag{9}$$

where I shall suppose

$$m_1 < 0, \quad m_2 > 0, \tag{9a}$$

$$m_{11} \geqq 0, \quad m_{22} \geqq 0, \quad m_{12} \leqq 0. \tag{9b}$$

Before discussing the postulated shape of the m function, let us take the last step:

$$\frac{\dot{w}}{w} = \lambda\Delta^* \quad (\lambda \text{ a positive constant, } \dot{w} \equiv dw/dt). \tag{10}$$

[17] Of course the firm will be tempted to pay the higher wage differential only to new workers—and only for a short time! But this tendency will be inhibited considerably if potential recruits know the long-run costs of joining a firm that engages in such sharp practices. I suppose, as an approximation, that new and old workers in a firm receive the same wage.

This assumes, as mentioned earlier, that each firm expects the wage rate paid by other firms to be constant at least for the duration of the wage negotiated. The rationale of (10), stated loosely, is that the average wage rate will rise (fall) if all firms want to pay a wage higher (lower) than other firms.[18] It is assumed here that firms in the aggregate adjust their wage only gradually in the direction of the average desired differential; otherwise v and u would be implied to adjust instantaneously to make $\Delta^* = 0$ continuously. The gradualness might come from the administrative and psychic cost of changing wage rates that causes wage rates to be changed only intermittently or periodically; if these wage negotiations are staggered across firms or across workers, then the average wage will move more or less smoothly as indicated. In addition, perhaps uncertainty of the firm that the "desired" wage differential, if instituted, would have the desired effect upon turnover costs will induce a cautious, gradual response in the individual firm's wage decision.

As for the postulated shape of the m function, the signs of the derivatives in (9a) are of course fundamental to the theory. The excess-demand theory, which is a special case, assumes that the second derivatives are zero with $m_2 = -m_1 = \text{constant} > 0$. My weaker restrictions on the second derivatives in (9b) are inessential; they affect only the curvature of the augmented Phillips curve which I shall derive. The inequality $m_{11} \geqq 0$, meaning that Δ^* decreases with the unemployment rate at a non-increasing rate, vacancy rate constant, is plausible if, as the data suggest (Eagly, 1965), the quit rate is likewise convex with respect to the unemployment rate. The inequality $m_{22} \geqq 0$ assumes "rising marginal costs" to the firm of filling vacancies by means other than raising its wage differential. Finally $m_{12} \leqq 0$ makes sense if it takes a larger increase of the firm's wage differential to facilitate the filling of some fraction of a given increment in its vacancies the smaller is the unemployment pool from which workers can conveniently be drawn. The curve labeled $m(u, v) = 0$ in Figure 1 gives the combinations of u and v that make $\Delta^* = 0$. Its slope, being $-m_2/m_1$, is necessarily positive, but the size of that slope and the curvature are indeterminate and of no qualitative consequence. To the right of this locus $\Delta^* > 0$, and to the left $\Delta^* < 0$.

In the United States and most other countries, satisfactory vacancy data are still unavailable. I shall couple the above model with a theory of labor turnover or employment dynamics, along lines suggested by Lipsey (1960), in order to derive testable implications of relations among easily observable data.

The absolute time rate of increase of the aggregate number of persons employed, denoted $\dot{N} = dN/dt$, consists of the number of persons hired

[18] Stability of the average wage is consistent with some positive differentials if there exist firms content with negative ones. What counts for the average wage movement is the weighted *average* desired differential, Δ^* (in relation to the ex post, actual, weighted average differential, say Δ, which necessarily equals zero).

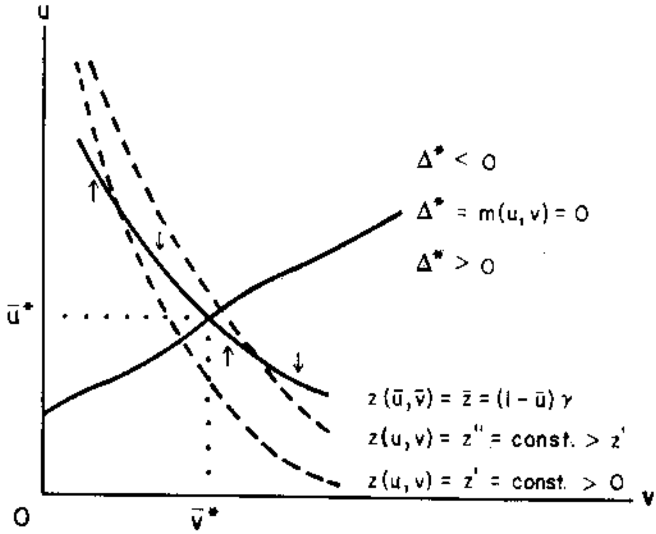

FIG. 1.—Relations between vacancy and unemployment rates

per unit time from the unemployment pool, denoted R, less the departures (due to death and retirement) per unit time of employed persons from the labor force, denoted D, and the quitting of employees to join the unemployed in search of new jobs, denoted Q. This accounting ignores involuntary terminations and layoffs, which I shall not treat, and it assumes that entrants to the labor force first enter the unemployment pool before being hired. Of course, the accessions and separations of employed persons who transfer directly from one firm to another cancel out and do not add to $\dot{N}$. That is,

$$\dot{N} = R - D - Q. \tag{11}$$

I shall make the variables on the right-hand side of (11) depend *in the aggregate* only upon unemployment (or employment), vacancies, and the labor supply. While the hire and quit rates of the individual firm depend upon its actual wage differential, the weighted average actual differential across all firms must be constant (being equal to zero), so one expects wage differentials to wash out in the aggregates.[19]

I shall suppose that D is proportional to employment, δ being the factor of proportionality. (This neglects any effect of a real wage change on people at the retirement margin.) To eliminate scale effects (rightly or not), I shall take new hires and quits to be homogeneous of degree one in

[19] Perhaps the dispersion of the wage differentials has some effect upon R and Q.

unemployment, vacancies, and the labor supply. Hence

$$\dot{N} = R(U, V, L) - \delta N - Q(U, V, L), \quad R(U, V, L) = LR(u, v, 1), \quad Q(U, V, L) = LQ(u, v, 1). \tag{12}$$

Equivalently, defining $z \equiv \dot{N}/L$,

$$z = R(u, v, 1) - \delta(1 - u) - Q(u, v, 1) = z(u, v); \quad u, v > 0, \tag{13}$$

where I shall suppose

$$z_1 > 0, \quad z_2 > 0 \tag{13a}$$

$$-z_2^{-2}\left\{\left[z_{11} + z_{12}\left(\frac{-z_1}{z_2}\right)\right]z_2 - \left[z_{21} + z_{22}\left(\frac{-z_1}{z_2}\right)\right]z_1\right\} > 0. \tag{13b}$$

Thus the absolute rate of change of employment per unit labor supply is a function of the same two variables that determine Δ^* and in so doing influence the rate of wage change.

What is the logic of the z function, in particular the role of the vacancy rate in that function? We ordinarily think of the level of labor input as determined by output which in turn depends upon aggregate demand and productivity. There probably is a fairly tight relationship between man-hours and output (given productivity); but N is measured by the number of persons employed. In a labor market that is at least moderately tight, the firm will respond *initially* to an increase of aggregate demand (which increases job vacancies) by lengthening hours worked per worker (including overtime), by more intensive use of "buffer" or "cushion" employees ("hoarded" labor), by calls for extraordinary efforts on the part of employees, and perhaps by raising prices to reduce output demanded. But these measures do not eliminate the job vacancies, and finding new employees to fill new jobs takes time.[20] Firms will choose to take time for two reasons: because marginal recruitment costs are positive, it may pay the firm to wait for suitable persons to present themselves for employment; and because there may be "rising marginal recruitment costs,"[21] it will pay the firm to smooth its recruitment efforts over time.

Now the properties of the z function. The assumptions on derivative signs in (13a) are, unlike those in (13b), fundamental to the theory. It is

[20] Some of the new employees wanted can be acquired virtually instantaneously so that the response of N to aggregate demand is not entirely the gradual or continuous response that I have postulated. Incidentally, since a raise of price will not appreciably reduce output demanded, prices will go on rising.

[21] That is, the additional recruitment or search costs necessary to increase by one the expected number of recruits per unit time may be greater if the firm is aiming at 500 recruits in a week than if it is aiming for only ten. This is a short-run cost curve in which we hold constant the size of the firm and its personnel office. Large firms are not implied to suffer disadvantages in recruitment.

assumed that, the unemployment rate constant, the higher the vacancy rate the greater is the rate at which firms will acquire unemployed workers, that is, $R_2 > 0$. A higher vacancy rate will induce more intensive recruitment, and it will increase the probability that any unemployed person contacting a firm will find a job open. This increase of accessions may itself induce more quits, as suggested in the paragraph preceding (6), so that $Q_2 > 0$ is possible. But it would be strange to find that the higher vacancy rate reduced employment growth on balance; any increase of quits will stimulate partially offsetting extra recruiting. Hence I postulate that $R_2 > Q_2 \geqq 0$, so that $z_2 = R_2 - Q_2 > 0$, for all u and v.

Clearly $R_1 > 0$ since, vacancy rate constant, the higher the unemployment rate the greater is the flow to the firm of unemployed workers who can fill open jobs and the easier is recruitment. Since an increase of unemployment discourages quitting, $Q_1 < 0$. Hence $z_1 = R_1 + \delta - Q_1 > 0$.

Consider the dashed curves labeled z = constant in Figure 1. Each depicts the locus of (u, v) combinations giving a particular value of z. The slope of such curves at any point is $-z_2/z_1 < 0$; as the unemployment rate is reduced an increase of the vacancy rate is required to keep z constant. These z contours as drawn display strict convexity or "diminishing marginal rate of substitution," meaning that as the unemployment rate is reduced the vacancy rate increases at an increasing rate along any contour. This convexity is the content of (13b).

The best rationale for this convexity is the presumption that $z_{21} = R_{21} - Q_{21} > 0$. This states that an increase of the vacancy rate has greater effect on employment growth the greater the unemployment rate. The primary basis for that assumption is that recruitment will be more difficult the smaller is unemployment (indeed totally unsuccessful in the aggregate at zero unemployment), so that $R_{21} > 0$. It is plausible also that an increase of the vacancy rate has less effect, if it has any, upon quits the less tight the labor market, so that $Q_{21} \leqq 0$. (Since $z_{12} = z_{21}$ an equivalent view is that changes of the unemployment rate have greater impact upon z the greater the vacancy rate.) Secondly, we should expect $z_{11} = R_{11} - Q_{11} \leqq 0$ on the two grounds that, vacancy rates constant, an increase of the *employment* rate reduces new hires at an increasing rate and that it increases quits at an increasing rate (or at least at non-decreasing rates).[22] Thirdly, and most controversially, it might be argued that $z_{22} = R_{22} - Q_{22} \leqq 0$. $R_{22} < 0$ could result from a rising marginal recruitment cost schedule; given the unemployment rate, the new hire rate (R) might even approach an upper bound as the vacancy rate increased without limit. My guess is that $Q_{22} \geqq 0$, but I know of no evidence or presumption in its favor. In any

[22] If quits *per employee* is linear in the employment rate, given the vacancy rate, then $Q(u, v, 1)$, that is, quits per unit labor supply, will be strictly convex with respect to the employment rate.

case, (13b) shows that the algebraic signs of second derivatives suggested here are merely *sufficient* for convexity of the z contours.[23]

We can now combine (9), (10), and (13) to obtain an augmented Phillips curve in terms of the easily observed variables u and z. Since z_2 is one-signed, (13) implicitly defines v as a single-valued function of u and z, say,

$$v = \psi(u, z), \tag{14}$$

when

$$\frac{\dot{w}}{w} = \lambda m[u, \psi(u, z)] = f(u, z), \tag{15}$$

which is our augmented Phillips curve. Since to every (u, z) pair there corresponds a unique v, there exists a derived Phillips-like relation between $\dot{w}/w$ and (u, z) pairs.

We can establish the properties of f after determining how v varies with u and z.

$$\begin{aligned}
\psi_1 &= \frac{-z_1[u, \psi(u, z)]}{z_2[u, \psi(u, z)]} < 0\,; \\
\psi_2 &= \frac{1}{z_2[u, \psi(u, z)]} > 0\,; \\
\psi_{11} &= -z_2^{-2}\left\{\left[z_{11} + z_{12}\left(\frac{-z_1}{z_2}\right)\right]z_2 - \left[z_{21} + z_{22}\left(\frac{-z_1}{z_2}\right)\right]z_1\right\} > 0\,; \\
\psi_{22} &= -z_2^{-3}z_{22} \geqq 0 \quad (?)\,; \\
\psi_{21} &= -z_2^{-2}\left[z_{21} + z_{22}\left(\frac{-z_1}{z_2}\right)\right] < 0 \quad (?).
\end{aligned} \tag{16}$$

The last two inequalities are based on the conjectures discussed in connection with (9b), while the first three inequalities follow from (13a) and (13b).

Now we can deduce the following restrictions on the augmented Phillips curve:

$$\begin{aligned}
f_1(u, z) &= \lambda(m_1 + m_2\psi_1) < 0\,; \\
f_{11}(u, z) &= \lambda(m_{11} + m_{12}\psi_1 + m_{22}\psi_1^2 + m_{22}\psi_{11}) > 0\,; \\
f_2(u, z) &= \lambda m_2\psi_2 > 0\,; \\
f_{22}(u, z) &= \lambda(m_{22}\psi_2^2 + m_2\psi_{22}) \geqq 0 \quad (?)\,; \\
f_{21}(u, z) &= \lambda[(m_{21} + m_{22}\psi_1)\psi_2 + m_2\psi_{21}] < 0 \quad (?).
\end{aligned} \tag{17}$$

The first result states that every constant-z Phillips curve is negatively sloped: Decreased unemployment directly adds pressure on wage differentials, and this effect is reinforced by the concomitant increase of vacancies

[23] It might be thought that the convexity of the R contours and convexity of the Q contours would suffice to imply convexity of the z contours, but the former two convexities are neither necessary nor sufficient for the latter convexity.

which is deducible from the constancy of z in the face of decreased unemployment. The second result states that this constant-z relation between the rate of wage change and the unemployment rate is strictly convex, as the Phillips curve is ordinarily drawn; as the unemployment rate is decreased by equal amounts the vacancy rate must increase at an increasing rate to keep z constant, by virtue of (13b), which implies $\psi_{11} > 0$, so that even in the simple excess-demand case (in which the second derivatives in [9b] are equal to zero) the rate of wage increase itself increases at an increasing rate. As for the third result, $f_2 > 0$, the higher is employment growth, the unemployment rate constant, the higher must be the vacancy rate and hence the greater the upward pressure on the money wage. Thus the association between high employment growth and high wage gains is consistent with the excess-demand or generalized-excess-demand theory of the Phillips curve. The convexity of this relation between wage change and z is not certain since it involves the problematical ψ_{22}. Finally, there is a negative interaction between u and z, meaning $f_{21} < 0$, if my guess is right that z_{21} is strongly positive; this interaction means that a given increase of z signifies a greater increase of the vacancy rate the smaller is the unemployment rate.

The variables u and z cannot go their own way for long since a high (low) z implies a falling (rising) u. There is, therefore, some interest in the "steady-state" Phillips curve that relates the rate of wage increase to alternative, *constant* values of the unemployment rate. Let us take the proportionate rate of growth of the labor supply to be a non-negative constant, γ. Then, corresponding to any *steady-state* unemployment rate, to be denoted $\bar{u}$, there is a steady $\bar{z}$ and a steady $\bar{v}$ which obey the relation

$$\bar{z} = z(\bar{u}, \bar{v}) = \frac{N}{L}\frac{\dot{N}}{N} = \frac{N}{L}\frac{\dot{L}}{L} = (1 - \bar{u})\gamma, \quad \gamma \geqq 0. \tag{18}$$

If $\gamma > 0$, then clearly $\bar{z}$ must be higher the smaller $\bar{u}$. This relation also yields a locus of steady-state $(\bar{u}, \bar{v})$ points, which is shown in Figure 1 by the solid, downward sloping curve intersecting (from below) the broken-line iso-z contours. This locus is negatively sloped and flatter than the z contours, for as steady-state $\bar{u}$ is decreased, $\bar{v}$ must increase not only enough to keep z constant but to increase z to the required level implied by (18). Referred to the *vertical axis*, the slope is

$$\frac{d\bar{v}}{d\bar{u}} = \frac{-(z_1 + \gamma)}{z_2} < 0, \tag{19}$$

and, at least for sufficiently small γ, the locus will be convex like the z contours:

$$\frac{d^2\bar{v}}{d\bar{u}^2} = -z_2^{-2}\left\{\left[z_{11} + z_{12}\left(\frac{-z_1 - \gamma}{z_2}\right)\right]z_2 - \left[z_{21} + z_{22}\left(\frac{-z_1 - \gamma}{z_2}\right)\right](z_1 + \gamma)\right\} > 0 \quad (?). \tag{20}$$

It is not surprising, therefore, that our steady-state Phillips curve, $f[\bar{u}, (1 - \bar{u})\gamma]$, is negatively sloped and steeper than the constant-z Phillips curves:

$$\frac{\partial f[\bar{u}, (1 - \bar{u})\gamma]}{\partial \bar{u}} = f_1 - f_2\gamma < 0. \tag{21}$$

Also we find

$$\frac{\partial^2 f[\bar{u}, (1 - \bar{u})\gamma]}{\partial \bar{u}^2} = f_{11} - f_{12}\gamma - (f_{21} - f_{22}\gamma)\gamma > 0 \quad (?), \tag{22}$$

so there is some presumption of convexity (and certainly for small enough γ).

I note in passing that the steady-state Phillips curve is higher the greater the labor force growth rate, that is, $\partial f/\partial\gamma > 0$ for $\bar{u} < 1$. The reason is that faster growth of the labor supply requires a larger z and hence a larger vacancy rate to hold steady any given unemployment rate. This is an interesting testable implication of the theory. (The relationship may help to explain the aforementioned improvement in Britain's Phillips curve.)

Are there direct tests of the above theory of the augmented Phillips curve?[24] Quarterly British vacancy data have been prepared by Dow and Dicks-Mireaux (1958). Their study shows a scatter diagram of U and V points which, after 1950 or so, cluster around a convex, negatively sloped curve like the z contours or the steady-state locus in Figure 1. This is encouraging support for the *long-run* implications of (13) and (18). But my theory denies a strict and simple short-run relation between the unemployment rate *level* and the vacancy rate level. (Otherwise, the unemployment rate would suffice as an indicator of generalized excess demand.) In its unadulterated form, the employment dynamics model here implies that unemployment and vacancy levels together determine the rate of change of employment and, hence, given γ, the *rate of change* of the unemployment rate. The differential equation is

$$-\dot{u} = z(u, v) - (1 - u)\gamma. \tag{23}$$

This says that if, at the prevailing u, v exceeds the corresponding $\bar{v}$ on the steady-state locus, so that $z > \bar{z} = (1 - u)\gamma$, then u will be falling (and vice versa if v is less than the corresponding $\bar{v}$). See the arrows in Figure 1.

The British data, despite being quarterly, offer a striking example that u can fall because v is high even though v is falling, which supports the emphasis on the level of v, rather than its rate of change, as a determinant of $\dot{u}$. After a sharp rise of vacancies that reduced unemployment, the latter went on falling in the second half of 1955 when vacancies had leveled off and proceeded to fall (Dow and Dicks-Mireaux, 1958, Fig. 1*B*, p. 3).

[24] All of the empirical evidence to be cited was consulted after I had arrived at an almost identical model in an earlier unpublished manuscript so that this evidence permits a real test of the model.

Indeed, the early postwar years in general showed a long-run trend of falling unemployment coinciding with falling vacancies. On the other hand, cyclical turning points usually occurred in the same quarter, so perhaps one should not totally neglect the rate of change of vacancies as a determinant of unemployment movements.

In the United States one has to make do with the Help-wanted Advertising Index, Series 46, in *Business Cycle Developments* (U.S. Department of Commerce). In a recent study of this index, Cohen and Solow (1967) in effect regressed the value of this index on the unemployment rate and the "new hire rate." Now (23) implies that v is a decreasing function both of u and $\dot{u}$, since points above the steady-state locus will be associated with falling u. It is of some interest, therefore, that the new hire rate which may be a proxy for $-\dot{u}$ entered positively in that regression and the unemployment rate negatively; further, study of the residuals showed vacancies to be underestimated by this regression in cyclical phases of falling unemployment.[25]

A hasty study of the monthly data on aggregate unemployment and vacancies in Australia also appears to give some support to the present model.[26] After dividing U and V by a geometrically rising series that approximates the growth of the labor supply, I used a standard program to deseasonalize the resulting unemployment and vacancy rates. One of the best regression results was the following:

$$\log v_t = 9.76 - \underset{(44.10)}{0.95} \log u_t - \underset{(2.40)}{0.35} \log (u_{t+1}/u_t), \quad \bar{R}^2 = .925, \quad \mathrm{DW} = 0.15, \tag{24}$$

where the numbers in parentheses are t-ratios and v_t and u_t denote an average of the seasonally adjusted percentage vacancy rate and unemployment rate, respectively, in month t and month $t + 1$ (multiplied by 100). Both coefficients have the predicted signs and are highly significant. The serial correlation is fearsome, but that is partly due to the monthly averaging. When only even-numbered observations were run, the Durbin-Watson statistic rose to 0.35 and the t-ratio for $\log (u_{t+1}/u_t)$ rose to 3.17, with no appreciable change in the coefficients. When the regression is turned around to make $\log (u_{t+1}/u_t)$ the dependent variable, the t-ratios

[25] Cohen and Solow (1967) wrote: "The residuals [from this regression] progressively underestimated [the help-wanted index] in the course of upswings and overestimated during downswings, the error getting worse in the course of each one-way movement" (p. 109). Apart from the progressivity, this constitutes additional support for the theory. As for the progressivity, the authors suggest that "formal advertising is treated as something of a last-resort method of recruitment." This means, I take it, that the help-wanted advertising index is not a totally satisfactory measure of job vacancies.

[26] I am grateful to Peter Burley of Princeton University for providing me with these data and to Arthur Donner and Steven Salop for carrying out these and other calculations made for this paper.

remained significant but $\bar{R}^2$ plummeted, perhaps because the rate of change of unemployment is subject to considerable measurement error. On the whole, I think these explorations offer some hope of very good results from a complete analysis.

I shall now try informally and briefly to open the model to some other factors. The "bottleneck" theory also helps to explain why wage increases should be associated with rapidly *increasing* employment. An economy adjusted to one level of aggregate demand, with its peculiar structure, cannot adapt instantaneously to a higher aggregate demand level with its new structure; certain types of labor will be in excess demand, and this will drive up the general wage index. Hansen's model (1957) emphasizes that excess supplies of other types of labor, even if they sum to a figure in excess of the total of excess demands, need not hold down the wage index if wages are stickier downward than upward. In the usual bottleneck theory, however, the resulting change in wage structure will dissolve the bottlenecks, so that a low *level* of unemployment is not *ultimately* or *persistently* inflationary. It takes another slump and the passage of time if major bottlenecks are to reappear. Such a theory, therefore, seems to fit in with "ratchet inflation" of the sort analyzed by Bronfenbrenner (1954).

Lipsey attributed the influence of $\dot{u}$ in his regressions to an aggregation phenomenon (1960, pp. 21–23). To the extent that each sector of the economy has a simple and strictly convex Phillips curve of its own, the simple macro Phillips curve will shift upward with an increase in the sectoral inequality of unemployment rates. Lipsey suggested that these inequalities are worse in upturns than in downturns, so that a negative $\dot{u}$ tends to be more inflationary than a positive $\dot{u}$ at the same u. In any case, changes in the structure of vacancy and unemployment rates may be important.

What about unions? As a starting point, one might suppose the union to maximize the welfare of its members. In that case the union's wage objectives will be determined by real income opportunities outside the union. It will examine the wage differential between union jobs and jobs that members could get elsewhere, weighing also the expected time required to get jobs elsewhere, hence unemployment rates and vacancy rates in the relevant areas and occupations. The average wage differential desired by unions thus depends upon our pervasive u and v. At sufficiently small unemployment rates or large vacancy rates, the unions, just like individuals and firms, desire incompatibly large wage differentials, and the general index of wage rates will therefore rise.[27] But this is only a possible

[27] This ties in somewhat with Keynes' (1936) emphasis on the relative wage: "Every trade union will put up some resistance to a cut in money-wages [since such reductions 'are seldom or never of an all-round character']. But . . . no trade union would dream of striking on every occasion of a rise in the cost of living" (pp. 14–15). See also Hicks (1955). I should think, however, that the desired relative wage is dependent on labor market conditions.

start. It is not clear to me how unions regard the interests of new members. And Paul Weinstein has suggested to me that the union leadership will be constrained in its wage policy by the need to support financially its administrative bureaucracy.

Finally, the explanation of the influence of the change of employment (or unemployment) upon wage increases is sometimes expectational. Ball (1964) suggests that firms and workers extrapolate the unemployment trend and set wages on the basis of the projected unemployment rate. Let us now try to introduce expectations into the model.

III. Expectations and Macroequilibrium

In Part II it was postulated that each firm expects other firms as a whole to hold their wage rates constant. In that case, it is natural for the firm to assume that an increase in its wage rates would assist it in attracting new employees and in discouraging quitting, since it would expect any increase of its wage to increase its wage differential. But in the general case the firm will have to forecast wage changes elsewhere in order to estimate the employment effects of its wage decision. This assumes that frequent wage negotiation with employees is sufficiently costly that wage contracts run for something like a year.

A simple derivation of the result I want—too glib a derivation as we shall see—might go like this. Let each firm expect with certainty that the average wage paid elsewhere will change at a certain proportionate rate over the life of the firm's wage contract. Consider now a firm whose immediate and prospective vacancy rate (v_i) in relation to labor market conditions (u and v) is such that, in the absence of wage changes elsewhere, it would want to keep its present wage rate to maintain its expected wage differential at its present actual level; this firm is in equilibrium in the sense that its actual wage differential equals its desired differential. But if the firm in fact expects the average wage elsewhere to be increasing at the rate of 2 per cent annually and it expects other firms to pass on the higher costs through a 2 per cent rise of prices annually, then it will want to raise its wage rates by 2 per cent annually; for it will calculate that it can raise its prices by 2 per cent without loss of customers and thus leave unchanged its real position, that is, its real sales, its product wage and vacancy rate, and its competitiveness in the labor market. As for the disequilibrium case, if its vacancy rate and labor market conditions are such that in the absence of expectation of wage changes elsewhere it would want to raise its wage by 1 per cent, say, it will, under the above expectations, want in fact to raise its wage by 3 per cent for the next year. Upon averaging over firms we are then led to the proposition that we must add the expected rate of wage change, denoted $\dot{w}^e/w$, to the rate of wage change that would occur under stationary wage expectations, in order to determine the actual rate of wage change per annum:

$$\frac{\dot{w}}{w} = \lambda\Delta^* + \frac{\dot{w}^e}{w} = f(u, z) + \frac{\dot{w}^e}{w}. \tag{25}$$

The result is quite natural. By "equilibrium," following Hayek, Lindahl, Harrod, and others (using varied terminology), we generally mean a path along which the relevant variables work out as people think they will. A necessary labor-market condition for what might be called a *macroequilibrium* in terms of the relevant averages and aggregates is therefore equality of the expected and actual rate of change of the average wage rate:

$$\frac{\dot{w}}{w} = \frac{\dot{w}^e}{w}. \tag{26}$$

Hence macroequilibrium entails

$$f(u, z) = \Delta^* = m(u, v) = 0, \tag{27}$$

meaning that "generalized excess demand," as measured by $m(u, v)$, be equal to zero. Any other result would be disturbing! But note that this equilibrium admits a rising or falling average money wage. Further, there is no clearing of the labor market in any ordinary sense.

This result needs interpretation and defense. First there is a matter of dating the variables. Imagine that wage negotiations are annual and are evenly staggered (across firms) over the year. Consider a firm negotiating at the beginning of the calendar year. Suppose it expects average wage rates *in the future* to rise steadily at the rate of 2 per cent over the year. Then if the wage index is 100 at the beginning of the year, the firm will expect the index to stand approximately at 101 by midyear. By raising its wage by just 1 per cent, the firm can expect to maintain on the average over its new contract its past average competitiveness with other employers over the old contract. Thus if the wage index stood at 100 throughout last year and our firm is content with its past wage differential, we appear to get only a 1 per cent wage rise resulting from a 2 per cent expected rise of the index. The resolution of this puzzle consists of defining $\dot{w}^e/w$ as the expected rate of change of the index from six months prior to the firm's wage negotiation to six months after the wage negotiation, so that it is centered on the date of the firm's wage decision. In our example, therefore, the "expected rate of wage change" so defined is really only 1 per cent. If, in the following year, the expected *future* rate of wage change (2 per cent) is unaltered and this year's expectations are borne out —so that the index will next year be expected to rise from approximately 101 (at last midyear) to approximately 103 (at the next midyear)—our firm must then raise its wage by 2 per cent if it expects to stay as competitive as before with other employers. This matter is possibly of some econometric significance, since the above example suggests that a perfect proxy for the expected *future* rate of wage change will tend to enter a regression equation resembling (25) with a less-than-unitary coefficient; it is only the expected rate of wage change as I defined it that is predicted to enter such an equation with a unitary coefficient.[28]

Why should the expected rate of wage change enter in (25) rather than expected price change? I believe the expectation of price increases affects money wages only through its effects on expected vacancy rates and the expected unemployment rate. *Given the latter*, a rise of the expected rate of inflation will have little or no effect upon the wage increase which a firm grants if it expects other firms to hold the line on the money-wage rates they pay; in particular, the threat of an employee expecting a rise of the cost of living to quit in search of another job will be empty if it is not expected that other firms' wages will rise with the cost of living. Whether Keynes was right that unions too are interested only in *relative* wages I do not know, but I gather that cost-of-living clauses are not very widespread in this country and have never ranked very high among union objectives.

If (25) is to be really satisfactory, however, it must hold when the expected price trend is flat as well as in the case (discussed above) where producers can expect to pass on their wage increase in higher prices with impunity. Probably (25) *is* too simple; a full analysis requires a theory of the optimal price dynamics of the firm. Yet I am prepared to defend it as a tolerable approximation along the following lines. Continue to abstract from productivity growth and consider a firm at wage-setting time. The vacancy rate of this firm, v_i, and the values of u and v which determine its desired differential must be taken as expected averages over the life of the wage contract. Though the firm will be concerned more with the near future than it will be with the less certain far future, let us imagine the firm thinks simply in terms of its mid-contract prospects, say v_i^e, u^e, and v^e, and its desired mid-contract differential, Δ_i^*, which is a function of these prospects. I shall evaluate the firm's v_i^e at the wage it expects it will need to maintain the competitiveness it enjoyed, as measured by its past mid-contract differential Δ_i, over the last contract period. Hence, if the desired differential $\Delta_i^* = k^i(u^e, v^e, v_i^e)$ is equal to the previous differential, Δ_i, *when the expected rate of wage change is zero*, it will not alter its wage rate; for in this situation maintenance of its former wage will yield it an expected

[28] The left-hand side variable is likewise the rate of change of the actual wage index expressed at annual rates. If wage negotiations are evenly distributed over the year, the firms setting wages in January, by raising their wage rates 1 per cent, will raise the index by one-twelfth of 1 per cent from its December level and hence by 1 per cent at an annual rate. Where annual wage negotiations are unevenly distributed over the year (producing some seasonality), one may want to work with the actual one-year rates of change of the index (for example, January-to-January), in which case the "expected rate of wage change" is an average of twelve figures centered (respectively) on each of the twelve months in the one-year interval.

vacancy rate at mid-contract with which it is content. As a second situation, suppose now that, *other things equal*, the firm expects a 1 per cent rate of wage increase (as defined earlier, from mid-contract to mid-contract). In this situation it does not expect to be able to raise its prices by an additional 1 per cent without loss of customers. Therefore when the firm evaluates its vacancy rate at the 1 per cent higher wage it will find its expected vacancy rate smaller in this second situation, so that its $\Delta_i^* = k^i(u^e, v^e, v_i^e)$ is less than its previous average wage differential, Δ_i. This means that while the firm may raise its wage it will raise it less than 1 per cent in order to reduce its expected differential. To the extent that this second situation is general among firms, we will have a smaller $m(u^e, v^e)$. Firms will recruit less so that z and hence $f(u, z)$ will both be smaller. Thus a *ceteris paribus* rise of $\dot{w}^e/w$ in (25), to the extent that businesses do not expect to be able to shift the expected wage costs onto buyers, will be partially offset by a resulting fall of z and $f(u, z)$ so that $\dot{w}/w$ is not implied to rise by an equal amount.

But other things, like productivity and the demand for the firm's product, need not be equal. As I argued earlier, if the firm expects to be able to raise its price in proportion to its wage rates without loss of prospective sales—because, say, other firms are expected to raise their prices in that proportion and aggregate demand is not expected to change—then neither the expected product wage implied by the firm raising its wage rate just enough to maintain its previous competitiveness nor the expected quantity of its output demanded (all at mid-contract) will change, so its expected vacancy rate, v_i^e, will not change; thus the firm will in this case match the expected rate of wage change, adding or subtracting the wage change it would have chosen under stationary expectations. Another example of interest is the expectation by the firm of growth in the marginal and average productivity of its labor together with expected growth of its output demanded (at present prices) at a rate equal to the expected rate of wage change. Such a change in the firm's situation will leave its expected vacancy rate unchanged from its previous mid-contract level, when this is evaluated at the wage expected to be necessary to keep its wage differential at its previous mid-contract level. Hence, the firm will raise its wage by just the amount of the expected rate of wage change if it likes its previous differential—by more (less) if that previous differential is too low (high). In all cases, the firm is imagined notionally to increase its wage by the amount it expects is necessary to keep its past average competitiveness, to make an optimal price adjustment, and then to evaluate its expected vacancy rate at the implied product wage and expected demand for its product; if the desired differential calculated at that hypothetical vacancy rate is equal to its past average differential, it goes ahead with the "competitive" wage increase; if the desired differential is greater (less), the firm will increase its wage by more (less) than the expected or competitive amount.

The mathematics of all this becomes simple if we shrink the contract period to zero to avoid dating complications. Suppose that each firm adjusts continuously its wage in such a way as to make the absolute rate of change of its expected wage differential, Δ_i^e, proportional to the difference between its desired differential and its present differential:

$$\dot{\Delta}_i^e = \lambda_i(\Delta_i^* - \Delta_i), \tag{28}$$

where

$$\Delta_i^e = \frac{w_i - w^e}{w^e},$$

$$\Delta_i^* = \frac{w_i^* - w^e}{w^e},$$

and $w^e = w$ at the current moment, though $\dot{w}^e = \dot{w}$ if and only if the average wage change is correctly forecast. Calculation of the derivative $\dot{\Delta}_i^e$ and its substitution in (28) yields

$$\frac{\dot{w}_i}{w_i} = \lambda_i \Delta_i^* \frac{w}{w_i} - \lambda_i \Delta_i \frac{w}{w_i} + \frac{\dot{w}^e}{w}. \tag{29}$$

For firms as a whole we have $\Delta_i = 0$ and $w/w_i = 1$ on the average. Hence, for the rate of change of the average wage in terms of average Δ^* and average λ we obtain (25). But the use of a continuous-time analysis which treats wage rate changes as costless really deprives the role of wage expectations of its rationale.[29]

I shall briefly point out some implications and needed qualifications of this model.

One implication seems to be that a guidepost policy can be successful if it causes firms to expect other firms to raise their wages at a lower rate. In this respect there seems to be some advantage in a numerical guidepost standard like 3.2 per cent wage growth.

The model has implications for the requirements of equilibrium. Our equilibrium condition (27) together with the differential equation (23) that links $\dot{u}$ to u and z imply that corresponding to every initial unemployment rate is an equilibrium time path, $u^*(t)$. Any such time path satisfies

$$f\left[u^*, (1 - u^*)\frac{\dot{L}}{L} - \dot{u}^*\right] = 0. \tag{30}$$

It is easy to show that if the rate of labor-supply growth, $\dot{L}/L$, is equal to a non-negative constant, γ, each equilibrium path (corresponding to each initial u) converges to a *steady-state equilibrium* in which $\dot{u}^* = 0$. The

[29] A continuous-time model with a set-up cost of changing the wage rate at any time—rather than periodic wage negotiations—might offer some interesting contrasts to the analysis here, though I would not expect differences in steady-state behavior.

steady-state equilibrium value of the unemployment rate, denoted by $\bar{u}^*$, is determined by

$$f[\bar{u}^*, (1 - \bar{u}^*)\gamma] = 0. \tag{31}$$

Corresponding to $\bar{u}^*$ is some steady-state equilibrium vacancy rate, $\bar{v}^*$, which is given by the relation $m(\bar{u}^*, \bar{v}^*) = 0$.

Consider now alternative steady-state equilibria corresponding to different rates of wage increase but having the same productivity growth. It is clear that each of these steady-state equilibria must have the same unemployment rate. This conclusion requires simply that γ, on which $\bar{u}^*$ depends, be invariant to the nominal trend of money-wage rates in any steady-state equilibrium. That requirement is satisfied if the labor supply is perfectly inelastic with respect to all economic variables. It is also satisfied if the growth of labor supply depends only upon real variables and the latter are invariant, in steady-state equilibrium, to the rate of change of nominal wage rates. (For example, constancy of steady-state markups over time would leave the rate of growth of the real wage independent of the nominal wage trend.) Thus the locus of steady-state equilibrium points in Figure 2 is a vertical (dashed) line at $\bar{u}^*$. This locus might be called the *equilibrium* steady-state Phillips curve.

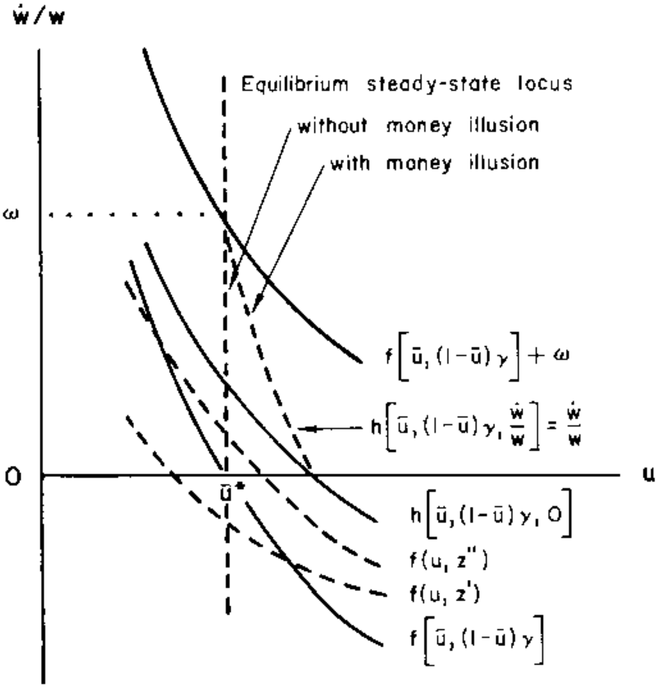

FIG. 2.—Augmented Phillips curves and equilibrium steady-state loci

Clearly this result fits the theory of anticipated inflation. For it implies that an economy experiencing and anticipating 10 per cent money-wage growth (and corresponding inflation rate) would not, in a steady state, have an unemployment rate different from what it would have if it were experiencing and anticipating a much smaller rate of wage increase.

What if higher money-wage growth in one steady state is matched by higher productivity growth? It is sometimes held that an economy can maintain a steady-state equilibrium—and thus a steady state with a stationary price trend (as well as any other trend)—with a smaller steady unemployment rate the faster its productivity growth. This is obvious on the usual Phillips curve analysis where no expectational variables are introduced; and it is also valid if the expected rate of wage change in my model is replaced by the expected rate of price change. But our theory denies this proposition if it is assumed that steady wage growth eventually generates the expectation of that growth. Then the difference in rates of wage increase consistent with price stability between rapid-productivity-growth and slow-productivity-growth situations does not permit a favorable difference in steady unemployment rates, since the difference in $\dot{w}/w$ will be matched by an equal difference in $\dot{w}^e/w$. Indeed the proposition in question could be reversed in a more general model: If rapid productivity growth and resulting obsolescence of plants strike firms unevenly and thus make greater demands for labor mobility and flexible skills, the steady-state equilibrium unemployment rate may very well be higher the faster is the growth of productivity. (But *given* productivity growth, $\bar{u}^*$ is still independent of the expected nominal wage trend.)

It is worth pointing out that because a rise of the rate of growth of the labor force will increase the value of z and hence the vacancy rate needed to maintain any given unemployment rate and because equilibrium $\bar{u}^*$ must then fall to accommodate a higher $\bar{v}^*$, the steady-state unemployment rate is higher the faster the labor supply grows. From (31) we calculate that

$$\frac{d\bar{u}^*}{d\gamma} = \frac{-(1 - \bar{u}^*)f_2}{f_1 - f_2\gamma} > 0. \qquad (32)$$

Thus rapid economic growth from any source appears to increase the equilibrium steady-state unemployment rate.

Given the rates of labor force and productivity growth, therefore, the model implies that $\bar{u}^*$ is a constant, independent of $\dot{w}/w$ and $\dot{w}^e/w$. It is clear from (30) and (31) that if the unemployment rate is maintained at any constant level other than $\bar{u}^*$ a disequilibrium will result, since every equilibrium path converges monotonically to $\bar{u}^*$. For example, if $u = \bar{u} < \bar{u}^*$, $\bar{u}$ a constant, then $f[\bar{u}, (1 - \bar{u})\gamma] > 0$, so that $\dot{w}/w > \dot{w}^e/w$. What are the consequences of such a disequilibrium? To answer this we need some theory of expectations. Suppose we adopt the adaptive-expectations theory, first used by Cagan (1956), according to which $\dot{w}^e/w$ tends toward $\dot{w}/w$. Then $u = \bar{u} < \bar{u}^*$ implies $\dot{w}^e/w$ will be rising. But every one-point increase of $\dot{w}^e/w$ makes $\dot{w}/w$ one point higher if $u = \bar{u}$ is maintained. As a consequence, $\dot{w}^e/w$ and hence $\dot{w}/w$ will be increasing without limit as long as $u = \bar{u}$. The result of this is hyperinflation. The same explosive spiral must eventually result if the unemployment rate, while possibly variable, is bounded below $\bar{u}^*$, that is, $u(t) \leqq \bar{u}^* - \epsilon$, for all t, $\epsilon =$ constant > 0.

Suppose we are convinced that steady, non-accelerating inflation at some moderate rate is possible in this country at a steady unemployment rate of 4 per cent. In the present model this implies $\bar{u}^*$ equals 4 per cent.[30] Is it plausible that, as the above model predicts, wages and prices would spiral upward at an ever accelerating rate if aggregate demand consistently maintained the unemployment rate at 3.5 per cent? One might argue that it is not plausible on the "money-illusion" ground that an unemployment rate as high as 4 per cent is consistent with a moderate and steady rate of inflation, because some of those firms which would like to reduce substantially their wage differentials prefer to accept below-optimal profits or even dismiss some employees rather than impose money-wage cuts on their employees, and because some employees would rather quit than suffer the indignity of a money-wage cut; this means that the average money wage can be rising at the expected rate of wage change even when the "true" average desired wage differential, Δ^*, is negative. But money-wage cuts are occasionally appropriate for a firm which wants a lower wage differential only when the expected rate of wage change is moderately low. On this argument, therefore, a 3.5 per cent unemployment rate might also be consistent with equilibrium if the expected rate of wage change were high enough that a firm could reduce its expected relative wage by the amount desired without having to impose a money-wage cut.

Formally, the introduction of this "money illusion" (or resistance to money wage cuts) necessitates the more general wage-change function,

$$\frac{\dot{w}}{w} = h\left(u, z, \frac{\dot{w}^e}{w}\right), \tag{33}$$

where, for those values of $1 - u$, z and $\dot{w}^e/w$ low enough to raise the wage-cut obstacle for one or more firms, the derivative $\partial h/\partial(\dot{w}^e/w)$ is less than one, increasing in both $\dot{w}^e/w$ and z and decreasing in u; for values of $1 - u$, z and $\dot{w}^e/w$ sufficiently large that the wage-cut constraint is not binding for any firm, the derivative $\partial h/\partial(\dot{w}^e/w)$ is a constant equal to one as in the original formulation.

This variant of the model implies that the locus of steady-state equilibrium points is vertical only for $\dot{w}^e/w$ equal to or exceeding some positive level, ω in Figure 2, that is sufficiently high to circumvent the money-illusion problem. As $\dot{w}^e/w$ is reduced by equal successive amounts, the

[30] Note that the unemployment rate required to keep average money-wage rates in pace with productivity in the American economy, perhaps 6 per cent, will exceed the American $\bar{u}^*$ if, as seems likely, the expected rate of change of the money wage exceeds the rate of growth of productivity.

steady-state curve $h[\bar{u}, (1 - \bar{u})\gamma, \dot{w}^e/w]$ shifts down by smaller amounts so that, in this range, the locus of steady-state equilibrium points, where $\dot{w}/w$ equals $\dot{w}^e/w$, is negatively sloped, meaning that the $\bar{u}$ necessary for equilibrium is a decreasing function of $\dot{w}^e/w$. A dashed curve in Figure 2 depicts this money-illusion version of the equilibrium steady-state Phillips curve.

This variant of the model admits the possibility that a 3.5 per cent unemployment rate may be a sustainable equilibrium level too, like 4 per cent, though only at a higher rate of wage increase. Nevertheless, there exists some unemployment rate, perhaps 3 per cent, such that maintenance of the unemployment rate at a level below that rate would require a disequilibrium accelerating spiral of wages and prices. Such a revision of the model appears to reinforce the earlier hypothesis that faster labor force growth worsens the unemployment-inflation trade-off if the faster labor force growth would tend to depress the rate of growth of real wage rates. It *could* reverse the earlier hypothesis that productivity growth increases the steady-state unemployment rate necessary for price stability (or any steady-state equilibrium) if productivity growth tended to raise the rate of growth of real wage rates.

Another qualification of the model may be appropriate, though probably it has only short-run significance. The above model takes expectations of wage change, vacancy rates, and so on, to be certain. One may feel intuitively that a mean expected wage increase of 5 per cent has less of an impact on the firm's wage increase than a 5 per cent increase that is expected with certainty, that in response to the former the firm will "hedge" with a less-than-competitive wage increase to reduce the variance of its prospective profits distribution at some cost to mean expected profits. Then $\dot{w}^e/w$ will have a less-than-unitary coefficient if changes in $\dot{w}^e/w$ are accompanied by increases of the dispersion of $\dot{w}^e/w$ (which die out if the new $\dot{w}^e/w$ stabilizes), even though the constant-dispersion coefficient is really unitary. If firms do behave in this manner, the slope of the equilibrium steady-state locus will be underestimated to the extent that high wage growth expectations are not intrinsically more uncertain than low wage growth expectations once they become habitual. Much as I would like to be able to justify this intuition, I find a rational basis for it altogether elusive thus far. In particular, from the point of view of employment effects alone, maintenance of a firm's competitiveness or even an increase of its competitiveness would seem to offer minimum risk of high recruitment expense and excessive quitting. On the other hand, firms may act on similar intuitions whether rational or not.

I have been considering modifications of the simple model that bear on its implication of explosive hyperinflation or hyperdeflation at all unemployment rates different from some unique steady-state equilibrium rate. I have registered skepticism regarding the hypothesis that the greater uncertainty temporarily attaching to extreme or outlying wage expectations serves to moderate the otherwise explosive wage change movements, thus lending the economy the *appearance* of non-explosiveness. Perhaps another factor that makes a 4 per cent unemployment rate or even a 3 per cent rate appear to be permanently sustainable without forever mounting inflation is that expectations are not always "adaptive" in the way usually specified. When the standard expectations model predicts a rate of wage increase of, say, 6 per cent per year, employers may "switch off" that model, suspend the adaptation of their expectation to events, and place their faith in Washington or Providence to prevent wage increases beyond, say, 5 per cent.[31] But such bounds on expectations would eventually give way if Washington broke faith by continuing to permit wage increases outside the bound; so the point relates only to the statistical appearance of non-explosiveness.

IV. Summary

A generalized excess-demand theory of the rate of change of the average money-wage rate has been developed for frictional labor markets that allocate heterogeneous jobs and workers without having perfect information and market clearance by auction. There are two explanatory variables: the vacancy rate and the unemployment rate. The unemployment rate and the rate of change of employment (per unit of labor supply) are shown to be joint proxies for the vacancy rate. Hence generalized excess demand can be regarded as a derived function of the unemployment rate and the rate of change of employment. This relationship is the augmented Phillips curve. Some of its properties are deduced. The steady-state Phillips curve that relates the rate of wage increase to the steady unemployment rate is also derived.

The expected rate of wage change is then added to the Phillips function—to the excess-demand term—to obtain the rate of wage increase under non-stationary expectations in a no-money-illusion world. Equilibrium entails equality between the actual and expected rates of wage change. The steady-state equilibrium locus is implied to be a vertical line at a unique steady-state equilibrium unemployment rate. This is consistent with the usual theory of anticipated inflation. But if there are downward money-wage rigidities, then, up to a point, every one percentage point increase of the expected rate of wage change produces *less* than a one percentage point increase of the actual rate of wage change. The steady-state equilibrium locus will then have the characteristic negative slope of the Phillips curve in the range of large unemployment rates. But at sufficiently small (steady) unemployment rates, equilibrium is impossible, and, under the adaptive expectations theory, an explosive hyperinflation will result.

[31] I believe I owe this point, or one very close to it, to G. L. Bach.

Statistical Appendix

For this occasion I have been able to carry out only a few experiments with American data. I have used a quarterly model which, upon summation over four quarters to avoid seasonality and to reduce noise and measurement error, yields a model where all variables are essentially four-quarter rates of change and four-quarter averages. The four-quarter rate of wage change, based on unpublished U.S. non-farm average hourly compensation data of the Bureau of Labor Statistics and the civilian and non-civilian "potential" labor force, were generously supplied by N. J. Simler. The variable $\bar{E}_t$ denotes the four-quarter average employment rate. I have usually worked with the level and rate of change of E rather than with the rate of change of employment per unit labor supply as in the model. Where appropriate, the variables are expressed as percentages. The regressions cover third-quarter 1953 to second-quarter 1964. Figures in parentheses are t-ratios.

A natural starting point is the regression

$$\frac{\dot{w}}{w_t} = \underset{(2.78)}{-3.55 + 0.71\bar{E}_t} - \underset{(2.76)}{0.66\bar{E}_{t-1}} + \underset{(6.73)}{0.73\frac{\dot{w}}{w_{t-1}}}, \quad \bar{R}^2 = .698, \tag{A.1}$$

where E is the global Simler-Tella adjusted employment rate. This can be interpreted as a *simple* Phillips curve combined with adaptive expectations or as an *augmented* Phillips curve in which $(\dot{w}/w)_{t-1}$ is simply extrapolated by firms. In the latter case it may make some sense to introduce $[(\dot{p}/p) - (\dot{w}/w)]_{t-1}$, where p is a price index, as an additional indicator of the discrepancy between the vacancy rate and its steady-state value in the following way:

$$\frac{\dot{w}}{w_t} = -9.26 + \underset{(3.11)}{1.11\bar{E}_t} - \underset{(3.14)}{1.00\bar{E}_{t-1}} + \underset{(1.58)}{0.21\left(\frac{\dot{p}}{p} - \frac{\dot{w}}{w}\right)_{t-1}} + \underset{(6.88)}{0.80\frac{\dot{w}}{w_{t-1}}}, \quad \bar{R}^2 = .709. \tag{A.2}$$

Use of the z-like rate of change variable, $C_t = (\bar{N}_t - \bar{N}_{t-1})/\bar{L}_t$, leads to a minor improvement in the fit:

$$\frac{\dot{w}}{w_t} = -10.92 + \underset{(1.65)}{0.13\bar{E}_t} + \underset{(3.20)}{1.06C_t} + \underset{(1.67)}{0.22\left(\frac{\dot{p}}{p} - \frac{\dot{w}}{w}\right)_{t-1}} + \underset{(6.83)}{0.79\frac{\dot{w}}{w_{t-1}}}, \quad \bar{R}^2 = .711. \tag{A.3}$$

Since the length of the work week, H, is also a good proxy for the vacancy rate, like C, it is not surprising that its introduction detracts from the power of C:

$$\frac{\dot{w}}{w_t} = -51.3 + \underset{(2.76)}{0.19\bar{E}_t} + \underset{(1.49)}{0.49C_t} + \underset{(3.56)}{8.61\bar{H}_t} + \underset{(3.17)}{0.40\frac{\dot{p}}{p_{t-1}}} + \underset{(3.57)}{0.45\frac{\dot{w}}{w_{t-1}}}, \quad \bar{R}^2 = .778. \tag{A.4}$$

This equation implies a very steep equilibrium steady-state Phillips curve.

On the other hand, the conjunction of the augmented Phillips curve and adaptive expectations yields

$$\frac{\dot{w}}{w_t} = -5.07 + \underset{(2.34)}{1.19\bar{E}_t} - \underset{(1.75)}{1.69\bar{E}_{t-1}} + \underset{(1.09)}{0.56\bar{E}_{t-2}} + \underset{(6.82)}{0.75}\frac{\dot{w}}{w_{t-1}}, \quad \bar{R}^2 = .700. \quad (A.5)$$

The E coefficients have the right signs and are largely significant. When the price change variable is introduced, $\bar{E}_{t-2}$ loses all significance and the twice-lagged price change variable has the wrong sign:

$$\frac{\dot{w}}{w_t} = -14.94 + \underset{(2.31)}{1.41\bar{E}_t} - \underset{(1.34)}{1.49\bar{E}_{t-1}} + \underset{(0.46)}{0.26\bar{E}_{t-2}} + \underset{(0.77)}{0.12}\left[\frac{\dot{p}}{p} - \frac{\dot{w}}{w}\right]_{t-1} + \underset{(1.29)}{0.18}\left(\frac{\dot{p}}{p} - \frac{\dot{w}}{w}\right)_{t-2} + \underset{(6.91)}{0.82}\frac{\dot{w}}{w_{t-1}}, \quad \bar{R}^2 = .718. \quad (A.6)$$

Introduction of the workweek did not appear to help.

Use of civilian non-agricultural employment to form a new employment rate, E', led to somewhat different results. While

$$\frac{\dot{w}}{w_t} = -3.96 + \underset{(3.05)}{0.88\bar{E}'_t} - \underset{(3.18)}{0.80\bar{E}'_{t-1}} + \underset{(1.14)}{0.13}\left(\frac{\dot{p}}{p} - \frac{\dot{w}}{w}\right)_{t-1} + \underset{(5.25)}{0.77}\frac{\dot{w}}{w_{t-1}}, \quad \bar{R}^2 = .712 \quad (A.7)$$

is not very different from (A.2), the following gives a smaller coefficient for $(\dot{w}/w)_{t-1}$ and a higher $\bar{R}^2$ than (A.5):

$$\frac{\dot{w}}{w_t} = -35.26 + \underset{(5.70)}{2.76\bar{E}'_t} - \underset{(5.35)}{4.31\bar{E}'_{t-1}} + \underset{(4.66)}{2.06\bar{E}'_{t-2}} + \underset{(5.49)}{0.59}\frac{\dot{w}}{w_{t-1}}, \quad \bar{R}^2 = .809. \quad (A.8)$$

The introduction of hours worked yields

$$\frac{\dot{w}}{w_t} = -41.5 + \underset{(4.11)}{2.48\bar{E}'_t} - \underset{(4.00)}{3.73\bar{E}'_{t-1}} + \underset{(3.75)}{1.79\bar{E}'_{t-2}} + \underset{(1.48)}{4.84\bar{H}_t} - \underset{(1.04)}{3.89\bar{H}_{t-1}} + \underset{(5.59)}{0.60}\frac{\dot{w}}{w_{t-1}}, \quad \bar{R}^2 = .810. \quad (A.9)$$

Introduction of the price change variables yields the mysterious equation

$$\frac{\dot{w}}{w_t} = -42.70 + \underset{(6.01)}{2.87}\bar{E}'_t - \underset{(5.35)}{4.22}\bar{E}'_{t-1} + \underset{(4.54)}{1.96}\bar{E}'_{t-2} - \underset{(0.39)}{0.05}\left(\frac{\dot{p}}{p} - \frac{\dot{w}}{w}\right)_{t-1}$$

$$+ \underset{(1.79)}{0.19}\left(\frac{\dot{p}}{p} - \frac{\dot{w}}{w}\right)_{t-2} + \underset{(4.86)}{0.59}\frac{\dot{w}}{w_{t-1}}, \quad \bar{R}^2 = .820, \quad \text{(A.10)}$$

or, equivalently, apart from rounding errors,

$$\frac{\dot{w}}{w_t} = -42.55 + \underset{(3.95)}{0.61}\bar{E}'_t + \underset{(5.80)}{2.25}(\bar{E}'_t - \bar{E}'_{t-1}) - \underset{(4.52)}{1.95}(\bar{E}'_{t-1} - \bar{E}'_{t-2})$$

$$- \underset{(0.39)}{0.05}\left(\frac{\dot{p}}{p} - \frac{\dot{w}}{w}\right)_{t-1} + \underset{(1.79)}{0.19}\left(\frac{\dot{p}}{p} - \frac{\dot{w}}{w}\right)_{t-2} + \underset{(4.85)}{0.60}\frac{\dot{w}}{w_{t-1}}, \quad \bar{R}^2 = .819. \quad \text{(A.11)}$$

Finally, for whatever curiosity value it may have, I computed

$$\frac{\dot{w}}{w_t} = -56.55 + \underset{(4.04)}{2.42}\bar{E}'_t - \underset{(3.48)}{3.38}\bar{E}'_{t-1} + \underset{(3.23)}{1.62}\bar{E}'_{t-2}$$

$$+ \underset{(1.64)}{6.14}\bar{H}_t - \underset{(0.91)}{3.55}\bar{H}_{t-1} - \underset{(0.21)}{0.04}\frac{\dot{p}}{p_{t-1}} + \underset{(1.07)}{0.21}\frac{\dot{p}}{p_{t-2}}$$

$$+ \underset{(4.71)}{0.66}\frac{\dot{w}}{w_{t-1}} - \underset{(1.65)}{0.22}\frac{\dot{w}}{w_{t-2}}, \quad \bar{R}^2 = .824. \quad \text{(A.12)}$$

The reader can calculate the equilibrium steady-state Phillips relations on the natural assumption that $\dot{p}/p = \dot{w}/w - \rho$ where ρ is invariant to the steady-state level of E.

I have not begun to test the many hypotheses which the present model suggests, such as the various non-linearities and interaction terms. Work of this sort probably requires more careful data construction. But I believe that several of the main features of the model have received some support from these empirical results.

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