

# National Markets and International Relative Prices

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## Abstract

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This paper examines the role of active price discrimination across national markets and sectoral reallocation in explaining the behavior of real exchange rates and the terms of trade. We first document the properties of international relative prices using bilateral data for the United States and Canada. We then present a dynamic, general equilibrium, two-sector monetary economy with both intra- and inter-industry trade and explore the model's qualitative and quantitative implications. In the environment we consider, firms set prices in their own currency and prices are fully flexible. We show that active price discrimination is capable of generating large and persistent movements in each of the relative price series we consider. Furthermore, we demonstrate that traditional comparative advantage effects arising from the two-sector structure of the economy significantly affect the cyclical properties of this model relative to single-sector environments.

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## I. Introduction

This paper examines the role of national markets and sectoral reallocation in explaining the behavior of real exchange rates and the terms of trade. The focus on national markets and real exchange rate movements is primarily motivated by the empirical work of Engel (1999,1993), Engel and Rogers (1998,1996), Goldberg and Knetter (1997), Rogers and Jenkins (1995), and others. In particular, Engel and Rogers' work suggests that movements in the relative prices of traded goods appear to account for nearly all of the movement in U.S. real exchange rates. They also conclude that the two most important explanations for movements in the relative prices of traded goods between the U.S. and Canada are differences in national markets and nominal price stickiness. Furthermore, they argue that nominal price stickiness alone cannot account for all of the movement in these prices. That is, national markets would lead to deviations in prices across these two countries even in the absence of price rigidities.

The focus on sectoral reallocation resulting from the incorporation of multiple traded goods within a country and its effect on the behavior of international prices is motivated by intuition suggesting that resource reallocation within a country in response to changing patterns of comparative advantage could have significant effects on relative price movements. Most of the recent open economy macroeconomic literature which examines the variability and persistence of the terms of trade generally assumes that a country produces only one traded good (and, perhaps, a non-traded good). Examples include Backus, Kehoe, and Kydland (1994), Backus and Crucini (2000), Boileau (1999), and Stockman and Tesar (1995). The assumption of a single traded sector within a country, however, stands in stark contrast to the pure theory of international trade and does not allow for traditional comparative advantage effects to influence relative price movements. In the economy studied in this paper, the incorporation of two traded sectors in an explicitly dynamic, stochastic environment allows for an examination of the effects of shocks on the pattern of comparative advantage across countries, resource reallocation, and relative price movements.<sup>1</sup>

Before presenting the theoretical economy, we document the properties of the relative price series of interest using bilateral price data for the United States and Canada. Our empirical results are consistent with what many others have found regarding international prices using multilateral data. In particular, movements in the bilateral terms of trade and the real exchange rate are highly variable and persistent. Real and nominal exchange rates are pro-cyclical while the terms of trade is countercyclical. All of the relative prices we consider are positively contemporaneously correlated with net exports. Finally, we also document that the nominal exchange rate is positively correlated

with both the real exchange rate and the bilateral terms of trade.

The observed high degree of correlation between real and nominal exchange rates has led many authors to examine environments in which sticky prices are the primary source of deviations from the law of one price in traded goods. (See, for example, Betts and Devereux (2000,1998) and Chari, Kehoe, and McGratten (2000).) In particular, this literature examines environments where producers set prices in the currency of the market where the goods are sold (local currency pricing) and those prices are rigid. Hence, when the nominal exchange rate changes and prices do not respond, the real exchange must change as well. Chari, Kehoe, and McGratten (2000) suggest that long periods of price stickiness (approximately 1 year) and preferences which are separable between consumption and leisure are required to explain the observed variability and persistence of movements in real exchange rates.

Obstfeld and Rogoff (2000), however, have criticized this approach arguing that the assumptions and predictions of these models are inconsistent with many other facts. In particular, they argue that general currency invoicing applies to contract of 90 days or less and so the time horizons which are needed in these models to generate movements in the real exchange rate which are consistent with what is observed are implausibly long. They also argue that the evidence on currency invoicing suggests a predominance of invoicing in exporters' home currencies. Finally, they provide evidence that the multilateral terms of trade and the nominal exchange rate are positively correlated for many countries which is inconsistent with models with sticky prices and local currency pricing.

In response to these criticisms and based on the empirical evidence provided by Engel and Rogers (1998,1996), Goldberg and Knetter (1997) and others, we examine a model which gives rise to deviations from the law of one price for traded goods and from purchasing power parity due to the presence of active price discrimination across national markets. In the environment we consider, prices are set in the exporter's currency and are fully flexible. The characteristics of markets in different countries vary over time and an assumption of market segmentation allows producers to price discriminate across markets according to those differences.

An example of an empirical study which provides evidence that firms set different prices in different markets according to market characteristics is provided by Aw (1993). He examined the relative importance of three sources of international price differences in the market for Taiwanese footwear: differences in marginal costs, differences in conduct, and differences in price elasticities of demand. His results indicate significant differences in both the price elasticities of demand and conduct. Goldberg and Knetter (1997) describe other studies which suggest that exporters engage

in conscious price discrimination across markets.

The remainder of the paper is organized as follows. Section 2 presents the empirical evidence for bilateral price movements between the United States and Canada. Section 3 presents the theoretical economy. Section 4 discusses our calibration of the model and Section 5 presents the quantitative implications of the model for relative prices and other aggregate moments of interest. Section 6 concludes and discusses directions for future research.

## **2. Properties of International Relative Prices**

In this section we present empirical evidence on international relative price series using bilateral price data for the United States and Canada. Data sources are given in the appendix.

### *2.1 Exchange Rates*

In this section, we examine the properties of real and nominal exchange rates between the U.S. and Canada. We use the the spot price of Canadian dollars to U.S. dollars as our measure of the nominal exchange rate. The real exchange rate can be measured in a variety of ways but is generally an indicator of the relationship between aggregate price indices across countries. The real exchange rate is used to measure deviations from purchasing power parity (PPP) across countries. In particular, relative PPP is said to hold when the real exchange rate is constant. In the presence of perfectly competitive markets, no non-traded goods, and no trade barriers, relative PPP is expected to hold. Many authors, however, (e.g. Isard (1977), Kravis and Lipsey (1978, 1977, 1974), Mussa (1986), and Rogoff (1996)) using various data sets and methods have presented evidence of large, persistent deviations from relative PPP.

We measure the real exchange rate between the U.S. and Canada as the ratio of a U.S. aggregate price index multiplied by the nominal exchange rate to a Canadian aggregate price index. We consider two measures of aggregate price indexes and report properties of the real exchange rate for both measures. First, we measure aggregate price indexes by implicit consumption deflators constructed using real and nominal consumption from the OECD Quarterly National Accounts. The second measure uses the CPI from each country as the aggregate price index. For the nominal exchange rate and both measures of the real exchange rate, we use quarterly data from 1981.1-1999.1, log and Hodrick- Prescott filter the rates and then compute statistics of deviations from trend. The results are given in Table 1 and indicate that nominal and real exchange rates over this time period are quite variable (almost twice as variable as GDP), are highly autocorrelated, and are weakly positively correlated with Canadian GDP. Furthermore, real and nominal exchange rates exhibit a high degree of cross-correlation. These results indicate large and persistent deviations from

PPP and are consistent with what other authors have found for different countries over different time periods.

The two most common explanations for observed deviations from PPP are deviations from the law of one price for traded goods across countries and the presence of non-traded goods. Engel (1999) Engel and Rogers (1998), Chari, Kehoe, and McGratten (2000,1998), and others, however, have provided evidence that very little of the movements in real exchange rates arises from fluctuations in the relative prices of non-traded to traded goods.

Furthermore, Engel (1993) examined the relative prices of similar goods across U.S. and Canadian markets and the relative prices of different goods sold within the U.S. His evidence indicated that the volatility of prices of similar goods across countries was generally greater than the volatility of different prices of goods sold within a country. This evidence suggests that models which seek to examine deviations from relative PPP should generate more movement in the relative price of the same good sold in two different countries than in the relative price of two different producers selling in the same market.

## 2.2 *Terms of Trade*

Another international relative price that has received attention in the open economy macroeconomics literature is the relative price of a country's imports to its exports, or its terms of trade. Backus, Kehoe and Kydland (BKK) (1995,1994), for example, using aggregate implicit import and export price deflators, have documented persistent and highly variable movements in the multilateral terms of trade for a variety of countries.

We focus on the *bilateral* terms of trade between the U.S. and Canada. We define the terms of trade between the U.S. and Canada to be the ratio of the Canadian import price index for imports from the U.S. to the Canadian export price index for exports to the U.S. We use Laspeyres fixed weighted price indices for each price series as our measure of price indices. Both series are in base year 1986 at quarterly frequencies from 1981.1-1999.1. We report relevant statistics for our measure of the terms of trade in Table 1. The results suggest that this measure of the terms of trade is approximately as variable as Canadian GDP. The terms of trade is positively autocorrelated but is not as persistent as the nominal or real exchange rate. Furthermore, the terms of trade is negatively correlated with GDP.

We also examine the relationship between these countries' bilateral terms of trade and net exports. We measure net exports as the ratio of Canadian exports to the U.S. minus Canadian imports from the U.S. in current prices to Canadian GNP in current prices. This data suggests that contemporaneously, the bilateral terms of trade and net exports are positively correlated for

these two countries over this time period. A further examination of the autocorrelation structure between these variables as was done in BKK (1994) indicates the same type of S-curve for our measure of the terms of trade as they found using multilateral measures of the terms of trade.

Recently, attention has also been focused on the relationship between the terms of trade and the nominal exchange rate. Obstfeld and Rogoff (2000) point to this correlation in their evaluation of local-currency pricing models of real exchange rates. Those models predict a negative correlation between these two variables whereas in the data examined by Obstfeld and Rogoff using multilateral terms of trade, the correlation is positive for most countries. Our data on bilateral terms of trade between the U.S. and Canada also indicates a positive correlation between these variables.

We now present a model with multiple traded sectors and active international price discrimination to examine these relative price movements. We characterize its quantitative properties for these relative prices series and discuss its implications for other international comovements. The model generates large and persistent movements in the real exchange rate and produces relative volatilities which are consistent with Engel's (1993) evidence on individual price movements. The model also produces significant movements in the terms of trade.

### 3. The Economy

We consider a two country, two sector monetary economy. One sector produces a homogeneous good for consumption and investment under constant returns to scale while the other sector produces differentiated goods for consumption under increasing returns to scale. All goods are traded. There are no nominal rigidities in the model and money is incorporated into the model primarily to allow us to examine the model's prediction for the relationship between real and nominal exchange rates.

We begin with the specification of consumer preferences and endowments. We focus on a representative consumer in each country who is endowed with capital and one unit labor. The preferences of the representative consumer in country  $j \in \{1, 2\}$  are characterized by an expected utility function of the form

$$E_o \sum_{t=0}^{\infty} \beta^t U(C_{jt}, l_{jt}, M_{jt}/P_{jt}), \quad (1)$$

where

$$U(C, l, M/P) = \frac{[(aC^\lambda + (1-a)(M/P)^\lambda)^{\frac{\gamma}{\lambda}} (1-l)^{1-\gamma}]^{1-\sigma}}{1-\sigma} \quad 0 < \gamma < 1, \quad \sigma > 0, \quad (2)$$

and

$$C_{jt} \equiv \left[ \int_0^n (c_{jt}^i)^{\rho_{jt}} di \right]^{\frac{\nu}{\rho_{jt}}} d_{jt}^{1-\nu} \quad 0 < \nu < 1. \quad (3)$$

Here  $c_{jt}^i$  is consumption of differentiated good  $i \in [0, n]$ ,  $d_{jt}$  is consumption of the homogeneous good,  $l_{jt}$  is employment in country  $j$ ,  $M_{jt}$  is nominal money balances of currency  $j$ , and  $P_{jt}$  is the price index in country  $j$ . The period utility function given by equation (2) is the same as the non-separable function used by Chari, Kehoe, McGratten (1998) to facilitate comparison with that study. The specification of composite consumption given by equation (3) is widely used in models of industrial organization, international trade, and growth which incorporate differentiated products.

Consumer preferences differ across countries with respect to the degree of substitution among differentiated products as governed by  $\rho_{jt} \in (0, 1)$ . These parameters are stochastic and as will be shown below, movements in this parameter are necessary to generate deviations from relative PPP across countries. Movements in this parameter act as a proxy for “changing market conditions” and provide motivation for firms to set different prices in the two different countries.<sup>2</sup>

To guarantee that the realizations of preference parameters are on the unit interval, we determine them as follows for  $j \in \{1, 2\}$ :

$$\rho_{jt} = \frac{e^{v_{jt}}}{1 + e^{v_{jt}}}, \quad (4.1)$$

where  $v_{jt}$  are stochastic variables. Let  $\hat{v}_t$  denote the vector of deviations of these parameters from their mean,  $\bar{v}$ . The stochastic process which governs these variables is given by:

$$\hat{v}_{t+1} = \Lambda \hat{v}_t + \epsilon_t^v, \quad (4.2)$$

where  $\epsilon^v = (\epsilon_1^v, \epsilon_2^v)'$  is distributed normally and independently over time with variance-covariance matrix  $V^v$ .

Production of the homogeneous good in country  $j$  is governed by the following constant returns to scale technology:

$$y_{jt} = z_{jt}^y (k_{jt}^y)^\theta (l_{jt}^y)^{1-\theta} \quad 0 < \theta < 1, \quad (5)$$

where  $k_{jt}^y$  is capital input,  $l_{jt}^y$  is labor input, and  $z_{jt}^y$  is a technology shock. Meanwhile, each differentiated good is produced by a single producer and the market structure is monopolistic competition without entry. The world-wide measure of differentiated goods is fixed at  $n$  where this is the number of producers which is consistent with zero profits in the deterministic steady state. A distinct set of differentiated goods is produced in each country and the measure of goods in each country is fixed at  $.5n$ ; that is, each country produces half of the differentiated goods. Let  $i \in [0, .5n]$  index the differentiated goods produced in country 1 and let  $i \in (.5n, n]$  index the differentiated goods produced in country 2. Production of differentiated good  $i$  in country  $j$  is

governed by the following increasing returns to scale technology:

$$m_t^i = z_{jt}^m (k_t^i)^\alpha (l_t^i)^{1-\alpha} - \phi \quad 0 < \alpha < 1, \quad \phi > 0, \quad (6)$$

where  $k_t^i$  is capital input,  $l_t^i$  is labor input, and  $z_{jt}^m$  is a technology shock to the differentiated goods sector in country  $j$ . Firms pay a fixed cost of production each period given by  $\phi$ . The Cobb-Douglas technologies specified here are widely used in open economy macroeconomic models.

Let  $z_t \equiv (z_{1t}^y, z_{2t}^y, z_{1t}^m, z_{2t}^m)'$  denote the vector of technology shocks and let  $\hat{z}_t$  denote deviations from their means,  $\bar{z}^y$  and  $\bar{z}^m$ . The technology shocks follow:

$$\hat{z}_{t+1} = \Gamma \hat{z}_t + \epsilon_t^z, \quad (7)$$

where  $\epsilon^z = (\epsilon_1^y, \epsilon_2^y, \epsilon_1^m, \epsilon_2^m)'$  is distributed normally and independently over time with variance-covariance matrix  $V^z$ .

Capital formation incorporates adjustment costs and additions to capital require inputs of the homogeneous good. Let  $k_{1t}^m = \int_0^{.5n} k_t^i di$  and  $k_{2t}^m = \int_{.5n}^n k_t^i di$  denote the total capital stock employed in the differentiated goods sector in country 1 and country 2 respectively. Then the aggregate capital stock in country  $j$ ,  $K_{jt}$ , must satisfy:

$$K_{jt} = k_{jt}^y + k_{jt}^m. \quad (8)$$

The law of motion for the aggregate capital stock in country  $j$  is given by

$$K_{jt+1} = (1 - \delta)K_{jt} + (x_{jt}/K_{jt})^\psi K_{jt} \quad 0 < \psi < 1, \quad (9)$$

where  $x_{jt}$  is investment of the homogeneous good in country  $j$ ,  $\delta$  is the depreciation rate and  $\psi$  determines the size of adjustment costs. It is well known that in open economy macroeconomic models, some mechanism such as capital adjustment costs is required to limit the volatility of investment flows across countries. For tractability and flexibility, we choose the specification given in equation (9).

Let  $q_{jt}$  denote the price of the homogeneous good denominated in units of currency  $j$  and let  $p_{jt}^i$  denote the price of differentiated good  $i$  sold in country  $j$  denominated in units of the currency of the producer of the good. That is, for  $i \in [0, .5n]$ ,  $p_{jt}^i$  is denoted in units of currency 1 while for  $i \in (.5n, 1]$ ,  $p_{jt}^i$  is denoted in units of currency 2. We chose this specification to be consistent with the evidence presented in Obstfeld and Rogoff (2000) which suggests that firms generally invoice in their own currency. Finally, let  $e_t$  denote the nominal exchange rate between the two currencies.

Finally, we specify the money supply process. These are given by the following for  $j = \{1, 2\}$ :

$$M_{jt} = \mu_{jt}M_{jt-1}, \quad (10)$$

where  $\mu_{jt}$  are stochastic processes and  $M_{j0}$  are given. New money balances of currency  $j$  are given to consumers in Country  $j$  via a lump-sum transfer given by  $T_{jt} = M_{jt} - M_{jt-1}$ . Let  $\mu \equiv (\mu_{1t}, \mu_{2t})$  denote the vector of money growth shocks. These stochastic processes are first-order Markov in deviations from their means and are given by:

$$\hat{\mu}_t = \Omega \hat{\mu}_{t-1} + \epsilon_t^\mu. \quad (11)$$

An equilibrium in this economy consists of prices and allocations such that consumers and producers are optimizing and all markets clear. In constructing the equilibrium, we first examine the problems of the representative consumers in each country.

Consumers have access to markets for a set of one-step-ahead contingent claims. These claims pay one unit of currency 1 in the next period, contingent on the aggregate state at that time. The aggregate state for the world economy at time  $t$  is denoted  $S_t$ , and is given by

$$S_t \equiv (K_{1t}, K_{2t}, \rho_{1t}, \rho_{2t}, z_{1t}^y, z_{2t}^y, z_{1t}^m, z_{2t}^m, \mu_{1t}, \mu_{2t}). \quad (12)$$

Given our previous specifications, the state vector is Markov, with transition probability density  $f(S_{t+1}, S_t)$ . We let  $\Sigma$  denote the state space. Let  $v(S_{t+1}, S_t)$  denote the price in state  $S_t$  of a contingent claim that pays off in state  $S_{t+1}$ .

For a consumer in country  $j$ , let  $b_j(S_t)$  denote the contingent claims holdings for state  $S_t$ , let  $k_{jt}$  denote per capita capital stock, let  $w_{jt}$  and  $r_{jt}$  denote wage and rental price of capital respectively in units of currency  $j$ , let  $\Pi_{jt}$  denotes profits from the differentiated good sector in country  $j$ , and let  $T_{jt}$  denote transfers of currency  $j$ . For ease of exposition, let  $\tilde{p}_{jt}^i$  denote the price of good  $i$  sold in country  $j$  and denominated in units of currency  $j$ . That is, for  $i \in [0, .5n]$ , let  $\tilde{p}_{1t}^i \equiv p_{1t}^i$  and let  $\tilde{p}_{2t}^i \equiv (p_{2t}^i/e_t)$  and for  $i \in (.5n, n]$ , let  $\tilde{p}_{1t}^i \equiv e_t p_{1t}^i$  and let  $\tilde{p}_{2t}^i \equiv p_{2t}^i$ .

With this notation, the budget constraint for a consumer in country 1 at time  $t$  in units of currency 1 is given by

$$\int_0^n \tilde{p}_{1t}^i c_{1t}^i di + q_{1t}(d_{1t} + x_{1t}) + M_{1t} + \int_{\Sigma} v(S_{t+1}, S_t) b_1(S_{t+1}) dS_{t+1} \leq w_{1t}l_{1t} + r_{1t}k_{1t} + \Pi_{1t} + b_1(S_t) + M_{1t-1} + T_{1t}. \quad (13)$$

Consumers in Country 2 face a symmetric budget constraint. The consumer in each country faces a two-stage maximization problem. In the first stage, consumers choose investment, leisure, contingent claim holdings, nominal money holdings, and expenditure on consumption, denoted  $E_{jt} \equiv \int_0^n \tilde{p}_{jt}^i c_{jt}^i di + q_{jt} d_{jt}$ , to maximize their expected discounted lifetime utility as given in equation (1). In solving this dynamic problem, consumers take prices and the aggregate capital stock, labor supply, and money supply as given.

In the second stage, consumers allocate their consumption expenditure across the differentiated goods and the homogeneous good. We first examine the second stage in which a representative consumer in country  $j$  faces the following static maximization problem:

$$\begin{aligned} \max_{d_j; c_j^i, i \in [0, n]} & \left[ \int_0^n (c_j^i)^{\rho_j} di \right]^{\frac{\nu}{\rho_j}} d_j^{1-\nu} \\ \text{subject to} & \int_0^n \tilde{p}_j^i c_j^i di + q_j d_j = E_j, \end{aligned}$$

where time subscripts have been dropped and prices and  $E_j$  are taken as given. The solution to this problem is characterized by the following:

$$c_j^i = \left[ \frac{\tilde{p}_j^i \int_0^n (c_j^{i'})^{\rho_j} di'}{\nu E_j} \right]^{\frac{1}{\rho_j - 1}} \quad d_j = \frac{(1 - \nu) E_j}{q_j}. \quad (14)$$

Using this equation and the definition of  $C_j$  given by equation (3), utility maximizing consumption levels must satisfy

$$C_j = \nu^\nu (1 - \nu)^{1-\nu} \left[ \int_0^n (\tilde{p}_j^i)^{\frac{\rho_j}{\rho_j - 1}} di \right]^{\frac{\nu(1-\rho_j)}{\rho_j}} q_j^{\nu-1} E_j. \quad (15)$$

This equation allows us to determine consumption and price indices for country  $j$  as follows:

$$C_j P_j = E_j, \quad (16)$$

where the consumption index,  $C_j$ , is given by equation (3) and the price index,  $P_j$ , is given by

$$P_j \equiv \frac{\left[ \int_0^n (\tilde{p}_j^i)^{\frac{\rho_j}{\rho_j - 1}} di \right]^{\frac{\nu(\rho_j - 1)}{\rho_j}} q_j^{1-\nu}}{\nu^\nu (1 - \nu)^{1-\nu}}. \quad (17)$$

We use these price indexes to construct real exchange rates in the model.

In the first stage, consumer  $j$  faces the following dynamic optimization problem:

$$\max_{E_{jt}; x_{jt}; l_{jt}; b_j(S_{t+1}); M_{jt}} \mathbf{E}_o \sum_{t=0}^{\infty} \beta^t U((E_{jt}/P_{jt}), l_{jt}, (M_{jt}/P_{jt})),$$

subject to (14.1) or (14.2) with  $\int_0^n \tilde{p}_{jt}^i c_{jt}^i di + q_{jt} d_{jt} = E_{jt}$

$$k_{jt+1} = (1 - \delta)k_{jt} + (x_{jt}/k_{jt})^\psi k_{jt}.$$

In solving this problem, the consumer takes prices, profits, and the aggregate labor supply and capital stock as given. The first-order conditions which characterize a solution to this problem are given in the appendix.

We now turn to the differentiated goods sector and drop time subscripts for presentation purposes. The technology for production of differentiated goods gives rise to the following cost function for producer  $i$  located in country  $j$ :

$$C^m(w_j, r_j, m^i) = Ar_j^\alpha w_j^{1-\alpha} \left[ \frac{m^i + \phi}{z_j^m} \right], \quad (18)$$

where  $A \equiv (\alpha/(1 - \alpha))^{-\alpha} + (\alpha/(1 - \alpha))^{1-\alpha}$  and  $i \in [0, .5n]$  for  $j = 1$  and  $i \in (.5n, n]$  for  $j = 2$ . Firms in this sector choose how much to supply to each country to maximize period by period profits, taking into account the demand functions given by equation (15). Let  $m_j^i$  denote sales in country  $j$  of producer  $i \in [0, n]$ . The problem for firm  $i \in [0, .5n]$  located in country 1 is given by:

$$\begin{aligned} \max_{m_1^i, m_2^i} \quad & p_1^i m_1^i + p_2^i m_2^i - C^m(w_1, r_1, m_1^i + m_2^i) \\ \text{subject to} \quad & \tilde{p}_j^i = \frac{(m_j^i)^{\rho_j - 1} \nu E_j}{\int_0^n m_j^{i' \rho_j} di'}. \end{aligned}$$

In solving this problem, the firm takes the denominator in the demand function as given. The solution to this problem is the familiar markup pricing rule given by:

$$p_j^i = \left( \frac{1}{\rho_j} \right) \left( \frac{Ar_1^\alpha w_1^{1-\alpha}}{z_1^m} \right) \quad (19)$$

Differentiated firms located in country 2 face a symmetric problem.

These equations imply that the economy will be characterized by a symmetric equilibrium in which all firms within a country have equal sales and prices in a particular market. Hence we let  $m_j^1 \equiv m_j^i$  and  $p_j^1 \equiv p_j^i$  for  $i \in [0, .5n]$  denote the sales and price respectively in country  $j$  of a representative firm located in country 1 and let  $m_j^2 \equiv m_j^i$  and  $p_j^2 \equiv p_j^i$  for  $i \in (.5n, n]$  denote the sales and price respectively in country  $j$  of a representative firm located in country 2.

We now define the relative price series which are the focus of the paper for our theoretical economy. It is important to note first that in the presence of preference shocks, the economy will

violate the law of one price for differentiated goods in that the same good will be sold for different prices in different locations. This can easily be seen from equation (19) which implies the following:

$$\frac{p_{2t}^1}{p_{1t}^1} = \frac{p_{2t}^2}{p_{1t}^2} = \frac{\rho_{1t}}{\rho_{2t}}. \quad (20)$$

We now turn to the second relative price we consider. The real exchange rate is defined as the ratio of the two countries' price indices (consumption deflators) converted to a common currency:

$$RER_t \equiv \frac{e_t P_{2t}}{P_{1t}} = \frac{\left[ .5n \left( p_{2t}^1 \frac{\rho_{2t}}{\rho_{2t}^{\rho_{2t}-1}} + (e_t p_{2t}^2) \frac{\rho_{2t}}{\rho_{2t}^{\rho_{2t}-1}} \right) \right]^{\frac{\nu(\rho_{2t}-1)}{\rho_{2t}}}}{\left[ .5n \left( p_{1t}^1 \frac{\rho_{1t}}{\rho_{1t}^{\rho_{1t}-1}} + (e_t p_{1t}^2) \frac{\rho_{1t}}{\rho_{1t}^{\rho_{1t}-1}} \right) \right]^{\frac{\nu(\rho_{1t}-1)}{\rho_{1t}}}}, \quad (21)$$

where we have used the equilibrium condition that the homogenous good satisfies the law of one price; i.e.  $e_t q_{2t} = q_{1t} \forall t$ . This expression makes it clear that in the absence of differences in preferences across countries, the real exchange rate will be constant and equal to one and absolute and relative purchasing power parity will hold. In the presence of preference shocks, however, the economy will generate movements in the real exchange rate and deviations from both absolute and relative PPP. It should also be noted that if preferences were non-stochastic but differed across countries (i.e.  $\rho_{1t} = \bar{\rho}_1 \neq \bar{\rho}_2 = \rho_{2t} \forall t$ ), then the real exchange rate would vary in response to technology shocks alone and both absolute and relative purchasing power parity would fail to hold.

Finally, we examine the bilateral terms of trade between these two countries. To do so, we construct export and import price indices for each country and define the terms of trade (for country 1) to be the ratio of country 1's import price index to its export price index. Here we demonstrate our method for constructing Laspeyres (fixed weight) export and import price indices. We cannot use steady-state values as weights as there is no trade of the homogeneous good in the steady state. Instead, in our simulations we simulate our economy for a long time horizon and take average relative quantities of exports and imports of the homogeneous good over that simulation as our weights. Let this average share of homogeneous goods in exports be denoted by  $\omega_E$  and the average share of homogeneous goods in imports be denoted by  $\omega_I$ . We let prices in the steady state be denoted with a bar. Then the Laspeyres export price index for country 1 at time  $t$  is given by

$$EP_t = \omega_E \left( \frac{e_t q_{2t}}{\bar{e} \bar{q}_2} \right) + (1 - \omega_E) \left( \frac{p_{2t}^1}{\bar{p}_2^1} \right) \quad (22.1)$$

while the import price index is

$$IP_t = \omega_I \left( \frac{q_{1t}}{\bar{q}_1} \right) + (1 - \omega_I) \left( \frac{e_t p_{1t}^2}{\bar{e} \bar{p}_1^2} \right) \quad (22.2)$$

The terms of trade for country 1 at time  $t$  are defined as follows:

$$TOT_t = \frac{IP_t}{EP_t}. \quad (23)$$

We seek to determine the qualitative and quantitative properties of these relative prices in this economy. To do so, we simulate the economy using the methods described by Blanchard and Kahn (1980) and compute average moments over the simulations.

#### 4. Calibration

The parameter values used for the benchmark economy are provided in Table 2. Whenever possible, parameter values were taken directly from Chari, Kehoe, and McGratten (1998) to facilitate comparison with their study and with the data. These include the parameters of the period utility function:  $\beta$ ,  $\gamma$ ,  $a$ ,  $\lambda$ , and  $\sigma$ ; the depreciation rate:  $\delta$ ; and the parameters governing the process for the growth rate of the money stock:  $\bar{\mu}$ ,  $\rho_\mu$ , and  $\sigma_\mu$ .

To choose values for capital shares in the production technologies, parameters for the technology shock processes, and the share of expenditures devoted to the homogeneous good, it is necessary to divide the economy into two sectors. To do this, we take the following approach. As a proxy for the differentiated sector, we use the manufacturing industry. We make this choice because many manufacturing industries are subject to increasing returns to scale and are imperfectly competitive to some degree. We use all remaining industries as a proxy for the homogeneous, perfectly competitive sector.

Using consumption data for Canada and the U.S. taken from the OECD quarterly national accounts for 1981-1996 and using the sector classification described above, we compute the share of differentiated goods in total consumption to be 44% for Canada and 39% for the U.S.. Hence in our benchmark case we use  $\nu = .42$  to approximate the share of expenditures devoted to goods produced in the non-competitive sector.

Regarding technology parameters, we compute average labor compensation for Canada and the U.S. using data from the OECD for 1981-1995 for the sectors defined above. Doing so yields capital share parameters of  $\theta = .45$  in the homogeneous sector and  $\alpha = .33$  in the differentiated goods sector for Canadian data and  $\theta = .50$  and  $\alpha = .32$  for U.S. data. Hence, in our benchmark simulations we use  $\theta = .48$  and  $\alpha = .32$ . The large value for capital share in the homogeneous sector results primarily from the inclusion of agriculture in that sector and we provide sensitivity analysis with respect to these parameter. The value of the fixed cost parameter  $\phi$  does not affect the quantitative results, and so, without loss of generality, we set it equal to 1.00. The adjustment

cost parameter for capital accumulation is set so that the volatility of investment relative to GDP in all cases considered is broadly consistent with the data.

For the technology shock process, we first construct Solow residuals for Canada for both the homogeneous and differentiated goods sectors. We construct this residual for each sector by taking the log of real GDP in a sector less employment in that sector multiplied by labor share in that sector. We linearly detrend the measured Solow residual and estimate by least squares the parameters of equation (7). This gives us our persistence parameters given by matrix  $\Gamma$  and our variance-covariance matrix  $V^z$ . As capital stock data was unavailable at this level of disaggregation, we did not use it in our estimate of the Solow residual. Unfortunately, quarterly GDP by sector is unavailable for the U.S., making it impossible to estimate feedback effects and/or the degree of correlations in innovations between the U.S. and Canada. Instead, we did not include feedback effects in the technology shocks across countries but we chose the correlation between the countries' innovations so that the correlation of real GDP across countries in the simulations was approximately consistent with the observed correlation. This is similar to the approach taken in Chari, Kehoe, and McGratten (2000) who chose the correlation in innovations to the money supplies so as to match the observed correlation in outputs.

The parameters of the process for the shocks which affect preferences given in (4.2) are calibrated so that the volatility of the real exchange rate relative to output is consistent with the data. This implies a standard deviation for  $v$  equal to .029 in the benchmark case. In addition, we set the diagonal values in the persistence matrix,  $\Lambda$  at .9 to generate persistence in the real exchange rate. We then examine the effect of this calibration on other moments of interest such as the variability of the terms of trade and consumption.

## 5. Results

We now examine the quantitative properties of the theoretical economy. Tables 3–4 present the results of numerical experiments. In those tables  $GDP$ ,  $C$ , and  $X$  refer to real GDP, consumption and investment, respectively, where the real series are obtained by evaluating relevant quantities at steady state prices. In addition,  $NX$  refers to the ratio of nominal net exports to nominal GNP.

We present results for a benchmark economy with parameters chosen according to the calibration described in the previous section. We also present an economy with no monetary shocks to determine the effect of movements in the money supply on relative prices in this model. For comparison purposes, the tables also give results for an economy characterized by constant returns to scale, perfect competition, and no preference shocks. In that economy there are three traded goods:

a homogeneous good produced by both countries, a differentiated good produced by country 1, and a differentiated good produced by country 2. The homogeneous good is used for consumption and investment purposes while the differentiated goods are used solely for consumption. All goods are priced at marginal cost. The parameters for that economy are as given in Table 2 except  $\phi = 0$  and  $\sigma_v = 0$ . The results for this economy are presented to clarify the effects of multiple traded goods and traditional comparative advantage on the properties of aggregate variables. This economy is labeled *Perfect Competition* in the tables.

Table 3 presents properties of the relative price series which are the primary focus of this study. That table documents the relative volatility of these variables, their cyclical properties, and their relationship with other aggregates. Our primary objective is to examine the effects of multiple traded goods (and traditional comparative advantage), monopolistic competition, and price discrimination across national markets on the properties of these international relative prices.

We first examine the behavior of the real exchange rate in our theoretical economy. Recall that the benchmark economy allows for variability in the preferences of consumers across national markets so that the volatility of the real exchange rate relative to GDP is broadly consistent with the data. The results of Table 4 indicate that when we allow for this level of variability in preferences, the volatility of consumption relative to output is consistent with the data. Hence, the incorporation of time-varying differences in national markets to rationalize movements in the real exchange rate does not lead to unreasonable volatility in other aggregates of interest. The model generates persistent movements in the real exchange rate but of a lower degree than that observed in the data. As the tables demonstrate, the elimination of monetary shocks has no effect on the moments of any real variables in the model and, in particular, does not affect the variability of the real exchange rate.

As was suggested in the introduction, the two most common explanations for deviations from PPP resulting from deviations from traded goods prices across countries are national markets and sticky prices. Chari, Kehoe, and McGratten (CKM) (1998,2000) examined real exchange rate movements in the presence of sticky prices and local currency pricing. They consider a model with the same utility function as we have used here (except with no preference shocks), the same forcing process for monetary shocks, and 6 quarters of price stickiness. The ratio of the standard deviation of the real exchange rate to the standard deviation of GDP in their benchmark economy was .48. A striking, but unsurprising, contrast between the model considered here with active price discrimination and the model with sticky prices considered by CKM is that our model does not generate a high degree of correlation between the real and nominal exchange rate. Sticky prices

will naturally generate a high positive correlation between these variables. These results suggest that a model with both national markets and sticky prices (or, perhaps, wages) may better explain the behavior of the real exchange rate.

The table also indicates that the model is capable of producing movements in the relative prices of individual goods which are broadly consistent with the evidence presented in Engel (1993). As discussed in Section 2, Engel's evidence indicated that the volatility of prices of similar goods across countries is generally greater than the volatility of different prices of goods sold within a country. In Table 3, we present the volatility of relative prices across countries measured by the ratio  $p_{1t}^1/p_{2t}^1$ . This is the ratio of the prices charged by a producer located in country 1 for goods sold in country 1 relative to goods sold in country 2, all in currency 1. We also present the volatility of relative prices across producers measured by the ratio  $p_{1t}^1/(e_t p_{1t}^2)$ . This is the ratio of the price charged by a producer located in country 1 for goods sold in country 1 relative to the price charged by a producer located in country 2 for goods sold in country 1, all in currency 1. The table indicates that for the cases with preference shocks, the relative price across countries is more variable than the relative price across producers, which is consistent with Engel's evidence.

We now turn to the behavior of the terms of trade. Table 3 indicates that the volatility of the terms of trade in the benchmark version of this economy is broadly consistent with the data and considerably more volatile relative to output than in Backus, Kehoe, and Kydland (1995,1994) and Stockman and Tesar (1995). In comparing the results of the benchmark economy with that of the economy characterized by no preference shocks or by perfect competition, one might conclude that the volatility of the terms of trade results only from the preference shocks and not from the multi-sector nature of the economy as we had previously speculated. However, this results from the fact that with this calibration there is very little trade of the homogeneous good across countries in this model. Indeed, the average share of exports (or imports) attributable to the homogeneous good in any period is only 3% (i.e.  $\omega_E = \omega_I = .03$  in equations (22.1)-(22.2)). Hence, as can be seen from those equations, nearly all of the movement in the terms of trade is due to movements in the relative marginal costs of differentiated producers located in the two countries. Indeed, Table 3 demonstrates this result: the variability in the terms of trade is virtually the same as the variability in the relative price across producers in the economy with no preference shocks.

In simulations of the economy characterized by perfect competition and with technology shocks to the homogeneous sector calibrated as in Backus, Kehoe, and Kydland (1994), the average share of trade in the homogeneous good is approximately 10% and the volatility of the terms of trade is more than *four* times that of output. This contrasts with a volatility in the terms of trade equal

to approximately one-third of output in the BKK (1994) benchmark economy. This difference is primarily attributable to the multi-sector nature of the economy studied here. Specifically, in this economy technology shocks alter the pattern of comparative advantage across countries, causing countries to reallocate resources across sectors. This reallocation, in turn, results in larger movements in relative prices than in models where countries specialize in a single traded good and where such reallocation is not possible.

Table 3 also shows that the terms of trade is negatively correlated with GDP in this model which is consistent with the data but in contrast to the properties of the single-sector model examined by Backus, Kehoe, and Kydland (BKK) (1994) who predict a positive correlation. Also, the terms of trade and net exports are positively contemporaneously correlated in this economy. This is also in contrast to the findings of BKK (1994) but consistent with the data. When we examine the auto-correlation structure between the terms of trade and net exports, however, the model does not appear generate an S-shaped function as in BKK (1994), although the function is asymmetric about zero. This is interesting because the intuition that BKK put forward for the S-shape function their economy generates relates to the dynamics of capital formation in a single sector economy. In particular, they focus on the reallocation of resources across countries (via investment) following a positive productivity shock to one country. Again, the key difference between our economy and their economy is that here each country produces more than one traded good. This implies that in our economy, a shock to a sector causes reallocations within as well as across countries, whereas in their economy, a shock causes a reallocation only across countries. This difference between the two environments generates very different responses in the trade balance in response to shocks and explains the variation in results across the two economies. Our results suggest that in an environment with multiple traded sectors, we need to look beyond the dynamics of capital formation to better understand the dynamic relationships between the terms of trade and net exports.

Another moment to consider which has received attention recently is the correlation between the terms of trade and the nominal exchange rate. Obstfeld and Rogoff (2000) have criticized sticky price models with local currency pricing because those models will generate a negative correlation between these two variables, whereas the data indicates a positive relationship. The reason for a negative correlation in those models is straightforward. The terms of trade under local currency pricing will be the ratio of a the price of a country's imports to the price of its exports multiplied by the nominal exchange rate. If prices do not move, then a rise in the nominal exchange rate must cause this ratio to fall and the nominal exchange rate and the terms of trade will be

negatively correlated. In the model considered in this paper with producer currency pricing and fully flexible prices, there is no a priori reason to suggest that these two variables will be negatively (or positively) correlated. The results of the numerical experiments indicate that in the benchmark economy, these two variables are virtually uncorrelated. This is still not consistent with the data which suggests a positive correlation between these variables but may indicate a promising direction for future research.

Lastly, we turn to the prediction of the model for other aggregates. Here we focus on the anomalies of the model. As Table 4 indicates investment is negatively correlated across countries. This reflects that property of the model that resources transfer to the country which is relatively more productive or which has a higher growth rate of the money supply. In contrast to single sector models, however, there is also transfer of resources across sectors within a country which mitigates these effects. So in comparing with the results of the model of Backus, Kehoe, Kydland (1995), we see that these aggregates are not as negatively correlated as in their model. We should also note, however, that in our calibration, we have allowed no feedback in technology or monetary shocks and we have set the correlations in innovations to technology shocks so that the correlation of real GDP across countries is broadly consistent with the data. As noted previously, more careful calibration of the relationship between technology shocks across sectors and countries is needed. Similar to many open economy real business cycle models with complete markets, this model produces consumption correlations which are considerably higher than GDP correlations which is inconsistent with the data. Our prior intuition suggested that the presence of preference shocks might be helpful in resolving this puzzle but the quantitative results clearly indicate that preferences are not variable enough to overcome the risk-sharing behavior in the economy which leads to high consumption correlations.

## 6. Conclusions and Extensions

In this paper, we provide evidence regarding the properties of bilateral relative price series between the U.S. and Canada. Consistent with what other authors have found, the data suggests that the bilateral terms of trade and nominal and real exchange rates are all persistent and variable. We present a multi-sector model with elements of monopolistic competition and price discrimination which is capable of generating variability in these relative prices. We also find that traditional comparative advantage effects (generally absent from existing open economy macroeconomic models) play a central role in generating some of the cyclical properties of the economy. It is important to note that these results are generated in an environment with prices set in the exporter's currency

and fully flexible prices. This addresses some of the criticisms raised by Obstfeld and Rogoff (2000) of a class of models which examines real exchange rate variability.

While our results clearly point to the importance of imperfect competition, price discrimination, and static comparative advantage in accounting for certain features of relative prices, there are aspects of our model that are not satisfactory. Perhaps most important is that the model produces less persistence in the real exchange rate than what is observed in the data and that real and nominal exchange rates are virtually uncorrelated in the benchmark economy. This clearly results from the fact that preference shocks are the only force behind generating deviations from the law of one price across countries. One conclusion is that this type of price discrimination should be coupled with nominal price rigidities to address the behavior real and nominal exchange rates. Another conclusion is that this form of imperfect competition which has received considerable attention in the international and growth literature is not adequate for studying international relative price movements. In contrast, a model in which firms produce differentiated products but interact *strategically* will produce deviations from the law of one price across countries in the presence of technology shocks alone and may be better suited for exploring variability in the real exchange rate.

## Footnotes

1. Furthermore, the multiple traded good structure allows the model to account for observed trade patterns which include both inter- and intra-industry trade (which have been studied extensively in static models of international trade incorporating increasing returns).
2. This modeling approach should be thought of as a reduced form for capturing changes in the pricing behavior of producers due to changes in demand, competitive conditions, local distribution costs, regulations, etc.

## Appendix I: Data Sources

### 1. *Terms of Trade*

The Laspeyres export (import) price indices for exports from (to) the U.S. to (from) Canada used to construct the terms of trade reported in Table 1 are at quarterly frequencies from 1981.1-1999.1, in base year 1986, and were taken from the CANSIM data base.

### 2. *Real Exchange Rate*

The Canadian and U.S. consumption deflators were constructed by taking the ratio of nominal to real (1996 prices) personal expenditures on goods and services at quarterly frequencies from 1981.1-1999.1 and were taken from the OECD quarterly accounts. The spot exchange rate is the Canadian/U.S. exchange rate at quarterly frequencies from 1981.1-1999.1, taken from the CANSIM data base.

The Canadian (base 1992) and U.S. CPIs (base 1982-84) were taken from the CANSIM data base.

### 3. *Real GDP*

Canadian real GDP (1992 prices) at quarterly frequencies from 1981.1-1999.1 was taken from the OECD quarterly national accounts.

### 4. *Net Exports*

Canadian Total exports to the U.S. and total imports from the U.S. in current dollars at quarterly frequencies from 1981.1-1999.1 was taken from the CANSIM data base. Canadian nominal GNP was obtained from the OECD quarterly national accounts.

### 5. *Consumption Shares*

The share of homogeneous goods in consumer expenditure,  $\nu$ , was calibrated using Canadian and U.S. nominal disaggregated private final consumption expenditure data at annual frequencies from 1981-1996 and was taken from the OECD National Accounts, Detailed Tables.

### 6. *Capital Shares*

Capital shares in each sector,  $\theta$  and  $\alpha$ , were calibrated using Canadian and U.S. Employee Compensation data at annual frequencies from 1981-1996 and was taken from the OECD National Accounts, Detailed Tables.

### 7. *Technology Shock Processes*

Solow residuals for each sector were constructed using Canadian real GDP at factor cost and number of employees at the industry level at quarterly frequencies from 1978-1997 and were taken from the CANSIM data base.

## Appendix II: Consumers' First-Order Conditions

Let the derivative of the utility function in country  $j$  at time  $t$  with respect to consumption be denoted  $U_{C_{jt}}$ , with respect to labor be denoted  $U_{l_{jt}}$ , and with respect to real money balances be denoted  $U_{m_{jt}}$ . Then, by substituting the constraints into the consumer's utility function, we derive the following first order conditions for a consumer in country  $j$ :

$$\left(\frac{U_{C_{jt}}}{P_{jt}}\right)w_{jt} + U_{l_{jt}} = 0 \quad (II.1)$$

$$\left(\frac{U_{C_{jt}}}{P_{jt}}\right)\left(\frac{-q_{jt}}{\psi}\right)\left(\frac{x_{jt}}{k_{jt}}\right)^{1-\psi} \quad (II.2)$$

$$+ \beta \mathbf{E}_t \left(\frac{U_{C_{jt+1}}}{P_{jt+1}}\right) \left[ r_{jt+1} - \frac{q_{jt+1}x_{jt+1}}{k_{jt+1}} + \left(\frac{q_{jt+1}}{\psi}\right)\left(\frac{x_{jt+1}}{k_{jt+1}}\right)^{1-\psi} \left[ \left(\frac{x_{jt+1}}{k_{jt+1}}\right)^\psi + 1 - \delta \right] \right] = 0$$

$$\frac{U_{m_{jt}} - U_{C_{jt}}}{P_{jt}} + \beta \mathbf{E}_t \frac{U_{C_{jt+1}}}{P_{jt+1}} = 0 \quad (II.3)$$

In addition, the consumer in country 1 faces the following first order condition for holdings of contingent claims  $\forall S_{t+1}$ :

$$\left(\frac{-U_{C_{1t}}}{P_{1t}}\right)v(S_{t+1}, S_t) + \beta \left(\frac{U_{C_{1t+1}}}{P_{1t+1}}\right)f(S_{t+1}, S_t) = 0, \quad (II.4)$$

while the consumer in country 2 faces

$$\left(\frac{-U_{C_{2t}}}{P_{2t}}\right)\left(\frac{v(S_{t+1}, S_t)}{e_t}\right) + \beta \left(\frac{U_{C_{2t+1}}}{P_{2t+1}}\right)\left(\frac{f(S_{t+1}, S_t)}{e_{t+1}}\right) = 0 \quad (II.5)$$

Combining equations (II.4) and (II.5), we derive

$$\left(\frac{U_{C_{1t}}}{U_{C_{1t+1}}}\right)\left(\frac{P_{1t+1}}{P_{1t}}\right) = \left(\frac{U_{C_{2t}}}{U_{C_{2t+1}}}\right)\left(\frac{P_{2t+1}}{P_{2t}}\right)\left(\frac{e_{t+1}}{e_t}\right) \quad \forall t. \quad (II.6)$$

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**TABLE 1: Canada/US Exchange Rates and Bilateral Terms of Trade**

<b>Moment</b>	<b>Nominal Exchange Rate</b>	<b>Real Exchange Rate Using Deflators</b>	<b>Real Exchange Rate Using CPIs</b>	<b>Terms of Trade</b>
<b>Std. Deviation</b>	3.12	3.24	3.39	1.77
<b>Std. Deviation Relative to GDP</b>	1.79	1.86	1.95	1.02
<b>Autocorrelation</b>	.88	.89	.89	.76
<b>Correlation with GDP</b>	.11	.25	.27	-.24
<b>Correlation with Net Exports</b>	.25	.22	.23	.15
<b>Correlation with Nominal Exchange Rate</b>	1.00	.98	.97	.63

Notes: Data is quarterly from 1981.1-1999.1. All data is logged except net exports and moments are computed from Hodrick-Prescott filtered data. The Real Exchange Rate using Deflators refers to the ratio of the US Consumer Price Deflator (base 1992) multiplied by the spot nominal exchange rate between the US and Canadian dollar and the Canadian Consumer Price Deflator (base 1992). Nominal and real consumption expenditures used to construct these deflators was obtained from the OECD quarterly accounts. The nominal exchange was obtained from Cansim. The Real Exchange Rate using CPIs refers to the ratio of the US CPI (base 1982-1984) multiplied by the spot nominal exchange rate between the US and Canadian dollar and the Canadian CPI (base 1992). CPIs were obtained from Cansim. The Terms of Trade refers to the ratio of the Laspeyres Canadian import price index for imports from the US to the Laspeyres Canadian export price index for exports to the US, obtained from Cansim. Net Exports refers to the ratio of nominal net exports from Canada to the US (Canadian exports to the US minus imports from the US) to Canadian nominal GDP, obtained from Cansim.

**TABLE 2: Benchmark Parameter Values**

**Preferences:**

$$\beta = .99, a = .73, \lambda = -1.56, \gamma = .32, \sigma = 2, \nu = .42$$

**Technologies :**

$$\theta = .48, \alpha = .32, \phi = 1.00, \delta = .026, \psi = .99$$

**Shock Processes:**

*Technology Shocks:*

$$\bar{z}_y = \bar{z}_m = 1$$

$$\Gamma = \begin{bmatrix} .946 & 0 & 0 & 0 \\ 0 & .946 & 0 & 0 \\ 0 & 0 & .915 & 0 \\ 0 & 0 & 0 & .915 \end{bmatrix}$$

$$V^z = \begin{bmatrix} (.0055)^2 & 0 & 0 & 0 \\ 0 & (.0055)^2 & 0 & 0 \\ 0 & 0 & (.0165)^2 & 0 \\ 0 & 0 & 0 & (.0165)^2 \end{bmatrix}$$

$$\text{corr}(\epsilon_1^y, \epsilon_2^y) = .98 \quad \text{corr}(\epsilon_1^m, \epsilon_2^m) = .98$$

*Preference Shocks:*

$$\bar{v} = 2.197 \rightarrow \bar{\rho} = .9$$

$$\Lambda = \begin{bmatrix} .9 & 0 \\ 0 & .9 \end{bmatrix}$$

$$V^v = \begin{bmatrix} (.018)^2 & 0 \\ 0 & (.018)^2 \end{bmatrix}$$

*Money Growth Shocks:*

$$\bar{\mu} = 1.015$$

$$\Omega = \begin{bmatrix} .57 & 0 \\ 0 & .57 \end{bmatrix}$$

$$V^\mu = \begin{bmatrix} (.0092)^2 & 0 \\ 0 & (.0092)^2 \end{bmatrix}$$

**TABLE 3: Properties of Relative Prices in Theoretical Economies**

Moment	Data	Benchmark	No Monetary Shocks	No Preference Shocks	Perfect Competition
Standard Deviations Relative to GDP					
Nominal Exch. Rate	1.79	3.67 (.80)	.13 (.03)	3.67 (.81)	3.78 (.86)
Real Exch. Rate	1.86	1.85 (.40)	1.89 (.41)	.00 (.00)	.00 (.00)
Terms of Trade	1.02	1.08 (.23)	1.11 (.24)	.38 (.08)	.47 (.10)
Relative Price Across Countries	–	1.13 (.24)	1.15 (.25)	.00 (.00)	.00 (.00)
Relative Price Within a Country	–	.34 (.07)	.35 (.08)	.39 (.09)	.48 (.10)
Autocorrelations					
Nominal Exch. Rate	.88	.70 (.08)	.65 (.09)	.70 (.09)	.70 (.09)
Real Exch. Rate	.89	.65 (.09)	.63 (.09)	– (–)	– (–)
Terms of Trade	.76	.65 (.09)	.65 (.09)	.66 (.09)	.66 (.09)
Correlation with GDP					
Nominal Exch. Rate	.11	-.02 (.21)	.12 (.20)	.00 (.22)	-.02 (.22)
Real Exch. Rate	.25	.15 (.21)	.15 (.20)	– (–)	– (–)
Terms of Trade	-.24	-.11 (.21)	-.11 (.21)	.14 (.21)	.11 (.21)

**TABLE 3: Properties of Relative Prices in Theoretical Economies (Continued)**

Moment	Data	Benchmark	No Monetary Shocks	No Preference Shocks	Perfect Competition
Correlation with Net Exports/GDP					
Nominal Exch. Rate	.25	.00 (.21)	-.15 (.20)	.03 (.20)	.03 (.21)
Real Exch. Rate	.22	-.21 (.20)	-.20 (.20)	- (-)	- (-)
Terms of Trade	.15	.39 (.16)	.38 (.16)	.67 (.11)	.70 (.10)
Correlation with Nominal Exch. Rate					
Real Exch. Rate	.98	.03 (.21)	.97 (.01)	- (-)	- (-)
Terms of Trade	.63	-.04 (.21)	-.93 (.03)	.00 (.21)	-.01 (.22)

**TABLE 4: Business Cycle Properties in Theoretical Economies**

Moment	Data	Benchmark	No Monetary Shocks	No Preference Shocks	Perfect Competition
Standard Deviation of GDP	1.74	1.01 (.15)	1.00 (.15)	1.00 (.15)	.98 (.15)
Standard Deviations Relative to GDP					
Consumption	.79	.78 (.10)	.78 (.10)	.78 (.11)	.79 (.11)
Investment	3.28	3.31 (.58)	3.31 (.56)	3.24 (.59)	3.24 (.58)
Employment	.72	.51 (.06)	.51 (.06)	.40 (.05)	.44 (.04)
Autocorrelations					
GDP	.87	.66 (.09)	.66 (.09)	.66 (.09)	.67 (.09)
Consumption	.85	.65 (.10)	.65 (.09)	.65 (.09)	.66 (.09)
Investment	.95	.64 (.10)	.64 (.10)	.64 (.10)	.65 (.10)
Employment	.91	.66 (.09)	.66 (.09)	.66 (.09)	.66 (.09)
Net Exports/GDP	.88	.62 (.10)	.61 (.10)	.62 (.10)	.63 (.09)
Cross Correlations					
GDP Across Countries	.88	.87 (.06)	.87 (.05)	.90 (.04)	.89 (.05)
Consumption Across Countries	.78	.97 (.01)	.98 (.01)	.98 (.01)	.98 (.01)
Investment Across Countries	-.01	-.16 (.20)	-.17 (.20)	-.11 (.21)	-.09 (.20)
Employment Across Countries	.53	.43 (.18)	.43 (.18)	.74 (.11)	.61 (.14)
Net Exports/GDP and GDP	-.36	.05 (.21)	.06 (.20)	.08 (.20)	.07 (.20)

*Notes to Tables 3-4:* Data statistics reported in Table 3 are taken from Table 1. Some data statistics reported in Table 4 are taken from Backus, Kehoe, and Kydland (1995) for Canada from 1970.1-1990.2. Statistics for the theoretical economy are based on Hodrick-Prescott filtered data and are averages over 1000 simulations of length 73 (the same number of quarters as in the data for statistics on relative prices.) Standard deviations of the moments over the 1000 trials are in parentheses. Except for the ratio of net exports to GDP, statistics refer to logarithms of variables.