International Relative Prices in the Presence of Deviations from the Laws of One Price

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Abstract

We explore the possibility that monopolistic competition, price discrimination, and multiple traded goods within a county may account for the following features of international price data: persistent and highly variable movements in the terms of trade; variability in the real exchange rate (*i.e.* deviations from relative purchasing power parity); and persistent, time-varying deviations from the Law of One Price across countries and across producers. We first document the properties of these relative price series using bilateral data for the United States and Canada. We then present a dynamic, general equilibrium, two-sector economy with both intra- and inter-industry trade and explore the model's qualitative and quantitative implications. In particular, we show that price discrimination is capable of generating persistent movements in each of the relative price series we consider. Furthermore, we demonstrate that traditional comparative advantage effects arising from the two-sector structure of the economy are essential in accounting for many of the model's cyclical properties.

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I. Introduction

This paper examines the role of international price discrimination and sectorial reallocation in explaining the behavior of real exchange rates and the terms of trade. The focus on price discrimination and real exchange rate movements is primarily motivated by the empirical work of Engel (1998,1993), Engel and Rogers (1998,1996), Goldberg and Knetter (1997), Rogers and Jenkins (1995), and others. In particular, Engel and Rogers' work suggests that movements in the relative prices of traded goods appears to account for nearly all of the movement in U.S. real exchange rates. They also conclude that the two most important explanations for movements in the relative prices of traded goods between the U.S. and Canada is pricing to market and nominal price stickiness. Furthermore, they argue that nominal price stickiness cannot account for all of the movement in these prices (i.e. national markets would lead to deviations in prices across these two countries even in the absence of price rigidities).[1]

The focus on sectorial reallocation resulting from the incorporation of multiple traded goods within a country and its effect on the behavior of international prices is motivated by intuition suggesting that resource reallocation within a country in response to changing patterns of comparative advantage could have significant effects on relative price movements. Most of the recent open economy macroeconomic literature which examines the variability and persistence of the terms of trade generally assumes that a country produces only one traded good (and, perhaps, a non-traded good). Examples include Backus, Kehoe, and Kydland (1994), Backus and Crucini (1998), Boileau (1999), and Stockman and Tesar (1995). The assumption of a single traded sector within a country, however, stands in stark contrast to the pure theory of international trade and does not allow for traditional comparative advantage effects to influence relative price movements. In the economy studied in this paper, the incorporation of two traded sectors in an explicitly dynamic, stochastic environment allows for an examination of the effects of shocks on the pattern of comparative advantage across countries, resource reallocation, and relative price movements.[2]

Before presenting the theoretical economy, we document the properties of four relative price series of interest using bilateral price data for the United States and Canada. Our empirical results are consistent with what many others have found regarding international prices. In particular,

[1] Chari, Kehoe, and McGratten (1998) examine the role of nominal price rigidities in explaining the behavior of the real exchange rate.

[2] Furthermore, the multiple traded good structure allows the model to account for observed trade patterns which include both inter- and intra-industry trade (which have been studied extensively in static models of international trade incorporating increasing returns).

movements in the terms of trade and the real exchange rate are highly variable and persistent. Our data also suggests variable and persistent movements in the relative price of traded goods. This is consistent with the empirical literature documenting deviations from the Law of One Price (which is surveyed by Goldberg and Knetter (1997)).

As suggested above, due to the presence of price discrimination and multiple traded goods within a country, the theoretical economy we explore is capable of generating variability in each of these relative price series. The model's quantitative predictions also include several interesting features, some of which are in striking contrast to previous studies. With regard to relative price movements, the model exhibits excessive volatility in the terms of trade whereas existing macroeconomic models typically generate too little volatility in this relative price. In addition, relative to the data, the model exhibits too little volatility in the real exchange rate and in the relative price of the same good across countries. This is not surprising as we expect nominal price rigidities (which are not captured by this model), as well as price discrimination, to be a part of the explanation for the behavior of these relative price series.

The remainder of the paper is organized as follows. Section 2 presents the empirical evidence for bilateral price movements between the United States and Canada. Section 3 presents the theoretical economy while Section 4 discusses the model's implications for other international comovements. Section 5 discusses our calibration of the model and Section 6 presents the quantitative implications of the model for relative prices and other aggregate moments of interest. Section 7 concludes and discusses directions for future research.

2. Properties of International Relative Prices

In this section we present empirical evidence on four types of international relative prices using bilateral price data for the United States and Canada. We describe each relative price and its properties in the data. Data sources are given in the appendix.

2.1 Terms of Trade

An international relative price that has received attention recently in the open economy macroeconomics literature is the relative price of a country's imports to its exports, or its terms of trade. Backus, Kehoe and Kydland (BKK) (1995, 1994), for example, using aggregate implicit import and export price deflators, have documented persistent and highly variable movements in the terms of trade for a variety of countries.

We focus on the *bilateral* terms of trade between the U.S. and Canada. We define the terms of trade between the U.S. and Canada to be the ratio of the Canadian import price index for imports from the U.S. to the Canadian export price index for exports to the U.S. We consider two alternative measures of these price indexes: Paasche current or varying weighted price indices and Laspeyres fixed weighted price indices. Both series are in base year 1986 at quarterly frequencies from 1981.1-1996.4. We log and Hodrick-Prescott filter each ratio and compute statistics of deviations from trend. This filter is used widely in open economy macroeconomic studies and emphasizes the medium and high frequency movements in the data. We report relevant statistics for the terms of trade in Table 1. Not surprisingly, the table indicates that the terms of trade constructed using Paasche indices is more variable and less persistent than that constructed using Laspeyres indices but both exhibit considerable variability. In addition, both series are negatively correlated with Canadian output. In comparing this bilateral measure with the aggregate measure used by BKK, we see that the Paasche measure exhibits similar volatility to their measure but is much less persistent while the Laspeyres measure is considerably less volatile and somewhat less persistent than their measure. (It should also be noted that their statistics are computed over the time series from 1955-1990 whereas ours are from 1981-1996.)

We also examine the relationship between these countries' bilateral terms of trade and net exports. We measure net exports as the ratio of Canadian exports to the U.S. minus Canadian imports from the U.S. in current prices to Canadian GNP in current prices. This data suggests that contemporaneously, the bilateral terms of trade and net exports are positively correlated for these two countries over this time period. A further examination of the autocorrelation structure between these variables indicates little relation between these two variables and no evidence of an S-curve as BKK document for the U.S. and other countries. If anything, this data may suggest the opposite, i.e. a positive correlation between the bilateral terms of trade and lagged net exports and a negative correlation with future net exports.

2.2 Real Exchange Rate

The real exchange rate between countries can be measured in a variety of ways but is generally an indicator of the relationship between aggregate price indices across countries. One common measure of the real exchange rate between a domestic and foreign country is the ratio of the domestic country's consumer price index multiplied by the spot exchange rate to the foreign country's consumer price index. The real exchange rate is often used to measure deviations from purchasing power parity (PPP) across countries. In particular, relative PPP is said to hold when the real exchange rate is constant. In the presence of perfectly competitive markets, no non-traded goods, and no trade barriers, relative PPP is expected to hold. Many authors, however, (e.g. Isard (1977), Kravis and Lipsey (1978, 1977, 1974), Mussa (1986), and Rogoff (1996)) using various data sets and methods have presented evidence of large, persistent deviations from relative PPP.

We focus on properties of the real exchange rate between the United States and Canada and interpret movements in this rate as evidence of deviations from relative PPP. We define the real exchange rate between these two countries as the ratio of the Canadian consumption deflator to the U.S. consumption deflator multiplied by the spot exchange rate. We construct the real exchange rate using quarterly data from 1981.1-1996.4, log and Hodrick-Prescott filter the rate and then compute statistics of deviations from trend. The results are given in Table 1 and indicate that the real exchange rate over this time period has shown considerable variability, is highly autocorrelated, is weakly positively correlated with Canadian GNP, and is positively correlated with net exports. Hence, as many others have found, this data suggests the presence of persistent departures from relative PPP and implies that any theoretical model of relative international prices should be capable of generating deviations from PPP. The two most common explanations for observed deviations from PPP are deviations from the Law of One Price across countries and non-traded goods. We now turn to evidence of deviations from the Law of One Price between the U.S. and Canada.

2.3 Relative Price of the Same Good Across Countries

An examination of relative prices across countries of a particular good is a study of the Law of One Price across countries, or what is commonly termed pricing to market. The Law of One Price across countries simply states that in the presence of perfectly competitive markets and no trade barriers, the same good should sell for the same price in two countries (when measured in the same currency). As with PPP, many authors (e.g. Engel (1998,1993), Engel and Rogers (1998,1996), Goldberg and Knetter (1997), Isard (1977), Knetter (1993,1989), Lapham (1995), Marston (1990), and Protopadakis and Stoll (1983, 1986)) have documented persistent, time-varying deviations from the Law of One Price across countries.

We examine these relative prices between the U.S. and Canada for goods produced in Canada by studying the time-series properties of the ratio of Canadian industrial price indices to Canadian export price indices for exports to the U.S. at the highest level of industry disaggregation possible. The Canadian industrial price index is a disaggregated index of prices of Canadian produced goods sold in Canada *and* abroad while the Canadian export price index is a disaggregated index of prices of Canadian produced goods sold in the U.S. Movements in their ratio are interpreted as evidence of deviations from the Law of One Price between the U.S. and Canada. Industry matches for these two series were available for 25 industries, quarterly from 1981.1-1996.4. We construct the ratio for each industry, log, and Hodrick-Prescott filter the ratio and report statistics for deviations from trend in

Table 2. The percent standard deviation ranges from .75%–16.40% for this set of industries and the median industry is characterized by a standard deviation in the relative price which is twice that of Canadian GNP. The table also indicates that these relative prices are positively autocorrelated and positively correlated with Canadian GNP for most industries. Hence, this data suggests that a theoretical model which focuses on relative price movements between countries should be capable of generating variability in the relative price of the same good sold in different countries. In addition, these results also suggest that deviations from the Law of One Price may account for some of the deviations from PPP observed in the data. This is consistent with the findings of Engel (1998,1993), Engel and Rogers (1998,1996), and Rogers and Jenkins (1995) who conclude that movements in the prices of similar goods across borders appears to be more significant for explaining real exchange rate movements than are movements in the relative price of non-traded to traded goods.

2.4 Relative Price of Similar Goods Sold in the Same Country

An examination of relative prices of two similar goods produced in different countries but sold in the same country is a study of the Law of One Price across producers. The Law of One Price across producers states that in the presence of perfectly competitive markets and no trade barriers, goods in the same industry, sold in the same location, but produced by firms located in different countries should sell for the same price. Although this has received less attention in the empirical literature than PPP or the Law of One Price across countries, researchers (e.g. Isard (1977), Lapham (1995)) have documented persistent, time-varying deviations from the Law of One Price across producers.

We examine these relative prices between U.S. and Canadian producers for goods sold in Canada by studying the time-series properties of the ratio of Canadian industrial price indices to Canadian import price indices for imports from the U.S. at the highest level of industry disaggregation possible. Movements in this ratio are interpreted as evidence of deviations from the Law of One Price between U.S. and Canadian producers. Industry matches for these two series were available for 24 industries, quarterly from 1981.1-1996.4. We construct the ratio for each industry, log and Hodrick-Prescott filter the ratio, and report statistics for deviations from trend in Table 3. The percent standard deviation ranges from 1.80%–12.10% for this set of industries and the median industry is characterized by a standard deviation in the relative price which is nearly twice that of Canadian GNP. The table also indicates that these relative prices are positively autocorrelated. There is, however, no clear pattern of correlation between these relative prices and Canadian GNP for these industries. Thus, this data suggests that a theoretical model which focuses on relative price movements between countries should be capable of generating variability in the relative price of similar goods sold in the same country but produced in different countries.

We now summarize our findings regarding relative price movements between the U.S. and Canada. The data exhibits highly variable and persistent movements in the bilateral terms of trade between these two countries and a positive contemporaneous correlation with net exports. The data also exhibits persistent deviations from relative PPP and both versions of the Law of One Price between the U.S. and Canada. The empirical regularities reported here suggest that a theoretical model focusing on relative international price movements should be consistent with variability in all of these relative price series. We now present a model which is capable of generating such relative price variability, characterize its quantitative properties, and discuss its implications for other international comovements.

3. The Economy

We consider a two country, two sector economy. One sector produces a homogeneous good for consumption and investment under constant returns to scale while the other sector produces differentiated goods for consumption under increasing returns to scale. All goods are traded.

We begin with the specification of consumer preferences and endowments. We focus on a representative consumer in each country who is endowed with one unit of labor and capital. The preferences of the representative consumer in country $j \in \{1, 2\}$ are characterized by an expected utility function of the form

$$\mathrm{E}_o \sum_{t=0}^{\infty} \beta^t U(C_{jt}, l_{jt}), \tag{1}$$

where

$$U(C, l) = \frac{\left[C^{\gamma}(1-l)^{1-\gamma}\right]^{1-\sigma}}{1-\sigma} \qquad 0 < \gamma < 1, \quad \sigma > 0, \tag{2}$$

and

$$C_{jt} \equiv \left[\int_0^n (c^i_{jt})^{\rho_{jt}} di\right]^{\frac{\nu}{\rho_{jt}}} d_{jt}^{1-\nu} \quad 0 < \nu < 1, \tag{3}$$

and c^i_{jt} is consumption of differentiated good $i \in [0, n]$, d_{jt} is consumption of the homogeneous good, and l_{jt} is employment in country j. The specification of composite consumption given by equation (3) is widely used in models of industrial organization, international trade, and growth which incorporate vertically differentiated products. The period utility function given by equation (2) is consistent with the specification of preferences often used in open economy macroeconomic models.

Consumer preferences differ across countries with respect to the degree of substitution among differentiated products as governed by $\rho_{jt} \in (0, 1)$. This parameter is stochastic, representing taste shocks, and, as will be shown below, movements in this parameter are necessary to generate deviations from relative PPP and from the Law of One Price across countries. Let $\rho_t \equiv (\rho_{1t}, \rho_{2t})'$ and let $\hat{\rho}_t$ denote deviations in these parameters from their (same) mean, $\overline{\rho}$. These preference parameters are governed by the following stochastic process:

$$\hat{\rho}_{t+1} = \Lambda \hat{\rho}_t + \epsilon_t^{\rho}. \tag{4}$$

where $\epsilon^{\rho} = (\epsilon_1^{\rho}, \epsilon_2^{\rho})'$ is distributed normally and independently over time with variance V^{ρ}.

Production of the homogeneous good in country j is governed by the following constant returns to scale technology:

$$y_{jt} = z_{jt}^{y} (k_{jt}^{y})^{\theta} (l_{jt}^{y})^{1-\theta} \qquad 0 < \theta < 1, \tag{5}$$

where k_{jt}^{y} is capital input, l_{jt}^{y} is labor input, and z_{jt}^{y} is a technology shock. Meanwhile, each differentiated good is produced by a single producer and the market structure is monopolistic competition without entry. The world-wide measure of differentiated goods is fixed at n. A distinct set of differentiated goods is produced in each country and the measure of goods in each country is fixed at $.5n$; that is, each country produces half of the differentiated goods. Let $i \in [0, .5n]$ index the differentiated goods produced in country 1 and let $i \in (.5n, n]$ index the differentiated goods produced in country 2. Production of differentiated good i in country j is governed by the following increasing returns to scale technology:

$$m_t^i = z_{jt}^{m} (k_t^i)^{\alpha} (l_t^i)^{1-\alpha} - \phi \qquad 0 < \alpha < 1, \quad \phi > 0, \tag{6}$$

where k_t^i is capital input, l_t^i is labor input, and z_{jt}^{m} is a technology shock to the differentiated goods sector in country j. Firms pay a fixed cost of production each period given by ϕ. The Cobb-Douglas technologies specified here are widely used in open economy macroeconomic models.

Let $z_t \equiv (z_{1t}^{y}, z_{2t}^{y}, z_{1t}^{m}, z_{2t}^{m})'$ denote the vector of technology shocks and let $\hat{z}_t$ denote deviations from their means, $\bar{z}^y$ and $\bar{z}^m$. The technology shocks follow:

$$\hat{z}_{t+1} = \Gamma \hat{z}_t + \epsilon_t^{z}, \tag{7}$$

where $\epsilon^{z} = (\epsilon_1^{y}, \epsilon_2^{y}, \epsilon_1^{m}, \epsilon_2^{m})'$ is distributed normally and independently over time with variance V^{z}.

Capital formation incorporates adjustment costs and additions to capital require inputs of the homogeneous good. Let $k_{1t}^{m} = \int_0^{.5n} k_t^i di$ and $k_{2t}^{m} = \int_{.5n}^{n} k_t^i di$ denote the total capital stock employed in the differentiated goods sector in country 1 and country 2 respectively. Then the aggregate capital stock in country j, K_{jt}, must satisfy:

$$K_{jt} = k_{jt}^{y} + k_{jt}^{m}. \tag{8}$$

The law of motion for the aggregate capital stock in country j is given by

$$K_{jt+1} = (1-\delta)K_{jt} + (x_{jt}/K_{jt})^{\psi} K_{jt} \qquad\qquad 0 < \psi < 1, \tag{9}$$

where x_{jt} is investment of the homogeneous good in country j, δ is the depreciation rate and ψ determines the size of adjustment costs. It is well known that in open economy macroeconomic models, some mechanism such as capital adjustment costs is required to limit the volatility of investment flows across countries. For tractability and flexibility, we choose the specification given in equation (9).

The world resource constraint for the homogeneous good is

$$\sum_{j=1}^{2}(d_{jt} + x_{jt}) = \sum_{j=1}^{2} y_{jt}. \tag{10}$$

The world resource constraint for differentiated good i is

$$\sum_{j=1}^{2} c_{jt}^{i} = m_{t}^{i}. \tag{11}$$

Let q_t denote the price of the homogeneous good and let p_{jt}^{i} denote the price of differentiated good i sold in country j. Then the value of net exports by country j, nx_{jt} is given by

$$nx_{1t} = q_t(y_{1t} - d_{1t} - x_{1t}) + \int_{o}^{.5n} p_{2t}^{i} c_{2t}^{i} di - \int_{.5n}^{n} p_{1t}^{i} c_{1t}^{i} di \tag{12.1}$$

and

$$nx_{2t} = q_t(y_{2t} - d_{2t} - x_{2t}) + \int_{.5n}^{n} p_{1t}^{i} c_{1t}^{i} di - \int_{0}^{.5n} p_{2t}^{i} c_{2t}^{i} di. \tag{12.2}$$

An equilibrium in this economy consists of prices and allocations such that consumers and producers are optimizing and all markets clear. In constructing the equilibrium, we first examine the problems of the representative consumers in each country. Consumers have access to markets for a complete set of one-step-ahead contingent claims. These claims pay one unit of consumption good in the next period, contingent on the aggregate state at that time. The aggregate state for the world economy at time t is denoted S_t, and is given by

$$S_t \equiv (K_{1t}, K_{2t}, \rho_{1t}, \rho_{2t}, z_{1t}^{y}, z_{2t}^{y}, z_{1t}^{m}, z_{2t}^{m}). \tag{13}$$

Given our previous specifications, the state vector is Markov, with transition probability density $f(S_{t+1}, S_t)$. We let Σ denote the state space. Let $v(S_{t+1}, S_t)$ denote the price in state S_t of a contingent claim that pays off in state S_{t+1}.

The budget constraint for the consumer in country j at time t is given by

$$\int_0^n p_{jt}^i c_{jt}^i di + q_t d_{jt} + q_t x_{jt} + \int_\Sigma v(S_{t+1}, S_t) b_j(S_{t+1}) dS_{t+1} \leq w_{jt} l_{jt} + r_{jt} k_{jt} + \Pi_{jt} + q_t b_j(S_t), \quad (14)$$

where $b_j(S_t)$ denotes the contingent claims holdings for state S_t, k_{jt} denotes per capita capital stock, w_{jt} denotes the wage, r_{jt} denotes the rental price of capital, and Π_{jt} denotes profits from the differentiated good sector in country j. In equilibrium, the aggregate capital stock, K_{jt}, must equal the per-capita capital stock, k_{jt}, in country j.

The consumer in each country faces a two-stage maximization problem. In the first stage, consumers choose investment, leisure, contingent claim holdings, and expenditure on consumption, denoted $E_{jt} \equiv \int_0^n p_{jt}^i c_{jt}^i di + q_t d_{jt}$, to maximize their expected discounted lifetime utility as given in equation (1), taking prices and the aggregate capital stock and labor supply as given. In the second stage, consumers allocate their consumption expenditure across the differentiated goods and the homogeneous good. We first examine the second stage in which consumer j faces the following static maximization problem:

$$\max_{d_j;\, c_j^i,\, i\in[0,n]} \left[\int_0^n (c_j^i)^{\rho_j} di\right]^{\frac{\nu}{\rho_j}} d_j^{1-\nu}$$

$$\text{subject to} \quad \int_0^n p_j^i c_j^i di + (q) d_j = E_j,$$

where time subscripts have been dropped and prices and E_j are taken as given. The solution to this problem is characterized by the following:

$$c_j^i = \left[\frac{p_j^i \int_o^n (c_j^{i'})^{\rho_j} di'}{\nu E_j}\right]^{\frac{1}{\rho_j - 1}} \qquad\qquad d_j = \frac{(1-\nu)E_j}{q}. \quad (15)$$

Using this equation and the definition of C_j given by equation (3), utility maximizing consumption levels must satisfy

$$C_j = \nu^\nu (1-\nu)^{1-\nu} \left[\int_o^n (p_j^i)^{\frac{\rho_j}{\rho_j - 1}} di\right]^{\frac{\nu(1-\rho_j)}{\rho_j}} q^{\nu - 1} E_j. \quad (16)$$

This equation allows us to determine consumption and price indices for country j as follows:

$$C_j P_j = E_j, \quad (17)$$

where the consumption index, C_j, is given by equation (3) and the price index, P_j, is given by

$$P_j \equiv \frac{\left[\int_o^n (p_j^i)^{\frac{\rho_j}{\rho_j - 1}} di\right]^{\frac{\nu(\rho_j - 1)}{\rho_j}} q^{1-\nu}}{\nu^\nu (1-\nu)^{1-\nu}}. \quad (18)$$

In the first stage, consumer j faces the following dynamic optimization problem:

$$\max_{E_{jt};x_{jt};l_{jt};b_j(S_{t+1})} \mathrm{E_o} \sum_{t=0}^{\infty} \beta^t U((E_{jt}/P_{jt}), l_{jt}),$$

$$\text{subject to} \quad E_{jt} + q_t x_{jt} + \int_{\Sigma} v(S_{t+1}, S_t) b_j(S_{t+1}) dS_{t+1} \leq w_{jt} l_{jt} + r_{jt} k_{jt} + \Pi_{jt} + q_t b_j(S_t)$$

$$k_{jt+1} = (1-\delta) k_{jt} + (x_{jt}/k_{jt})^{\psi} k_{jt}.$$

In solving this problem, the consumer takes prices, profits, and the aggregate labor supply and capital stock as given. Let the derivative of the utility function in country j at time t with respect to consumption be denoted $U_{C_{jt}}$ and the derivative with respect to labor be denoted $U_{l_{jt}}$. Then, by substituting the constraints into the consumer's utility function, we derive the following first order conditions:

$$l_{jt}: \quad U_{C_{jt}}(w_{jt}/P_{jt}) + U_{l_{jt}} = 0 \tag{19}$$

$$\begin{aligned} k_{jt+1}: \quad & U_{C_{jt}}(1/P_{jt})(-q_t/\psi)\left[\left(\frac{x_{jt}}{k_{jt}}\right)\right]^{1-\psi} \\ & +\beta \mathrm{E}_t \left\{ U_{C_{jt+1}}(1/P_{jt+1}) \left[r_{jt+1} - \frac{q_{t+1} x_{jt+1}}{k_{jt+1}} + (q_{t+1}/\psi) \left(\frac{x_{jt+1}}{k_{jt+1}}\right)^{1-\psi} \left[\left(\frac{x_{jt+1}}{k_{jt+1}}\right)^{\psi} + 1 - \delta \right] \right] \right\} = 0 \end{aligned} \tag{20}$$

$$b_j(S_{t+1}): \quad U_{C_{jt}}(-1/P_{jt}) v(S_{t+1}, S_t) + \beta U_{C_{jt+1}}(q_{t+1}/P_{jt+1}) f(S_{t+1}, S_t) = 0 \tag{21}$$

Since the Euler equation (21) holds in both countries, we have

$$\frac{U_{C_{1t}} P_{1t+1}}{U_{C_{1t+1}} P_{1t}} = \frac{U_{C_{2t}} P_{2t+1}}{U_{C_{2t+1}} P_{2t}} \qquad \forall t. \tag{22}$$

Using the functional form for $U(C,l)$ specified in (2), (22) can be written as

$$\left(\frac{U(C_{1t}, l_{1t})}{U(C_{1t+1}, l_{1t+1})}\right)\left(\frac{C_{1t+1}}{C_{1t}}\right)\left(\frac{P_{1t+1}}{P_{1t}}\right) = \left(\frac{U(C_{2t}, l_{2t})}{U(C_{2t+1}, l_{2t+1})}\right)\left(\frac{C_{2t+1}}{C_{2t}}\right)\left(\frac{P_{2t+1}}{P_{2t}}\right). \tag{23}$$

This equation characterizes an equilibrium in which all country specific risk has been eliminated. Combining it with an initial condition in the deterministic steady-state of no borrowing and lending determines a relation between the value of marginal utilities with respect to composite consumption across countries which must hold every period. This condition is specified and discussed in Section 4 below as it has important implications for international comovements.

We now turn to the differentiated goods sector and drop time subscripts for presentation purposes. The technology for production of differentiated goods gives rise to the following cost function for producer i located in country j:

$$C^m(w_j, r_j, m^i) = Ar_j^\alpha w_j^{1-\alpha}\left[\frac{m^i + \phi}{z_j^m}\right], \tag{24}$$

where $A \equiv (\alpha/(1-\alpha))^{-\alpha} + (\alpha/(1-\alpha))^{1-\alpha}$ and $i \in [0, .5n]$ for $j = 1$ and $i \in (.5n, n]$ for $j = 2$. Firms in this sector choose how much to supply to each country to maximize period by period profits, taking into account the demand functions given by equation (15). Let m_j^i and p_j^i denote sales and the price respectively in country j of producer $i \in [0, n]$. The problem for firm $i \in [0, .5n]$ is given by:

$$\max_{m_1^i, m_2^i} \; p_1^i m_1^i + p_2^i m_2^i - C^m(w_1, r_1, m_1^i + m_2^i)$$

$$\text{subject to} \quad p_j^i = \frac{(m_j^i)^{\rho_j - 1}\nu E_j}{\int_0^n m_j^{i'\rho_j} di'}.$$

In solving this problem, the firm takes the denominator in the demand function as given. The solution to this problem is the familiar markup pricing rule given by:

$$p_j^i = \left(\frac{1}{\rho_j}\right)\left(\frac{Ar_1^\alpha w_1^{1-\alpha}}{z_1^m}\right) \qquad for \quad i \in [0, .5n]. \tag{25.1}$$

Differentiated firms located in country 2 face a symmetric problem and the solution to their maximization problem gives rise to the following pricing equation:

$$p_j^i = \left(\frac{1}{\rho_j}\right)\left(\frac{Ar_2^\alpha w_2^{1-\alpha}}{z_2^m}\right) \qquad for \quad i \in [.5n, n]. \tag{25.2}$$

The equations imply that the economy will be characterized by a symmetric equilibrium in which all firms within a country have equal sales and prices in a particular market. Hence we let $m_j^1 \equiv m_j^i$ and $p_j^1 \equiv p_j^i$ for $i \in [0, .5n]$ denote the sales and price respectively in country j of a representative firm located in country 1 and let $m_j^2 \equiv m_j^i$ and $p_j^2 \equiv p_j^i$ for $i \in (.5n, n]$ denote the sales and price respectively in country j of a representative firm located in country 2. With this notation, the aggregate conditional factor demands for the differentiated goods sector in country j are given by:

$$k_j^m = .5n\left(\frac{m_1^j + m_2^j + \phi}{z_j^m}\right)\left(\frac{\alpha}{1-\alpha}\frac{w_j}{r_j}\right)^{1-\alpha}, \tag{26.1}$$

$$l_j^m = .5n\left(\frac{m_1^j + m_2^j + \phi}{z_j^m}\right)\left(\frac{\alpha}{1-\alpha}\frac{w_j}{r_j}\right)^{-\alpha}. \tag{26.2}$$

Returning to the specification of equilibrium, in the homogeneous good sector, constant returns to scale and perfect competition imply that factors will be paid their value marginal product:

$$w_j = q(1-\theta)z_j^y\left(\frac{k_j^y}{l_j^y}\right)^{\theta}, \tag{27.1}$$

$$r_j = q\theta z_j^y\left(\frac{k_j^y}{l_j^y}\right)^{\theta-1}. \tag{27.2}$$

We now define the four relative price series which are the focus of the paper for our theoretical economy. We first define the bilateral terms of trade between these two countries. To do so, we construct export and import price indices for each country and define the terms of trade (for country 1) to be the ratio of country 1's import price index to its export price index.

Each period a country will be exporting its differentiated goods and importing the other country's differentiated goods. In addition, depending on the realization of shocks, a country may be a net exporter or a net importer of the homogeneous good in any given time period. The net exports of the homogeneous good for country j at time t are given by:

$$h_{jt} \equiv y_{jt} - d_{jt} - x_{jt}. \tag{28}$$

Normalizing the price of the homogeneous good in the steady-state to 1 and denoting steady-state variables with a bar gives the price of the homogeneous good at time t as follows:

$$q_t \equiv \frac{\beta^t U_{C_{1t}}\bar{P}}{U_{\bar{C}_1}P_{1t}} = \frac{\beta^t U_{C_{2t}}\bar{P}}{U_{\bar{C}_2}P_{2t}}, \tag{29}$$

where the second inequality follows from the condition given by equation (23). Here $\bar{P}$ is the steady-state version of the price index given by equation (18) (which is common across countries when the preference shock parameter, ρ, has the same mean across countries).

As in the data, we consider two methods for constructing export and import price indices: Paasche and Laspeyres. The first is to construct Paasche variable weight indices with date t relative expenditures as weights and the steady-state as the base year. In these definitions, we let $\bar{p}$ denote the price of a differentiated good in the steady-state (the base year). Under this method, the export price index for country j at time t is given by

$$EP_{jt} = \begin{cases} \left[\frac{q_t h_{jt}}{q_t h_{jt} + .5np_{it}^j m_{it}^j}\right] q_t + \left[\frac{.5np_{it}^j m_{it}^j}{q_t h_{jt} + .5np_{it}^j m_{it}^j}\right]\left(\frac{p_{it}^j}{\bar{p}}\right) \quad i \neq j, & \text{if } h_{jt} > 0 \\ \left(\frac{p_{it}^j}{\bar{p}}\right) \quad i \neq j, & \text{otherwise} \end{cases} \tag{30.1}$$

while the import price index is

$$IP_{jt} = \begin{cases} \left[\frac{-q_t h_{jt}}{-q_t h_{jt} + .5np^i_{jt} m^i_{jt}}\right] q_t + \left[\frac{.5np^i_{jt} m^i_{jt}}{-q_t h_{jt} + .5np^i_{jt} m^i_{jt}}\right]\left(\frac{p^i_{jt}}{\bar{p}}\right) \quad i \neq j, & \text{if } h_{jt} < 0 \\ \left(\frac{p^i_{jt}}{\bar{p}}\right) \quad i \neq j, & \text{otherwise.} \end{cases} \tag{30.2}$$

The second method is to construct Laspeyres fixed weight indices. We cannot use steady-state values as weights as there is no trade of the homogeneous good in the steady state. Instead, in our simulations we simulate our economy for a long time horizon and take average relative quantities of exports and imports over that simulation as our weights. Let this average share of homogeneous goods in exports be denoted by ω_E and the average share of homogeneous goods in imports be denoted by ω_I. Then the Laspeyres export price index for country j at time t is given by

$$EP_{jt} = \omega_E q_t + (1 - \omega_E)\left(\frac{p^j_{it}}{\bar{p}}\right) \tag{31.1}$$

while the import price index is

$$IP_{jt} = \omega_I q_t + (1 - \omega_I)\left(\frac{p^i_{jt}}{\bar{p}}\right) \tag{31.2}$$

In either case, the terms of trade for country j at time t are defined as follows:

$$TOT_{jt} = \frac{IP_{jt}}{EP_{jt}}. \tag{32}$$

Note that the terms of trade using the Laspeyres export and import price indices will fluctuate only because of price movements, whereas, using the Paasche indices, it will fluctuate because of movements in *both* prices and weights.

The second relative price we consider, the real exchange rate, is defined as the ratio of the two countries' price indices:

$$e_t \equiv \frac{P_{2t}}{P_{1t}} = \frac{\left[.5n\left(p^1_{2t}{}^{\frac{\rho_{2t}}{\rho_{2t}-1}} + p^2_{2t}{}^{\frac{\rho_{2t}}{\rho_{2t}-1}}\right)\right]^{\frac{\nu(\rho_{2t}-1)}{\rho_{2t}}}}{\left[.5n\left(p^1_{1t}{}^{\frac{\rho_{1t}}{\rho_{1t}-1}} + p^2_{1t}{}^{\frac{\rho_{1t}}{\rho_{1t}-1}}\right)\right]^{\frac{\nu(\rho_{1t}-1)}{\rho_{1t}}}}. \tag{33}$$

This expression makes it clear that in the absence of differences in preferences across countries, the real exchange rate will be constant and equal to one and absolute and relative purchasing power parity will hold. In the presence of preference shocks, however, the economy will generate movements in the real exchange rate and deviations from both absolute and relative PPP. It should also be noted that if preferences were non-stochastic but differed across countries (i.e. $\rho_{1t} = \bar{\rho}_1 \neq$

$\bar{\rho}_2 = \rho_{2t}\ \forall t$), then the real exchange rate would vary in response to technology shocks alone and both absolute and relative purchasing power parity would fail to hold.

To examine the model's implications for the relative price across countries of goods produced in the same country, we must define a country's producer price index for differentiated goods for comparison with our measure in the data. We define the producer price index for goods produced in country j as the fixed weighted average of prices charged in country j and prices charged in country i. Now, on average, one-half of any differentiated good will be sold in country 1 and one-half in country 2 in any time period. With these average weights, the producer price index for goods produced in country j at time t is given by:

$$PPI_{jt} = (.5/\bar{p})(p_{1t}^j + p_{2t}^j) \tag{34}$$

We can now define our measure of deviations from the Law of One Price across countries (DAC) for goods produced in country j as the ratio of the producer price index in country j to the price index for exports from country j to country $i \neq j$ as follows:

$$DAC_{jt} = \frac{PPI_{jt}}{(p_{it}^j/\bar{p})} = \frac{.5(p_{1t}^j + p_{2t}^j)}{p_{it}^j} = \frac{.5(\rho_{1t} + \rho_{2t})}{\rho_{jt}}. \tag{35}$$

This expression makes it clear that deviations from the Law of One Price across countries will be generated in this economy if and only if the relative substitutability of differentiated goods across countries varies over time; i.e. only in the presence of preference shocks.

The final relative price we examine is the relative price across goods produced in different countries but sold in the same country. We define our measure of deviations from the Law of One Price across producers (DAP) for goods sold in country j as the ratio of the producer price index in country j to the price index for imports from country i into country $j \neq i$ as follows:

$$DAP_{jt} = \frac{PPI_{jt}}{(p_{jt}^i/\bar{p})} = \frac{.5(p_{1t}^j + p_{2t}^j)}{p_{jt}^i} = \left(\frac{r_{jt}}{r_{it}}\right)^{\alpha}\left(\frac{w_{jt}}{w_{it}}\right)^{1-\alpha}\left(\frac{z_{jt}^m}{z_{it}^m}\right)\left(\frac{(\rho_{1t} + \rho_{2t})}{\rho_{it}}\right). \tag{36}$$

Therefore, deviations from the Law of One Price across producers will be generated in this economy if the realizations of technology shocks to either sector differ across countries over time *or* in the presence of preference shocks.

We seek to determine the qualitative and quantitative properties of these relative prices in this economy. To do so, we simulate the economy using the methods described by Blanchard and Kahn (1980) and compute moments from simulations for the four relative prices. The full equilibrium system of equations for the economy is presented in the appendix. Before presenting the simulation results, however, we turn to a characterization of the economy's steady-state and briefly explore the model's implications for other international comovements.

4. Steady State and Relative Consumption and Prices

In the deterministic steady state in which countries are identical except with respect to the set of differentiated goods they are producing, all prices (goods and factors) will be equalized across countries. Countries will trade differentiated goods only and there will be no trade in the homogeneous good. We normalize the number of firms in the industry so that firms in the differentiated sector earn zero profits in the steady state. Using the pricing rules given by equations (25.1)–(25.2) and the consumer demand functions given by (15), we set the number of firms as follows:

$$n = \frac{2\nu\bar{E}(1-\bar{\rho})}{\phi\bar{\rho}\bar{p}}, \tag{37}$$

where $\bar{E}$ is steady state consumption expenditures in each country and $\bar{p}$ is the steady state price of a differentiated good.

Because of factor and goods price equalization in the steady state, we have

$$\left(\frac{U_{\bar{C}_1}}{\bar{P}_1}\right) = \left(\frac{U_{\bar{C}_2}}{\bar{P}_2}\right). \tag{38}$$

Hence the condition given by (23) implies that the following must hold at each time t:

$$\left(\frac{U_{C_{1t}}}{P_{1t}}\right) = \left(\frac{U_{C_{2t}}}{P_{2t}}\right). \tag{39}$$

This relationship simply states that the marginal utilities with respect to total consumption expenditure, $E_{jt} = C_{jt}P_{jt}$, must be equated across countries in an equilibrium with complete markets. Using the form of the utility function, this can be written as

$$\frac{C_{1t}}{C_{2t}} = \left(\frac{U(C_{1t}, l_{1t})}{U(C_{2t}, l_{2t})}\right)\left(\frac{P_{2t}}{P_{1t}}\right). \tag{40}$$

Consider for a moment the case with no disutility of working, i.e. suppose $U(C_{jt}) = C_{jt}^{1-\sigma}/(1-\sigma)$. In this case, equation (40) becomes

$$\left(\frac{C_{1t}}{C_{2t}}\right) = \left(\frac{P_{2t}}{P_{1t}}\right)^{\frac{1}{\sigma}} = e_t^{\frac{1}{\sigma}}. \tag{41}$$

Now e_t as given in equation (33) will vary with preference shocks and may vary with technology shocks. Therefore, equation (41) illustrates that in this economy without non-traded goods (including leisure), price discrimination will lead to correlations in consumption indices across countries that are less than one. This is true even in an economy with only technology shocks if there is a constant difference in the elasticity of substitution among differentiated goods across countries. This occurs as risk sharing incentives lead expenditures to be perfectly correlated but the price indices at which those expenditures are deflated to produce real consumption varies across countries and across time. Hence price differences across countries arising from price discrimination may be part of the explanation for the relatively low observed cross country consumption correlations.

Equations (40) and (41) are also useful for examining the model's predictions regarding the relationship between consumption ratios across countries and the real exchange rate. Consider first the case with no leisure given by equation (41). As in the endowment model with non-traded goods explored by Backus and Smith (1993), this model predicts a monotonic relationship between consumption ratios and real exchange rates. Backus and Smith, using a large set of countries, showed that the data does not support such a relationship. However the model with leisure given by equation (40) does not predict such a straightforward relationship between consumption ratios and the real exchange rate. Using the consumer's first order condition for labor, equation (40) can be written as:

$$\frac{C_{1t}}{C_{2t}} = \left(\frac{w_{1t}}{w_{2t}}\right)^{\frac{(\gamma-1)(1-\sigma)}{\sigma}} e_t^{\frac{\gamma(1-\sigma)+\sigma}{\sigma}}. \tag{42}$$

Hence the relationship between consumption ratios and the real exchange rate depends upon the relationship between relative wages across countries. Thus the monotonicity result derived in Backus and Smith (1993) depends upon the fact that they consider an endowment economy and the inclusion of leisure may be an important feature in explaining the non-monotonicity result that they found in the data.

5. Calibration

The parameter values used for the benchmark economy are provided in Table 4. Those values that are not specific either to love-of-variety preferences or to the two sector structure of the economy are chosen to be consistent with previous studies. These include the discount factor, β, the composite consumption share, γ, the degree of risk aversion, σ, and the depreciation rate, δ.

To choose values for capital shares in the production technologies, parameters for the technology shock processes, and the share of expenditures devoted to the homogeneous good, it is necessary to divide the economy into two sectors. To do this, we take the following approach. As a proxy for the homogeneous good sector, we select industries which could be broadly classified as being characterized by perfectly competitive markets or as non-traded. As our theoretical model has no role for non-traded consumption goods, we choose to place them in this sector to be consistent with open economy macroeconomic models which only have traded goods produced under perfect competition. In Canadian and U.S. data for the variables of interest, these industries include agriculture, mining, electricity, gas and water, construction, finance, insurance, real estate, and other services. Remaining industries are grouped to form a proxy for the non-competitive sector in our theoretical economy, and include manufacturing, wholesale trade, retail trade, and transport and communication. Since this classification is somewhat arbitrary, we provide sensitivity analysis for those parameters which are sensitive to the chosen sectorial classification.

Using consumption data for Canada and the U.S. taken from the OECD quarterly national accounts for 1981-1993 and using the sector classification described above, we compute the share of differentiated goods in total consumption to be 62% for Canada and 56% for the U.S.. Hence in our benchmark case we use $\nu = .60$ to approximate the share of expenditures devoted to goods produced in the non-competitive sector and provide sensitivity analysis for this parameter. It is not obvious how to set the mean of the elasticity of substitution among differentiated goods, $\overline{\rho}$. Thus, for our benchmark economy, we set it equal to .30 to be consistent with the benchmark value for the elasticity of substitution between foreign and domestic goods used in Backus, Kehoe, and Kydland (1994), and we perform sensitivity analysis with respect to this parameter.

Regarding technology parameters, we compute average labor compensation for Canada using data from the OECD for 1981-1991 for the sectors defined above. Doing so yields capital share parameters of $\theta = .57$ in the homogeneous sector and $\alpha = .30$ in the differentiated goods sector (U.S. data provides similar values). The large value for capital share in the homogeneous sector results primarily from the inclusion of agriculture in that sector and we provide sensitivity analysis with respect to that parameter. The value of the fixed cost parameter ϕ does not affect the quantitative results, and so, without loss of generality, we set it equal to 1.00. The adjustment cost parameter for capital accumulation is set so that investment is approximately three times as variable as GNP in the benchmark case which is broadly consistent with the data. This requires a very high level of adjustment costs with $\psi = .70$. The effect of adjustment costs on the properties of the model is significant and we also examine a case with much lower adjustment costs.

For the technology shock process, we first construct Solow residuals for Canada for both the homogeneous and differentiated goods sectors. We construct this residual for each sector by taking the log of output in a sector less employment in that sector multiplied by labor share in that sector. Unfortunately, similar data is unavailable for the U.S., making it impossible to estimate feedback effects and/or the degree of correlations in innovations between the U.S. and Canada. Given this, we chose to eliminate feedback and correlated innovations from our benchmark case and later examine cases in which these effects are present. We first difference the measured Solow residual and estimate by least squares the parameters of equation (7). This gives us our persistence parameters given by matrix Γ and our variance matrix V^z. As capital stock data was unavailable at this level of disaggregation, we did not use it in our estimate of the Solow residual.

The parameters of the preference shock process given in (4) are calibrated so that our consumption composite is approximately 80% as volatile as GNP which is broadly consistent with Canadian and U.S. data. This implies a standard deviation for ρ equal to .005. In addition, we set the diagonal values in the persistence matrix, Λ at .98 to generate persistence in the real exchange rate. We choose this approach as preference shocks are the driving force behind deviations from PPP and the Law of One Price across countries and we seek to determine the volatility of these relative prices when the variability of the consumption composite is broadly consistent with the data.

6. Results

We now examine the quantitative properties of the theoretical economy. Tables 5–7 present the results of numerical experiments. In those tables GNP and X refer to real GNP and investment, respectively, where the real series are obtained by deflating the nominal series in country j by the consumer price index in country j, P_j. NX refers to the ratio of nominal net exports to nominal GNP. In Table 5, TOT refers to the terms of trade constructed using Laspeyres price indices as given by equation (31). The terms of trade constructed using Paasche price indices give nearly identical results and so is not reported here. In Tables 6–7, Z refers to the aggregate Solow residual. All other variables are consistent with notation defined in the theoretical model.

The tables present results for the economy described in Section 3 for a variety of parameter settings. The experiment labeled *Feedback* allows for feedback among the technology shocks by letting the off-diagonal elements within a sector in matrix A^z equal .03; that is we allow feedback within each sector across countries but not across sectors. We also allow innovations in correlations within a sector across countries by setting $Corr(\epsilon_1^y, \epsilon_2^y) = Corr(\epsilon_1^m, \epsilon_2^m) = .4$. This experiment is labeled *Correlated Innovations* in the tables.

In addition, the tables give results for an economy characterized by constant returns to scale, perfect competition, and no preference shocks. In that economy there are three traded goods: a homogeneous good produced by both countries, a differentiated good produced by country 1, and a differentiated good produced by country 2. The homogeneous good is used for consumption and investment purposes while the differentiated goods are used solely for consumption. All goods are priced at marginal cost. The parameters for that economy are as given in Table 4 except $\phi = 0$ and $\sigma_\rho = 0$. The results for this economy are presented to clarify the effects of multiple traded goods and traditional comparative advantage on the properties of aggregate variables. This economy is labeled *Perfect Competition* in the tables.

Table 5 presents properties of the variables which are the primary focus of this study, the four types of relative price movements. That table documents the volatility of these variables, their cyclical properties, and their relationship with net exports. Our primary objective is to examine the effects of multiple traded goods (and traditional comparative advantage), monopolistic competition, and price discrimination on the properties of these international relative prices.

We first examine the behavior of the terms of trade. Table 5 indicates that the terms of trade in this economy is considerably more volatile relative to output than in Backus, Kehoe, and Kydland (1995, 1994) and Stockman and Tesar (1995) and, in fact, is *more* volatile relative to output than in the data that we examine.[3] Note that this is the case even in the absence of preference shocks and for the economy with perfect competition. We can therefore attribute the primary cause of the volatility in the terms of trade to the multi-sector nature of the economy. Specifically, technology shocks alter the pattern of comparative advantage across countries, causing countries to reallocate resources across sectors. This reallocation, in turn, results in larger movements in relative prices than in models where countries specialize in a single traded good and where such reallocation is not possible.

Table 5 also shows that, for most parameter settings, the terms of trade is weakly positively correlated with output, which is consistent with what other authors have found but at odds with our data findings. Note that the exceptions are when the only source of shocks is technology shocks to the homogeneous good sector ($\sigma_\rho = \sigma_x = 0$) and when the importance of differentiated goods is reduced ($\nu = .2$ or the case of perfect competition when there are only two differentiated goods). Also, for most parameter settings, the terms of trade and net exports are positively contemporaneously correlated in this economy, in contrast to the findings of BKK (1994) but consistent with the data. Note, again, however, that when the importance of the differentiated goods sector is diminished, net exports and the terms of trade are negatively contemporaneously correlated.

The intuition behind these results is the following. A positive technology shock to the homo-

[3] Backus, Kehoe, and Kydland (1994), using implicit price deflators for imports and exports to measure the terms of trade, report the relative standard deviation of the terms of trade to be approximately 1.60 for both the U.S. and Canada.

geneous good sector in country 1 gives that country a comparative advantage in production of the homogeneous good. That country, then, increases its production of the homogeneous good and becomes a net exporter of that good and decreases production of its differentiated goods and becomes a net importer of those goods. This causes an increase in the relative price of its differentiated goods and a fall in its terms of trade. Due to its increased productivity, country 1's aggregate output and aggregate net exports increase. Hence, with shocks only to the homogeneous sector, the terms of trade is negatively correlated with both aggregate output and net exports. On the other hand, a positive technology shock to the differentiated goods sector in country 1 leads to an increase in output of its differentiated goods, a reduction in their price, and a rise in its terms of trade. Again, aggregate output and aggregate net exports in country 1 increase in response to its increased productivity. Thus, with shocks only to the differentiated goods sector, the terms of trade is positively correlated with both aggregate output and net exports. The results indicate that when both shocks are present, shocks to the differentiated goods sector dominate movements in the terms of trade (when those goods are relatively important in consumption.)

In addition, in most cases, if we were to graph the autocorrelation structure between net exports and the terms of trade for this economy as we did for the data in Figures 1, and 2, we would see that the model produces either a tent-shaped autocorrelation function or a v-shaped autocorrelation function. In particular, the model does not generate an S-shaped function as in BKK (1994). Again, the key difference between our economy and their economy is that here each country produces more than one traded good. This implies that in our economy, a shock to a sector causes reallocations within as well as across countries, whereas in their economy, a shock causes a reallocation only across countries. This difference between the two environments generates very different responses in the trade balance in response to shocks and explains the variation in results across the two economies.

We turn now to the real exchange rate (P_2/P_1) and the relative price of goods across countries (DAC). As expected, the model is capable of generating deviations from relative PPP (variability in the real exchange rate) and deviations from the Law of One Price across countries in the presence of preference shocks. Table 5 also indicates that the degree of variability in the real exchange rate is quite sensitive to changes in the parameters of the model. In general, however, when the model is calibrated so that aggregate consumption volatility is broadly consistent with the data, the relative variability of the real exchange rate and the relative price of goods across countries is significantly lower than in the data.[4] This result is not surprising as all deviations from PPP and the Law of

[4] It should be noted that the smoothness of the real exchange rate is exacerbated by our assuming that preferences

One Price in this model are a result of price discrimination, yet we expect nominal price rigidities to also have a role in explaining the volatility of the real exchange rate. In a model with preferences similar to those used in this model and in which all deviations from PPP arise from nominal price rigidities, Chari, Kehoe, and McGratten (1998) generate relative volatilities of the real exchange rate between .4 and .5. These results suggest that both price discrimination and nominal price rigidities are important in explaining the observed variability of the real exchange rate.

Regarding our final relative price, the relative price of similar goods produced in different countries but sold in the same country (DAP), we see that the model is capable of generating deviations from the Law of One Price across producers. The degree of variability of this relative price is not significantly affected by the parameters of the model and is generally slightly below what we observe in the data. The model also predicts that this variable should be negatively correlated with aggregate output (when the differentiated goods sector is relatively important) whereas there is no clear pattern present in the data.

Finally, it should also be noted that autocorrelation coefficients of the relative price series are not presented here as they are insensitive to the experiments performed. All price series exhibited autocorrelation coefficients of approximately .70 but persistence in these series is not surprising given the persistence properties of the exogenous technology and preference shocks.

We now briefly explore some of the other implications of the model as presented in Tables 6–7, focusing on the major anomalies of the theoretical economy. The results suggest that there are two primary anomalies regarding the cyclical properties of the economy: net exports are positively correlated with aggregate output and consumption is negatively correlated with output for some parameter settings. These findings contrast with much of the existing literature as well as the data. As explained below, these anomalies could be interpreted as new puzzles for open economy macroeconomic models as they result primarily from the traditional comparative advantage effects present in this multi-sector model.

An explanation for why net exports are positively correlated with aggregate output in the model can be found in the multi-sector nature of the economy and the size of adjustment costs needed to make the economy well-behaved. In particular, in response to a positive technology shock in country 1, resources are shifted towards production of the goods of comparative advantage in *both* countries. Thus, investment rises in both countries to take advantage of changes in comparative

are identical in the two countries in the steady state. If preferences differed across countries in the steady state (i.e. $\bar{\rho}_1 \neq \bar{\rho}_2$), then the real exchange rate would exhibit more variability as it would also be affected by technology shocks.

advantage. Country 1 runs a trade surplus due to its increased productivity and does not simply borrow to invest as it would in the BKK models.

The negative correlation between aggregate consumption and output can best be understood by considering the economy with constant returns in both sectors and perfect competition. In that economy, a shock to homogeneous goods production in country 1 causes consumers in that country to increase their consumption of the homogeneous good and the differentiated good produced in the other country while decreasing consumption of the differentiated good produced in their own country (because of a rise in its relative price). The fall in consumption of their own differentiated good causes aggregate consumption to fall while real GNP rises. On the other hand, a shock to differentiated goods' production in country 1 causes consumers in that country to decrease consumption of the homogeneous good (both because of a rise in its relative price and because of an increase in investment) which causes aggregate consumption to fall while real GNP rises. With both shocks present, consumption is counter-cyclical. As Table 6 indicates, the introduction of preference shocks can lead to pro-cyclical consumption as they affect markups which affect consumption patterns. Specifically, an increase in markups in country 1 leads to a shift in consumption by consumers in that country away from differentiated goods and toward homogeneous goods. This results in an increase in aggregate consumption and output in that country implying pro-cyclical consumption in the presence of preference shocks.

Table 7 presents cross country correlations in aggregate variables. As discussed in Section 4 and as Stockman and Tesar (1995) suggest, the presence of preference shocks will lower cross-country consumption correlations. However, consumption is always more highly correlated than output in the model in contrast to the data although the spread between the correlations is less than in most models with complete markets and no non-traded consumption goods.

We also find that employment, investment and output are positively correlated across countries, which is consistent with the data but contrasts with the BKK models. In the BKK models, resources are shifted towards the country that experiences a favorable shock, making employment, investment, and output negatively correlated across countries. This incentive also exists in our model, but is diminished by the incentive to reallocate resources within each country towards the goods of comparative advantage. Thus, output and investment can be positively or negatively correlated across countries, depending on the relative importance of the comparative advantage effects. Our results indicate that traditional comparative advantage effects are quite important and lead to a positive correlation in these variables across countries. Note also that positive correlations in investment are enhanced by the high adjustment costs necessary to calibrate the relative volatility of investment.

7. Conclusions

In this paper, we provide evidence regarding the properties of relative price series between the U.S. and Canada. Consistent with what other authors have found, the data suggests that the bilateral terms of trade, the real exchange rate, relative prices across countries, and relative prices across producers are all persistent and variable. We present a multi-sector model with elements of monopolistic competition and price discrimination which is capable of generating variability in all of these relative prices. We also find that traditional comparative advantage effects (generally absent from existing open economy macroeconomic models) play a central role in generating some of the cyclical properties of the economy. In particular, these effects are responsible for generating positive cross-country correlations in output and investment. They are also essential in accounting for several discrepancies between the theory and the data and may present new anomalies for this area of research: excessive volatility in the terms of trade, counter-cyclical consumption, and pro-cyclical net exports.

While our results clearly point to the importance of imperfect competition, price discrimination, and static comparative advantage in accounting for certain features of relative prices, there are certain aspects of our model that are not entirely satisfactory. Perhaps most important is that the model produces significantly less variability in the real exchange rate and in relative prices across countries than what is observed in the data. This clearly results from the fact that preference shocks are the only force behind generating deviations from the Law of One Price across countries. One conclusion is that this type of price discrimination should be coupled with nominal price rigidities to address the observed volatility of the real exchange rate. Another conclusion is that this form of imperfect competition which has received considerable attention in the international and growth literature is not adequate for studying international relative price movements. In contrast, a model in which firms produce differentiated products but interact *strategically* will produce deviations from the Law of One Price across countries in the presence of technology shocks alone and may be better suited for exploring variability in the real exchange rate.

APPENDIX

1. Data Sources

1. *Terms of Trade*

   The Paasche and Laspeyres export (import) price indices for exports from (to) the U.S. to (from) Canada used to construct the terms of trade reported in Table 1 are at quarterly frequencies from 1981.1-1994.2, in base year 1986, and were taken from the CANSIM data base.

2. *Real Exchange Rate*

   The Canadian consumption deflator was constructed by taking the ratio of nominal to real (1986 prices) personal expenditures on goods and services at quarterly frequencies from 1981.1-1994.2 and were taken from the CANSIM data base. The spot exchange rate is the Canadian/U.S. exchange rate at quarterly frequencies from 1981.1-1994.2, taken from the CANSIM data base. The U.S. consumption deflator at quarterly frequencies from 1981.1-1994.2 was taken from CITIBASE.

3. *Relative Price of Goods Across Countries and Across Producers*

   The Canadian Industrial Product Price Indices, the Canadian Laspeyres Export Price Indices for exports to the US for major industrial groups, and the Canadian Laspeyres Import Price Indices for imports from the US for major industrial groups are at quarterly frequencies from 1981.1-1994.2, in base year 1986 and were taken from the CANSIM data base.

4. *Real Output*

   Canadian real GNP (1986 prices) at quarterly frequencies from 1981.1-1992.4 was taken from the International Financial Statistics data base.

5. *Net Exports*

   Canadian Total exports to the U.S. and total imports from the U.S. in current dollars at quarterly frequencies from 1981.1-1994.2 was taken from the CANSIM data base. Canadian nominal GNP was obtained from the International Financial Statistics data base.

6. *Consumption Shares*

   The share of homogeneous goods in consumer expenditure, ν, was calibrated using Canadian and U.S. nominal disaggregated private final consumption expenditure data at annual frequencies from 1981-1993 and was taken from the OECD National Accounts, Detailed Tables.

6. *Capital Shares*

   Capital shares in each sector, θ and α, were calibrated using Canadian and U.S. Employee Compensation data at annual frequencies from 1981-1991 and was taken from the OECD National Accounts, Detailed Tables.

7. *Technology Shock Processes*

   Solow residuals for each sector were constructed using Canadian real GDP at factor cost and number of employees at the industry level at quarterly frequencies from 1976.-1996.4 and were taken from the CANSIM data base.

2. Equilibrium System

Because prices in this economy decline at rate β^t, we write our system of equilibrium equations with prices at time t divided by β^t. Let a ˆ over a nominal variable denote that variable divided by β^t; ex: $\hat{E}_{jt} \equiv E_{jt}/\beta^t$. The dynamic equilibrium is characterized by the following system of non-linear first order difference equations in two state variables (K_{1t}, K_{2t}) and eleven jump variables $(\hat{q}_t, l^y_{1t}, l^y_{2t}, k^y_{1t}, k^y_{2t}, m^1_{1t}, m^1_{2t}, m^2_{1t}, m^2_{2t}, x_{1t}, x_{2t})$. We simulate the economy by taking a first order Taylor series approximation of this non-linear system around the steady state and simulating the approximate economy using the methods described in Blanchard and Kahn (1980). The equilibrium system consists of nine atemporal equations and four intertemporal equations and is specified as follows:

$$\frac{U(C_{1t}, l_{1t})}{\hat{E}_{1t}} = \frac{U(C_{2t}, l_{2t})}{\hat{E}_{2t}} \tag{E1}$$

$$\frac{(1-\nu)(\hat{E}_{1t} + \hat{E}_{2t})}{\hat{q}_t} + x_{1t} + x_{2t} = y_{1t} + y_{2t} \tag{E2}$$

$$K_{jt} = k^y_{jt} + k^m_{jt} \qquad j = 1, 2 \tag{E3}$$

$$\hat{p}^j_{it} = \frac{2\nu \hat{E}_{it} m^{j\ \rho_{it}-1}_{it}}{n(m^{1\ \rho_{it}}_{it} + m^{2\ \rho_{it}}_{it})} \qquad j = 1, 2; i = 1, 2 \tag{E4}$$

$$\frac{U(C_{1t}, l_{1t})}{\hat{E}_{1t}} = \frac{U(\bar{C}, \bar{l})}{\bar{E}_1} \tag{E5}$$

$$K_{jt+1} = (1-\delta)K_{jt} + \left(\frac{x_{jt}}{K_{jt}}\right)^{\psi} K_{jt} \qquad j = 1, 2 \tag{E6}$$

$$\left(\frac{\hat{q}_t U(C_{jt}, l_{jt})}{\hat{E}_{jt}}\right)\left(\frac{-1}{\psi}\right)\left(\frac{x_{jt}}{K_{jt}}\right)^{1-\psi}$$

$$+\beta\left(\frac{U(C_{jt+1}, l_{jt+1})}{\hat{E}_{jt+1}}\right)\left[\hat{r}_{jt+1} - \frac{\hat{q}_{t+1}x_{jt+1}}{K_{jt+1}} + \left(\frac{\hat{q}_{t+1}}{\psi}\right)\left(\frac{x_{jt+1}}{K_{jt+1}}\right)^{1-\psi}\left[\left(\frac{x_{jt+1}}{K_{jt+1}}\right)^{\psi} + 1 - \delta\right]\right] = 0 \qquad j = 1, 2 \tag{E7}$$

where

$$\hat{E}_{jt} = \left(\frac{\gamma}{1-\gamma}\right)\hat{w}_{jt}(1 - l_{jt}) \qquad j = 1, 2$$

$$\hat{w}_{jt} = \hat{q}_t(1-\theta)z^y_{jt}\left(\frac{k^y_{jt}}{l^y_{jt}}\right)^{\theta} \qquad j = 1, 2$$

$$\hat{r}_{jt} = \hat{q}_t \theta z^y_{jt}\left(\frac{k^y_{jt}}{l^y_{jt}}\right)^{\theta-1} \qquad j = 1, 2$$

$$\hat{p}^j_i = \left(\frac{1}{\rho_i}\right)\left(\frac{A\hat{r}^{\alpha}_{jt}\hat{w}^{1-\alpha}_{jt}}{z^m_{jt}}\right) \qquad j = 1, 2; i = 1, 2$$

$$k_{jt}^{m} = (.5n)\left(\frac{m_{1t}^{j} + m_{2t}^{j} + \phi}{z_{jt}^{m}}\right)\left(\frac{\alpha\hat{w}_{jt}}{(1-\alpha)\hat{r}_{jt}}\right)^{1-\alpha} \qquad j = 1, 2$$

$$l_{jt}^{m} = (.5n)\left(\frac{m_{1t}^{j} + m_{2t}^{j} + \phi}{z_{jt}^{m}}\right)\left(\frac{\alpha\hat{w}_{jt}}{(1-\alpha)\hat{r}_{jt}}\right)^{-\alpha} \qquad j = 1, 2$$

$$l_{jt} = l_{jt}^{y} + l_{jt}^{m} \qquad j = 1, 2$$

$$C_{jt} = \frac{\hat{E}_{jt}}{\hat{P}_{jt}} \qquad j = 1, 2$$

$$\hat{P}_{jt} = \frac{\left[.5n\left(\hat{p}_{jt}^{1\,\frac{\rho_{jt}}{\rho_{jt}-1}} + \hat{p}_{jt}^{2\,\frac{\rho_{jt}}{\rho_{jt}-1}}\right)\right]^{\frac{\nu(\rho_{jt}-1)}{\rho_{jt}}} \hat{q}_{t}^{1-\nu}}{\nu^{\nu}(1-\nu)^{1-\nu}} \qquad j = 1, 2$$

$$y_{jt} = z_{jt}^{y} k_{jt}^{y\;\theta} l_{jt}^{y\;1-\theta} \qquad j = 1, 2$$

The first equation represents the condition on marginal utilities given by equation (39) in the text which must hold in this economy with complete markets. Equation (E2) represents the equilibrium pricing equation for homogeneous goods given by equation (29) in the text. Equation (E3) is the resource constraint for the homogeneous good. Equation (E4) is capital market clearing and (E5) is the demand function for differentiated goods from the consumers' problems. Equation (E6) is the law of motion for the aggregate capital stocks and (E7) are the first order conditions for investment from the consumers' problems.

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TABLE 1: Canada/US Terms of Trade, Real Exchange Rates, and Net Exports

Variable	Std. Dev. (%)	Ratio of Std. Dev. to that of Output	Auto-Correlation	Corr with Output	Corr with Net Exports
Paasche TOT	2.51	1.19	.25	-.06	.21
Laspeyres TOT	1.66	.79	.71	-.30	.27
Real Exchange Rate	3.59	1.73	.86	.19	.24
Net Exports	.17	.08	.49	-.39	1.00

Notes: Data is quarterly from 1981.1-1994.4. All data is logged except net exports and moments are computed from Hodrick-Prescott filtered data. Paasche TOT refers to the ratio of the Paasche Canadian import price index for imports from the US to the Paasche Canadian export price index for exports to the US. Laspeyres TOT refers to the ratio of the Laspeyres Canadian import price index for imports from the US to the Laspeyres Canadian export price index for exports to the US. Real Exchange Rate refers to the ratio of the US Consumer Price Deflator (base 1986) multiplied by the spot exchange rate between the US and Canada and the Canadian Consumer Price Deflator (base 1986). Net Exports refers to the ratio of nominal net exports from Canada to the US (Canadian exports to the US minus imports from the US) to Canadian nominal GNP.

TABLE 2: Relative Export and Producer Prices

	Industry	Std. Dev. (%)	Ratio of Std. Dev. to that of Output	Auto-Correlation	Corr with Output
	Median	**4.17**	**1.99**	**.49**	**.12**
1	Meat Products	5.16	2.45	.66	.42
2	Fish Products	4.51	2.14	.71	.26
3	Alcoholic Beverages	5.80	2.76	.43	-.04
4	Rubber, Leather, Plastics	4.16	1.98	.27	-.15
5	Textiles	5.92	2.82	.44	.23
6	Lumber	3.95	1.88	.30	.02
7	Wood Pulp	5.32	2.53	.49	.23
8	Newsprint Paper	3.44	1.64	.75	.32
9	Aluminum Products	2.83	1.35	.24	-.25
10	Copper Products	3.89	1.85	.11	.27
11	Nickel Products*	15.95	7.59	.73	.57
12	Industrial Machinery	.82	.39	.67	.01
13	Agricultural Machinery	.75	.36	.40	.18
14	Office Equipment	16.40	7.80	.73	.29
15	Passenger Automobiles*	.92	.44	.45	.12
16	Trucks, Chassis, and Tractors	1.31	.62	.44	.18
17	Motor Vehicle Parts	.77	.37	.70	.25
18	Electric and Communication Equip	3.04	1.44	.79	.06
19	Petroleum and Coal Products	6.31	3.00	.24	.02
20	Inorganic Chemicals	9.84	4.68	.47	-.12
21	Organic Chemicals	8.21	3.91	.49	.26
22	Fertilizers*	6.39	3.03	.52	-.19
23	Paper Products*	4.17	1.99	.58	-.41
24	Iron and Steel	15.24	7.25	.51	-.08
25	Metal Fabricated Products	3.61	1.72	.74	-.11

Notes: Data is quarterly from 1981.1-1996.4 with the exception of industries designated by an * which are from 1981.1-1995.2. Moments are computed from logged, Hodrick-Prescott filtered ratios of the Canadian Industrial Producer Price Index to the Price Index for Exports to the U.S.

TABLE 3: Relative Import and Producer and Import Prices

	Industry	Std. Dev. (%)	Ratio of Std. Dev. to that of Output	Auto-Correlation	Corr with Output
	Median	**4.07**	**1.93**	**.61**	**-.03**
1	Meat Products	3.63	1.73	.31	-.40
2	Fish Products	6.29	2.99	.17	-.10
3	Dairy Products	9.44	4.49	.44	-.15
4	Sugar Preparations*	12.10	5.76	.57	.46
5	Fodder and Feed	4.25	2.02	.44	.33
6	Beverages	9.58	4.55	.10	-.16
7	Tobacco*	10.94	5.20	.17	.06
8	Non-metallic Minerals	4.77	2.27	.66	-.06
9	Textile Materials	2.89	1.38	.57	.08
10	Organic Chemicals	8.22	3.91	.70	.40
11	Plastic Materials	3.65	1.73	.73	.32
12	Petroleum & Coal Products	5.13	2.44	.33	-.02
13	Agricultural Machinery	3.35	1.81	.61	.22
14	Agricultural Machinery	3.80	1.81	.61	.22
15	Passenger Automobiles*	3.42	1.63	.83	-.15
16	Trucks & Other Vehicles	1.80	.85	.31	-.04
17	Motor Vehicle Parts	1.98	.94	.82	.06
18	TVs, Radios, & Phonographs	5.69	2.71	.40	.11
19	Communication Equipment	3.88	1.84	.63	-.31
20	Apparel	4.88	2.32	.61	-.32
21	Printed Matter	3.59	1.71	.83	-.13
22	Household Furnishings	3.71	1.76	.76	-.16
23	Metal Working Machinery*	3.64	1.62	.75	.11
24	Office Equipment	10.11	4.81	.70	.37

Notes: Data is quarterly from 1981.1-1996.4 with the exception of industries designated by an * which are from 1981.1-1995.2. Moments are computed from logged, Hodrick-Prescott filtered ratios of the Canadian Industrial Producer Price Index to the Price Index for Imports from the U.S.

TABLE 4: Benchmark Parameter Values

Preferences:

Discount Factor: $\beta = .99$

Composite Consumption Share: $\gamma = .34$

Risk Aversion: $\sigma = 2.00$

Differentiated Consumption Share: $\nu = .60$

Mean of Elasticity of Substitution: $\bar{\rho} = .30$

Technologies :

Capital Share in Homogeneous Sector: $\theta = .57$

Capital Share in Differentiated Sector: $\alpha = .30$

Fixed Cost: $\phi = 1.00$

Depreciation Rate: $\delta = .025$

Capital Adjustment Cost: $\psi = .70$

Shock Processes:

Technology Shocks:

$$\Gamma = \begin{bmatrix} .923 & 0 & 0 & 0 \\ 0 & .923 & 0 & 0 \\ 0 & 0 & .959 & 0 \\ 0 & 0 & 0 & .959 \end{bmatrix}$$

$$V^z = \begin{bmatrix} (.0080)^2 & 0 & 0 & 0 \\ 0 & (.0080)^2 & 0 & 0 \\ 0 & 0 & (.0125)^2 & 0 \\ 0 & 0 & 0 & (.0125)^2 \end{bmatrix}$$

Preference Shocks:

$$\Lambda = \begin{bmatrix} .98 & 0 \\ 0 & ..98 \end{bmatrix}$$

$$V^\rho = \begin{bmatrix} (.005)^2 & 0 \\ 0 & (.005)^2 \end{bmatrix}$$

TABLE 5: Properties of Relative Prices in Theoretical Economies

	Standard Deviation (%)	Ratio of Standard Deviation to that of GNP				Correlation with GNP				Correlation
Economy	GNP	TOT	P_2/P_1	DAC	DAP	TOT	P_2/P_1	DAC	DAP	(nx,TOT)
Data	2.10	.79	1.73	1.99	1.93	-.30	.19	.12	-.03	.27
Benchmark	1.77	1.51	.82	.25	1.50	.18	.45	.45	-.11	.48
	(.27)	(.30)	(.14)	(.04)	(.30)	(.19)	(.17)	(.16)	(.19)	(.17)
$\sigma_\rho = 0$	1.49	1.76	.00	.00	1.81	.09	–	–	-.09	.46
	(.21)	(.27)	(.00)	(.00)	(.29)	(.22)	–	–	(.17)	(.18)
$\sigma_y = 0$	1.59	1.52	.89	.27	1.50	.36	.48	.48	-.28	.98
	(.19)	(.27)	(.13)	(.04)	(.27)	(.18)	(.13)	(.13)	(.18)	(.02)
$\sigma_x = 0$	1.31	1.10	1.16	.36	.91	.14	.63	.63	.08	-.37
	(.27)	(.27)	(.22)	(.07)	(.21)	(.18)	(.14)	(.13)	(.18)	(.13)
$\sigma_\rho = 0$	.79	1.24	.00	.00	1.28	-.80	–	–	.80	-1.00
$\sigma_x = 0$	(.12)	(.16)	(.00)	(.00)	(.16)	(.08)	–	–	(.08)	(.00)
$\psi = .95$	2.07	1.28	.09	.21	1.25	.04	.03	-.03	-.04	.47
	(.32)	(.29)	(.02)	(.05)	(.27)	(.18)	(.17)	(.17)	(.19)	(.15)
$\nu = .2$	1.66	1.47	.10	.27	1.44	-.14	.07	-.07	.13	-.13
	(.23)	(.28)	(.02)	(.06)	(.24)	(.15)	(.15)	(.15)	(.15)	(.16)
$\nu = .9$	2.95	.95	1.09	.15	.92	.38	.60	.60	-.31	.82
	(.38)	(.18)	(.16)	(.02)	(.18)	(.17)	(.13)	(.13)	(.19)	(.07)
$\bar{\rho} = .8$	1.84	1.17	.33	.23	1.16	.25	.20	-.20	-.30	.72
	(.18)	(.21)	(.05)	(.03)	(.23)	(.17)	(.17)	(.17)	(.16)	(.10)
$\theta = .4$	1.76	1.53	.04	.26	1.49	.03	.03	-.04	-.04	.27
	(.21)	(.23)	(.01)	(.05)	(.23)	(.20)	(.25)	(.25)	(.19)	(.17)
Feedback	1.82	1.55	.85	.26	1.51	.26	.43	.43	-.20	.52
	(.28)	(.33)	(.15)	(.05)	(.33)	(.21)	(.15)	(.15)	(.21)	(.19)
Correlated	1.91	1.11	.75	.23	1.04	.20	.44	.44	-.12	.44
Innovations	(.29)	(.19)	(.10)	(.03)	(.17)	(.17)	(.13)	(.13)	(.18)	(.14)
Perfect	1.67	1.40	.00	.00	1.44	-.15	–	–	.15	-.25
Competition	(.21)	(.24)	(.00)	(.00)	(.25)	(.17)	–	–	(.17)	(.18)

TABLE 6: Other Properties in Theoretical Economies

	Ratio of Standard Deviation to that of GNP					Correlation with GNP				
Economy	C	X	L	Z	NX	C	X	L	Z	NX
Data	.85	2.80	.86	.74	.78	.83	.52	.69	.84	-.26
Benchmark	.82 (.14)	3.07 (.55)	.65 (.11)	.22 (.03)	.32 (.06)	.38 (.16)	.72 (.09)	.50 (.14)	.77 (.08)	.47 (.14)
$\sigma_\rho = 0$	.24 (.04)	3.60 (.36)	.64 (.10)	.08 (.01)	.37 (.06)	-.45 (.24)	.91 (.04)	.85 (.07)	.28 (.22)	.43 (.19)
$\sigma_y = 0$	.94 (.19)	3.17 (.49)	.59 (.11)	.15 (.02)	.27 (.05)	.45 (.14)	.70 (.12)	.31 (.17)	.76 (.08)	.33 (.20)
$\sigma_x = 0$	1.18 (.19)	2.36 (.42)	.75 (.16)	.19 (.04)	.28 (.03)	.63 (.12)	.43 (.14)	.23 (.15)	.49 (.15)	.65 (.10)
$\sigma_\rho = 0$ $\sigma_x = 0$	.36 (.09)	2.99 (.39)	.88 (.05)	.11 (.02)	.44 (.05)	-.19 (.19)	.82 (.08)	.96 (.02)	-.74 (.10)	.80 (.08)
$\psi = .95$	1.00 (.09)	6.59 (1.74)	1.24 (.07)	.24 (.03)	.56 (.12)	-.75 (.08)	.92 (.04)	.88 (.04)	-.55 (.11)	-.27 (.24)
$\nu = .2$	.93 (.13)	4.34 (.74)	1.22 (.08)	.11 (.01)	.35 (.07)	-.63 (.08)	.86 (.04)	.91 (.03)	-.67 (.08)	.47 (.23)
$\nu = .9$	.93 (.07)	1.46 (.15)	.28 (.05)	.22 (.01)	.27 (.05)	.90 (.03)	.69 (.10)	-.22 (.17)	.97 (.01)	.47 (.15)
$\bar{\rho} = .8$	.47 (.14)	4.09 (.69)	.83 (.06)	.08 (.01)	.47 (.08)	-.54 (.15)	.83 (.06)	.94 (.02)	-.49 (.13)	.50 (.11)
$\theta = .4$	.43 (.06)	5.13 (.64)	.87 (.08)	.19 (.03)	.39 (.06)	-.39 (.19)	.86 (.05)	.86 (.07)	-.33 (.19)	.47 (.16)
Feedback	.82 (.15)	3.56 (1.79)	.64 (.11)	.14 (.02)	.31 (.07)	.36 (.16)	.72 (.09)	.46 (.16)	.52 (.14)	.44 (.18)
Correlated Innovations	.78 (.20)	3.34 (.56)	.66 (.13)	.14 (.03)	.23 (.04)	.28 (.16)	.79 (.09)	.59 (.14)	.51 (.16)	.39 (.19)
Perfect Competition	.43 (.07)	4.08 (.59)	.76 (.05)	.04 (.01)	.30 (.06)	-.75 (.09)	.92 (.03)	.97 (.01)	-.03 (.18)	.48 (.20)

TABLE 7: Cross-Country Correlations in Theoretical Economies

	Cross-Country Correlations				
Economy	GNP	C	X	L	Z
Data	.76	.49	-.01	.53	.75
Benchmark	.21 (.19)	.67 (.10)	.88 (.05)	.58 (.14)	.35 (.18)
$\sigma_\rho = 0$	.45 (.17)	.80 (.10)	.94 (.02)	.46 (.12)	.46 (.11)
$\sigma_y = 0$	.41 (.18)	.72 (.10)	.94 (.02)	.90 (.06)	.58 (.14)
$\sigma_x = 0$	-.25 (.16)	.69 (.11)	.63 (.14)	.46 (.18)	.59 (.14)
$\sigma_\rho = 0$ $\sigma_x = 0$	-.31 (.20)	.77 (.11)	.71 (.12)	.10 (.23)	.77 (.10)
$\psi = .95$	.66 (.11)	.98 (.02)	.59 (.13)	.92 (.03)	.85 (.05)
$\nu = .2$	.43 (.13)	.99 (.01)	.97 (.01)	.83 (.05)	.96 (.02)
$\nu = .9$	.13 (.16)	.56 (.11)	.51 (.12)	.19 (.18)	.40 (.15)
$\bar{\rho} = .8$	.39 (.13)	.66 (.08)	.95 (.02)	.65 (.10)	.38 (.16)
$\theta = .4$	.47 (.13)	.93 (.03)	.98 (.01)	.69 (.06)	-.77 (.06)
Feedback	.27 (.16)	.67 (.10)	.90 (.04)	.64 (.11)	.50 (.14)
Correlated Innovations	.46 (.14)	.68 (.11)	.94 (.02)	.78 (.08)	.62 (.10)
Perfect Competition	.54 (.14)	.96 (.03)	.96 (.01)	.81 (.07)	.89 (.07)

Notes to Tables 5-7: Data statistics reported in Table 5 are taken from Tables 1-3. Data statistics reported in Tables 6-7 are taken from Backus, Kehoe, and Kydland (1995) for Canada from 1970.1-1990.2. Statistics for the theoretical economy are based on Hodrick-Prescott filtered data and are averages over fifty simulations of length 100. Standard deviations of the moments over the fifty trials are in parentheses. Except for the ratio of net exports to output, statistics refer to logarithms of variables.