Import Protection as Export Destruction

Hiroyuki Kasahara[*] Beverly Lapham[†]

December 2005

Incomplete

Abstract

This paper develops a dynamic, stochastic industry model of heterogeneous firms to examine the effects of trade liberalization on resource reallocation, industry productivity, and welfare in the presence of import and export complementarities. The model highlights mechanisms whereby import policies affect exports and export policies affect imports. We first present a simplified version of the model and use that theoretical model to develop an empirical model which we structurally estimate using Chilean plant-level manufacturing data. A comparison of our estimates of transition probabilities across export/import status with the data suggests a need for a dynamic approach. We then examine a dynamic version and structurally estimate that model using the same data set. The estimated model is used to perform counterfactual experiments regarding different trading regimes to assess the positive and normative effects of barriers to trade in import and export markets. These experiments suggest that the welfare gain due to trade is substantial and because of import and export complementarities, policies which inhibit the importation of foreign intermediates can have a large adverse effect on the exportation of final goods.

[*]University of Western Ontario

[†]Queen's University; The author gratefully acknowledges financial support from the Social Sciences Humanities Council of Canada.

1 Introduction

This paper develops a dynamic stochastic industry model of heterogeneous firms to examine the effects of trade liberalization on resource reallocation, industry productivity, and welfare in the presence of import and export complementarities. We use the theoretical model to develop an empirical model which we estimate using Chilean plant-level manufacturing data. The estimated model is then used to perform counterfactual experiments regarding different trading regimes to assess the positive and normative effects of barriers to trade in import and export markets.

The theoretical trade literature with increasing returns typically identifies two effects on productivity of firms as trade increases: the *scale effect* as surviving firms increase output and produce at lower average cost and the *selection effect* as firms are forced to exit. As Melitz (2003) has shown in a model with heterogeneous exporting firms, the least productive firms will typically exit due to the selection effect. Thus resources will be reallocated to more productive firms, and aggregate productivity will rise.

Indeed, empirical work suggests that there is a substantial amount of resource reallocation across firms within an industry following trade liberalization and these shifts in resources do contribute to productivity growth in the sector. Pavcnik (2002) uses Chilean data and finds that such reallocations contribute to productivity growth after trade liberalization in that country. Trefler (2004) estimates these effects in Canadian manufacturing following the U.S.-Canada free trade agreement using plant- and industry-level data and finds significant increases in productivity among both importers and exporters. Empirical evidence also suggests that relatively more productive firms are more likely to export (see, for example, Bernard and Jensen(1999), Aw, Chung, and Roberts (2000), and Clerides, Lack and Tybout (1998)).

In this paper we provide empirical evidence that suggests that whether or not a firm is *importing* intermediates for use in production may also be important for explaining differences in plant performance (see also, Kasahara and Rodrigue 2004). Our data suggests that firms which are both importing and exporting tend to be larger and more productive than firms that are active in either market, but not both. Hence, the impact of trade on resource reallocation across firms which are importing may be as important as shifts across exporting firms.

To simultaneously address the empirical regularities concerning importers, we begin by extending the model of Melitz (2003) to incorporate imported intermediate goods. In this environment, we allow final goods producers to differ with regard to both their productivity and their fixed cost of importing. We also incorporate complementarities in the fixed costs of importing and exporting. In the model, the use of foreign intermediates increases firm's productivity but, due to the presence of fixed costs of importing, only inherently high productive firms importing intermediates. Trade liberalization which lowers restrictions on the importation of intermediates increases aggregate productivity because some inherently productive firms start importing and achieve within-plant productivity gains. This, in turn, leads to a resource reallocation from less productive to more productive importing firms. Furthermore, productivity gains among highly productive firms through imported intermediates may allow some of them to start exporting, leading to a resource reallocation in addition to that emphasized by Melitz(2003). Similarly, events that encourage exporting (e.g., liberalization in trading partners or export subsidies) may well have an impact on firm's decision to import since newly exporting firms would have a higher incentive to start importing. Thus, the model identifies an important mechanism whereby import tariff policy affects aggregate exports and whereby export subsidies affect aggregate imports.

To quantitatively examine the impact of trade on aggregate productivity and welfare, we estimate a stochastic industry equilibrium model of exports and imports using a panel of Chilean manufacturing plants. Our estimates suggest significant complementarities in both sunk and fixed costs of importing and exporting. Furthermore, the basic observed patterns of productivity across firms with different import and export status is well captured by the estimated model. We also perform a variety of counterfactual experiments to examine the effects of trade policies. The experiments suggest that the welfare gain due to exposure to trade is found to be substantial. Another important finding is that because of import and export complementarities, policies which inhibit the importation of foreign intermediates can have a large adverse effect on the exportation of final goods. In addition, the experiments indicate that the equilibrium price response plays a major role in redistributing resources from less productive firms to more productive firms. In particular, experiments based on a partial equilibrium model that ignores the equilibrium price response provide fairly different estimates of the impact of trade on aggregate productivity.

Not surprisingly the static model is not well-suited to capture the observed transition probabilities of plants across different import and export categories. With this in mind, we extend the basic empirical model to incorporate dynamic elements and estimate this model with the same data set. The fundamental insights gained from the static estimation continue to hold in the dynamic model. In addition, we find evidence of dynamic complementarities in fixed costs of importing and exporting. That is, a firm which is importing in the current period faces a cost advantage to begin exporting in subsequent periods. We also demonstrate that the dynamic model is broadly consistent with the transition probabilities across import and export status categories.

The remainder of the paper is organized as follows. Section 2 presents empirical evidence on the static and dynamic distribution of importers and exporters and their performance using Chilean manufacturing plant-level data. Section 3 presents a theoretical model with import and export complementarities. Section 4 presents a static empirical model based on the theoretical model developed in the previous section. Section 5 discusses the data used and the results of the structural estimation of the static model. Sections 6 and 7 present a dynamic extension of the empirical model and the estimation methodology for the dynamic model. Section 8 presents the dynamic empirical results and Section 9 concludes.

2 Empirical Motivation

In this section we briefly describe Chilean plant-level data and provide summary statistics to characterize patterns and trends of plants which may or may not participate in international markets.

2.1 Data

We use the Chilean manufacturing census for 1990-1996. In the data set, we observe the number of blue collar workers and white collar workers, the value of total sales, the value of export sales, and the value of imported materials. The export/import status of a firm is identified from the data by checking if the value of export sales and/or the value of imported materials are zero or positive. The value of the revenue from the home market is computed as (the value of total sales)-(the value of export sales). We use the manufacturing output price deflator to convert the nominal value into the real value. The entry/exiting decisions can be identified in the data by looking at the number of workers across years. We use unbalanced panel data of 7234 plants for 1990-1996, including all the plants that have been observed at least one year between 1990

Table 1: Exporters and Importers in Chile for 1990-1996 (% of Total)

	1990	1991	1992	1993	1994	1995	1996	1990-96 ave.
Exporters	8.7	9.4	9.2	9.2	8.5	9.8	8.7	9.1
Importers	12.8	11.9	13.1	13.2	13.2	12.2	11.6	12.6
Ex/Importers	8.2	9.6	10.7	12.0	13.1	12.4	12.7	11.2
Exports by Exporters	43.1	42.9	49.3	42.5	34.0	39.3	39.3	41.5
Exports by Ex/Importers	56.9	57.1	50.7	57.5	66.0	60.7	60.7	58.5
Imports by Importers	35.2	31.7	31.5	28.7	21.2	22.0	25.4	28.0
Imports by Ex/Importers	64.8	68.3	68.5	71.3	78.8	78.0	74.6	72.0
Output by Exporters	17.9	16.3	23.4	19.0	15.1	20.3	17.6	18.5
Output by Importers	16.7	12.8	14.9	15.3	13.8	14.3	13.5	14.5
Output by Ex/Importers	38.8	44.5	40.5	44.1	50.1	45.4	48.3	44.5
No. of Plants	4722	4628	4938	5084	5040	5123	5455	4999

Notes: Exporters refers to plants that export but do not import. Importers refers to plants that import but do not export. Ex/Importers refers to plants that both export and import.

and 1996.

2.2 Importers and Exporters Distribution and Performance

Table 1 provides several important basic facts about exporters and importers. The fraction of plants that are engaged in trade is relatively small but has increased over time as shown in the first three rows of Table 1. The fraction of plants which were involved in international trade increased from 29.7% in 1990 to 33% in 1996 while plants that both export and import grew from 8.2% to 12.7% over this time period. Furthermore, as shown in the fourth through seventh rows of Table 1, plants that both export and import account for a larger fraction of exports and imports than their counterparts which only export or only import. Overall, this table indicates that plants that engage in both exporting and importing are increasingly common and play an important role in determining the volume of trade.

This table also demonstrates the relative importance in manufacturing activities of plants which engage in international trade. In particular, the percentage of total output accounted for by these firms increased from 73.4% in 1990 to 79.4% in 1996. is apparent from the sixth to the eighth rows of Table 1. In addition, plants that both export and import became increasingly important in accounting for total output: they constitute only 12.7 percent of the sample but account for 48.3 percent of total output in 1996.

We now turn to measures of plant performance and their relationships with export and import status. While the differences in a variety of plant attributes between exporters and non-exporters are well-known (e.g., Bernard and Jensen, 1999), few previous empirical studies have discussed how plant performance measures depend on import status. Table 2 presents estimated premia in various performance measures according to export and import status. Following Bernard and Jensen (1999), Columns 1-3 of Table 2 report the export and import premia estimated from a pooled ordinary least squares regression using the data from 1990-1996:

$$\ln X_{it} = \alpha_0 + \alpha_1 d_{it}^x (1 - d_{it}^m) + \alpha_2 d_{it}^m (1 - d_{it}^x) + \alpha_3 d_{it}^x d_{it}^m + Z_{it}\beta + \epsilon_{it},$$

where X_{it} is a vector of plant attributes (employment, sales, labor productivity, wage, non-production worker ratio, and capital per worker). Here, d_{it}^x is a dummy for year t's export status, d_{it}^m is a dummy for year t's import status, Z includes industry dummies at the four-digit ISIC level, year dummies, and total employment to control for size.[1] The export premium α_1 is the average percentage difference between exporters and non-exporters among plants that do not import foreign intermediates. The import premium α_2 is the average percentage difference between importers and non-importers among plants that do not export. Finally, α_3 captures the percentage difference between plants that neither export nor import and plants that both export and import.

The results show that there are substantial differences not only between exporters and non-exporters but also between importers and non-importers. The export premia among non-importers are positive and significant for all characteristics except for the ratio of non-production workers to total workers as shown in column 1. The import premia among non-exporters are positive and significant for all characteristics in column 2, suggesting the importance of import status in explaining plant performance even after controlling for export status. Comparing columns 1-2 with column 3, plants that are both exporting and importing tend to be larger and be more productive than plants that are engaged in either export or import but not both.[2] The point estimates suggest that the magnitude of the performance gap for various characteristics across different export/import status are substantial.

[1] Regional dummies are available only for a subset of samples and hence we did not include them as controls.

[2] Since export status is positively correlated with import status, the magnitude of the export premia estimated without controlling for import status is likely to be overestimated by capturing the import premia.

Table 2: Premia of Exporter and Importer

	Pooled OLS: 1990-1996			Long-Difference: 1990-1996		
Export/Import Status	Exporters	Importers	Ex/Importers	Exporters	Importers	Ex/Importers
Total Employment	0.915	0.636	1.463	0.155	0.060	0.214
	(0.019)	(0.016)	(0.018)	(0.040)	(0.026)	(0.034)
Total Sales	0.300	0.551	0.763	0.121	0.106	0.216
	(0.019)	(0.013)	(0.017)	(0.036)	(0.025)	(0.030)
Value Added per Worker	0.313	0.513	0.710	0.122	0.086	0.174
	(0.022)	(0.015)	(0.019)	(0.054)	(0.043)	(0.048)
Average Wage	0.194	0.338	0.435	0.088	0.067	0.092
	(0.011)	(0.009)	(0.010)	(0.024)	(0.019)	(0.023)
Non-Production/Total Workers	0.012	0.229	0.353	0.056	0.025	0.098
	(0.014)	(0.011)	(0.013)	(0.044)	(0.033)	(0.041)
Capital per Worker	0.458	0.489	0.759	0.115	0.130	0.309
	(0.023)	(0.017)	(0.021)	(0.060)	(0.050)	(0.055)
No. of Observations	33721			3248		

Notes: Standard errors are in parentheses. "Total Employment" reports the estimates for exporter/importer premia from regression excluding the logarithm of total employment from a set of regressors.

Columns 4-6 of Table 2 report the export and import premia estimated from a long-difference regression to control for plant fixed effects using the data from 1990 and 1996:

$$\begin{aligned}\ln X_{i,96} - \ln X_{i,90} \quad &= \quad \tilde{\alpha}_0 + \alpha_1[d^x_{i,96}(1-d^m_{i,96}) - d^x_{i,90}(1-d^m_{i,90})] + \alpha_2[d^m_{i,96}(1-d^x_{i,96}) - d^m_{i,90}(1-d^x_{i,90})] \\ &\quad +\alpha_3(d^x_{i,96}d^m_{i,96} - d^x_{i,90}d^m_{i,90}) + (Z_{i,96} - Z_{i,90})\beta + \eta_i.\end{aligned}$$

The results based on long-difference regressions in columns 4-6 show the similar patterns to those based on the pooled OLS in columns 1-3 although the standard errors are now much larger. Notably, all the point estimates for column 6 are larger than those reported in columns 4-5, suggesting that plants that are both exporting and importing are larger and more productive than other plants.

2.3 Importers and Exporters Dynamics

Table 3 shows the transition dynamics of export/import status in the sample as well as plant exiting and entry. The first four rows and columns in Table 3 present the empirical transition probability of export/import status conditioned that plants continue in operation. The results show substantial persistence in plant export/import status. Plants that neither export nor im-

Table 3: Transition Probabilities of Export and Import Status and Entry and Exit

	Export/Import Status at $t+1$ conditioned on Staying						
	(1) No-Export /No-Import	(2) Export /No-Import	(3) No-Export /Import	(4) Export /Import	(2)+(4) Export	(3)+(4) Import	Exit at $t+1^a$
No-Export/No-Import at t	0.927	0.024	0.042	0.007	0.031	0.048	0.082
Export/No-Import at t	0.147	0.677	0.013	0.163	0.841	0.176	0.070
No-Export/Import at t	0.188	0.017	0.699	0.096	0.113	0.795	0.035
Export/Import at t	0.025	0.101	0.070	0.804	0.905	0.874	0.022
New Entrants at t^b	0.753	0.096	0.100	0.051	0.147	0.151	0.126
Empirical Dist. in 1990-96^c	0.686	0.086	0.120	0.108	0.194	0.228	0.068

Note: a). "Exit at $t+1$" is defined as plants that are observed at t but not observed at $t+1$ in the sample. b). "New Entrants t" is defined as plants that are not observed at $t-1$ but observed at t in the sample, of which row represents the empirical distribution of export/import status at t as well as the probability of not being observed (i.e., exit) at $t+1$. c). "Actual Dist. in 1990-1996" is the empirical distribution of Export/Import Status in 1990-1996.

port, categorized as "No-Export/No-Import," are very likely (with 92.7% probability) to neither export nor import next period. Plants that both export and import keep the same status next period with a relatively high probability of 80.4%. Plants that are either exporting or importing (but not both) keep the same status with probabilities of 67.7% and 69.9%, respectively, which is also substantial. The existence of persistence in export/import status suggests the presence of sunk costs to exporting and importing. [3]

Table 3 also indicates that the probability of exporting next period depends on import status in this period even after controlling for export status. Among non-exporters, in the first and the third rows of the fifth column of the table, the probability of exporting next period for importers is 11.3 percent, which is substantially higher than for non-importers, 3.1 percent. Among exporters, the probability of exporting next period for importers is higher by 6.4 percent than that for non-importers. Similarly, the probability of importing next period is higher among exporters than non-exporters even after controlling for importing status. The differences in the probability of importing next period for exporters are higher than for non-exporters by 12.8 percent among importers and by 7.9 percent among non-importers.

Plants that are engaged in export activities and/or import activities are also more likely to

[3] Unobserved plant-specific characteristics may also lead to persistence in export/import status.

survive as shown in the last column of Table 3. While the exiting probability for plants that are both exporting and importing is only 2.2 percent, plants that are either exporting or importing (but not both) are more likely to exit next period with probabilities of 7.0 percent and 3.5 percent, respectively. Plants that are neither exporting nor importing have the highest exiting probability of 8.2 across different export/import status.

Finally, the fifth row of Table 3 reports the empirical distribution of export/import status as well as the probability of exiting next period among new entrants. Comparing the empirical distribution of export/import status among all plants reported in the sixth row, new entrants are less likely to export or import than incumbents; the probabilities of exporting and importing among new entrants are, respectively, 14.7 percent and 15.1 percent, while the (unconditional) probabilities of exporting and importing among all plants are, respectively, 19.4 percent and 22.8 percent. New entrants face an exiting probability of 12.6 percent, which is substantially higher even relative to the exiting probability for plants that are neither exporting nor importing, 8.2 percent.

We now present a static model with heterogeneous firms which is based on Melitz(2003). We incorporate both exporting of final goods and importing of intermediate goods and allow for fixed import and export cost complementarities. We use this environment to examine the impact of trade liberalization on resource reallocation and productivity in the presence of such complementarities.

3 A Model of Exports and Imports

In this section we extend the trading environment studied by Melitz (2003) to include importing of intermediates by heterogeneous final goods producers. We demonstrate that the positive effects of trade on aggregate productivity and welfare due to resource reallocation from less to more productive firms highlighted by Melitz (2003) is strengthened by the presence of trade in intermediates. We also explore the effects of the prohibition of imports on export activity and show that restricting imports leads to a decline in the fraction of operating firms which engage in exporting.

3.1 Environment

The world is comprised of $N+1$ countries. Within each country there is a set of final goods producers and a set of intermediate goods producers. In the open economy, if a final good producer chooses to export, it will export its good to N countries and if it chooses to import intermediates, it will import from N countries.

3.1.1 Consumers

There is a representative consumer who supplies labour inelastically at level L. The consumer's preferences over consumption of a continuum of final goods are given by:

$$U = \left[\int_{\omega \in \Omega} q(\omega)^{\rho} d\omega \right]^{1/\rho},$$

where ω is an index over varieties, and $0 < \rho < 1$. The elasticity of substitution is given by $\sigma = 1/(1-\rho) > 1$.

Letting $p(\omega)$ denote the price of variety ω, we can derive optimal consumption of variety ω to be

$$q(\omega) = Q \left[\frac{p(\omega)}{P} \right]^{-\sigma}, \tag{1}$$

where P is a price index given by

$$P = \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)}, \tag{2}$$

and Q is a consumption index with $Q = U$. We also denote aggregate expenditure as $R = PQ$ and have

$$r(\omega) = R \left[\frac{p(\omega)}{P} \right]^{1-\sigma}, \tag{3}$$

where $r(\omega)$ is expenditure on variety ω with $R = \int_{\omega \in \Omega} r(\omega) d\omega$.

3.1.2 Production

We first describe the final-good sector which is characterized by a continuum of monopolistically competitive firms selling horizontally differentiated goods. Final goods firms sell to domestic consumers and in the trading environment choose whether or not to also export their goods to foreign consumers. In production, final goods producers employ labor, domestically produced intermediates, and in the trading environment, choose whether or not to also use imported intermediates.

There is an unbounded measure of ex ante identical potential entrants. Upon entering, an entrant pays a fixed entry cost, f_e, which is the same for all entrants. Each new entrant then draws a firm-specific productivity parameter, φ, from a continuous cumulative distribution $G(\varphi)$. A firm's productivity remains at this level throughout its operation. After observing φ, a firm decides whether to immediately exit or stay in the market. All final goods producers must pay a fixed production cost, f, each period to continue in operation. In addition, in each period, a firm is forced to exit with probability ξ.

In the open economy, firms must also pay fixed costs associated with importing intermediates and exporting their product in any period that they choose to be active in those markets. Before making their import and export decisions, firms draw a firm-specific shock to the fixed cost importing. This shock is denoted ϵ and is identically and independently distributed across firms and across time with a continuous cumulative distribution $H(\epsilon)$ and density $h(\epsilon)$ defined over $[\underline{\epsilon}, \bar{\epsilon}]$ with zero mean. The total fixed cost per import market for a firm which is importing but not exporting equals $f_m + \epsilon > 0$. A firm that is exporting but not importing incurs a non-stochastic cost of $f_x > 0$ each period for each export market. Finally, a firm that is both exporting and importing incurs a fixed cost equal to $\zeta(f_x + f_m + \epsilon)$ for each market, where $0 < \zeta \leq 1$ determines the degree of complementarity in fixed costs between exporting and importing. [4]

We let $d^x \in \{0, 1\}$ denote a firm's export decision where $d^x = 0$ implies that a firm does not export their good and let $d^m \in \{0, 1\}$ denote a firm's import decision where $d^m = 0$ implies that a firm does not use imported intermediates. Finally, let $d = (d^x, d^m)$ denote a final good producer's export/import status. With this notation, we can write the total per-period fixed cost of a firm that chooses d and draws ϵ as

$$F(d, \epsilon) = f + N\zeta^{d^x d^m}[d^x f_x + d^m(f_m + \epsilon)]. \tag{4}$$

The technology for a firm with productivity level φ and import status d^m is given by:

$$q(\varphi, d^m) = \varphi l(\varphi)^\alpha \left[\int_{j=0}^{n(d^m)} x(j, \varphi)^{\frac{\gamma-1}{\gamma}} dj \right]^{\frac{(1-\alpha)\gamma}{\gamma-1}}$$

where $l(\varphi)$ is labor input, $x(j, \varphi)$ is the input of intermediate variety j, α is the labor share,

[4] In Appendix 1 we derive lower bounds on the values for f_x and $f_m + \bar{\epsilon}$ and upper bounds on $f_m + \underline{\epsilon}$ which guarantee that there is a positive measure of firms in each export/import category in the open economy equilibrium. These restrictions are similar to the condition imposed by Melitz(2003) which ensures that his economy is characterized by partitioning of firms by export status.

and $\gamma > 1$ is an elasticity of substitution between any two intermediate inputs. The variable $n(d^m)$ denotes the range of intermediate inputs which are employed and it depends on the firm's import status. We normalize the measure of intermediates produced within a country to one (so $n(0) = 1$). If a firm imports foreign intermediate inputs, then it imports from all countries and the measure of intermediates it employs equals $n(1) = N + 1$.

In the intermediate goods industry, there is a continuum of firms, each producing a different variety indexed by j. As discussed above, the measure of varieties in this sector is fixed and is normalized to one in each country. Anyone can access the blueprints of the intermediate production technology for all varieties and there is free entry. Firms have identical linear technologies in labor input: $x(j) = l(j)$. These conditions imply that in the symmetric equilibrium, all intermediates will have the same price and that price will equal the wage which we normalize to one.

We now examine the optimization problems of final goods producers. In the symmetric equilibrium, inputs of all intermediates which are used will be equal so $x(j, \varphi) = x(\varphi)$ for all j. In this case, production is given by

$$q(\varphi, d^m) = a(\varphi, d^m) l(\varphi)^{\alpha} [n(d^m) x(\varphi)]^{1-\alpha}, \tag{5}$$

where $a(\varphi, d^m) = \varphi n(d^m)^{\frac{1-\alpha}{\gamma-1}}$. We will refer to this term as the firm's total factor productivity. Note that the firm's total factor productivity depends not only on inherent productivity, φ, but also on the range of varieties of intermediates a firm employs $n(d^m)$, which in turn depends on the firm's import decision. Recalling the normalization that $n(0) = 1$, we can write

$$a(\varphi, d^m) = \varphi \lambda^{d^m} \tag{6}$$

where $\lambda = n(1)^{\frac{1-\alpha}{\gamma-1}} = (N + 1)^{\frac{1-\alpha}{\gamma-1}} > 1$, implying that $a(\varphi, 1) > a(\varphi, 0)$. Thus, a firm which imports intermediates will have higher total factor productivity than if it does not import. Note that this increase in productivity results from increasing returns to variety in the production function. This approach allows us to incorporate an import premium and is motivated by the empirical findings presented in Section 2 and in Kasahara and Rodrigue (2005). An alternative approach would include incorporating vertically differentiated inputs with foreign inputs of higher quality to generate an import premium. The approach taken here has the advantage of tractability and is widely used in models of trade with differentiated products (see Ethier (1982), for example).

It is well-known that the form of preferences implies that final goods producers will price at a constant markup equal to $1/\rho$ over marginal cost. Hence, using the final goods technology and recalling that all intermediates are priced at the wage which equals one, we have the following pricing rule for final goods sold in the home market for a producer with productivity φ and import status d^m:

$$p^h(\varphi, d^m) = \left(\frac{1}{\rho}\right)\left(\frac{1}{\Gamma a(\varphi, d^m)}\right), \tag{7}$$

where $\Gamma \equiv \alpha^\alpha(1-\alpha)^{1-\alpha}$. Note that the prices in the closed economy equilibrium are just given by equation (7) with $d^m = 0$.

We also assume that there are iceberg exporting costs so that $\tau > 1$ units of goods has to be shipped abroad for 1 unit to arrive at its destination. The pricing rule for final goods sold in the foreign market then is given by:

$$p^f(\varphi, d^m) = \tau p^h(\varphi, d^m) \tag{8}$$

The total revenue of a final good producer depends on inherent productivity and export/import status. Using equation (3), revenue from sales in the home market can be written as

$$r^h(\varphi, d^m) = R\left(P\rho\Gamma a(\varphi, d^m)\right)^{\sigma-1} \tag{9}$$

and revenue from foreign sales per country of export is given by

$$r^f(\varphi, d) = d^x \tau^{1-\sigma} r^h(\varphi, d^m). \tag{10}$$

Hence, total revenue for a firm with inherent productivity φ and export/import status d is given by

$$r(\varphi, d) = r^h(\varphi, d^m) + N r^f(\varphi, d), \tag{11}$$

or

$$r(\varphi, d) = (1 + d^x N \tau^{1-\sigma}) r^h(\varphi, d^m). \tag{12}$$

Thus, using equations (6), (9), and (12), we can determine revenue for a firm with inherent productivity φ and export/import status d relative to a firm with the same productivity parameter who is neither exporting nor importing:

$$r(\varphi, d) = b_x^{d^x} b_m^{d^m} r(\varphi, 0, 0), \tag{13}$$

where $b_x \equiv 1 + N\tau^{1-\sigma}$ and $b_m \equiv \lambda^{\sigma-1}$.

Turning to profits, we see that the pricing rule of firms implies that profits of a final good producer with inherent productivity φ, export/import status d, and fixed import cost shock ϵ can be written as

$$\pi(\varphi, d, \epsilon) = \frac{r(\varphi, d)}{\sigma} - F(d, \epsilon) \tag{14}$$

In what follows, we explore the equilibria of four economies: the closed economy and three trading economies. Let autarkic equilibrium variables be denoted with a subscript A and note that in this equilibrium firms can only choose $d = (0, 0)$. We denote equilibrium variables in the full trading equilibrium with both importing and exporting with a subscript T. It is also useful to consider variants of the economy which are characterized by partial trade. In particular, our open economy with $\zeta = b_m = 1$ is equivalent to the open economy studied by Melitz (2003) with exporting of final goods but no importing of intermediates and we denote this economy with an X subscript. We also examine an economy with importing of intermediate goods but no exporting of final goods by evaluating our open economy with $\zeta = b_x = 1$. We denote equilibrium variables of this economy with an I subscript.

Thus, equilibrium levels of the aggregate price index and aggregate revenue in economy $S \in \{A, T, X, I\}$ are denoted P_S and R_S respectively. Evaluating equation (9) at these equilibrium values in the relevant economies and using this in equation (12) allows us to determine equilibrium revenue functions for final goods producers in each economy. We denote these revenue functions as $r_S(\varphi, d)$ for $S \in \{A, T, X, I\}$. Similarly, we can derive profit functions, $\pi_S(\varphi, d, \epsilon)$ for each economy from equation (14).

3.2 Exit, Export, and Import Decisions

3.2.1 Exit Decision

We focus on stationary equilibria in which aggregate variables remain constant over time. Under the assumptions of no discounting, that the productivity level for a firm is constant throughout its life, and that the fixed import cost shocks are independent across time, a final goods firm faces a static optimization problem. In the closed economy, after observing its productivity, a firm will choose to exit if its period profits are negative. In the open economy, after observing its productivity, a firm will choose to exit if its expected period profits are negative where the expectation is taken over the stochastic component of the fixed cost of importing, ϵ.

Each firm's value function in economy $S \in \{A, T, X, I\}$ is given by

$$V_S(\varphi) = max\left\{0, \sum_{t=0}^{\infty}(1-\xi)^t E_{\epsilon_t}\left(\underset{d_t\in\{0,1\}^2}{max}\pi_S(\varphi, d_t, \epsilon_t)\right)\right\} = max\left\{0, E_\epsilon\left(\underset{d\in\{0,1\}^2}{max}\frac{\pi_S(\varphi, d, \epsilon)}{\xi}\right)\right\}. \tag{15}$$

In this equation, the second equality follows because ϵ is independently distributed over time. Now since profits are strictly increasing in φ, there exists a φ_S^* such that a firm will exit if $\varphi < \varphi_S^*$ where φ_S^* is characterized by

$$E_\epsilon\left(\underset{d\in\{0,1\}^2}{max}\frac{\pi_S(\varphi_S^*, d, \epsilon)}{\xi}\right) = 0. \tag{16}$$

or

$$\int_{\underline{\epsilon}}^{\bar{\epsilon}} \underset{d\in\{0,1\}^2}{max}\pi_S(\varphi_S^*, d, \epsilon)h(\epsilon)d\epsilon = 0. \tag{17}$$

For tractability, we impose lower bounds on $\underline{\epsilon}$ so that firms with the marginal productivity for operation in the three open economies will choose to neither import nor export. These bounds are presented in Appendix 1. Under these bounds, we have the following proposition which characterizes the cutoff productivities in the four economies considered.

Proposition 1

If the cutoff productivities for operation in each economy are unique, then these cutoff productivities satisfy $r_S(\varphi_S^*, 0, 0) = \sigma f$, for $S \in \{A, T, X, I\}$.

The proof of this and all remaining propositions are presented in Appendix II. Below, we will prove that the cutoff productivities are unique.

Note that in each economy, we can relate all other firms' revenues to the revenues of the marginal firm for operation in that economy. Using equations (6), (9), and (13), we can derive in the closed economy $\forall\ \varphi$:

$$r_A(\varphi, 0, 0) = \left[\frac{\varphi}{\varphi_A^*}\right]^{\sigma-1}\sigma f. \tag{18}$$

While in the full trading economy, we can write $\forall\ (\varphi, d)$

$$r_T(\varphi, d) = b_x^{d^x} b_m^{d^m}\left[\frac{\varphi}{\varphi_T^*}\right]^{\sigma-1}\sigma f. \tag{19}$$

Similarly, for the partial trading economies, we have $\forall\ (\varphi, d^x)$

$$r_X(\varphi, d^x, 0) = b_x^{d^x}\left[\frac{\varphi}{\varphi_X^*}\right]^{\sigma-1}\sigma f. \tag{20}$$

and $\forall\ (\varphi, d^m)$

$$r_I(\varphi, 0, d^m) = b_m^{d^m} \left[\frac{\varphi}{\varphi_I^*}\right]^{\sigma-1} \sigma f. \tag{21}$$

3.2.2 Export and Import Decisions

For the full trading economy, we now consider the export and import decisions for firms which choose not to exit. Recall that firms make exit decisions before observing ϵ but make export and import decisions after observing ϵ. Define the following:

$$\Phi(\varphi) \equiv \left(\frac{\varphi}{\varphi_T^*}\right)^{\sigma-1} \left(\frac{f}{N}\right), \tag{22}$$

For convenience, we can reference firms of different productivity levels by Φ where the dependence on φ is understood. We will refer to this variable as relative productivity. Thus, using equations (14) and (19), we can write profits in terms of Φ:

$$\hat{\pi}(\Phi, d, \epsilon) = N \left(b_x^{d^x}\right) \left(b_m^{d^m}\right) \Phi - F(d, \epsilon) \tag{23}$$

To obtain the export and import decision rules as a function of a firm's productivity and fixed import cost, we define the following variables.

Let $\Phi_x^{d^m}(\epsilon)$ be implicitly defined by $\hat{\pi}(\Phi_x^{d^m}(\epsilon), 1, d^m, \epsilon) = \hat{\pi}(\Phi_x^{d^m}(\epsilon), 0, d^m, \epsilon)$ or

$$\Phi_x^{d^m}(\epsilon) = \frac{\zeta^{d^m} f_x + d^m(\zeta^{d^m} - 1)(f_m + \epsilon)}{b_m^{d^m}(b_x - 1)}. \tag{24}$$

So a firm with import status d^m, fixed import cost shock ϵ, and relative productivity $\Phi_x^{d^m}(\epsilon)$ will be indifferent between exporting and not exporting. Let $\Phi_m^{d^x}(\epsilon)$ be implicitly defined by $\hat{\pi}(\Phi_m^{d^x}(\epsilon), d^x, 1, \epsilon) = \hat{\pi}(\Phi_m^{d^x}(\epsilon), d^x, 0, \epsilon)$ or

$$\Phi_m^{d^x}(\epsilon) = \frac{\zeta^{d^x}(f_m + \epsilon) + d^x(\zeta^{d^x} - 1) f_x}{b_x^{d^x}(b_m - 1)}. \tag{25}$$

So a firm with export status d^x, fixed import cost shock ϵ, and relative productivity $\Phi_m^{d^x}(\epsilon)$ will be indifferent between importing and not importing. Let $\Phi_{xm}(\epsilon)$ be implicitly defined by $\hat{\pi}(\Phi_{xm}(\epsilon), 1, 1, \epsilon) = \hat{\pi}(\Phi_{xm}(\epsilon), 0, 0, \epsilon)$ or

$$\Phi_{xm}(\epsilon) = \frac{\zeta(f_x + f_m + \epsilon)}{(b_x b_m - 1)}. \tag{26}$$

So a firm with fixed import cost shock ϵ, and relative productivity $\Phi_{xm}(\epsilon)$ will be indifferent between participating in both exporting and importing markets and not participating in either market.

These variables allow us to determine the firms' choices of d depending on their Φ and their ϵ. If we let $\theta \equiv f_m + \epsilon$, where $\theta \in (f_m + \underline{\epsilon}, f_m + \bar{\epsilon}) \equiv (\underline{\theta}, \bar{\theta})$, then we can graph each of the variables defined in equations (24), (25), and (26) above as a function of θ, and determine firms' export and import choices depending on their relative productivity, Φ, and the random component of their fixed import cost, ϵ. We first consider the case with no complementarities in fixed export and import costs, $\zeta = 1$. Figure 1 graphs cutoff functions for this case and shows the four regions of possible export/import status. Note that $\Phi(\varphi_T^*) = \frac{f}{N}$ so active firms are those with $\Phi \geq \frac{f}{N}$. As the figure demonstrates, the space of (Φ, θ) is partitioned into four areas according to firms' export and import choices. Firms with relatively low productivity and low fixed cost of importing will choose to import but not export. Firms with relatively low productivity and higher fixed cost of importing will choose to neither import nor export. Firms with relatively high productivity and relatively high fixed cost of importing will choose to export but not import. Finally, firms with relatively high productivity will choose to both import and export.

By examining the equations for the different regions, we can also determine the effect of complementarities in the fixed costs of importing and exporting. Recall that a decrease in ζ represents an increase in complementarities. Examination of equations (24)-(26) shows that a decrease in ζ will shift down and decrease the slopes of $\Phi_m^1(\cdot)$, $\Phi_x^1(\cdot)$, and $\Phi_{xm}(\cdot)$. As can be seen from Figure 2, each of these changes would serve to increase the measure of firms choosing to both export and import and decrease the measure of firms in each of the other three areas. This is intuitive as an increase in the complementarities in fixed costs of importing and exporting should increase the fraction of firms which choose to engage in both activities.

3.3 Aggregation

In this section, we derive a weighted average of firm productivity levels for each of the four types of economies. As in Melitz(2003), these averages will also represent aggregate productivity in each environment because they summarize the information in the distribution of productivities which are relevant for aggregate variables.

Let $\nu_S(\varphi_S^*, d)$ denote the fraction of successful firms that have export/import status equal to d in economy $S \in \{A, T, X, I\}$. These fractions are presented in Appendix I. Furthermore, let $M_S(\varphi_S^*)$ denote the equilibrium mass of operating firms in economy S. With these variables, we can derive the total mass of firms with export/import status equal to d in each economy as

$$M_S(\varphi_S^*, d) = \nu_S(\varphi_S^*, d) M_S(\varphi_S^*) \tag{27}$$

Hence, the total number of varieties available to a consumer in each equilibrium is given by $M_S^C(\varphi_S^*) = M_S(\varphi_S^*) + N[M_S(\varphi_S^*, 1, 0) + M_S(\varphi_S^*, 1, 1)]$, where $M_S(\varphi_S^*)$ varieties are purchased from home producers and $N(M_S(\varphi_S^*, 1, 0) + M_S(\varphi_S^*, 1, 1))$ are purchased from foreign producers (imported final goods).

We demonstrate in Appendix I that the aggregate price index given by equation (2) for each economy can be written as a function of a weighted average of the productivities of all firms (home and foreign) from which a consumer purchases final goods. Let $\tilde{b}_S(\varphi_S^*, d)$ denote this weighted average across firms choosing export/import status d in economy $S \in \{A, T, X, I\}$ from which the consumer purchases final goods. The price index in economy S as a function of these average productivities is given by:

$$P_S = \left[\frac{1}{\Gamma\rho}\right] \left[\sum_{d \in \{0,1\}^2} M_S(\varphi_S^*, d) \tilde{b}_S(\varphi_S^*, d)^{\sigma-1}\right]^{\frac{1}{1-\sigma}} \tag{28}$$

or

$$P_S = M_S^C(\varphi_S^*)^{\frac{1}{1-\sigma}} p \left(\left[\frac{1}{M_S^C(\varphi_S^*)} \sum_{d \in \{0,1\}^2} M_S(\varphi_S^*, d) \tilde{b}_S(\varphi_S^*, d)^{\sigma-1}\right]^{\frac{1}{1-\sigma}} \right) \tag{29}$$

This equation implies that the aggregate price level can be written as a function of the number of varieties available to the consumer ($M_S^C(\varphi_S^*)$) and a weighted average of productivities of all operating firms given by:

$$\bar{b}_S(\varphi_S^*) = \left[\frac{1}{M_S^C(\varphi_S^*)} \left(\sum_{d \in \{0,1\}^2} M_S(\varphi_S^*, d) \tilde{b}_S(\varphi_S^*, d)^{\sigma-1}\right)\right]^{\frac{1}{\sigma-1}} \tag{30}$$

This measure of average productivity summarizes the effects of the distribution of productivity levels and fixed import costs on aggregate outcomes.

Hence, we can write the aggregate price level in each economy as

$$P_S = \frac{M_S^C(\varphi_S^*)^{\frac{1}{1-\sigma}}}{\Gamma\rho \bar{b}_S(\varphi_S^*)}. \tag{31}$$

Furthermore, using equations (3), (18), (19), (20), (21), (30), and the average productivity measures defined in Appendix I, we can write aggregate revenue in each economy as

$$R_S = M_S^C(\varphi_S^*) r_S(\bar{b}_S(\varphi_S^*), 0, 0) \tag{32}$$

3.4 Autarkic and Trading Equilibria

As the above analysis suggests, all variables in the stationary equilibrium for each economy can be determined once we determine the cutoff variable for operation, φ_S^*. We now seek to characterize the equations which determine these cutoff variables in each of the four economies under consideration. Let average profits within each group of firms according to export/import status in economy S be denoted $\bar{\pi}_S(\varphi_S^*, d)$. These average profit functions are derived in Appendix I. Note that the average profits for each group are derived under the equilibrium condition that expected profits for the marginal firm equal zero.

Thus, for each economy we have our first equilibrium equation in two unknowns: average overall profit and the cutoff productivity:

$$\bar{\pi}_S(\varphi_S^*) = \sum_{d\in\{0,1\}^2} \nu_S(\varphi_S^*, d)\bar{\pi}_S(\varphi_S^*, d). \tag{33}$$

The second equilibrium equation for each economy is given by the free-entry condition which guarantees that the ex-ante value of an entrant must be equal zero:

$$(1 - G(\varphi_S^*))\left(\frac{\bar{\pi}_S(\varphi_S^*)}{\xi}\right) - f_e = 0. \tag{34}$$

Combining equations (33) and (34) determines the equilibrium cutoff productivity for exit in each economy, φ_S^*:

$$\sum_{d\in\{0,1\}^2} \nu_S(\varphi_S^*, d)\bar{\pi}(\varphi_S^*, d) = \frac{\xi f_e}{(1 - G(\varphi_S^*))}. \tag{35}$$

Proposition 2

The four equilibria cutoff productivities, φ_A^*, φ_X^*, φ_I^*, and φ_T^* exist and are unique.

The following proposition examines the impact of various forms of trade on the cutoff productivity level for operation.

Proposition 3

(i.) $\varphi_A^* < \varphi_X^* < \varphi_T^*$.

(ii.) $\varphi_A^* < \varphi_I^* < \varphi_T^*$.

We also note that equation (34) and this proposition implies that average profits have a similar ranking across the different economies. This proposition implies that opening trade in either final goods or intermediates or both causes firms with lower productivity to exit. In the economy with no importing, this result is identical to that identified by Melitz(2003) where the exportation of final goods generates a resource reallocation from less productive firms to more productive firms. In the economy with only importing, there is also exit of less productive firms and this, along with the importing of foreign intermediates in the presence of increasing returns to variety, causes aggregate productivity to increase. In the full trading equilibrium, both effects are at work and the reallocation from less productive firms to more productive firms is intensified and aggregate productivity increases.

We now state a number of propositions which examine the impact of moving from autarky to an economy with trade on this measure of welfare and other aggregate variables of interest.

Proposition 4

(i.) $M_A > M_X > M_T$

(ii.) $M_A > M_I > M_T$

This result is also similar to Melitz (2003) and results as the supply of labor is fixed but more productive firms now demand more labour so some firms must exit. This is an example of a *selection effect* as discussed in the trade literature with increasing returns and free entry (see Krugman (1979), for example.) Our environment identifies an additional mechanism arising from the presence of imported intermediates that strengthens the selection effect discussed by Melitz (2003). Note also that the number of varieties available to the consumer in the open economies may be higher or lower than the number of varieties available to the consumer in autarky.

We are also interested in the normative effects of trade and we can use the equilibrium aggregate price index in each equilibrium to calculate welfare per worker:

$$W_S = \frac{1}{P_S}. \tag{36}$$

In moving from autarky to an economy with trade in final goods, consumer welfare is impacted by two effects. The number of varieties available to the consumer changes and aggregate productivity is higher. In the trading economy with no trade in final goods but trade in intermediates, consumer welfare is only affected by the latter effect. The aggregate productivity gain impacts positively on welfare. If the number of varieties available to the consumer is higher in trade then welfare is also enhanced by this effect but if it falls then welfare is negatively impacted. However, the next proposition states that the increase in welfare from the productivity gain dominates and welfare is higher in any of the trading economies than in autarky It also demonstrates that full trade generates higher welfare than partial trade.

Proposition 5

(i.) $W_A < W_X < W_T$

(ii.) $W_A < W_I < W_T$

We now examine the effect of trade on firms' revenues and profits. As in Melitz(2003), we compare revenues and profits of a firm with a given level of productivity between autarky and trade. Recalling that $R_S = L \ \forall S \in \{A, T, X, I\}$, we note that a firm's market share within the home country just equals its revenue in each equilibrium. Thus, comparing revenues is also a comparison of market shares. The next proposition argues that trade shifts market shares from firms which do not engage in trade in the open economy to firms which do. In what follows let $r_A(\varphi)$ and $\pi_A(\varphi)$ denote the revenue and profits of a firm in autarky with inherent productivity φ.

Proposition 6

Let $\bar{b}_x$ and $\bar{b}_m$ be defined by $\varphi_T^*(\bar{b}_x) = b_m^{\frac{1}{\sigma-1}}\varphi_A^*$ and $\varphi_T^*(\bar{b}_m) = b_x^{\frac{1}{\sigma-1}}\varphi_A^*$. Consider a firm with productivity φ, then comparing revenue of this firm between autarky and trade, we have the following results:

(i.) $r_T(\varphi, 0, 0) < r_A(\varphi) < r_T(\varphi, 1, 1)$

(ii.) $r_A(\varphi) \lesseqgtr r_T(\varphi, 1, 0)$ as $b_m \lesseqgtr \bar{b}_m$

(iii.) $r_A(\varphi) \lesseqgtr r_T(\varphi, 0, 1)$ as $b_x \lesseqgtr \bar{b}_x$

In the proof of this proposition we demonstrate that $\bar{b}_x > 1$ and $\bar{b}_m > 1$. Thus when $\zeta = b_m = 1$ (an economy with no intermediate imports) firms which choose to export in the open economy have higher revenues than they did in autarky (this result is analogous to the revenue comparison in Melitz(2003)). Similarly, when $\zeta = b_x = 1$ (no final goods exports), firms which choose to import in the open economy have higher revenues than they did in autarky. In the economy with both exports and imports, (ii.) implies that if the returns to importing are large enough, then a firm which chooses to export but not import in the open economy will lose revenue. The intuition for this result is that a firm which chooses to only export is at a disadvantage relative to its domestic and foreign competitors who are importing intermediates and gaining in productivity. If the returns to importing are large (b_m large) then the presence of these competitors leads to lower market shares for a firm which is only exporting. We have a similar effect for firms which are only importing if the returns to exporting are high (b_x large).

The next proposition compares the profits for firms between autarky and trade.

Proposition 7

(i.) $\pi_T(\varphi, 0, 0, \epsilon) < \pi_A(\varphi) \qquad \forall \varphi, \;\forall \epsilon$

(ii.) $\exists\ \bar{\varphi}_T(\epsilon) > \varphi_T^*$ such that $\pi_T(\varphi, 1, 1, \epsilon) \lesseqgtr \pi_A(\varphi)$ as $\varphi \lesseqgtr \bar{\varphi}_T(\epsilon), \qquad \forall\ \epsilon$

(iii.) For $b_m \leq \bar{b}_m$, $\exists\ \bar{\varphi}_X > \varphi_T^*$ such that $\pi_T(\varphi, 1, 0, \epsilon) \lesseqgtr \pi_A(\varphi)$ as $\varphi \lesseqgtr \bar{\varphi}_X, \qquad \forall\ \epsilon$

For $b_m > \bar{b}_m$, $\pi_T(\varphi, 1, 0, \epsilon) < \pi_A(\varphi), \qquad \forall\ \varphi, \forall\ \epsilon$

(iv.) For $b_x \leq \bar{b}_x$, $\exists\ \bar{\varphi}_M(\epsilon) > \varphi_T^*$ such that $\pi_T(\varphi, 0, 1, \epsilon) \lesseqgtr \pi_A(\varphi)$ as $\varphi \lesseqgtr \bar{\varphi}_M(\epsilon), \qquad \forall\ \epsilon$

For $b_x > \bar{b}_x$, $\pi_T(\varphi, 0, 1, \epsilon) < \pi_A(\varphi), \qquad \forall\ \varphi, \forall\ \epsilon$

The first statement in this proposition follows because firms which neither export nor import in the open economy lose revenue so must have lower profits in trade than in autarky. The second part of this proposition states that trade allows the most efficient firms (higher productivity and lower fixed costs of importing) to fully engage in trade and gain market share and profits. However, we know that for a firm with $\epsilon_1 < \epsilon < \epsilon_2$ and $\varphi = \tilde{\varphi}_{xm}(\epsilon)$ that this firm will fully engage in trade but will have lower profits in trade than in autarky. Hence, only a subset of firms which choose to both import and export in the open economy will have higher profits than in autarky.

Part (iii.) of the proposition suggests the possibility of a similar partitioning among firms which only export in the open economy. If the returns to importing are not too high, then all firms which choose to only export in the open economy will have higher market share but possibly some of those firms will have lower profits while others have higher profits. Part (iv.) of the proposition states a similar result for firms which only import in the open economy. Figures 4-6 exhibit the revenue and profit comparisons contained in Propositions 6 and 7.

3.5 Import Restrictions

We now briefly examine the effects on export activity of import restrictions. In this environment, the importation of intermediates makes firms more productive because of the increasing returns to variety in production. This may allow a larger fraction of firms to cover the fixed cost associated with exporting and allow them to enter the export market. Thus, a restriction on imports may decrease export activity and hence, import protection may act as export destruction in this environment.

Recall from equation (6) that a firm's total factor productivity is given by

$$a(\varphi, d^m) = \varphi \lambda^{d^m} \tag{37}$$

where $\lambda = n(1)^{\frac{1-\alpha}{\gamma-1}} > 1$. We model import restrictions as a decrease in the number of import markets to which firms have access – that is a decrease in $n(1)$ (which we previously assumed equaled the number of a country's trading partners). Hence an import restriction will be modeled as a decrease in λ and a resulting decrease in $b_m = \lambda^{\sigma-1}$. In what follows, we seek to determine the effect of a decrease in b_m on a measure of export activity in the full trading equilibrium.

Proposition 8

In the full trading equilibrium, the fraction of firms which export and the average revenue and market share of exporting firms is increasing in b_m.

Figure 3 demonstrates the effect on the fraction of exporters when there are no fixed cost complementarities ($\zeta = 1$) for the extreme case in which imports are prohibited ($b_m \to 1$). The hatched area in that figure shows the fraction of exporting firms which stop exporting when imports are prohibited, and, hence the export destruction due to import protection.

It should also be clear that restricting imports of intermediates will lower aggregate productivity in this environment. This will result from a direct effect and an indirect effect. The direct effect is clear from the above discussion and equation (6), which demonstrates that importing intermediates increases productivity because of increasing returns to variety in production. As Proposition 3 demonstrates, there will also be an indirect negative effect on aggregate productivity because restricting imports decreases the average cutoff productivity for operation so resources will be reallocated from more productive firms toward less productive firms and aggregate productivity will fall.

4 Structural Estimation of Static Model

4.1 The Environment

In this section, we develop an empirical model based on the theoretical model in the previous section. We add additional shocks to the model but the fundamental environment remains the same. In particular we introduce a stochastic fixed cost of exporting, similar to the fixed cost of importing in the model above to retain symmetry between these two activities. We also incorporate a cost shocks associated with exiting to improve the estimation.

We make the following distributional assumptions:

- The logarithm of plant-specific productivity upon entry is drawn from $N(0, \sigma_\varphi^2)$.[5] Productivity is constant after the initial draw.

- The cost shocks associated with the export/import decision, denoted by $\epsilon_t^d(d)$ for $d \in \{0,1\}^2$, are independently drawn from the identical extreme-value distribution with mean zero and scale parameter ϱ^d. Let $\epsilon_t^d \equiv (\epsilon_t^d(0,0), \epsilon_t^d(1,0), \epsilon_t^d(0,1), \epsilon_t^d(1,1))$.

- The cost shocks associated with the exiting decision, denoted by $\epsilon_t^\chi = (\epsilon_t^\chi(0), \epsilon_t^\chi(1))$, are independently drawn from the identical extreme-value distribution with mean zero and scale parameter ϱ^χ.

The discount factor is given by $\beta \in (0,1)$. The timing of the incumbent's decision with the productivity φ within each period is as follows. At the beginning of every period, a firm faces a possibility of a large negative shock that leads it to exit with an exogenous probability ξ. Then, a firm draws the (additional) choice-dependent idiosyncratic cost shocks associated with exiting decisions, $\epsilon_t^\chi = (\epsilon_t^\chi(0), \epsilon_t^\chi(1))$. Given the realizations, the firm decides whether it exits from the market or continues to operate. If the firm decides to exit, it receives the terminal value of $\epsilon_t^\chi(0)$. If the firm decides to continue to operate, then it will draw the choice-dependent idiosyncratic cost shocks associated with export and import, $\epsilon_t^d(d)$ for $d \in \{0,1\}^2$. After observing $\epsilon_t^d(d)$, the firm makes export and import decisions. The Bellman's equation for an incumbent firm with the productivity φ is written as

$$
\begin{aligned}
V(\varphi) &= \int \max\{\epsilon^{\chi'}(0), W(\varphi) + \epsilon^{\chi'}(1)\} dH^\chi(\epsilon^{\chi'}) \\
W(\varphi) &= \int \max_{d \in \{0,1\}^2} \left\{\pi(\varphi, d) + \beta(1-\xi)V(\varphi) + \epsilon^{d'}(d)\right\} dH^d(\epsilon^{d'}),
\end{aligned}
$$

or, using the properties of the extreme-value distributed random variables [c.f., Rust (1987)], the Bellman's equation is rewritten as

$$
\begin{aligned}
V(\varphi) &= \varrho^\chi \ln(\exp(0) + \exp(W(\varphi)/\varrho^\chi)) \\
W(\varphi) &= \varrho^d \ln\left(\sum_{d'} \exp\left([\pi(\varphi, d') + \beta(1-\xi)V(\varphi)]/\varrho^d\right)\right).
\end{aligned}
\tag{38}
$$

[5]The mean of initial productivity draws is set to zero in order to achieve the identification.

With the solution to the functional equation (54), the conditional choice probabilities of exiting and export/import decisions follow the Nested Logit formula. In particular, taking into account the exogenous exiting probability of ξ, the probability of exiting ($\chi = 0$) and staying ($\chi = 1$) is given by:

$$P(\chi = 1|\varphi) = (1 - \xi)\frac{\exp(W(\varphi)/\varrho^{\chi})}{\exp(0) + \exp(W(\varphi)/\varrho^{\chi})}, \tag{39}$$

and $P(\chi = 0|\varphi) = 1 - P(\chi = 1|\varphi)$. *Conditional on* $\chi = 1$ (i.e., continuously operating), the choice probabilities of d are given by the multinomial logit formula:

$$P(d|\varphi, \chi = 1) = \frac{\exp([\pi(\varphi, d) + \beta(1 - \xi)V(\varphi)]/\varrho^{d})}{\sum_{d' \in \{0,1\}^2} \exp([\pi(\varphi, d') + \beta(1 - \xi)V(\varphi)]/\varrho^{d})}, \tag{40}$$

4.2 Stationary Equilibrium

As before, we focus on a stationary equilibrium in which the distribution of φ is constant over time and let the stationary distribution of φ among incumbents is denoted by $\mu(\varphi)$.

The expected value of an entering firm is given by $\int V(\varphi')g_0(\varphi')d\varphi'$, where $V(\cdot)$ is given in (54). Under free entry, this value must be equal to the fixed entry cost f_e:

$$\int V(\varphi')g_{\varphi}(\varphi')d\varphi' = f_e,$$

where $g_{\varphi}(\varphi) = \phi(\varphi/\sigma_{\varphi})/\sigma_{\varphi}$. In equilibrium, this free entry condition has to be satisfied.

The stationarity requires that, for each productivity-type φ, the number of exiting firms is equal to the number of *successful* new entrants so that:

$$MP(\chi = 1|\varphi)\mu(\varphi) = M_e P(\chi = 0|\varphi)g_{\varphi}(\varphi),$$

where M is a total mass of incumbents; M_e is a mass of new entrants. This implies that the stationary distribution μ can be computed as:

$$\mu(\varphi) = \frac{M_e}{M}\frac{P(\chi = 1|\varphi)}{P(\chi = 0|\varphi)}g_{\varphi}(\varphi),$$

where, using $\int d\mu(\varphi) = 1$, we may derive

$$\frac{M_e}{M} = \frac{1}{\int \frac{P(\chi=1|\varphi)}{P(\chi=0|\varphi)}g_{\varphi}(\varphi)d\varphi}.$$

4.3 Likelihood Function

The total revenue and the export revenue are assumed to be measured with errors, denoted by ω_{it} and ω_{it}^f, where ω_{it} is a measurement error for total revenue and ω_{it}^f is a measurement error for export revenue. We also consider labor augmented technological change at the annual rate of α_t. Modifying the revenue functions by incorporating measurement errors $(\omega_{it}, \omega_{it}^f)$ and time trend α_t, we specify the logarithm of the *observed* total and export revenues as:

$$\ln r_{it} = \alpha_0 + \alpha_t t + \log[1 + \exp(\alpha_x) d_{it}^x] + \alpha_m d_{it}^m + \ln \varphi_i + \omega_{it} \tag{41}$$

$$\ln r_{it}^f = \alpha_0 + \alpha_t t + \alpha_x + \alpha_m d_{it}^m + \ln \varphi_i + \omega_{it}^f, \tag{42}$$

where ω_{it} is measurement error in total revenue and ω_{it} is measurement error in export revenue.

Importantly, equations (41)-(42) are *reduced-form* specifications; we have the following relationships between reduced-form parameters and structural parameters:[6]

$$\begin{aligned} \alpha_0 &= \ln[\alpha^\alpha (1-\alpha)^{1-\alpha} R P^{\sigma-1}], \\ \alpha_x &= \ln[N \tau^{1-\sigma}], \\ \alpha_m &= (\sigma - 1) \ln \lambda. \end{aligned}$$

Note that, since α_0, α_x, and α_m are not structural parameters, they could be affected by policy changes. In particular, any policy change that will affect the aggregate price P will lead to a change in α_0. As we discuss later, counterfactual policy experiments we conduct in this paper explicitly take into account for equilibrium price responses using our knowledge on the relationship between the reduced form parameter α_0 and the aggregate price P.

Given these specifications for revenues, firm's profit is

$$\pi(\varphi_i, d_{it}) = (1/\sigma) r(\varphi_i, d_{it}) - F(d_{it}), \tag{43}$$

where

$$\begin{aligned} r(\varphi_i, d_{it}) &= [1 + \exp(\alpha_x) d_{it}^x] \exp(\alpha_0 + \alpha_m d_{it}^m + \ln \varphi_i) \\ F(d_{it}) &= f + \zeta_f^{d_{it}^x d_{it}^m} (f_x d_{it}^x + f_m d_{it}^m). \end{aligned}$$

[6] Also, with abuse of notation, we replace $(\sigma - 1) \ln \varphi$ by $\ln \varphi$ since $(\sigma - 1)$ cannot be separately identified from the variance of $\ln \varphi$.

Conditioning on the value of φ_i, we may compute the estimates of ω_{it} and ω_{it}^f as

$$\begin{aligned}\tilde{\omega}_{it}(\varphi_i) &= \ln r_{it} - \alpha_0 - \alpha_t t - \log[1 + \exp(\alpha_x) d_{it}^x] - \alpha_m d_{it}^m - \ln \varphi_i \\ \tilde{\omega}_{it}^f(\varphi_i) &= \ln r_{it}^f - \alpha_0 - \alpha_t t - \alpha_x - \alpha_m d_{it}^m - \ln \varphi_i.\end{aligned}$$

We assume that ω_{it} and ω_{it}^f are jointly drawn from $N(0, \Sigma_\omega)$. Let $\sigma_1^2 = Var(\omega)$, $\sigma_2^2 = Var(\omega^f)$, $\sigma_{12} = Cov(\omega, \omega^f)$, and $\rho = \frac{\sigma_{12}}{\sigma_1 \sigma_2}$. Then, the density function for ω_{it} is given by

$$g_\omega(\omega_{it}) = \frac{1}{\sigma_1} \phi\left(\frac{\omega_{it}}{\sigma_1}\right),$$

while the density function for ω_{it}^f conditional on ω_{it} is

$$g_\omega^f(\omega_{it}^f | \omega_{it}) = \frac{1}{\sqrt{1-\rho^2}\sigma_2} \phi\left(\frac{\omega_{it}^f - (\sigma_{12}/\sigma_2^2) \o mega_{it}}{\sqrt{1-\rho^2}\sigma_2}\right),$$

where $\rho^2 = \frac{\sigma_{12}^2}{\sigma_1^2 \sigma_2^2}$. The joint density function for ω and ω^f is given by $g_\omega(\omega_{it}) g_\omega^f(\omega_{it} | \omega_{it}^f)$.

Among the incumbents firms, φ_i must be distributed according to $\mu(\varphi)$. On the other hand, among new entrants, the probability density function of φ is given by

$$g_e(\varphi) = \frac{P(\chi = 1|\varphi)}{\int P(\chi = 1|\varphi') g_\varphi(\varphi') d\varphi'} g_\varphi(\varphi).$$

The parameter vector to be estimated is $\theta = (\alpha_0, \alpha_x, \alpha_m, \sigma_\varphi^2, f, f_x, f_m, \zeta_f, \varrho^\chi, \varrho^d, igma_1, \sigma_2, o)$. The discount factor $\tilde{\beta}$ is not estimated and is set to 0.96. The parameters are estimated by the method of Maximum Likelihood.

Conditioning on φ_i, the likelihood contribution from the observation of plant i for $t > T_{i,0}$ is computed as:

$$L_{it}(\theta|\varphi_i) = \begin{cases} P(\chi_{it} = 0|\varphi_i) & \text{for } \chi_{it} = 0, \\ P(\chi_{it} = 1|\varphi_i) P(d_{it}|\varphi_i, \chi_{it} = 1) g_\omega(\tilde{\omega}_{it}(\varphi_i)) & \text{for } \chi_{it} = 1 \text{ and } d_{it}^x = 0, \\ \underbrace{P(\chi_{it} = 1|\varphi_i)}_{\text{Staying}} \underbrace{P(d_{it}|\varphi_i, \chi_{it} = 1)}_{\text{Export/Import}} \underbrace{g_\omega(\tilde{\omega}_{it}(\varphi_i)) g_\omega^f(\tilde{\omega}_{it}^f(\varphi_i)|\tilde{\omega}_{it}(\varphi_i))}_{\text{Revenue}} & \text{for } \chi_{it} = 1 \text{ and } d_{it}^x = 1. \end{cases}$$

For the initial period of $t = T_{i,0}$, we have the observation of plants that stay in the market so that the likelihood is conditioned on $\chi_{it} = 1$,

$$L_{it}(\theta|\varphi_i) = \begin{cases} P(d_{it}|\varphi_i, \chi_{it} = 1) g_\omega(\tilde{\omega}_{it}(\varphi_i)) & \text{for } d_{it}^x = 0 \text{ and } t = T_{i,0} \\ \underbrace{P(d_{it}|\varphi_i, \chi_{it} = 1)}_{\text{Export/Import}} \underbrace{g_\omega(\tilde{\omega}_{it}(\varphi_i)) g_\omega^f(\tilde{\omega}_{it}^f(\varphi_i)|\tilde{\omega}_{it}(\varphi_i))}_{\text{Revenue}} & \text{for } d_{it}^x = 1 \text{ and } t = T_{i,0} \end{cases}$$

The likelihood contribution from plant i conditioned on φ_i is

$$L_i(\theta|\varphi_i) = \prod_{T_{i,0}}^{T_{i,1}} L_{it}(\theta|\varphi_i).$$

Since we do not observe φ_i, we integrate out the unobserved φ_i to compute the likelihood contribution from plant i observation. The distribution of φ_i crucially depends on whether a plant is observed in the initial sample period or not. If plant i is observed in the initial sample period, we integrate out φ_i using the stationary distribution $\mu(\varphi)$ while, if plant i enters into the sample after the initial sample period, we use the distribution of initial draws upon successful entry $g_e(\varphi)$. Thus,

$$L_i(\theta) = \begin{cases} \int L_i(\theta|\varphi')\mu(\varphi')d\varphi' & \text{for } T_{i,0} = 1990, \\ \int L_i(\theta|\varphi')g_e(\varphi')d\varphi' & \text{for } T_{i,0} > 1990. \end{cases}$$

The parameter vector θ can be estimated by maximizing the logarithm of likelihood function

$$\mathcal{L}(\theta) = \sum_{i=1}^{N} \ln L_i(\theta). \tag{44}$$

Evaluation of the log-likelihood involves solving computationally intensive dynamic programming problem that approximates the Bellman equation (54) by discretization of state space. For each candidate parameter vector θ, we solve the discretized version of (54) and then obtain the choice probabilities, (55) and (56), as well as the stationary distribution from the associated policy function. Once the choice probabilities and the stationary distribution are obtained for a particular candidate parameter vector θ, then we may evaluate the log-likelihood function (61). Repeating this process, we can maximize (61) over the parameter vector space of θ to find the estimate.

5 Data and Results for Static Model

5.1 Data

We use the Chilean manufacturing census for 1990-1996. We focus on the following five observable variables: d^x_{it}, d^m_{it}, χ_{it}, r_{it}, and r^f_{it}, where i represents plant's identification and t represents the year t. In the data set, we observe the number of blue workers and white workers, the value of total sales, the value of export sales, and the value of imported materials. We use the real values of total sales and export sales, respectively, for r_{it} and r_{it}^f where the manufacturing output price deflator is used to convert the nominal value into the real value. The export/import status, (d_{it}^m, d_{it}^x), is identified from the data by checking if the value of export sales and/or the value of imported materials are zero or positive. The entry/exiting decisions, χ_{it}, can be identified in the data by looking at the number of workers across years. We use unbalanced panel data of 7234 plants for 1990-1996, including all the plants that has been observed at least one year between 1990 and 1996: $\{\{d_{it}^x, d_{it}^m, \chi_{it}, r_{it}^h, r_{it}^f\}_{t=T_{i,0}}^{T_{i,1}}\}_{i=1}^{7234}$. Here, $T_{i,0}$ is the first year in which firm i appears in the data, which is either 1990 or the year in which firm i entered between 1991 and 1996. $T_{i,1}$ is the last year in which firm i appears in the data, which is either 1996 or the year in which firm i exited between 1991 and 1995.

5.2 Parameter Estimates

Table 4 presents the maximum likelihood estimates of the empirical models and their asymptotic standard errors, which are computed using the outer product of gradients estimator. The parameters are evaluated in the unit of million US dollars in 1990. The standard errors are generally small.

The estimate of α_m is 0.064, indicating that importing materials from abroad has a substantial impact, a 6.4 percent increase, on the total revenues. The estimate of α_x implies that, when a plant exports, the exporting revenue is a relatively small fraction, 6.6[=exp(-2.723)] percent, of the domestic revenue.

The estimated per-period fixed cost of operating in the market is $\hat{f} = 4.382/\sigma$ million US dollars. If, say, $\sigma = 4$, then the estimated per-period fixed cost is approximated equal to 110 thousand dollars. The estimated per-period fixed costs for export and import are also substantial: $\hat{f}_x = 1.049/\sigma$ and $\hat{f}_m = 0.892/\sigma$. The parameter determining the degree of complementarity in the per-period fixed cost associated with export and import, ζ_f, is estimated as 0.694, indicating that a firm can save more than 30 percent of per-period fixed cost associated with trade by engaging in both export and import activities.

The estimated magnitudes of the shocks associated with the exiting decision and the export/import decisions are large relative to the per-period profit. The estimate of $\rho_d = 0.462/\sigma$, implying the standard error of $\frac{\pi}{\sqrt{6}} \times 0.462/\sigma = 1.026/\sigma$ in export/import cost shocks. This is more than the "average" incumbent's profit from domestic sales.[7] The estimate of ρ_χ is much larger than ρ_d and implies that the standard error of the shocks associated with the exiting decision is $\frac{\pi}{\sqrt{6}} \times 81.234/\sigma = 180.45/\sigma$. This is more than 200 times as large as the "average" incumbent's profit from domestic sales. This implausible estimate might be viewed as an evidence for the mis-specification—the model's specification is not rich enough so that the large standard error of the shocks associated with exiting is required to explain the observed exiting choices.[8]

The exogenous exiting probability is estimated as 0.13 percent, implying that a large and highly productive firm faces a small probability of exiting due to a negative shock. The fixed entry cost, which is estimated from the free entry condition $\hat{f}_e = \int \hat{V}(\varphi, (0,0))\hat{g}_0(\varphi')d\varphi'$, is $210.27/\sigma$, which is 288 times as large as the domestic profit for the "average" incumbent.

In the model, the higher productivity firms are more likely to survive than the lower productivity firms. To examine how important such a selection mechanism to determine aggregate productivity, the first to the fourth rows of Table 5 compare the mean of productivity across different groups of firms. The average productivity level among successful new entrants is 1.6 percent higher than the average productivity level of the initial draws from $g_0(\varphi)$, indicating that those who initially drew the relatively higher productivity are more likely to succeed in entering into the market. Over time, the selection leads to a larger impact on the average productivity. The average productivity among incumbents at the steady state is almost three times as high as the average productivity across initial draws. When we also include the effect of import, as shown in the fourth row, the average productivity at the steady state is more than 4.5 times as high as the average of initial draws.

Table 6 compares the actual and the predicted distribution of export/import status as well as the actual and the predicted average productivity and market shares across different export/import states. As shown in the first row, the cross-sectional pattern of export and import are replicated by the estimated model pretty well. In the actual data, while a 68.6 percent of the firms are neither exporting nor importing, their market shares account only for a 22.7

[7]The average productivity is $\ln\phi = 0.478$. Then, the "average" incumbent's profit from domestic sales is computed as $0.7313/\sigma[= (1/\sigma)\exp(\hat{\alpha}_0 + 0.478)]$.

[8]This large standard error of shocks associated with exiting/staying choices may explain the relatively small magnitude of profit computed as $\exp(\alpha_0 + \ln\phi)$. Due to a large potential positive shocks for staying, the expected value of shocks *conditional on* staying is large.

Table 4: Maximum Likelihood Estimates

Parameters	Estimates	
α_0	-0.791	(0.004)
α_m	0.064	(0.002)
α_x	-2.723	(0.025)
σ_0	1.090	(0.002)
σf	4.382	(0.228)
σf_x	1.049	(0.029)
σf_m	0.892	(0.023)
ζ_f	0.694	(0.004)
$\sigma \rho_d$	0.462	(0.012)
$\sigma \rho_\chi$	81.234	(2.159)
σ_h	0.341	(0.001)
σ_f	1.914	(0.013)
ρ	0.012	(0.001)
α_t	0.038	(0.001)
ξ	0.0013	(0.0001)
f_e	210.274	
log-likelihood	-79389.21	
No. of Plants	7234	

Notes: Standard errors are in parentheses. The parameters are evaluated in the unit of million US dollars in 1990.

Table 5: The Mean of Productivity

Mean of φ at Entry Trial	1.000
Mean of φ at Successful Entry	1.016
Mean of φ at Steady State	2.948
Mean of $\exp(\alpha_m d^m)\varphi$ at Steady State	4.508

Notes: The reported numbers are relative to the productivity level at entry. In particular, the original numbers are divided by the mean of φ at entry (i.e., $\int \varphi g_0(\varphi) d\varphi$).

Table 6: Productivity and Market Shares by Export/Import Status (Actual vs. Predicted)

	Export/Import Status					
	(1) No-Export	(2) Export	(3) No-Export	(4) Export	(2)+(4)	(3)+(4)
Actual	/No-Import	/No-Import	/Import	/Import	Export	Import
Dist. of Ex/Im Status	0.686	0.086	0.120	0.108	0.194	0.228
Average of $\ln \varphi$	0.035	1.532	1.434	2.596	2.118	1.978
Market Share	0.227	0.187	0.145	0.441	0.628	0.586
Predicted						
Dist. of Ex/Im Status	0.678	0.090	0.126	0.106	0.196	0.232
Average of $\ln \varphi$	0.188	0.674	0.680	2.176	1.501	1.380
Market Share	0.066	0.156	0.158	0.620	0.776	0.777

percent of total outputs. On the other hand, only a 10.8 percent of the firms are both exporting and importing but they account for 44.1 percent of total output. The estimated model qualitatively replicates this pattern but the predicted magnitude of the market share is bit away from the actual magnitude: the model under-predicts the market share for the firms that are neither exporting nor importing while it over-predicts the market share for the exporting and importing firms. As the actual data suggests (in the second row), relatively small number of exporters and importers account for large market shares because they tend to be more productive and hence employ more workers relative to non-exporters and non-importers. This basic observed pattern on the productivity across different export and import status is also captured by the estimated model although the estimated model over-predicts the average productivity among non-exporters and non-importers while it under-predicts the average productivity among exporters and importers.

Table 7 compares the actual and the predicted transition probability of export/import status and entry/exit together. Here, the estimated model fails to replicate the actual transition pattern of export and import; in particular, the estimated model does not capture the observed persistence in export/import status which is the prominent feature of the actual transition probability of export/improt status. What are the missing elements from the current model that may explain the persistence in export/import status? There are, at least, two possible extensions. First, the current model does not incorporate the sunk start-up cost of exporting as well as importing.[9] The model with the sunk start-up cost for exporting and importing

[9]See Roberts and Tybout (1997) for the empirical evidence for sunk start-up cost for exporting. Kasahara and Rodrigue (2005) provides the evidence for sunk start-up cost for importing.

Table 7: Distribution of Export/Import Status and Entry/Exit (Actual vs. Predicted)

Actual	(1) No-Export /No-Import	(2) Export /No-Import	(3) No-Export /Import	(4) Export /Import	(2)+(4) Export	(3)+(4) Import	Exit at $t+1^a$
No-Export/No-Import at t	0.927	0.024	0.042	0.007	0.031	0.048	0.082
Export/No-Import at t	0.147	0.677	0.013	0.163	0.841	0.176	0.070
No-Export/Import at t	0.188	0.017	0.699	0.096	0.113	0.795	0.035
Export/Import at t	0.025	0.101	0.070	0.804	0.905	0.874	0.022
Predicted							
No-Export/No-Import at t	0.722	0.088	0.124	0.066	0.154	0.189	0.075
Export/No-Import at t	0.660	0.093	0.131	0.116	0.209	0.247	0.066
No-Export/Import at t	0.659	0.093	0.131	0.116	0.210	0.248	0.066
Export/Import at t	0.402	0.095	0.134	0.369	0.464	0.503	0.035

Note: a). "Exit at $t+1$" is defined as plants that are observed at t but not observed at $t+1$ in the sample. b). "New Entrants t" is defined as plants that are not observed at $t-1$ but observed at t in the sample, of which row represents the empirical distribution of export/import status at t as well as the probability of not being observed (i.e., exit) at $t+1$.

may replicate the observed persistence in export/import status. Second, the current model only incorporates the heterogeneity in terms of productivity while, in reality, there are other sources of heterogeneity; some firms are inherently more likely to export or import even after controlling for productivity. For instance, the transportation cost (τ) might be different, depending on what kinds of products a firm is exporting. Ignoring heterogeneity other than productivity may be the reasons for the model's inability to capture the observed persistence of export/import status. In the later sections, we estimate the models that incorporate sunk costs and/or other sources of unobserved heterogeneity.

5.3 Counterfactual Experiments

While some of the structural parameters are not identified from the empirical model (crucially, we cannot identify σ), we may solve for the change in the equilibrium aggregate price as a result of counterfactual experiments as follows.

Denote the equilibrium aggregate price under the parameter θ by $P(\theta)$. Under the estimated parameter $\hat{\theta}$, we may compute the estimate of fixed entry cost as: $\hat{f}_e = \int V(\varphi';\hat{\theta})g_{\varphi}(\varphi';\hat{\theta})d\varphi'$, where $V(\varphi;\theta)$ is the fixed point of the Bellman's equation (54) under the parameter θ and $g_{\varphi}(\varphi;\theta)$ is the probability density function of the initial productivity under θ.

Suppose that we are interested in a counterfactual experiment characterized by a counterfactual parameter $\tilde{\theta}$ that is different from the estimated parameter $\hat{\theta}$. Note that the following relationships hold between α_0 and the aggregate price P:

$$\hat{\alpha}_0 \quad = \quad \ln[\alpha^{\alpha}(1-\alpha)^{1-\alpha} R P(\hat{\theta})^{\sigma-1}]$$

Then, we may write the estimated profit function, (53), evaluated at the counterfactual aggregate price $P(\tilde{\theta})$ as:

$$\begin{aligned}
&\hat{\pi}(\varphi_i, d_{it}; P(\tilde{\theta})) \\
&= \frac{1}{\sigma} \exp[(\sigma-1)\ln(P(\tilde{\theta})/P(\hat{\theta}))][1+\exp(\hat{\alpha}_x) d_{it}^x] \exp(\hat{\alpha}_0 + \hat{\alpha}_m d_{it}^m + \ln \varphi_i), \\
&= \frac{1}{\sigma} \exp[\ln(K(\tilde{\theta})/K(\hat{\theta}))][1+\exp(\hat{\alpha}_x) d_{it}^x] \exp(\hat{\alpha}_0 + \hat{\alpha}_m d_{it}^m + \ln \varphi_i),
\end{aligned}$$

where $K(\theta) = RP(\theta)^{\sigma-1}$ is the demand shifter under the parameter θ.

Then, we may compute the equilibrium price changes in the demand shifters, $\ln(K(\tilde{\theta})/K(\hat{\theta})) = (\sigma-1)\ln(P(\tilde{\theta})/P(\hat{\theta}))$. Specifically, the equilibrium price under the counterfactual parameter $\tilde{\theta}$ is determined by the free entry condition:

$$\hat{f}_e = \int V(\varphi'; \tilde{\theta}, P(\tilde{\theta})) g_{\varphi}(\varphi'; \tilde{\theta}) d\varphi',$$

where the dependence of the value function V on the aggregate price $P(\tilde{\theta})$ is explicitly indicated.

We may quantify the impact of counterfactual experiments on the welfare level by examining how much the equilibrium aggregate price level P changes as a result of counterfactual experiments since the aggregate price level P is inversely related to the welfare level W.[10]

To quantitatively investigate the impact of international trade, we conduct the four counterfactual experiments with the following counterfactual parameters:

(1) No Export: $f_x \to \infty$ and $\alpha_x \to -\infty$.

(2) No Import: $f_m \to \infty$ and $\alpha_m = 0$.

(3) Autarky: $f_x, f_m \to \infty$, $\alpha_x \to -\infty$, and $\alpha_m = 0$.

(4) No Complementarity: $\zeta_f = 1$.

Table 8 presents the results of counterfactual experiments using the estimated model. To examine the importance of equilibrium response to quantify the impact of counterfactual policies,

[10]To see this, note that the income is constant at the level of L. From the budget constraint $PQ = L$ and the definition of aggregate product $W = Q = \left[\int_{\omega\in\Omega} q(\omega)^{\rho} d\omega\right]^{1/\rho}$, the utility level is equal to $W = P^{-1}L$.

Table 8: Counterfactual Experiments

		Counterfactual Experiments			
		(1)	(2)	(3)	(4)
	Trade	No Export	No Import	Autarky	No Comp.
With Equilibrium Price Effect					
$\Delta(\sigma-1)\ln P$	0.000	0.064	0.079	0.123	0.018
Exiting Rates at Entry Trial (%)	7.823	7.841	7.847	7.861	7.827
ln(Average φ) at Steady State	0.000	-0.026	-0.029	-0.051	-0.003
A Fraction of Exporters	0.196	0.000	0.151	0.000	0.147
A Fraction of Importers	0.232	0.191	0.000	0.000	0.188
Market Shares of Exporters	0.552	0.000	0.423	0.000	0.425
Market Shares of Importers	0.592	0.482	0.000	0.000	0.486
Without Equilibrium Price Effect					
Exiting Rates at Entry Trial (%)	7.823	7.889	7.907	7.956	7.840
ln(Average φ) at Steady State	0.000	-0.005	-0.002	-0.011	0.003
A Fraction of Exporters	0.196	0.000	0.142	0.000	0.144
A Fraction of Importers	0.232	0.183	0.000	0.000	0.185
Market Shares of Exporters	0.552	0.000	0.407	0.000	0.422
Market Shares of Importers	0.592	0.469	0.000	0.000	0.482

Note: "Average φ" at Steady State is a productivity average using the plants' combined revenues (or market shares) as weights: $\int \sum_d \varphi^{\sigma-1} \frac{r(\varphi,d)P(d|\varphi)}{\int \sum_{d'} r(\varphi')P(d'|\varphi')d\mu(\varphi')} d\mu(\varphi)$.

the results both with and without equilibrium aggregate price response are reported. According to the experiment, moving from autarky to trade may decrease the equilibrium aggregate price by a $12.3/(\sigma-1)$ percent. This implies that, say if $\sigma = 4$, exposure to trade increases the real income by a (12.3/4=)3.1 percent, leading to a substantial increase in welfare. When a country opens up its economy, more productive firms start exporting and importing, which in turn increases the aggregate labor demand and hence leads to an increase in the real wage or a decrease in the aggregate final goods price.

Under no equilibrium response, the exiting rates at entry trial increase from 7.82 to 7.96 percent by moving from trade to autarky; without equilibrium price response, the expected profit is lower in autarky than in trade because there is no opportunity to export/import, leading to higher exit rates in autarky than in trade. The equilibrium price adjustment, at least partly, offsets this effect as we see by comparing the exiting rates in autarky between "With Equilibrium Price Effect" and "With Equilibrium Price Effect." When the economy moves from trade to autarky, an increase in the equilibrium aggregate price increases the expected profit increases, which in turn lowers the exiting rates from 7.96 to 7.86.

The impact of trade on aggregate productivity—measured by a productivity average using the plants' market shares as weights—can be understood by comparing "ln(Average φ) at Steady State" between trade and autarky. Moving from trade to autarky leads to a 5.1 decrease in aggregate productivity at the steady state. Here, by comparing between with and without the equilibrium price responses, we notice the importance of equilibrium price response to quantitatively explain the impact of trade on aggregate productivity; without equilibrium price response, the impact of trade on aggregate productivity is only 1.1 percent as opposed to 5.1 percent.

The counterfactual experiments under no export or no import (but not both) highlight the interaction between aggregate export and aggregate import in the presence of heterogenous firms. According to the estimated model, when the economy moves from trade to no import, a fraction of *exporters* declines from a 19.6 percent to a 15.1 percent; when the economy moves from trade to no export, a fraction of *importers* declines from a 23.2 percent to a 19.1 percent. In terms of market shares, moving from trade to no import leads to a decrease in the total market shares of *exporters* from 55.2 to 42.3 percent while moving from trade to no export leads to a decrease in the total market shares of *importers* from 59.2 to 48.2 percent. Thus, policies that prohibits the import of foreign materials could have a large negative impact on the export of final consumption goods, or vice-versa. The similar results hold even without equilibrium price effect and thus the equilibrium price response is little to do with these results. Rather, it is due to the complementarity between export and import within both revenue function $r(\cdot)$ and sunk-cost function $F(\cdot)$.

To examine the role of complementarity between export and import in the sunk-cost function—relative to the role played by the complementarity in the revenue function—we conducted what would happen to a fraction of importers and/or a fraction of exporters had there been no complementarity between export and import in the sunk cost function. The results are striking. Eliminating the complementarity between export and import in the sunk cost function has essentially the same effect on export and import, respectively, as restricting to no import and no export. Thus, the results from the experiments show that it is the complementarity in the sunk cost function, rather than the complementarity in the revenue function, that determines the impact of exporting policies (e.g., export subsidies) on intermediate imports or the impact of importing policies (e.g., import tariffs) on exports.

6 The Dynamic Model

6.1 The Environment

In this section, we modify the above environment to introduce interesting dynamics into the model to facilitate structural estimation of the model in the next section. Preferences and production technologies are unchanged but we modify the processes governing productivity. Firms continue to pay a fixed cost of entry and a fixed per-period production cost but we modify the fixed costs associated with exporting and importing. We begin with a description of productivity draws. Upon entering, an entrant draws an initial productivity from a continuous cumulative distribution $G_0(\cdot)$ and the density function $g_0(\cdot)$. After the initial realization, productivity at time t, φ_t, follows a first-order Markov process, where the distribution of φ_{t+1} conditional on φ_t is given by $G(\cdot|\varphi_t)$ with density function $g(\cdot|\varphi_t)$.

There are both per-period fixed costs and one-time sunk costs associated with exporting and importing:

$$F(d_t, d_{t-1}) = f + \zeta_f^{d_t^x d_t^m}(f_x d_t^x + f_m d_t^m) + \zeta_c^{d_t^x d_t^m}[\zeta_{cx}^{d_{t-1}^m} c_x d_t^x (1 - d_{t-1}^x) + \zeta_{cm}^{d_{t-1}^x} c_m d_T^m (1 - d_{t-1}^m)].$$

The second term on the right hand side, $\zeta_f^{d_t^x d_t^m}(f_x d_t^x + f_m d_t^m)$, captures per-period fixed cost while the third term, $\zeta_c^{d_t^x d_t^m}(\zeta_{cx}^{d_{t-1}^m} c_x d_t^x(1 - d_{t-1}^x) + \zeta_{cm}^{d_{t-1}^x} c_m d_t^m(1 - d_{t-1}^m))$, captures one-time sunk cost associated with exporting and importing. The parameter $0 < \zeta_f < 1$ determines the degree of complementarity between current export and import in per-period fixed cost. Similarly, the parameter $0 < \zeta_c < 1$ captures the degree of complementarity between current export and import in one-time sunk cost. While the parameter $0 < \zeta_{cx} < 1$ captures the dynamic effect of past import status on the current sunk cost of exporting, $0 < \zeta_{cm} < 1$ captures the dynamic effect of past export status on the current sunk cost of importing. If $\zeta_{cx} < 1$, then the past importing experience reduces the one-time sunk cost of exporting and hence increases the probability of exporting this period.

A firm's net profit depends on the current productivity φ_T and current and past export/import status (d_T, d_{t-1}):

$$\pi(\varphi_t, d_t, d_{t-1}) = \frac{r(\varphi_t, d_t)}{\sigma} - F(d_t, d_{t-1}).$$

Denote the exiting decision by $\chi \in \{0, 1\}$ where $\chi_T = 1$ indicates that a firm is operating at t and $\chi_T = 0$ implies that a firm exits at the beginning of period t.

There are choice-dependent idiosyncratic cost shocks associated with exiting decisions as well as export/import decisions. If a firm chooses $\chi_t \in \{0, 1\}$, then it has to pay the additional cost of $\epsilon_t^{\chi}(\chi_t)$ associated with the exiting choice χ_T, where $\epsilon^{\chi} = (\epsilon^{\chi}(0), \epsilon^{\chi}(1))$ is drawn from the cumulative distribution $H_{\chi}(\cdot)$. Similarly, if a firm chooses $d_t = (d_t^x, d_t^m) \in \{0, 1\}^2$ this period, then it has to pay $\epsilon_t^d(d_t)$, where $\epsilon^d = (\epsilon^d(0, 0), \epsilon^d(1, 0), \epsilon^d(0, 1), \epsilon^d(1, 1))$ is drawn from the cumulative distribution $H_d(\cdot)$.

The discount factor is given by $\beta \in (0, 1)$. The timing of the incumbent's decision with the state (φ_{t-1}, d_{t-1}) within each period is as follows. At the beginning of every period, a firm faces a possibility of a large negative shock that leads it to exit with an exogenous probability ξ. If the firm survives, it draws φ_t from the cumulative distribution $G(\cdot|\varphi_{t-1})$ and the additional choice-dependent idiosyncratic cost shocks associated with exiting decisions, $\epsilon_T^{\chi} = (\epsilon_T^{\chi}(0), \epsilon_T^{\chi}(1))$, from $H_{\chi}(\cdot)$. Given the realizations, the firm decides whether it exits from the market or continues to operate. If the firm decides to exit, it receives the terminal value of $\epsilon_T^{\chi}(0)$. If the firm decides to continue to operate, then it will draw the choice-dependent idiosyncratic cost shocks associated with export and import, $\epsilon_T^d(d)$ for $d \in \{0, 1\}^2$. After observing $\epsilon_T^d(d)$,the firm makes export and import decisions. The Bellman's equation is written as:

$$\begin{aligned} V(\varphi_t, d_{t-1}) &= \int \max\{\epsilon^{\chi'}(0), W(\varphi_t, d_{t-1}) + \epsilon^{\chi'}(1)\} dH_{\chi}(\epsilon^{\chi'}) \\ W(\varphi_t, d_{t-1}) &= \int \max_{d_t \in \{0,1\}^2} \left\{ \tilde{W}(\varphi_T, d_T, d_{t-1}) + \epsilon^{d'}(d_t) \right\} dH_d(\epsilon^{d'}), \end{aligned} \tag{45}$$

where

$$\tilde{W}(\varphi_T, d_T, d_{t-1}) = \pi(\varphi_t, d_t, d_{t-1}) + \beta(1 - \xi) \int V(\varphi', d_t) G(d\varphi'|\varphi_t).$$

The policy function associated with this Bellman's equation (45), together with the exogenous process of φ_t, determines the transition function of (φ_t, d_t). For a firm with the state (φ_t, d_{t-1}), the exiting probability is:

$$P(\chi_t = 0|\varphi_t, d_{t-1}) = \xi + (1 - \xi) \int 1(\epsilon^{\chi'}(0) > W(\varphi_t, d_{t-1}) + \epsilon^{\chi'}(1)) dH_{\chi}(\epsilon^{\chi'}), \tag{46}$$

where $\chi_{it} \in \{0, 1\}$ represents the exiting/staying decision. The probability of choosing $d_{it} = (i, j)$ for $(i, j) \in \{0, 1\}^2$ conditional on the survival is

$$P(d_t = (i, j)|\varphi_t, d_{t-1}, \chi_t = 1) = \int 1 \left[(i, j) = argmax_{d_T \in \{0,1\}^2} \left\{ \tilde{W}(\varphi_T, d_T, d_{t-1}) + \epsilon^{d'}(d_t) \right\} \right] dH_d(\epsilon^{d'}). \tag{47}$$

6.2 Stationary Equilibrium

As before, we focus on a stationary equilibrium in which the joint distribution of (φ, d) is constant over time and let the stationary distribution of (φ, d) among incumbents is denoted by $\mu(\varphi, d)$. Equations (**??**)-(**??**) continue to hold in this dynamic economy.

New entrants are assumed to have no past export/import experience so that $d_{t-1} = (0, 0)$. The probability of successful entry among new entrants is $p_{in} = \int P(\chi_t = 1|\varphi', d_{t-1} = (0, 0))dG_0(\varphi')$. The expected value of an entering firm is given by $\int V(\varphi', d_T = (0, 0))dG_0(\varphi')$, where $V(\cdot)$ is given in (45). Under free entry, this value must be equal to the fixed entry cost f_e:

$$\int V(\varphi', d_T = (0, 0))dG_0(\varphi') = f_e. \tag{48}$$

In equilibrium, this free entry condition has to be satisfied.

Given the stationary distribution, the average probability of exiting among incumbent is $\delta = \int P(\chi_t = 0|\varphi', d')d\mu(\varphi', d')$. The stationarity requires that the number of exiting firms is equal to the number of *successful* new entrants:

$$\delta M = p_{in} M_e,$$

where M_e is the mass of new entrants.

The evolution of the probability measure among incumbents has to take account of both the transition of states among survivors and entry/exit processes. Define the probability that a firm with the state (φ_{t-1}, d_{t-1}) continues in operation at t with the state (φ_t, d_t) is

$$P(\varphi_t, d_t, \chi_T = 1|\varphi_{t-1}, d_{t-1}) = P(d_t|\varphi_t, d_{t-1}, \chi_t = 1)P(\chi_t = 1|\varphi_t, d_{t-1})g(\varphi_T|\varphi_{t-1}).$$

The probability that a new successful entrant will operate with the state (φ_t, d_t) is:

$$P_e(\varphi_T, d_T) = P(d_t|\varphi_t, d_{t-1} = (0, 0), \chi_T = 1)g_0(\varphi_t). \tag{49}$$

Again, new entrants have no past export/import experience so that $d_{t-1} = (0, 0)$ because they have no past export/import experience.

Then, the transition of the measure $\mu(\varphi, d)$ is determined by:

$$\mu_t(\varphi, d) = (T\mu_{t-1})(\varphi, d) \equiv \int P(\varphi, d, \chi = 1|\varphi', d')d\mu_{t-1}(\varphi', d') + \delta P_e(\varphi, d). \tag{50}$$

Here, we impose the equilibrium condition that the mass of entrants is equal to the mass of exits so that $\delta P_e(\varphi, d)$ represents the probability measure of new entrants with the state (φ, d).[11] The stationary distribution, μ, is a fixed point of the operator T: $\mu = T\mu$.

A stationary equilibrium consists of an aggregate price P, an aggregate revenue R, a mass of incumbents M, and a probability distribution of incumbents μ such that

- Given the the aggregates P and R (and hence the demand function (**??**)), a firm solves the Bellman equation (45).
- Free Entry condition (48) holds.
- The probability distribution μ is a fixed point of the operator T induced by a firm's policy function: $T(\mu) = \mu$.
- The mass of incumbents is a constant over time: $\delta M = p_{in} M_e$.
- Aggregate budget constraint holds: $R = L$.[12]

6.3 Algorithm for Computing a Stationary Equilibrium

Define the demand shifter $K \equiv RP^{\sigma-1}$ (See the demand function (**??**)).

1. For fixed value of $K = K_j$, we may compute the revenue function r and solve the Bellman equation (45) and obtain the choice probabilities, (46)-(47).
2. Compute $J(K_j) = (\int V(\varphi', (0,0)) dG_0(\varphi') - f_e)^2$.
3. Repeat Step 1-2 to find K^* such that $J(K^*) = 0$.
4. Using (46)-(47), compute a stationary distribution μ by computing a fixed point of the operator T.

[11]Denote the mass of incumbents at t by M_T. The mass of incumbents with the state (φ_{t-1}, d_{t-1}) at $t-1$ is $M_{t-1}\mu_{t-1}(\varphi_{t-1}, d_{t-1})$. Among them, the mass $P(\chi_T = 0|\varphi_{t-1}, d_{t-1})M_{t-1}\mu_{t-1}(\varphi_{t-1}, d_{t-1})$ exits from the market, and only the mass of $P(\varphi_T, d_t, \chi_t = 1|\varphi_{t-1}, d_{t-1})M_{t-1}\mu_{t-1}(\varphi_{t-1}, d_{t-1})$ will *survive and reach the state* (φ_T, d_t) at t. The mass of incumbents at t with the state (φ_T, d_t) who are not new entrants can be computed by summing up $P(\varphi_T, d_t, \chi_t = 1|\varphi_{t-1}, d_{t-1})M_{t-1}\mu_{t-1}(\varphi_{t-1}, d_{t-1})$ over the distribution $\mu_{t-1}(\varphi_{t-1}, d_{t-1})$: $M_{t-1}\int P(\varphi_T, d_t, \chi_{it} = 1|\varphi', d')d\mu_{t-1}(\varphi', d')$. On the other hand, the mass of successful new entrants with the state (φ_T, d_t) is $p_{in}M_e P_e(\varphi_T, d_T)$. Therefore, the transition of the distribution of incumbent's state is written as $M_T\mu_t(\varphi, d) = M_{t-1}\int P(\varphi, d, \chi = 1|\varphi', d')d\mu_{t-1}(\varphi', d') + p_{in}M_{e,t}P_e(\varphi, d)$. The stationary equilibrium requires that (i) $p_{in}M_{e,t} = \delta M_{t-1}$ and (ii) $M_{t-1} = M_t$. Then, we get $\mu_t(\varphi, d) = \int P(\varphi, d, \chi = 1|\varphi', d')d\mu_{t-1}(\varphi', d') + \delta P_e(\varphi, d)$.

[12]This condition implies labor market clearing.

5. Compute the aggregate price index as $P = (K^*/R)^{\frac{1}{\sigma-1}} = (K^*/L)^{\frac{1}{\sigma-1}}$.

6. Compute M from (**??**) as $M = \frac{R}{\int r(\varphi', d')d\mu(\varphi', d')} = \frac{L}{\int r(\varphi', d')d\mu(\varphi', d')}$.

7 Structural Estimation of Dynamic Model

7.1 Empirical Specification

Use the subscript i to represent plant i and the subscript t to represent year t. To develop an estimable structural model, we make the following distributional assumptions:

- We assume that $\ln \varphi_{it}$ follows an AR(1) process: $\ln \varphi_{it} = \psi \ln \varphi_{it-1} + \omega_{it}$, where $\psi \in (0,1)$ and ω_{it} is independently drawn from $N(0, \sigma_\omega^2)$. That is, $g(\varphi_{it}|\varphi_{it-1}) = \frac{1}{\sigma_\omega}\phi((\ln \varphi_{it} - \psi \ln i_{it-1})/\sigma_\omega)$.

- The logarithm of the initial productivity upon entry is drawn from $N(0, \sigma_0^2)$ so that $g_0(\varphi_{it}) = \frac{1}{\sigma_0}\phi(\ln \varphi_{it}/\sigma_0)$.[13]

- $\epsilon_{it}^\chi(0)$ and $\epsilon_{it}^\chi(1)$ are independently drawn from the identical extreme-value distribution with mean zero and scale parameter ϱ^χ.

- $\epsilon_{it}^d(d)$'s for $d \in \{0,1\}^2$ are independently drawn from the identical extreme-value distribution with mean zero and scale parameter ϱ^d.

We also assume that there is an idiosyncratic shock to export revenue, denoted by η_t, independently drawn from $N(-0.5\sigma_\eta^2, \sigma_\eta)$ and its density function is given by $g_\eta(\eta) = \phi(\eta/\sigma_\eta)/\sigma_\eta$.[14] For simplicity, we assume that this idiosyncratic shock is observed only after the current year's export decision is made so that η_{it} does not affect the firm's export decision. The inclusion of export-revenue specific shocks is necessary to deal with the feature of the data that a substantial variation in export revenue even after controlling for domestic revenue. We also consider labor augmented technological change at the annual rate of α_t.

Modifying the revenue functions, we specify the logarithm of the domestic revenue and the export revenue as:

$$\ln r_{it}^h \quad = \quad \alpha_0 + \alpha_t t + \alpha_m d_{it}^m + \ln \varphi_{it} \qquad (51)$$

[13]The mean of initial productivity draws is set to zero in order to achieve the identification. $\ln \varphi_0$ cannot be separately identified from α_0^h and α_0^f.

[14]Assuming that the mean of η is equal to $-0.5\sigma_\eta^2$, we get $E[\exp(\eta)] = 1$.

$$\ln r_{it}^{f} = \alpha_0 + \alpha_t t + \alpha_x + \alpha_m d_{it}^{m} + \ln \varphi_{it} + \eta_{it}. \tag{52}$$

Importantly, they are **reduced-form** specifications; we have the following relationships between reduced-form parameters and structural parameters:[15]

$$\begin{aligned}
\alpha_0 &= \ln[\alpha^{\alpha}(1-\alpha)^{1-\alpha} R P^{\sigma-1}], \\
\alpha_x &= \ln[N\tau^{1-\sigma}], \\
\alpha_m &= (\sigma-1)\ln\lambda.
\end{aligned}$$

Note that α_0 crucially depends on the aggregate price P. This means that the values of α_0 will not be invariant with respect to policy changes as long as policy changes induce the equilibrium response to the aggregate price P. When we conduct counterfactual policy experiments later, we will pay a particular attention on the role of the equilibrium price response.

Given these specifications for revenues, firm's profit—after detrending and taking an expectation with respect to η_{it}—is

$$\pi(\varphi_{it}, d_{it}, d_{it-1}) = \frac{1}{\sigma}(1+\exp(\alpha_x)d_{it}^{x})\exp(\alpha_0 + \alpha_m d_{it}^{m} + \ln\varphi_{it}) - F(d_{it}, d_{it-1}). \tag{53}$$

Note that, by detrending and redefining the discount factor as $\tilde{\beta} = \beta(1-\xi)e^{\alpha_t}$, the firm's dynamic optimization problem becomes stationary.

Using the properties of the extreme-value distributed random variables (See the Appendix), the Bellman's equation (45) is rewritten as:

$$\begin{aligned}
V(\varphi_{it}, d_{it-1}) &= \varrho^{\chi}\ln(\exp(0) + \exp(W(\varphi_{it}, d_{it-1})/\varrho^{\chi})) \\
W(\varphi_{it}, d_{it-1}) &= \varrho^{d}\ln\left(\sum_{d_{it}}\exp\left([\pi(\varphi_{it}, d_{it}, d_{it-1}) + \tilde{\beta}\int V(\varphi', d_{it})G(d\varphi'|\varphi_{it})]/\varrho^{d}\right)\right).
\end{aligned} \tag{54}$$

With the solution to the functional equation (54), the conditional choice probabilities of exiting and export/import decisions follow the Nested Logit formula. In particular, the choice probability of exiting and staying conditional on the state (φ_{it}, d_{it-1})—which corresponds to (46)—is given by:

$$P(\chi_{it}|\varphi_{it}, d_{it-1}) = (1-\chi_{it})\xi + (1-\xi)\frac{\exp(\chi_{it}W(\varphi_{it}, d_{it-1})/\varrho^{\chi})}{\exp(0) + \exp(W(\varphi_{it}, d_{it-1})/\varrho^{\chi})}. \tag{55}$$

[15] Also, with abuse of notation, we replace $(\sigma-1)\ln\varphi$ by $\ln\varphi$ since $(\sigma-1)$ cannot be separately identified from the variance of $\ln\varphi$.

Conditional on choosing $\chi_{it} = 1$ (i.e., continuously operating), the choice probabilities of d_{it}, corresponding to (47), are given by the multinomial logit formula:

$$P(d_{it}|\varphi_{it}, d_{it-1}, \chi_{it} = 1) = \frac{\exp([\pi(\varphi_{it}, d_{it}, d_{it-1}) + \tilde{\beta} \int V(\varphi_{it+1}, d_{it})G(d\varphi_{it+1}|\varphi_{it})]/\varrho^d)}{\sum_{d' \in \{0,1\}^2} \exp([\pi(\varphi_{it}, d', d_{it-1}) + \tilde{\beta} \int V(\varphi_{it+1}, d')G(d\varphi_{it+1}|\varphi_{it})]/\varrho^d)}, \tag{56}$$

To achieve the identification, we normalize the profit function by $1/\sigma$; the various components of the fixed cost as well as the scale parameter ϱ^d are estimated up to the scale of σ.

The probability that a new successful entrant will operate with the state (φ_t, d_t), corresponding to (49), is:

$$P_e(\varphi_{it}, d_{it}) = P(d_{it}|\varphi_{it}, d_{it-1} = (0,0), \chi_{it} = 1)P(\varphi_{it}|\chi_{it} = 1), \tag{57}$$

where $P(d_{it}|\varphi_{it}, d_{it-1} = (0,0), \chi_{it} = 1)$ is given in (56) and $P(\varphi_{it}|\chi_{it} = 1)$ can be computed, using (55), as:

$$P(\varphi_{it}|\chi_{it} = 1) = \frac{P(\chi_{it} = 1|\varphi_{it}, d_{it-1} = (0,0))g_0(\varphi_{it})}{\int P(\chi_{it} = 1|\varphi', d_{it-1} = (0,0))g_0(\varphi')d\varphi'}.$$

To obtain the transition function of the state, define the probability that a firm with the state $(\varphi_{it-1}, d_{it-1})$ continues in operation at t with the state (φ_{it}, d_{it}) as:

$$P(\varphi_{it}, d_{it}, \chi_{it} = 1|\varphi_{it-1}, d_{it-1}) = P(d_{it}|\varphi_{it}, d_{it-1}, \chi_{it} = 1)P(\chi_{it} = 1|\varphi_{it}, d_{it-1})g(\varphi_{it}|\varphi_{it-1}).$$

Then, the transition of the measure $\mu(\varphi, d)$ is determined by (50).

7.2 Econometric Approach

The parameter vector to be estimated is $\theta = (\alpha_t, \alpha_0, \alpha_x, \alpha_m, \sigma_\omega^2, \sigma_0^2, f, f_x, f_m, \zeta_f, c_x, c_m, \zeta_c, \varrho^\chi, \varrho^d, \xi)$. The discount factor $\tilde{\beta}$ is not estimated and is set to 0.95. The parameters are estimated by the method of Maximum Likelihood.

Given the data and the parameter vector θ, we may compute the estimates of $\ln \varphi_{it}$ as:

$$\begin{aligned} \ln \tilde{\varphi}_{it}(\theta) &= \ln r_{it}^h - \alpha_0 - \alpha_t t - \alpha_m d_{it}^m \\ \tilde{\eta}_{it}(\theta) &= \ln r_{it}^f - \alpha_0 - \alpha_t t - \alpha_x d_{it}^x - \alpha_m d_{it}^m - \ln \tilde{\varphi}_{it}(\theta) \end{aligned}$$

Denote the first year and the last year in which firm i appears in the data by $T_{i,0}$ and $T_{i,1}$, respectively. The unbalanced plant-level data spans from 1990 to 1996. Thus, $T_{i,0}$ is either 1990 or the year in which firm i entered between 1991 and 1996. $T_{i,1}$ is either 1996 or the year in which firm i exited between 1991 and 1995.

For $t > T_{i,0}$, we can observe the past state variables $(d_{it-1},\ \tilde{\varphi}_{it-1})$; we may compute the likelihood contribution of the current observation $(d_{it}, \tilde{\varphi}_{it}, \tilde{\eta}_{it})$ conditioned on the observable past state variable $(d_{it-1},\ \tilde{\varphi}_{it-1})$ as

$$L_{it}(\theta) = \begin{cases} P(\chi_{it} = 1|\tilde{\varphi}_{it}, d_{it-1})P(d_{it}|\tilde{\varphi}_{it}, d_{it-1}, \chi_{it} = 1)g(\tilde{\varphi}_{it}|\tilde{\varphi}_{it-1})g_{\eta}(e\eta_{it})^{d^x_{it}}, & \text{for } \chi_{it} = 1 \\ \int P(\chi_{it} = 0|\varphi', d_{it-1})g(\varphi'|\tilde{\varphi}_{it-1})d\varphi', & \text{for } \chi_{it} = 0 \end{cases} \tag{58}$$

Each of the terms is explained as follows. First, $P(d_{it}, \chi_{it} = 1|\tilde{\varphi}_{it}, d_{it-1}) = P(\chi_{it} = 1|\tilde{\varphi}_{it}, d_{it-1})$ $P(d_{it}|\tilde{\varphi}_{it}, d_{it-1}, \chi_{it} = 1)$ is the likelihood of observing $(d_{it}, \chi_{it} = 1)$ conditioned on the current productivity $\tilde{\varphi}_{it}$ and the past export/import decision d_{it-1}. $g(\tilde{\varphi}_{it}|\tilde{\varphi}_{it-1})$ is the likelihood of observing $\tilde{\varphi}_{it}$ conditioning on the past state variable. If a firm is exporting (i.e., $d^x_{it} = 1$), then the likelihood of observing $\tilde{\eta}_{it}$ is also computed as $g_{\eta}(\tilde{\eta}_{it})$. The likelihood $L_{it}(\theta)$ for $\chi_{it} = 1$ is, therefore, the probability of observing $(d_{it}, \tilde{\varphi}_{it}, \tilde{\eta}_{it})$ conditioned on the past state variables $(d_{it-1},$ $\tilde{\varphi}_{it-1})$. In the case of $\chi_{it} = 0$ (i.e., firm i exists at t), we do not observe the current productivity shock $\tilde{\varphi}_{it}$. So, in order to compute the probability of exiting conditioned on $(d_{it-1},\ \tilde{\varphi}_{it-1})$, we integrate out $P(\chi_{it} = 0|\varphi_{it}, d_{it-1})$ over unobserved current shock φ_{it} using the conditional density function $g(\varphi_{it}|\varphi_{it-1})$.

For $t = T_{i,0}$, there are two cases. The first case is that firm i enters into the market after 1991 (i.e., $T_{i,0} > 1990$). The second case is that firm i has operated in 1990.

Consider the first case of $T_{i,0} > 1990$. The likelihood of observing $(d_{it}, \tilde{\varphi}_{it}, \tilde{\eta}_{it})$ for $t = T_{i,0}$ is written as

$$L_{it}(\theta) = P_e(\tilde{\varphi}_{it}, d_{it})g_{\eta}(\tilde{\eta}_{it})^{d^x_{it}}, \tag{59}$$

where $P_e(\varphi, d)$ is given in equation (57). For a new successful entrant, productivity shock φ_{it} is distributed according to $P_e(\tilde{\varphi}_{it}|\chi_{it} = 1)$, which is the distribution of initial draws conditioned on the successful entry.

Next, consider the case of $T_{i,0} = 1990$. While the evaluation of the choice-probabilities (55)-(56) as well as the probability of observing φ_{it} requires the past state variables $(d_{it-1},$ $\varphi_{it-1})$, we do not observe these past state variables at $t = 1990$ in this case. Note, however, that the theory suggests that the past state variables $(d_{it-1},\ \varphi_{it-1})$ is distributed according to a stationary distribution of $(d_{it-1},\ \varphi_{it-1})$, implied by the policy function associated with the parameter vector θ. Thus, we may use the stationary distribution to integrate out the unobserved past state variables to compute the likelihood of the initial observations. The likelihood is

$$L_{it}(\theta) = \int P(\chi_{it} = 1 | \tilde{\varphi}_{it}, d') P(d_{it} | \tilde{\varphi}_{it}, d', \chi_{it} = 1) g(\tilde{\varphi}_{it} | \varphi') g_\eta(\tilde{\eta}_{it})^{d^x_{it}} d\mu(\varphi', d'). \tag{60}$$

In sum, the likelihood contribution from the observation of firm i at t is computed as

$$L_{it}(\theta) = \begin{cases} P_e(\tilde{\varphi}_{it}, d_{it}) g_\eta(\tilde{\eta}_{it})^{d^x_{it}} & \text{for } t = T_{i,0} > 1990, \\ \int P(\chi_{it} = 1 | \tilde{\varphi}_{it}, d') P(d_{it} | \tilde{\varphi}_{it}, d', \chi_{it} = 1) g(\tilde{\varphi}_{it} | \varphi') g_\eta(\tilde{\eta}_{it})^{d^x_{it}} d\mu(\varphi', d') & \text{for } t = T_{i,0} = 1990, \\ P(\chi_{it} = 1 | \tilde{\varphi}_{it}, d_{it-1}) P(d_{it} | \tilde{\varphi}_{it}, d_{it-1}, \chi_{it} = 1) g(\tilde{\varphi}_{it} | \tilde{\varphi}_{it-1}) g_\eta(e\eta_{it})^{d^x_{it}}, & \text{for } \chi_{it} = 1 \text{ and } t > T_{i,0}, \\ \int P(\chi_{it} = 0 | \varphi', d_{it-1}) g(\varphi' | \tilde{\varphi}_{it-1}) d\varphi' & \text{for } \chi_{it} = 0 \text{ and } t > T_{i,0}. \end{cases}$$

The parameter vector θ can be estimated by maximizing the logarithm of likelihood function

$$\mathcal{L}(\theta) = \sum_{i=1}^{N} \sum_{t=T_{i,0}}^{T_{i,1}} \ln L_{it}(\theta). \tag{61}$$

Evaluation of the log-likelihood involves solving computationally intensive dynamic programming problem that approximates the Bellman equation (54) by discretization of state space. For each candidate parameter vector θ, we solve the discretized version of (54) and then obtain the choice probabilities, (55) and (56), as well as the stationary distribution from the associated policy function. Once the choice probabilities and the stationary distribution are obtained for a particular candidate parameter vector θ, then we may evaluate the log-likelihood function (61). Repeating this process, we can maximize (61) over the parameter vector space of θ to find the estimate.

In practice, we use the Gauss-Hermit quadrature grids to discretize the space of φ so that the integral in (54) can be approximately evaluated using the Hermit quadrature formula (c.f., Tauchen and Hussey, 1991; Stinebrickner, 2000). Another important issue is that, while the conditional choice probabilities, (55) and (56), can be evaluated only on the set of finite quadrature grids, we need to evaluate the conditional choice probabilities on the *realized* value of φ_{it} which is not necessarily on the set of grid points. We evaluate the conditional choice probabilities (55) and (56) at φ_{it} that is not on the quadrature grids by using cubic spline interpolation. The appendix discusses the approximation method in greater detail. To maximize the log-likelihood function (61), we first use the simplex method of Nelder and Mead to reach the neighborhood of the optimum and then use the BFGS quasi-Newton method.

Table 9: Maximum Likelihood Estimates

Parameters	Estimates	S.E.
α_t	0.086	(0.002)
α_0	-1.305	(0.026)
α_m	0.060	(0.004)
α_x	-2.201	(0.018)
σ_η	2.516	(0.014)
σ_ω	0.422	(0.000)
σ_0	1.320	(0.016)
σf	7.141	(0.784)
σf_x	3.168	(0.179)
σf_m	2.546	(0.149)
ζ_f	0.392	(0.020)
σc_x	48.765	(1.978)
σc_m	44.255	(1.753)
ζ_c	0.871	(0.009)
ζ_{cx}	0.960	(0.015)
ζ_{cm}	0.938	(0.016)
$\sigma \rho_d$	10.441	(0.408)
$\sigma \rho_\chi$	10.571	(1.300)
ψ	0.990	(0.001)
ξ	0.038	(0.001)
f_e	128.380	
$\sigma c_x + \sigma E(\epsilon_d \mid d_t^x = 1, d_{t-1}^x = 0)$	14.950	
$\sigma c_m + \sigma E(\epsilon_d \mid d_t^m = 1, d_{t-1}^m = 0)$	14.012	
log-likelihood	-69870.82	
No. of Plants	7234	

Notes: Standard errors are in parentheses. The parameters are evaluated in the unit of million US dollars in 1990.

8 Results for Dynamic Model

8.1 Parameter Estimates

Table 9 presents the maximum likelihood estimates of the empirical models and their asymptotic standard errors, which are computed using the outer product of gradients estimator. The parameters are evaluated in the unit of million US dollars in 1990. The standard errors are generally small.

The estimate of α_t implies that the revenue is growing at the annual rate of 8.6 percent. The estimate of α_m is 0.06, indicating that importing materials from abroad has a substantial impact, a 6.0 percent increase, on the total revenues. The standard error for export revenue is estimated as 2.51, which seems to be high. This high estimate might be due to our specification for export revenue function.[16]

The estimated per-period fixed cost of operating in the market is $\hat{f} = 7.14/\sigma$ million dollars. Thus, if, say, $\sigma = 4$, the estimated per-period fixed cost is approximately equal to 1.8 million US dollars. On the other hand, this per-period fixed cost is more than eight times as large as the profit from domestic sales for the "average" incumbent with the average productivity $\ln \varphi = 1.09$, $(\exp(\alpha_0 + 1.09)/\sigma =)0.81/\sigma$. Taken for face value, the result indicates that many firms put up with the profit loss for a potential big success in the future—a possible realization of a high value of φ. The estimated per-period fixed costs for export and import are also substantial: $\hat{f}_x = 3.17/\sigma$ and $\hat{f}_m = 2.55/\sigma$. The parameter determining the degree of complementarity in the per-period fixed cost associated with export and import, ζ_f, is estimated as 0.392, indicating that a firm can save more than a half of per-period fixed cost associated with trade by engaging in both export and import activities.

The estimated one-time sunk costs associated with export and import are also large: $\hat{c}_x = 48.77/\sigma$ and $\hat{c}_m = 44.26/\sigma$. We have to be careful, however, in interpreting these numbers since a firm is also getting a random shock $\epsilon(d)$. Since a firm tends to start exporting when a firm draws a lower random shock $\epsilon_d(d^x = 1)$, the average value of ϵ_d a firm is paying is low and tends to be negative. If we take into account the random shock, the average sunk cost paid among the firms that start exporting (or importing) is computed as $\hat{c}_x + \hat{E}(\epsilon_d(d_t^x = 1)|d_t^x = 1, d_{t-1}^x = 0) = 14.95/\sigma$ (or $\hat{c}_m + \hat{E}(\epsilon_d(d_t^m = 1)|d_t^m = 1, d_{t-1}^m = 0) = 14.01/\sigma$), which is substantially lower than $\hat{c}_x = 48.77/\sigma$ (or $\hat{c}_m 44.26/\sigma$). This means that, on average, a firm pays the sunk start-up cost of exporting (importing) that is about 18 (14) times as large as the average incumbent's annual profit from domestic sales.

The parameter estimate $\hat{\zeta}_c = 0.871$ indicates that a firm can save more than 12 percent of one-time sunk cost for export and import if it simultaneously exports and imports. On the other hand, $\hat{\zeta}_{cx} = 0.960$ and $\hat{\zeta}_{cm} = 0.938$ imply that the past import experience reduces the sunk cost for exporting by 4.0 percent while the past export experience reduces the sunk cost for importing by 6.2 percent.

The estimated magnitudes of the shocks associated with the exiting decision and the ex-

[16] The current specification might not be rich enough to capture the process of export revenue shocks.

Table 10: The Mean of Productivity

Mean of φ at Entry Trial	1.000
Mean of φ at Successful Entry	1.101
Mean of φ at Steady State	5.154
Mean of $\varphi \exp(d^m \alpha_m)$ at Steady State	5.345

Notes: The reported numbers are relative to the productivity level at entry. In particular, the original numbers are divided by the mean of φ at entry (i.e., $\int \varphi g_0(\varphi) d\varphi$).

port/import decisions are large relative to the per-period profit. While the estimate of $\rho_d = 10.44/\sigma$, implying the standard error of $\frac{\pi}{\sqrt{6}} \times 10.44/\sigma = 23.19/\sigma$ in export/import cost shocks, which is more than fourteen times as large as the "average" incumbent's profit. This indicates that a firm faces a large uncertainty in the magnitude of the one-time fixed cost for exporting and importing.

The estimate of the AR(1) coefficient for the φ process, ψ, is 0.990 so that φ follows a highly persistent process. The exogenous exiting probability is estimated as 3.8 percent, implying that even a large and highly productive firm faces a non-negligible probability of exiting due to a negative shock. The fixed entry cost, which is estimated from the free entry condition $\hat{f}_e = \int \hat{V}(\varphi, (0,0)) \hat{g}_0(\varphi') d\varphi'$, is $128.380/\sigma$, which is 158 times as large as the domestic profit for the "average" incumbent.

In the model, the higher productivity firms are more likely to survive than the lower productivity firms. To examine how important such a selection mechanism to determine aggregate productivity, the first to the fourth rows of Table 10 compare the mean of productivity across different groups of firms. The average productivity level among successful new entrants is 10.1 percent higher than the average productivity level of the initial draws from $g_0(\varphi)$, indicating that those who initially drew the relatively higher productivity are more likely to succeed in entering into the market. Over time, the selection leads to a larger impact on the average productivity. The average productivity at the steady state is more than five times higher than the average productivity across initial draws. When we also include the effect of import, the average productivity at the steady state increases by additional 19 percent.

Table 11 compares the actual and the predicted transition probability of export/import status and entry/exit together. The transition pattern of export and import are replicated by

Table 11: Transition Probability of Export/Import Status and Entry/Exit (Actual vs. Predicted)

	Export/Import Status at $t+1$ conditioned on Staying						
Actual	(1) No-Export /No-Import	(2) Export /No-Import	(3) No-Export /Import	(4) Export /Import	(2)+(4) Export	(3)+(4) Import	Exit at $t+1^a$
No-Export/No-Import at t	0.927	0.024	0.042	0.007	0.031	0.048	0.082
Export/No-Import at t	0.147	0.677	0.013	0.163	0.841	0.176	0.070
No-Export/Import at t	0.188	0.017	0.699	0.096	0.113	0.795	0.035
Export/Import at t	0.025	0.101	0.070	0.804	0.905	0.874	0.022
New Entrants at t^b	0.753	0.096	0.100	0.051	0.147	0.151	0.126
Predicted							
No-Export/No-Import at t	0.915	0.030	0.046	0.009	0.039	0.054	0.078
Export/No-Import at t	0.175	0.670	0.012	0.144	0.813	0.156	0.047
No-Export/Import at t	0.195	0.008	0.700	0.097	0.105	0.797	0.048
Export/Import at t	0.025	0.103	0.094	0.778	0.881	0.872	0.040
New Entrants at t	0.926	0.026	0.041	0.007	0.033	0.048	0.113

Note: a). "Exit at $t+1$" is defined as plants that are observed at t but not observed at $t+1$ in the sample. b). "New Entrants t" is defined as plants that are not observed at $t-1$ but observed at t in the sample, of which row represents the empirical distribution of export/import status at t as well as the probability of not being observed (i.e., exit) at $t+1$.

the estimated model pretty well. The predicted invariant distribution is also very close to the empirical distribution for 1990-1996. On the other hand, the estimated models do not predict export/import decisions among new entrants well.[17]

Table 12 compares the actual and the predicted average productivity and market shares across different export/import states. In the actual data, while a 68.6 percent of the firms are neither exporting nor importing, their market shares account only for a 22.8 percent of total outputs. On the other hand, only a 10.8 percent of the firms are both exporting and importing but they account for 44.1 percent of total output. As shown in Table 12, this pattern in the market share is well replicated by the estimated models. As the actual data suggests (in the third row), relatively small number of exporters and importers account for large market shares because they tend to be more productive and hence employ more workers relative to non-exporters and non-importers. This basic observed pattern on the productivity across different export and import status is also captured by the estimated model as shown in the last row. The estimated model indicates that the exporting and the importing firms are more productive on average than the non-exporting and the non-importing firms since the more productive firms, expecting the

[17] One possible reason is the presence of unobserved heterogeneity. In the future, we will incorporate unobserved heterogeneity into the model.

Table 12: Productivity and Market Shares by Export/Import Status (Actual vs. Predicted)

		Export/Import Status					
		(1) No-Export /No-Import	(2) Export /No-Import	(3) No-Export /Import	(4) Export /Import	(2)+(4) Export	(3)+(4) Import
Actual	Dist. of Ex/Im Status	0.686	0.086	0.120	0.108	0.194	0.228
	Market Shares	0.228	0.187	0.145	0.441	0.627	0.586
	Average of $\ln\varphi$	0.484	1.337	1.474	2.381	1.911	1.898
Predicted	Dist. of Ex/Im Status	0.659	0.092	0.128	0.121	0.213	0.249
	Market Shares	0.254	0.118	0.101	0.527	0.645	0.628
	Average of $\ln\varphi$	0.763	1.507	1.262	2.349	1.984	1.789

higher returns from exporting and importing, are more willing to pay the one-time fixed costs of exporting and importing and hence are more likely to become exporters and importers.

8.2 Counterfactual Experiments

While some of the structural parameters are not identified from the empirical model (crucially, we cannot identify σ), we may solve for the change in the equilibrium aggregate price as a result of counterfactual experiments as follows.

Denote the equilibrium aggregate price under the parameter θ by $P(\theta)$. Under the estimated parameter $\hat{\theta}$, we may compute the estimate of fixed entry cost as: $\hat{f}_e = \int V(\varphi', (0,0); \hat{\theta}) g_0(\varphi'; \hat{\theta}) d\varphi'$, where $V(\varphi, d; \theta)$ is the fixed point of the Bellman's equation (54) under the parameter θ and $g_0(\varphi; \theta)$ is the probability density function of the initial productivity under θ.

Suppose that we are interested in a counterfactual experiment characterized by a counterfactual parameter $\tilde{\theta}$ that is different than the estimated parameter $\hat{\theta}$. Note that the following relationships hold between α_0 and the aggregate price P:

$$\hat{\alpha}_0 \quad = \quad \ln(\alpha^{\alpha}(1-\alpha)^{1-\alpha} R P(\hat{\theta})^{\sigma-1}).$$

Then, we may write the estimated profit function, (53), evaluated at the counterfactual aggregate price $P(\tilde{\theta})$ as:

$$\begin{aligned}
&\hat{\pi}(\varphi_t, d_t, d_{t-1}; P(\tilde{\theta})) \\
&= \frac{1}{\sigma} \exp[(\sigma-1)\ln(P(\tilde{\theta})/P(\hat{\theta}))](1+\exp(\hat{\alpha}_x) d_t^x) \exp(\hat{\alpha}_0 + \hat{\alpha}_m d_t^m + \ln\varphi_t) - \hat{F}(d_t, d_{t-1}), \\
&= \frac{1}{\sigma} \exp[\ln(K(\tilde{\theta})/K(\hat{\theta}))](1+\exp(\hat{\alpha}_x) d_t^x) \exp(\hat{\alpha}_0 + \hat{\alpha}_m d_t^m + \ln\varphi_t) - \hat{F}(d_t, d_{t-1}),
\end{aligned}$$

where $K(\theta) = RP(\theta)^{\sigma-1}$ is the demand shifter under the parameter θ.

Then, we may compute the equilibrium changes in the demand shifters, $\ln(K(\tilde{\theta})/K(\hat{\theta})) = (\sigma-1)\ln(P(\tilde{\theta})/P(\hat{\theta}))$. Specifically, we may use the same algorithm for computing a stationary equilibrium using the free entry condition under the counterfactual parameter $\tilde{\theta}$:

$$\hat{f}_e = \int V(\varphi'; \tilde{\theta}, P(\tilde{\theta})) g_0(\varphi'; \tilde{\theta}) d\varphi',$$

where the dependence of the value function V on the aggregate price $P(\tilde{\theta})$ is explicitly indicated.

We may quantify the impact of counterfactual experiments on the welfare level by examining how much the equilibrium aggregate price level P changes as a result of counterfactual experiments since the aggregate price level P is inversely related to the welfare level W.[18]

To quantitatively investigate the impact of international trade, we conduct the four counterfactual experiments with the following counterfactual parameters:

(1) No Export: $f_x, c_x \to \infty$ and $\alpha_x \to -\infty$.

(2) No Import: $f_m, c_m \to \infty$ and $\alpha_m = 0$.

(3) Autarky: $f_x, c_x, f_m, c_m \to \infty$, $\alpha_x \to -\infty$, and $\alpha_m = 0$.

(4) No Complementarity: $\zeta_f = \zeta_c = \zeta_{cx} = \zeta_{cm} = 1$.

Table 13 presents the results of counterfactual experiments using the estimated model. To examine the importance of equilibrium response to quantify the impact of counterfactual policies, the results both with and without equilibrium aggregate price response are presented. According to the experiment, moving from autarky to trade may decrease the equilibrium aggregate price by a $19.4/(\sigma-1)$ percent. This implies that, say if $\sigma = 4$, exposure to trade increases the real income by a (19.4/4=)4.85 percent, leading to a substantial increase in welfare. When a country opens up its economy, more productive firms start exporting and importing, which in turn increases the aggregate labor demand and hence leads to an increase in the real wage or a decrease in the aggregate final goods price.

Under no equilibrium response, the exiting rates at entry trial increase from 13.55 to 22.88 percent by moving from trade to autarky; without equilibrium price response, the profit is lower

[18] To see this, note that the income is constant at the level of L. From the budget constraint $PQ = L$ and the definition of aggregate product $W = Q = \left[\int_{\omega\in\Omega} q(\omega)^\rho d\omega\right]^{1/\rho}$, the utility level is equal to $W = P^{-1}L$.

Table 13: Counterfactual Experiments

		Counterfactual Experiments			
		(1)	(2)	(3)	(4)
	Trade	No Export	No Import	Autarky	No Comp.
With Equilibrium Price Effect					
Welfare measured by $(\sigma - 1)\ln P$	0.000	0.131	0.128	0.194	0.063
Exiting Rates at Entry Trial (%)	13.55	14.23	14.57	14.99	13.81
ln(Average φ) at Steady State	0.000	-0.024	-0.003	-0.042	0.014
A Fraction of Exporters	0.213	0.000	0.093	0.000	0.092
A Fraction of Importers	0.249	0.135	0.000	0.000	0.135
Market Shares of Exporters	0.645	0.000	0.471	0.000	0.470
Market Shares of Importers	0.628	0.420	0.000	0.000	0.433
Without Equilibrium Price Effect					
Exiting Rates at Entry Trial (%)	13.55	19.39	19.75	22.88	16.25
ln(Average φ) at Steady State	0.000	-0.013	0.009	-0.021	0.019
A Fraction of Exporters	0.213	0.000	0.091	0.000	0.091
A Fraction of Importers	0.249	0.131	0.000	0.000	0.133
Market Shares of Exporters	0.645	0.000	0.450	0.000	0.459
Market Shares of Importers	0.628	0.397	0.000	0.000	0.422

Note: "Average φ" at Steady State is a productivity average using the plants' combined revenues (or market shares) as weights: $\int \sum_d \varphi^{\sigma-1} \frac{r(\varphi,d)}{\int \sum_{d'} r(\varphi',d')d\mu(\varphi',d')} d\mu(\varphi,d)$.

in autarky than in trade because there is no opportunity to export/import; the lower profit leads to higher exit rates of 22.88 percent in autarky. By comparing the exiting rates in autarky between "With Equilibrium Price Effect" and "With Equilibrium Price Effect," we notice that such a partial equilibrium effect is quantitatively misleading. When the economy moves from trade to autarky, the equilibrium aggregate price increases by 19.4 percent; this equilibrium price adjustment in turn lowers the exiting rates from 22.88 percent to 14.99 percent.

The impact of trade on aggregate productivity—measured by a productivity average using the plants' market shares as weights—can be understood by comparing "ln(Average φ) at Steady State" between trade and autarky. Moving from trade to autarky leads to a 4.2 decrease in aggregate productivity at the steady state. Here, by comparing between with and without the equilibrium price responses, we notice the importance of equilibrium price response to quantitatively explain the impact of trade on aggregate productivity; without equilibrium price response, the impact of trade on aggregate productivity is only 2.1 percent as opposed to 4.2 percent.

The counterfactual experiments under no export or no import (but not both) highlight the interaction between aggregate export and aggregate import in the presence of heterogenous firms. According to the estimated model, when the economy moves from trade to no import, a fraction of *exporters* declines from a 21.2 percent to a 8.2 percent; when the economy moves from trade to no export, a fraction of *importers* declines from a 24.6 percent to a 12.5 percent. In terms of market shares, moving from trade to no import leads to a decrease in the total market shares of *exporters* from 64.5 to 45.0 percent while moving from trade to no export leads to a decrease in the total market shares of *importers* from 62.8 to 39.7 percent. Thus, policies that prohibits the import of foreign materials could have a large negative impact on the export of final consumption goods, or vice-versa. The similar results hold even without equilibrium price effect and thus the equilibrium price response is little to do with these results; rather, it is due to the complementarity between export and import within both revenue function $r(\cdot)$ and sunk-cost function $F(\cdot)$.

The counterfactual experiments under no export or no import (but not both) highlight the interaction between aggregate export and aggregate import in the presence of heterogenous firms. According to the estimated model, when the economy moves from trade to no export, a fraction of *importers* declines from a 23.2 percent to a 19.1 percent; when the economy moves from trade to no import, a fraction of *exporters* declines from a 19.6 percent to a 15.1 percent. In terms of market shares, moving from trade to no export leads to a decrease in the total market shares of *importers* from 55.2 to 42.3 percent while moving from trade to no import leads to a decrease in the total market shares of *exporters* from 59.2 to 48.2 percent. Thus, policies that prohibits the import of foreign materials could have a large negative impact on the export of final consumption goods, or vice-versa. The similar results hold even without equilibrium price effect and thus the equilibrium price response is little to do with these results. Rather, it is due to the complementarity between export and import within both revenue function $r(\cdot)$ and sunk-cost function $F(\cdot)$.

To examine the role of complementarity between export and import in the sunk-cost function—relative to the role played by the complementarity in the revenue function—we conducted what would happen to a fraction of importers and/or a fraction of exporters had there been no complementarity between export and import in the sunk cost function. The results are striking. Eliminating the complementarity between export and import in the sunk cost function has essentially the same effect on export and import, respectively, as restricting to no import and no export. A fraction of exporters is 8.2 percent under no import while eliminating the complementarity in sunk-cost function decreases a fraction of exporters from 21.2 percent to 8.7

percent. Similarly, eliminating complementarity decreases a fraction of importers from 24.6 percent to 12.4 percent, which is almost identical to a fraction of importers under no export. Thus, it is the complementarity in the sunk cost function, rather than the complementarity in the revenue function, that determines the impact of exporting policies (e.g., export subsidies) on intermediate imports or the impact of importing policies (e.g., import tariffs) on exports.

9 Conclusions

We have developed and estimated a dynamic, stochastic, industry model of import and export with heterogeneous firms. The analysis highlights interactions between imports of intermediate goods and exports of final goods. In doing so, we have identified a potential mechanism whereby import policy can affect exports and export policy can affect imports.

References

[1] Aw, B., S. Chung, and M.J. Roberts (2000) "Productivity and Turnover in the Export Market: Micro-level Evidence from the Republic of Korea and Taiwan(China)," *World Bank Economic Review*, 14, 65-90.

[2] Baily, Martin N., Hulten, Charles, and Campbell, David (1992) "Productivity Dynamics in Manufacturing Plants," *Brookings Papers on Economic Activity: Microeconomics*, 67-119.

[3] Bernard, Andrew B. and J. Bradford Jensen (1999) "Exceptional Exporter Performance: Cause, Effect, and or Both?" *Journal of International Economics*, 47: 1-25.

[4] Bernard, Andrew B., Jonathan Eaton, J. Bradford Jensen, and Samuel Kortum (2003) "Plants and Productivity in International Trade," *American Economic Review*, 93(4): 1268-1290.

[5] Clerides, Sofronis, Lach, Saul, and James R. Tybout (1998) "Is Learning by Exporting Important? Micro-Dynamic Evidence from Colombia, Mexico, and Morocco," *Quarterly Journal of Economics* 113: 903-947.

[6] Coe, David T. and Elhanan Helpman (1995) "International R&D Spillovers," *European Economic Review*, 39: 859-887.

[7] Coe, David T., Elhanan Helpman, and Alexander W. Hoffmaister (1997) "North-South R&D Spillovers," *Economic Journal*, 107: 134-149.

[8] Ethier, W. (1982) "National and International Returns to Scale in the Modern Theory of International Trade," *American Economic Review*, 72, No. 3, 389-405.

[9] Helpman, Elhanan, Marc J. Melitz, and Stephen R. Yeaple (2004) "Export versus FDI with Heterogneous Firms" *American Economic Review*, 94: 300-316.

[10] Hummels, David, Ishii, Jun, and Yi, Kei-Mu (2001) "The Nature and Growth of Vertical Specialization in World Trade" *Journal of International Economics* 54: 75-96.

[11] Kasahara, Hiroyuki, and Joel Rodrigue (2004) "Does the Use of Imported Intermediates Increase Productivity? Plant-Level Evidence," mimeo.

[12] Krugman, Paul (1979) "Increasing Returns, Monopolistic Competition, and International Trade," *Journal of International Economics*, 9: 469-79.

[13] Lui, L. (1993) "Entry, Exit, and Learning in the Chilean Manufacturing Sector," *Journal of Development Economics*, 42: 217-242.

[14] Melitz, Mark J.(2003) "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71: 1695-1725.

[15] Pavcnik, Nina (2002) "Trade Liberalization, Exit, and Productivity Improvements: Evidence From Chilean Plants," Review of Economic Studies, 69(1): 245-276.

[16] Roberts, Mark and Tybout, Robert (1997) "Decision to Export in Columbia: An Empirical Model of Entry with Sunk Costs" *American Economic Review*, 87: 545-564.

[17] Trefler, Dan (2004) "The Long and Short of the Canada-U.S. Free Trade Agreement," *American Economic Review*, 94(4): 870-895.

[18] Yi, Kei-Mu "Can Vertical Specialization Explain the Growth of World Trade?" *Journal of Political Economy*, 111(1): 52-102.

10 Appendix I - Model Derivations

10.1 Bounds

The restrictions on fixed costs are as follows:

$$(C1) \qquad f_m + \underline{\epsilon} < \frac{\zeta f_x (b_m - 1)}{b_m(b_x - \zeta) + (\zeta - 1)}$$

$$(C2) \qquad f_m + \bar{\epsilon} > \frac{f_x[b_x(b_m - \zeta) + (\zeta - 1)]}{\zeta(b_x - 1)}$$

$$(C3) \qquad f_x > \left(\frac{f}{N}\right)(b_x - 1)$$

$$(C4) \qquad f_m + \underline{\epsilon} > \left(\frac{f}{N}\right)(b_m - 1)$$

$$(C5) \qquad \zeta(f_x + f_m + \underline{\epsilon}) > \left(\frac{f}{N}\right)(b_x b_m - 1)$$

First note that because a firm's revenue will be highest if the firm both exports and imports and because revenue is increasing in φ and φ is unbounded, there will be a set of firms (those with high φ) which will both export and import. Thus, we do not need to impose any additional restrictions to guarantee that a positive measure of firms will choose to engage in both types of trade. In contrast, we must restrict the fixed costs of exporting and importing to guarantee a positive measure of firms in each of the three remaining export/import categories. Consider each restriction in turn. The first condition ensures that the minimum fixed cost of importing is low enough so that some firms will choose to import but not export. The second restriction implies that the maximum fixed cost of importing is large enough so that some firms will choose to export but not import. The third condition guarantees that the fixed costs of exporting are high enough so that some firms will neither export nor import. The derivation of these bounds is given below.

The last two additional conditions, (C4)-(C5) are imposed to simplify the derivation of the cutoff productivity in the open economy and imply that the profits earned by a firm with the cutoff productivity are maximized by neither importing nor exporting. These conditions are similar to that imposed by Melitz(2003) on the minimum level of fixed costs of exporting to ensure partitioning of firms into exporters and non-exporters in his economy.

We note that Figure 2 allows us to derive the bounds given above on the random component of fixed costs of importing to ensure a positive measure of firms in each export/import category. Let θ_1 satisfy $\Phi_m^0(\theta_1) = \Phi_x^1(\theta_1) = \Phi_{xm}(\theta_1)$ or

$$\theta_1 \equiv \frac{\zeta f_x(b_m - 1)}{b_m(b_x - \zeta) + (\zeta - 1)}. \tag{62}$$

Let θ_2 satisfy $\Phi_m^1(\theta_2) = \Phi_x^0(\theta_2) = \Phi_{xm}(\theta_2)$ or

$$\theta_2 = \frac{f_x[b_x(b_m - \zeta) + (\zeta - 1)]}{\zeta(b_x - 1)}. \tag{63}$$

Note that $\theta_1 < \theta_2$. Now condition (C1) above implies that $\underline{\theta} < \theta_1$ which we can see from the figure implies a positive measure of firms choosing to import but not export. Condition (C2) implies that $\bar{\theta} > \theta_2$ so that there is a positive measure of firms choosing to export but not import.

We can derive similar conditions for partitioning firms in the economies with partial trade. In the economy with no importing ($b_m = \zeta = 1$), equation (24) implies that firms with relative productivity $\Phi \in \left[\frac{f}{N}, \frac{f_x}{b_x - 1}\right)$ will not export while firms with $\Phi \geq \frac{f_x}{b_x - 1}$ will export (which is identical to the partitioning derived by Melitz(2003)). Similarly, in the economy with no exporting ($b_x = \zeta = 1$), equation (25) implies that a firm with total fixed costs of importing equal to θ will not import if their relative productivity satisfies $\Phi \in \left[\frac{f}{N}, \frac{\theta}{b_m - 1}\right)$ and will import otherwise.

10.2 Notation

For the full trading economy, using the cutoff functions in terms of Φ defined in equations (24)-(26), we can define cutoffs functions in terms of φ as follows:

$$\tilde{\varphi}_i^j(\varphi_T^*, \epsilon) \equiv \left(\frac{N\Phi_i^j(\epsilon)}{f}\right)^{\frac{1}{\sigma-1}} \varphi_T^* \quad i \in \{x, I\}, \;\; j \in \{0, 1\} \tag{64}$$

$$\tilde{\varphi}_{xm}(\varphi_T^*, \epsilon) \equiv \left(\frac{N\Phi_{xm}(\epsilon)}{f}\right)^{\frac{1}{\sigma-1}} \varphi_T^* \tag{65}$$

Using equations (62) and (63), we can also define $\epsilon_j \equiv \theta_j - f_m$ for $j = 1, 2$.

For the economy with no importing, we denote the cutoff productivity for exporting as

$$\tilde{\varphi}_X(\varphi_X^*) \equiv \left(\frac{Nf_x}{f(b_x - 1)}\right)^{\frac{1}{\sigma-1}} \varphi_X^*. \tag{66}$$

Similarly, for the economy with no exporting, we denote the cutoff productivity for importing for a firm with random importing cost equal to ϵ as

$$\tilde{\varphi}_I(\varphi_I^*, \epsilon) \equiv \left(\frac{N(f_m + \epsilon)}{f(b_m - 1)}\right)^{\frac{1}{\sigma-1}} \varphi_I^*. \tag{67}$$

10.3 Firm Fractions

For each economy, the fraction of firms in each export/import status are given below.

$$\nu_A(\varphi_A^*, 0, 0) = 1 \qquad \nu_A(\varphi_A^*, 1, 0) = \nu_A(\varphi_A^*, 0, 1) = \nu_A(\varphi_A^*, 1, 1) = 0 \tag{68}$$

$$\nu_X(\varphi_X^*, 0, 0) = \left(\frac{1}{1 - G(\varphi_X^*)}\right) [G(\tilde{\varphi}_X(\varphi_X^*)) - G(\varphi_X^*)] \tag{69}$$

$$\nu_X(\varphi_X^*, 1, 0) = \left(\frac{1}{1 - G(\varphi_X^*)}\right) [1 - G(\tilde{\varphi}_X(\varphi_X^*))] \tag{70}$$

$$\nu_X(\varphi_X^*, 0, 1) = \nu_X(\varphi_X^*, 1, 1) = 0 \tag{71}$$

$$\nu_I(\varphi_I^*, 0, 0) = \left(\frac{1}{1 - G(\varphi_I^*)}\right) \left[\int_{\underline{\epsilon}}^{\bar{\epsilon}} \left(\int_{\varphi_I^*}^{\tilde{\varphi}_I(\varphi_I^*, \epsilon)} g(z)dz\right) h(\epsilon)d\epsilon\right] \tag{72}$$

$$\nu_I(\varphi_I^*, 0, 1) = \left(\frac{1}{1 - G(\varphi_I^*)}\right) \left[\int_{\underline{\epsilon}}^{\bar{\epsilon}} \left(\int_{\tilde{\varphi}_I(\varphi_I^*, \epsilon)}^{\infty} g(z)dz\right) h(\epsilon)d\epsilon\right] \tag{73}$$

$$\nu_I(\varphi_I^*, 1, 0) = \nu_I(\varphi_I^*, 1, 1) = 0 \tag{74}$$

$$\nu_T(\varphi_T^*, 0, 0) = \left(\frac{1}{1 - G(\varphi_T^*)}\right) \left[\int_{\underline{\epsilon}}^{\epsilon_1} \left(\int_{\varphi_T^*}^{\tilde{\varphi}_m^0(\varphi_T^*, \epsilon)} g(\varphi)d\varphi\right) h(\epsilon)d\epsilon + \int_{\epsilon_1}^{\epsilon_2} \left(\int_{\varphi_T^*}^{\tilde{\varphi}_{xm}(\varphi_T^*, \epsilon)} g(\varphi)d\varphi\right) h(\epsilon)d\epsilon \right.$$
$$\left. \int_{\epsilon_2}^{\bar{\epsilon}} \left(\int_{\varphi_T^*}^{\tilde{\varphi}_x^0(\varphi_T^*, \epsilon)} g(\varphi)d\varphi\right) h(\epsilon)d\epsilon\right] \tag{75}$$

$$\nu_T(\varphi_T^*, 0, 1) = \left(\frac{1}{1 - G(\varphi_T^*)}\right) \left[\int_{\underline{\epsilon}}^{\epsilon_1} \left(\int_{\tilde{\varphi}_m^0(\varphi_T^*, \epsilon)}^{\tilde{\varphi}_x^1(\varphi_T^*, \epsilon)} g(\varphi)d\varphi\right) h(\epsilon)d\epsilon\right] \tag{76}$$

$$\nu_T(\varphi_T^*, 1, 0) = \left(\frac{1}{1 - G(\varphi_T^*)}\right) \left[\int_{\epsilon_2}^{\bar{\epsilon}} \left(\int_{\tilde{\varphi}_x^0(\varphi_T^*, \epsilon)}^{\tilde{\varphi}_m^1(\varphi_T^*, \epsilon)} g(\varphi)d\varphi\right) h(\epsilon)d\epsilon\right] \tag{77}$$

$$\nu_T(\varphi_T^*,1,1) = \left(\frac{1}{1-G(\varphi_T^*)}\right)\left[\int_{\underline{\epsilon}}^{\epsilon_1}\left(\int_{\tilde{\varphi}_x^1(\varphi_T^*,\epsilon)}^{\infty} g(\varphi)d\varphi\right)h(\epsilon)d\epsilon + \int_{\epsilon_1}^{\epsilon_2}\left(\int_{\tilde{\varphi}_{xm}(\varphi_T^*,\epsilon)}^{\infty} g(\varphi)d\varphi\right)h(\epsilon)d\epsilon \right.$$
$$\left. + \int_{\epsilon_2}^{\bar{\epsilon}}\left(\int_{\tilde{\varphi}_m^1(\varphi_T^*,\epsilon)}^{\infty} g(\varphi)d\varphi\right)h(\epsilon)d\epsilon\right] \tag{78}$$

10.4 Aggregate Price Indexes and Average Productivities

In the autarkic equilibrium, $(S = A)$ equations (2), (6), and (7) imply that the aggregate price index is given by

$$P_A = \left[\frac{1}{\rho A}\right]\left[\left(\frac{M_A(\varphi_A^*)}{1-G(\varphi_A^*)}\right)\int_{\varphi_A^*}^{\infty}\varphi^{\sigma-1}g(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}} \tag{79}$$

or

$$P_A = M_A(\varphi_A^*)^{\frac{1}{1-\sigma}}p\left(\left[\frac{1}{M_A(\varphi_A^*)(1-G(\varphi_A^*))}\left(\int_{\varphi_A^*}^{\infty}\varphi^{\sigma-1}g(\varphi)d\varphi\right)\right]^{\frac{1}{\sigma-1}}\right) \tag{80}$$

Thus, our measures of average productivity in autarky are given by

$$\tilde{b}_A(\varphi_A^*,0,0) = \left[\frac{1}{(1-G(\varphi_A^*))}\left(\int_{\varphi_A^*}^{\infty}\varphi^{\sigma-1}g(\varphi)d\varphi\right)\right]^{\frac{1}{\sigma-1}} \tag{81}$$

$$\tilde{b}_A(\varphi_A^*,1,0) = \tilde{b}_A(\varphi_A^*,0,1) = \tilde{b}_A(\varphi_A^*,1,1) = 0, \tag{82}$$

where we have adopted the convention that average productivity for an export/import category with a zero measure of firms equals zero.

In the economy with no importing, $(S = X)$ equations (2), (6), (7), and (8) imply that the aggregate price index is given by

$$P_X = \left[\frac{1}{\rho A}\right]\left[\left(\frac{M_X(\varphi_X^*)}{1-G(\varphi_X^*)}\right)\left(\int_{\varphi_X^*}^{\tilde{\varphi}_X(\varphi_X^*)}\varphi^{\sigma-1}g(\varphi)d\varphi + \int_{\tilde{\varphi}_X(\varphi_X^*)}^{\infty}(1+N\tau^{\sigma-1})\varphi^{\sigma-1}g(\varphi)d\varphi\right)\right]^{\frac{1}{1-\sigma}}, \tag{83}$$

or

$$P_X = \left[\frac{1}{\rho A}\right]\left[\left(\frac{M_X(\varphi_X^*,0,0)}{\nu_X(\varphi_X^*,0,0)(1-G(\varphi_X^*))}\right)\left(\int_{\varphi_X^*}^{\tilde{\varphi}_X(\varphi_X^*)}\varphi^{\sigma-1}g(\varphi)d\varphi\right)\right.$$
$$\left. + \left(\frac{M_X(\varphi_X^*,1,0)b_x}{\nu_X(\varphi_X^*,1,0)(1-G(\varphi_X^*))}\right)\left(\int_{\tilde{\varphi}_X(\varphi_X^*)}^{\infty}\varphi^{\sigma-1}g(\varphi)d\varphi\right)\right]^{\frac{1}{1-\sigma}}. \tag{84}$$

Thus, our measures of average productivity in the partial trading economy with exporting

$$\tilde{b}_X(\varphi_X^*, 0, 0) = \left[\left(\frac{1}{\nu_X(\varphi_X^*, 0, 0)(1 - G(\varphi_X^*))}\right) \int_{\varphi_X^*}^{\tilde{\varphi}_X(\varphi_X^*)} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}} \tag{85}$$

$$\tilde{b}_X(\varphi_X^*, 1, 0) = \left[\left(\frac{b_x}{\nu_X(\varphi_X^*, 1, 0)(1 - G(\varphi_X^*))}\right) \int_{\tilde{\varphi}_X(\varphi_X^*)}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}} \tag{86}$$

$$\tilde{b}_X(\varphi_X^*, 0, 1) = \tilde{b}_X(\varphi_X^*, 1, 1) = 0. \tag{87}$$

In the economy with no exporting, ($S = I$) equations (2), (6), and (7) imply that the aggregate price index is given by

$$P_I = \left[\frac{1}{\rho A}\right] \left[\left(\frac{M_I(\varphi_I^*, 0, 0)}{1 - G(\varphi_I^*)}\right) \left(\int_{\underline{\epsilon}}^{\bar{\epsilon}} \left(\int_{\varphi_I^*}^{\tilde{\varphi}_I(\varphi_I^*, \epsilon)} \varphi^{\sigma-1} g(\varphi) d\varphi\right) h(\epsilon) d\epsilon\right.\right.$$

$$\left.\left. + \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left(\int_{\tilde{\varphi}_I(\varphi_I^*, \epsilon)}^{\infty} \lambda^{\sigma-1} \varphi^{\sigma-1} g(\varphi) d\varphi\right) h(\epsilon) d\epsilon\right)\right]^{\frac{1}{\sigma-1}} \tag{88}$$

Thus, our measures of average productivity in the partial trading economy with importing are

$$\tilde{b}_I(\varphi_I^*, 0, 0) = \left[\left(\frac{1}{\nu_I(\varphi_I^*, 0, 0)(1 - G(\varphi_I^*))}\right) \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left(\int_{\varphi_I^*}^{\tilde{\varphi}_I(\varphi_I^*, \epsilon)} \varphi^{\sigma-1} g(\varphi) d\varphi\right) h(\epsilon) d\epsilon\right]^{\frac{1}{\sigma-1}} \tag{89}$$

$$\tilde{b}_I(\varphi_I^*, 0, 1) = \left[\left(\frac{b_m}{\nu_I(\varphi_I^*, 0, 1)(1 - G(\varphi_I^*))}\right) \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left(\int_{\tilde{\varphi}_I(\varphi_I^*, \epsilon)}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi\right) h(\epsilon) d\epsilon\right]^{\frac{1}{\sigma-1}} \tag{90}$$

$$\tilde{b}_I(\varphi_I^*, 1, 0) = \tilde{b}_I(\varphi_I^*, 1, 1) = 0. \tag{91}$$

Using the same approach as in the previous three economies and using Figure 2, we have the following average productivities across groups of firms in the full trading equilibrium

$$\tilde{b}_T(\varphi_T^*, 0, 0) = \left[\left(\frac{1}{\nu_T(\varphi_T^*, 0, 0)(1 - G(\varphi_T^*))}\right) \left(\int_{\underline{\epsilon}}^{\epsilon_1} \left(\int_{\varphi_T^*}^{\tilde{\varphi}_m^0(\varphi_T^*, \epsilon)} \varphi^{\sigma-1} g(\varphi) d\varphi\right) h(\epsilon) d\epsilon\right.\right.$$

$$\left.\left. + \int_{\epsilon_1}^{\epsilon_2} \left(\int_{\varphi_T^*}^{\tilde{\varphi}_{xm}(\varphi_T^*, \epsilon)} \varphi^{\sigma-1} g(\varphi) d\varphi\right) h(\epsilon) d\epsilon \int_{\epsilon_2}^{\bar{\epsilon}} \left(\int_{\varphi_T^*}^{\tilde{\varphi}_x^0(\varphi_T^*, \epsilon)} \varphi^{\sigma-1} g(\varphi) d\varphi\right) h(\epsilon) d\epsilon\right)\right]^{\frac{1}{\sigma-1}} \tag{92}$$

$$\tilde{b}_T(\varphi_T^*, 0, 1) = \left[\left(\frac{b_m}{\nu_T(\varphi_T^*, 0, 1)(1 - G(\varphi_T^*))}\right) \left(\int_{\underline{\epsilon}}^{\epsilon_1} \left(\int_{\tilde{\varphi}_m^0(\varphi_T^*, \epsilon)}^{\tilde{\varphi}_x^1(\varphi_T^*, \epsilon)} \varphi^{\sigma-1} g(\varphi) d\varphi\right) h(\epsilon) d\epsilon\right)\right]^{\frac{1}{\sigma-1}} \tag{93}$$

$$\tilde{b}_T(\varphi_T^*, 1, 0) = \left[\left(\frac{b_x}{\nu_T(\varphi_T^*, 1, 0)(1 - G(\varphi_T^*))} \right) \left(\int_{\epsilon_2}^{\bar{\epsilon}} \left(\int_{\tilde{\varphi}_x^0(\varphi_T^*, \epsilon)}^{\tilde{\varphi}_m^1(\varphi_T^*, \epsilon)} \varphi^{\sigma-1} g(\varphi) d\varphi \right) h(\epsilon) d\epsilon \right) \right]^{\frac{1}{\sigma-1}} \quad (94)$$

$$\tilde{b}_T(\varphi_T^*, 1, 1) = \left[\left(\frac{b_x b_m}{\nu_T(\varphi_T^*, 1, 1)(1 - G(\varphi_T^*))} \right) \left(\int_{\underline{\epsilon}}^{\epsilon_1} \left(\int_{\tilde{\varphi}_x^1(\varphi_T^*, \epsilon)}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right) h(\epsilon) d\epsilon \right.\right.$$

$$\left.\left. + \int_{\epsilon_1}^{\epsilon_2} \left(\int_{\tilde{\varphi}_{xm}(\varphi_T^*, \epsilon)}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right) h(\epsilon) d\epsilon + \int_{\epsilon_2}^{\bar{\epsilon}} \left(\int_{\tilde{\varphi}_m^1(\varphi_T^*, \epsilon)}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right) h(\epsilon) d\epsilon \right) \right]^{\frac{1}{\sigma-1}} \quad (95)$$

10.5 Average Profits

Using equation (14), average profits across firms in autarky equals

$$\bar{\pi}_A(\varphi_A^*, 0, 0) = \left(\frac{1}{\nu_A(\varphi_A^*, 0, 0)} \right) \left(\frac{1}{1 - G(\varphi_A^*)} \right) \int_{\varphi_A^*}^{\infty} \left(\frac{r_A(\varphi, 0, 0)}{\sigma} - f \right) g(\varphi) d\varphi \quad (96)$$

or, using equations (18) and (68)

$$\bar{\pi}_A(\varphi_A^*, 0, 0) = {\varphi_A^*}^{1-\sigma} f \int_{\varphi_A^*}^{\infty} \left(\frac{\varphi^{\sigma-1}}{1 - G_A^*} \right) g(\varphi) d\varphi - f \quad (97)$$

or, using (18) and (81),

$$\bar{\pi}_A(\varphi_A^*, 0, 0) = \frac{r_A(\bar{b}_A(\varphi_A^*, 0, 0), 0, 0)}{\sigma} - f \quad (98)$$

Finally, since the other export/import categories have measure zero firms in autarky, we set $\bar{\pi}_A(\varphi_A^*, 1, 0) = \bar{\pi}_A(\varphi_A^*, 0, 1) = \bar{\pi}_A(\varphi_A^*, 1, 1) = 0$.

In the economy with no importing, using (14), we have

$$\bar{\pi}_X(\varphi_X^*, 0, 0) = \left(\frac{1}{\nu_X(\varphi_X^*, 0, 0)} \right) \left(\frac{1}{1 - G(\varphi_X^*)} \right) \int_{\varphi_X^*}^{\tilde{\varphi}_X(\varphi_X^*)} \left(\frac{r_X(\varphi, 0, 0)}{\sigma} - f \right) g(\varphi) d\varphi \quad (99)$$

or using (20) and (69)

$$\bar{\pi}_X(\varphi_X^*, 0, 0) = \left(\frac{1}{\nu_X(\varphi_X^*, 0, 0)} \right) {\varphi_X^*}^{1-\sigma} f \int_{\varphi_X^*}^{\tilde{\varphi}_X(\varphi_X^*)} \left(\frac{\varphi^{\sigma-1}}{1 - G_X^*} \right) g(\varphi) d\varphi - f \quad (100)$$

or using (20) and (85),

$$\bar{\pi}_X(\varphi_X^*, 0, 0) = \left(\frac{r_X(\bar{b}_X(\varphi_X^*, 0, 0), 0, 0)}{\sigma} \right) - f. \quad (101)$$

In a similar manner, we can derive

$$\bar{\pi}_X(\varphi_X^*, 1, 0) = \left(\frac{r_X(\bar{b}_X(\varphi_X^*, 1, 0), 0, 0)}{\sigma} \right) - (f + N f_x). \quad (102)$$

Finally, $\bar{\pi}_X(\varphi_X^*, 0, 1) = \bar{\pi}_X(\varphi_X^*, 1, 1) = 0$.

In the economy with no exporting, using similar methods to those for autarky or the no importing economy, we can derive

$$\bar{\pi}_I(\varphi_I^*, 0, 0) = \left(\frac{r_I(\bar{b}_I(\varphi_I^*, 0, 0), 0, 0)}{\sigma}\right) - f. \tag{103}$$

Using equation (14), we write

$$\bar{\pi}_I(\varphi_I^*, 0, 1) = \left(\frac{1}{\nu_I(\varphi_I^*, 0, 1)(1 - G(\varphi_I^*))}\right) \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left(\int_{\tilde{\varphi}_I(\varphi_I^*, \epsilon)}^{\infty} \left(\frac{r_I(\varphi, 0, 1)}{\sigma} - (f + N(f_m + \epsilon))\right) g(\varphi) d\varphi\right) h(\epsilon) d\epsilon \tag{104}$$

or using (21), (73) and (90),

$$\bar{\pi}_I(\varphi_I^*, 0, 1) = \left(\frac{r_I(\bar{b}_I(\varphi_I^*, 0, 1), 0, 0)}{\sigma}\right) - (f + N f_m) \tag{105}$$

$$- \left(\frac{N}{\nu_I(\varphi_I^*, 0, 1)(1 - G(\varphi_I^*))}\right) \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left(\int_{\tilde{\varphi}_I(\varphi_I^*, \epsilon)}^{\infty} \epsilon \;\; g(\varphi) d\varphi\right) h(\epsilon) d\epsilon, \tag{106}$$

or

$$\bar{\pi}_I(\varphi_I^*, 0, 1) = \left(\frac{r_I(\bar{b}_I(\varphi_I^*, 0, 1), 0, 0)}{\sigma}\right) - (f + N(f_m + \bar{\epsilon}_I(0, 1))), \tag{107}$$

where $\bar{\epsilon}_I(0, 1)$ is the average random component of fixed import costs across firms which import in this economy. Finally, $\bar{\pi}_I(\varphi_I^*, 1, 0) = \bar{\pi}_I(\varphi_I^*, 1, 1) = 0$.

Using similar methods, we can derive average profits for each group of firms in the full trading economy:

$$\bar{\pi}_T(\varphi_T^*, 0, 0) = \left(\frac{r_T(\bar{b}_T(\varphi_T^*, 0, 0), 0, 0)}{\sigma}\right) - f \tag{108}$$

$$\bar{\pi}_T(\varphi_T^*, 1, 0) = \left(\frac{r_T(\bar{b}_T(\varphi_T^*, 1, 0), 0, 0)}{\sigma}\right) - (f + N f_x) \tag{109}$$

$$\bar{\pi}_T(\varphi_T^*, 0, 1) = \left(\frac{r_T(\bar{b}_T(\varphi_T^*, 0, 1), 0, 0)}{\sigma}\right) - (f + N f_m)$$

$$- \left(\frac{N}{\nu_T(\varphi_T^*, 0, 1)}\right) \int_{\underline{\epsilon}}^{\epsilon_1} \left(\int_{\tilde{\varphi}_m^0(\varphi_T^*, \epsilon)}^{\tilde{\varphi}_x^1(\varphi_T, \epsilon)} \epsilon \;\; g(\varphi) d\varphi\right) h(\epsilon) d\epsilon, \tag{110}$$

or

$$\bar{\pi}_T(\varphi_T^*, 0, 1) = \left(\frac{r_T(\bar{b}_T(\varphi_T^*, 0, 1), 0, 0)}{\sigma}\right) - (f + N(f_m + \bar{\epsilon}_T(0, 1))), \tag{111}$$

where $\bar{\epsilon}_T(0,1)$ is the average random component of fixed import costs across firms who import but do not export.

$$\bar{\pi}_T(\varphi_T^*,1,1) = \left(\frac{r_T(\bar{b}_T(\varphi_T^*,1,1),0,0)}{\sigma}\right) - (f + \zeta N(f_x + f_m))$$
$$- \left(\frac{\zeta N}{\nu_T(\varphi_T^*,1,1)}\right)\left[\int_{\underline{\epsilon}}^{\epsilon_1}\left(\int_{\tilde{\varphi}_x^1(\varphi_T^*,\epsilon)}^{\infty} \epsilon \; g(\varphi)d\varphi\right)h(\epsilon)d\epsilon.\right.$$
$$+\int_{\epsilon_1}^{\epsilon_2}\left(\int_{\tilde{\varphi}_{xm}(\varphi_T^*,\epsilon)}^{\infty} \epsilon \; g(\varphi)d\varphi\right)h(\epsilon)d\epsilon$$
$$\left.+\int_{\epsilon_2}^{\bar{\epsilon}}\left(\int_{\tilde{\varphi}_m^1(\varphi_T^*,\epsilon)}^{\infty} \epsilon \; g(\varphi)d\varphi\right)h(\epsilon)d\epsilon\right], \tag{112}$$

or

$$\bar{\pi}_T(\varphi_T^*,1,1) = \left(\frac{r_T(\bar{b}_T(\varphi_T^*,1,1),0,0)}{\sigma}\right) - (f + \zeta N(f_x + f_m + \bar{\epsilon}_T(1,1))), \tag{113}$$

where $\bar{\epsilon}_T(1,1)$ is the average random part of fixed import costs across firms which both import and export.

10.6 Equilibrium Mass of Firms

For notational convenience, we drop the equilibrium cutoff productivity arguments in the functions below. In each of the four economies average revenue across all firms is given by

$$\bar{r}_S = \sum_{d\in\{0,1\}^2} \nu_S(d) r_S(\bar{b}_S(d),0,0). \tag{114}$$

and aggregate revenue is given by $R_S = \bar{r}_S M_S = L$, where the last inequality follows from the property that GNP equals payments to labor. Thus, in each economy the equilibrium mass of operating firms must be given by

$$M_S = \frac{L}{\bar{r}_S}. \tag{115}$$

Average profits in each economy can be written as

$$\bar{\pi}_S = \nu_S(0,0)\left(\frac{r_S(\bar{b}_S(0,0),0,0))}{\sigma} - f\right) + \nu_S(1,0)\left(\frac{r_S(\bar{b}_S(1,0),0,0))}{\sigma} - (f + Nf_x)\right)$$
$$+\nu_S(0,1)\left(\frac{r_S(\bar{b}_S(0,1),0,0))}{\sigma} - (f + N(f_m + \bar{\epsilon}_S(0,1)))\right)$$
$$+\nu_S(1,1)\left(\frac{r_S(\bar{b}_S(1,1),0,0))}{\sigma} - (f + \zeta N(f_x + f_m + \bar{\epsilon}_S(1,1)))\right), \tag{116}$$

where $\bar{\epsilon}_I(0,1)$, $\bar{\epsilon}_T(0,1)$, and $\bar{\epsilon}_T(1,1)$ are defined in the previous section and all other $\bar{\epsilon}_S(d)$ (those for categories with measure zero of firms) are set to zero. This implies that

$$\bar{\pi}_S = \frac{\bar{r}_S}{\sigma} - f - \nu_S(1,0)Nf_x - \nu_S(0,1)N(f_m + \bar{\epsilon}_S(0,1)) - \nu_S(1,1)\zeta N(f_x + f_m + \bar{\epsilon}_S(1,1)). \quad (117)$$

Hence, using equation (115), we have the equilibrium mass of firms in economy S:

$$M_S = \frac{L}{\sigma[\bar{\pi}_S + f + \nu_S(1,0)Nf_x + \nu_S(0,1)N(f_m + \bar{\epsilon}_S(0,1)) + \nu_S(1,1)\zeta N(f_x + f_m + \bar{\epsilon}_S(1,1))]}. \quad (118)$$

11 Appendix II: Proofs of Propositions

Proof of Proposition 1:

The cutoff productivity for operation in the autarkic equilibrium, φ_A^*, must satisfy the closed economy version of equation (17):

$$\int_{\underline{\epsilon}}^{\overline{\epsilon}} \pi_A(\varphi_A^*, 0, 0, \epsilon)h(\epsilon)d\epsilon = 0. \tag{119}$$

or using equations (4) and (14), we have

$$r_A(\varphi_A^*, 0, 0) = \sigma f, \tag{120}$$

where $r_A(\cdot)$ is given by equation 12 evaluated at $d = (0, 0)$. For the full trading economy we must show that $(0, 0) = argmax_{d\in\{0,1\}^2}\, \pi(\varphi_T^*, d, \epsilon)\ \forall\ \epsilon$ when $r(\varphi_T^*, 0, 0) = \sigma f$. It can easily be shown that when $r(\varphi_T^*, 0, 0) = \sigma f$, then $\forall\ \epsilon \in (\underline{\epsilon}, \overline{\epsilon})$ condition (C3) implies that $\pi(\varphi_T^*, 0, 0, \epsilon) > \pi(\varphi_T^*, 1, 0, \epsilon)$, condition (C4) implies that $\pi(\varphi_T^*, 0, 0, \epsilon) > \pi(\varphi_S^*, 0, 1, \epsilon)$, and (C5) implies $\pi(\varphi_T^*, 0, 0, \epsilon) > \pi(\varphi_T^*, 1, 1, \epsilon)$. Thus, the cutoff condition given by equation (17) becomes $\int_{\underline{\epsilon}}^{\overline{\epsilon}} \pi(\varphi_T^*, 0, 0, \epsilon)h(\epsilon)d(\epsilon) = 0$ which is clearly satisfied at our candidate φ_T^*. For the economy with no trade in intermediates we must show that $(0) = argmax_{d^x\in\{0,1\}}\, \pi(\varphi_X^*, d^x, 0, \epsilon)\ \forall\ \epsilon$ when $r(\varphi_X^*, 0, 0) = \sigma f$. It can easily be shown that when $r(\varphi_X^*, 0, 0) = \sigma f$, then $\forall\ \epsilon \in (\underline{\epsilon}, \overline{\epsilon})$ condition (C3) implies that $\pi(\varphi_X^*, 0, 0, \epsilon) > \pi(\varphi_X^*, 1, 0, \epsilon)$. Thus, the cutoff condition given by equation (17) becomes $\int_{\underline{\epsilon}}^{\overline{\epsilon}} \pi(\varphi_X^*, 0, 0, \epsilon)h(\epsilon)d(\epsilon) = 0$ which is clearly satisfied at our candidate φ_X^*.

For the economy with no trade in final goods we must show that $(0) = argmax_{d^m\in\{0,1\}}\, \pi(\varphi_X^*, 0, d^m, \epsilon)$ $\forall\ \epsilon$ when $r(\varphi_X^*, 0, 0) = \sigma f$. It can easily be shown that when $r(\varphi_I^*, 0, 0) = \sigma f$, then $\forall\ \epsilon \in (\underline{\epsilon}, \overline{\epsilon})$ condition (C4) implies that $\pi(\varphi_I^*, 0, 0, \epsilon) > \pi(\varphi_I^*, 0, 1, \epsilon)$. Thus, the cutoff condition given by equation (17) becomes $\int_{\underline{\epsilon}}^{\overline{\epsilon}} \pi(\varphi_I^*, 0, 0, \epsilon)h(\epsilon)d(\epsilon) = 0$ which is clearly satisfied at our candidate φ_I^*.

Proof of Proposition 2:

(i.) φ_A^* exists and is unique.

Define the following function of φ:

$$\gamma_A(\varphi) \equiv \varphi^{1-\sigma} f \int_{\varphi}^{\infty} z^{\sigma-1} g(z)dz - f[1 - G(\varphi)]. \tag{121}$$

Note that $\gamma_A(\cdot) > 0 \,\forall\, \varphi$ because

$$\gamma_A(\varphi) = f\left[\int_{\varphi}^{\infty}\left(\frac{z}{\varphi}\right)^{\sigma-1} g(z)dz - (1 - G(\varphi))\right], \tag{122}$$

which is clearly positive for $\sigma > 1$. The equilibrium cutoff level in autarky, φ_A^* satisfies

$$\gamma_A(\varphi_A^*) = \xi f_e. \tag{123}$$

Following Melitz, we will show that $\gamma_A(\varphi)$ is monotonically decreasing from ∞ to 0 on (0,∞). The derivative of this function is given by

$$\gamma_A'(\varphi) = (1-\sigma)\left[\frac{\gamma_A(\varphi) + f[1 - G(\varphi)]}{\varphi}\right] < 0, \tag{124}$$

where the inequality follows from $\sigma > 1$. The elasticity is

$$\frac{\gamma_A'(\varphi)\varphi}{\gamma_A(\varphi)} = (1-\sigma)\left[1 + \frac{f[1 - G(\varphi)]}{\gamma_A(\varphi)}\right] < (1-\sigma) < 0. \tag{125}$$

Since the elasticity is negative and bounded away from zero, $\gamma_A(\varphi)$ is decreasing to 0 as φ goes to ∞. Also, we see by inspection that $lim_{\varphi\to 0}\,\gamma_A(\varphi) = \infty$. This implies that the solution to equation (123) exists and is unique.

Turning to the economies with trade, we define the following functions for $i \in \{1, 2, 3\}$ and $j \in \{0, 1, 2, 3, 4, X, I\}$:

$$\begin{aligned}\delta_i(\varphi, \tilde{\varphi}_j(\varphi, \epsilon), \epsilon_L, \epsilon_H) \equiv \varphi^{1-\sigma} B_i \int_{\epsilon_L}^{\epsilon_H}\left(\int_{\tilde{\varphi}_j(\varphi,\epsilon)}^{\infty} z^{\sigma-1} g(z)dz\right) h(\epsilon)d\epsilon \\ - \int_{\epsilon_L}^{\epsilon_H}\left(C_i(\epsilon)\int_{\tilde{\varphi}_j(\varphi,\epsilon)}^{\infty} g(z)dz\right) h(\epsilon)d\epsilon,\end{aligned} \tag{126}$$

where

$$B_1 = f(b_x - 1) \qquad B_2 = f(bm - 1) \qquad B_3 = f(b_x - 1)(b_m - 1), \tag{127}$$

$$C_1(\epsilon) = Nf_x \qquad C_2(\epsilon) = N(f_m + \epsilon) \qquad C_3(\epsilon) = N(\zeta - 1)(f_x + f_m + \epsilon), \tag{128}$$

$$\begin{gathered}\tilde{\varphi}_o(\varphi, \epsilon) \equiv \tilde{\varphi}_x^1(\varphi, \epsilon) \qquad \tilde{\varphi}_1(\varphi, \epsilon) \equiv \tilde{\varphi}_m^o(\varphi, \epsilon) \qquad \tilde{\varphi}_2(\varphi, \epsilon) \equiv \tilde{\varphi}_{xm}(\varphi, \epsilon) \\ \tilde{\varphi}_3(\varphi, \epsilon) \equiv \tilde{\varphi}_m^1(\varphi, \epsilon) \qquad \tilde{\varphi}_4(\varphi, \epsilon) \equiv \tilde{\varphi}_x^o(\varphi, \epsilon),\end{gathered} \tag{129}$$

and $\tilde{\varphi}_X(\varphi, \epsilon)$ and $\tilde{\varphi}_I(\varphi, \epsilon)$ are given by equations (66) and (67) respectively.

(ii.) φ_X^* exists and is unique.

Define the following function of φ:

$$\gamma_X(\varphi) = \gamma_A(\varphi) + \delta_1(\varphi, \tilde{\varphi}_X(\varphi), \underline{\epsilon}, \bar{\epsilon}). \tag{130}$$

Note that $\gamma_X > 0 \; \forall \; \varphi$ because $\gamma_A(\varphi) > 0$ and

$$\delta_1(\varphi, \tilde{\varphi}_X, \underline{\epsilon}, \bar{\epsilon}) = N f_x \left[\int_{\tilde{\varphi}}^{\infty} \left(\frac{z}{\tilde{\varphi}} \right)^{\sigma - 1} - (1 - G(\tilde{\varphi})) \right], \tag{131}$$

which is clearly positive for $\sigma > 1$. The equilibrium cutoff level in economy X, φ_X^*, satisfies

$$\gamma_X(\varphi_X^*) = \xi f_e. \tag{132}$$

We will argue that $\gamma_X(\cdot)$is monotonically decreasing from ∞ to 0 on (0,∞), proving existence and uniqueness of the cutoff productivity in the economy with no importing. Above, we argued that the first function on the right-hand side of (163) has this property. The elasticity of the second function on the right-hand side with respect to φ is given by

$$\frac{\partial \ln \delta_1}{\partial \ln \varphi} = (1 - \sigma) \left[1 + \frac{\nu_X(\varphi, 1, 0)(1 - G(\varphi_X^*)) N f_x}{\delta_1} \right] < (1 - \sigma) < 0. \tag{133}$$

Since the elasticity is negative and bounded away from zero, δ_1 is decreasing to 0 as φ goes to ∞. Also, we see by inspection that $lim_{\varphi \to 0} \, \delta_1 = \infty$. Thus, $\gamma_X(\cdot)$ is the sum of two functions which are decreasing from ∞ to 0 on $(0, \infty)$ and so φ_X^* exists and is unique.

(iii.) φ_I^* exists and is unique.

Define the following function of φ:

$$\gamma_I(\varphi) = \gamma_A(\varphi) + \delta_2(\varphi, \tilde{\varphi}_I(\varphi), \underline{\epsilon}, \bar{\epsilon}). \tag{134}$$

Note that $\gamma_M(\varphi) > 0 \; \forall \; \varphi$ because $\gamma_A(\varphi) > 0$ and

$$\delta_2(\varphi, \tilde{\varphi}_I, \underline{\epsilon}, \bar{\epsilon}) = \int_{\underline{\epsilon}}^{\bar{\epsilon}} N(f_m + \epsilon) \left(\int_{\tilde{\varphi}_I(\varphi, \epsilon)}^{\infty} \left[\left(\frac{z}{\tilde{\varphi}_I(\varphi, \epsilon)} \right)^{\sigma - 1} - 1 \right] g(z) dz \right) h(\epsilon) d\epsilon \tag{135}$$

which is clearly positive when $\sigma > 1$. The equilibrium cutoff level in economy M, φ_I^*, satisfies

$$\gamma_I(\varphi_I^*) = \xi f_e. \tag{136}$$

We will argue that $\gamma_I(\cdot)$is monotonically decreasing from ∞ to 0 on (0,∞), proving existence and uniqueness of the cutoff productivity in the economy with no exporting. Above, we argued that the first function on the right-hand side of (164) has this property. The elasticity of the second function on the right-hand side with respect to φ is given by

$$\frac{\partial \ln \delta_2}{\partial \ln \varphi} = (1-\sigma)\left[1 + \frac{\nu_I(\varphi, 0, 1)(1 - G(\varphi_I^*))N(f_m + \bar{\epsilon}_I(0, 1)}{\delta_2}\right] < (1-\sigma) < 0. \tag{137}$$

Since the elasticity is negative and bounded away from zero, δ_2 is decreasing to 0 as φ goes to ∞. Also, we see by inspection that $lim_{\varphi \to 0}\, \delta_2 = \infty$. Thus, $\gamma_I(\cdot)$ is the sum of two functions which are decreasing from ∞ to 0 on $(0, \infty)$ and so φ_I^* exists and is unique.

(iv.) φ_T^* exists and is unique.

Define the following function of φ:

$$\gamma_1(\varphi) \equiv \sum_{i \in \{1,3\}} \delta_i(\varphi, \tilde{\varphi}_0(\varphi, \epsilon), \underline{\epsilon}, \epsilon_1) \tag{138}$$

$$\gamma_2(\varphi) \equiv \delta_2(\varphi, \tilde{\varphi}_1(\varphi, \epsilon), \underline{\epsilon}, \epsilon_1) \tag{139}$$

$$\gamma_3(\varphi) \equiv \sum_{i \in \{1,2,3\}} \delta_i(\varphi, \tilde{\varphi}_2(\varphi, \epsilon), \epsilon_1, \epsilon_2) \tag{140}$$

$$\gamma_4(\varphi) \equiv \sum_{i \in \{2,3\}} \delta_i(\varphi, \tilde{\varphi}_3(\varphi, \epsilon), \epsilon_2, \bar{\epsilon}) \tag{141}$$

$$\gamma_5(\varphi) \equiv \delta_1(\varphi, \tilde{\varphi}_4(\varphi, \epsilon), \epsilon_2, \bar{\epsilon}) \tag{142}$$

Using arguments similar to those used in parts (ii.) and (iii.) above, we can show that each of the $\gamma_i(\varphi)$ functions are positive-valued. We define the following function of φ

$$\gamma_T(\varphi) \equiv \gamma_A(\varphi) + \sum_{i=1}^{5} \gamma_i(\varphi) \tag{143}$$

and note that it is positive-valued. Through manipulation of equation (35), we can show that the equilibrium cutoff productivity level in trade, φ_T^* satisfies

$$\gamma_T(\varphi_T^*) = \xi f_e. \tag{144}$$

We will argue that this function is monotonically decreasing from ∞ to 0 on $(0, \infty)$ implying that φ_T^* exists and is unique. We will do so by showing that each function in the sum on the right-hand side of (143) has this property.

We already showed in part (i.) of this proof that $\gamma_A(\cdot)$ has this property. Let $\hat{\gamma}_j$ denote the elasticity of γ_j with respect to φ for $j = 1, ..., 5$. Dropping the arguments of functions, we can derive

$$\hat{\gamma}_1 = (1-\sigma)\left[1 + \frac{\sum_{i\in\{1,3\}} \int_{\underline{\epsilon}}^{\epsilon_1} \left(C_i \int_{\tilde{\varphi}_0}^{\infty} g(z)dz\right) h(\epsilon)d\epsilon}{\gamma_1}\right] < (1-\sigma) < 0. \tag{145}$$

Hence, since the elasticity of this function is negative and bounded away from 0, it must be monotonically decreasing on $(0, \infty)$. By inspection, we also see that the limit of this function as $\varphi \to 0$ equals ∞ because $lim_{\varphi\to 0}\tilde{\varphi}_0(\varphi, \epsilon) = 0$ for any ϵ.

Similarly, we have

$$\hat{\gamma}_2 = (1-\sigma)\left[1 + \frac{\int_{\bar{\epsilon}}^{\epsilon_1} \left(C_2 \int_{\tilde{\varphi}_1}^{\infty} g(z)dz\right) h(\epsilon)d\epsilon}{\gamma_2}\right] < (1-\sigma) < 0, \tag{146}$$

$$\hat{\gamma}_3 = (1-\sigma)\left[1 + \frac{\sum_{i=1}^{3} \int_{\epsilon_1}^{\epsilon_2} \left(C_i \int_{\tilde{\varphi}_2}^{\infty} g(z)dz\right) h(\epsilon)d\epsilon}{\gamma_3}\right] < (1-\sigma) < 0, \tag{147}$$

$$\hat{\gamma}_4 = (1-\sigma)\left[1 + \frac{\sum_{i=2}^{3} \int_{\epsilon_2}^{\bar{\epsilon}} \left(C_i \int_{\tilde{\varphi}_3}^{\infty} g(z)dz\right) h(\epsilon)d\epsilon}{\gamma_4}\right] < (1-\sigma) < 0, \tag{148}$$

and

$$\hat{\gamma}_5 = (1-\sigma)\left[1 + \frac{\int_{\epsilon_2}^{\bar{\epsilon}} \left(C_1 \int_{\tilde{\varphi}_4}^{\infty} g(z)dz\right) h(\epsilon)d\epsilon}{\gamma_5}\right] < (1-\sigma) < 0. \tag{149}$$

Furthermore, the limit of each of these functions as $\varphi \to 0$ equals ∞. Thus, each of these functions are monotonically decreasing from ∞ to 0 on $(0, \infty)$. These results imply that $\gamma_T(\varphi)$ is the sum of functions which decrease monotonically from ∞ to 0 on $(0, \infty)$ and, so, must also have this property. Thus, the φ_T^* which satisfies equation (144) exists and is unique.

Proof of Proposition 3:

The equilibrium equations given by (123), (132), (136), and (144) for the respective economies imply

$$\begin{aligned}
\xi f_e &= \gamma_A(\varphi_A^*) \\
&= \gamma_A(\varphi_X^*) + \delta_1(\varphi_X^*, \tilde{\varphi}_X(\varphi_X^*), \underline{\epsilon}, \bar{\epsilon}) \\
&= \gamma_A(\varphi_I^*) + \delta_2(\varphi_I^*, \tilde{\varphi}_I(\varphi_I^*), \underline{\epsilon}, \bar{\epsilon}). \\
&= \gamma_A(\varphi_T^*) + \sum_{i=1}^{5} \gamma_i(\varphi_T^*).
\end{aligned}$$

Now since $\delta_1(\cdot)$, $\delta_2(\cdot)$, and $\gamma_i(\cdot)$ for $i \in \{1, 5\}$ are positive-valued and since $\gamma_A(\varphi)$ is a decreasing function, we have $\varphi_A^* < \varphi_X^*$, $\varphi_A^* < \varphi_I^*$, and $\varphi_A^* < \varphi_T^*$.

Note that $\varphi_X^* = \varphi_T^*$ when $b_m = \zeta = 1$ and $\varphi_I^* = \varphi_T^*$ when $b_x = \zeta = 1$. We will demonstrate that φ_T^* is increasing in b_m, increasing in b_x, and decreasing in ζ. Now a movement from the economy with no importing to the economy with full trade can be represented by an increase in b_m and, possibly, a decrease in ζ. Thus, if we show that the cutoff productivity in the full trading equilibrium is increasing in b_m and decreasing in ζ, then we have the result that $\varphi_X^* < \varphi_T^*$. Using a similar argument for increases in b_x, we have the result that $\varphi_I^* < \varphi_T^*$.

Let the elasticity of a function $f(z)$ with respect to z be written as $\hat{f}^z$. Then equation (144) implies

$$\hat{\varphi}_T^{*z} = -\left(\frac{\hat{\gamma}_T^z}{\hat{\gamma}_T^{\varphi}}\right) \tag{150}$$

From equation (143), we can derive the following elasticities:

$$\begin{aligned}
\hat{\gamma}_T^{\varphi} = &\left[\frac{(1-\sigma)\varphi^{1-\sigma} f}{\gamma_T}\right]\Bigg[\int_{\varphi}^{\infty} z^{\sigma-1} g(z)dz \\
&+ \int_{\underline{\epsilon}}^{\epsilon_1}\left(b_m(b_x-1)\int_{\tilde{\varphi}_0}^{\infty} z^{\sigma-1}g(z)dz + (b_m-1)\int_{\tilde{\varphi}_1}^{\infty} z^{\sigma-1}g(z)dz\right)h(\epsilon)d\epsilon \\
&+ \int_{\epsilon_1}^{\epsilon_2}\left((b_m b_x - 1)\int_{\tilde{\varphi}_2}^{\infty} z^{\sigma-1}g(z)dz\right)h(\epsilon)d\epsilon \\
&+ \int_{\epsilon_2}^{\bar{\epsilon}}\left(b_x(b_m-1)\int_{\tilde{\varphi}_3}^{\infty} z^{\sigma-1}g(z)dz + (b_x-1)\int_{\tilde{\varphi}_4}^{\infty} z^{\sigma-1}g(z)dz\right)h(\epsilon)d\epsilon\Bigg] \\
&< 0
\end{aligned} \tag{151}$$

$$\begin{aligned}
\hat{\gamma}_T^{b_m} = &\left[\frac{b_I\varphi^{1-\sigma} f}{\gamma_T}\right]\Bigg[\int_{\underline{\epsilon}}^{\epsilon_1}\left((b_x-1)\int_{\tilde{\varphi}_0}^{\infty} z^{\sigma-1}g(z)dz + \int_{\tilde{\varphi}_1}^{\infty} z^{\sigma-1}g(z)dz\right)h(\epsilon)d\epsilon \\
&+ \int_{\epsilon_1}^{\epsilon_2}\left(b_x\int_{\tilde{\varphi}_2}^{\infty} z^{\sigma-1}g(z)dz\right)h(\epsilon)d\epsilon \\
&+ \int_{\epsilon_2}^{\bar{\epsilon}}\left(b_x\int_{\tilde{\varphi}_3}^{\infty} z^{\sigma-1}g(z)dz\right)h(\epsilon)d\epsilon\Bigg] \\
&> 0
\end{aligned} \tag{152}$$

$$\begin{aligned}
\hat{\gamma}_T^{b_x} = &\left[\frac{b_x\varphi^{1-\sigma} f}{\gamma_T}\right]\Bigg[\int_{\underline{\epsilon}}^{\epsilon_1}\left(b_m\int_{\tilde{\varphi}_0}^{\infty} z^{\sigma-1}g(z)dz\right)h(\epsilon)d\epsilon \\
&+ \int_{\epsilon_1}^{\epsilon_2}\left(b_m\int_{\tilde{\varphi}_2}^{\infty} z^{\sigma-1}g(z)dz\right)h(\epsilon)d\epsilon \\
&+ \int_{\epsilon_2}^{\bar{\epsilon}}\left((b_m-1)\int_{\tilde{\varphi}_3}^{\infty} z^{\sigma-1}g(z)dz + \int_{\tilde{\varphi}_4}^{\infty} z^{\sigma-1}g(z)dz\right)h(\epsilon)d\epsilon\Bigg]
\end{aligned}$$

$$> 0 \tag{153}$$

$$\begin{aligned}\hat{\gamma}_T^{\zeta} = \left[\frac{-\zeta N}{\gamma_T}\right] \Bigg[& \int_{\underline{\epsilon}}^{\epsilon_1} \left(\int_{\tilde{\varphi}_0}^{\infty} (f_x + f_m + \epsilon) g(z) dz \right) h(\epsilon) d\epsilon \\ & + \int_{\epsilon_1}^{\epsilon_2} \left(\int_{\tilde{\varphi}_2}^{\infty} (f_x + f_m + \epsilon) g(z) dz \right) h(\epsilon) d\epsilon \\ & + \int_{\epsilon_2}^{\bar{\epsilon}} \left(\int_{\tilde{\varphi}_3}^{\infty} (f_x + f_m + \epsilon) g(z) dz \right) h(\epsilon) d\epsilon \Bigg] \\ < 0 \end{aligned} \tag{154}$$

Hence, using equation (150) and the signs of these elasticities, we have

$$\hat{\varphi}_T^{*b_m} > 0 \qquad \hat{\varphi}_T^{*b_x} > 0 \qquad \hat{\varphi}_T^{*\zeta} < 0 \tag{155}$$

This proves the remaining parts of the proposition.

Proof of Proposition 4:

Inspection of equations (**??**), (**??**), (**??**), and (**??**) and the result that $\bar{\pi}_A < \bar{\pi}_X < \bar{\pi}_T$ and $\bar{\pi}_A < \bar{\pi}_I$ shows that $M_A > M_X$, $M_A > M_I$, and $M_A > M_T$.

Equation (115) implies that if we can show that $\bar{r}_X < \bar{r}_T$ and $\bar{r}_I < \bar{r}_T$, then we have the remaining results. To demonstrate these average revenue rankings, it is enough to show that $\bar{r}_T$ is increasing in b_m, increasing in b_x, and decreasing in ζ.

Changing the variable of integration in the average revenue calculations from φ to Φ allows us to write average profits in the full trading equilibrium as

$$\begin{aligned}\bar{r}_T = \left(\frac{\sigma N^{\frac{\sigma}{\sigma-1}} f^{\frac{1}{1-\sigma}} \varphi_T^*}{(\sigma-1)(1-G(f/N))} \right) \Bigg[& \int_{\underline{\epsilon}}^{\epsilon_1} \left(\int_{f/N}^{\Phi_1} \Phi^{\frac{1}{\sigma-1}} g(\Phi) d\Phi + b_I \int_{\Phi_1}^{\Phi_0} \Phi^{\frac{1}{\sigma-1}} g(\Phi) d\Phi + b_x b_I \int_{\Phi_0}^{\infty} \Phi^{\frac{1}{\sigma-1}} g(\Phi) d\Phi \right) h(\epsilon) d\epsilon \\ & + \int_{\epsilon_1}^{\epsilon_2} \left(\int_{f/N}^{\Phi_2} \Phi^{\frac{1}{\sigma-1}} g(\Phi) d\Phi + b_x b_I \int_{\Phi_2}^{\infty} \Phi^{\frac{1}{\sigma-1}} g(\Phi) d\Phi \right) h(\epsilon) d\epsilon \\ & \int_{\epsilon_2}^{\bar{\epsilon}} \left(\int_{f/N}^{\Phi_4} \Phi^{\frac{1}{\sigma-1}} g(\Phi) d\Phi + b_x \int_{\Phi_4}^{\Phi_3} \Phi^{\frac{1}{\sigma-1}} g(\Phi) d\Phi + b_x b_I \int_{\Phi_3}^{\infty} \Phi^{\frac{1}{\sigma-1}} g(\Phi) d\Phi \right) h(\epsilon) d\epsilon \Bigg]\end{aligned}$$

Now since, $\hat{\varphi}_T^{*b_m} > 0$, $b_x > 1$, $b_m > 1$, and $\frac{\partial \Phi_j}{\partial b_m} \leq 0 \; \forall j$, then the derivative of the above expression for average revenue with respect to b_m is positive. Similarly, because $\frac{\partial \Phi_j}{\partial b_x} \leq 0 \; \forall j$, the derivative with respect to b_x is positive. Finally, since $\frac{\partial \Phi_j}{\partial \zeta} \geq 0 \; \forall j$, the derivative of average revenue in the trading equilibrium with respect ζ is decreasing. These results imply that average revenue is higher in the full trading equilibrium than in either of the partial trading equilibrium and proves the proposition.

Proof of Proposition 5:

Welfare in this economy is the inverse of the price index. Hence, in autarky, equation (**??**) aggregate welfare is given by

$$W_A = \frac{1}{P_A} = M_A^{\frac{1}{\sigma-1}} A\rho\bar{b}_A(\varphi_A^*) \tag{156}$$

Furthermore, equations (**??**) and (13) and recalling that $R_A = wL = L$ allows us to derive

$$\bar{b}_A(\varphi_A^*) = \left[\frac{L}{M_A\sigma f}\right)^{\frac{1}{\sigma-1}} \varphi_A^* \tag{157}$$

Substituting this into equation (156) gives

$$W_A = A\rho\left[\frac{L}{\sigma f}\right]^{\frac{1}{\sigma-1}} \varphi_A^* \tag{158}$$

Similarly, from equation (31) in the trading equilibrium, we have

$$W_T = \frac{1}{P_T} = M_C{}^{\frac{1}{\sigma-1}} A\rho\bar{b}_T(\varphi_T^*) \tag{159}$$

Furthermore, equations (32) and (13) and recalling that $R_T = wL = L$ allows us to derive

$$\bar{b}_T(\varphi_T^*) = \left[\frac{L}{M_C\sigma f}\right]^{\frac{1}{\sigma-1}} \varphi_T^* \tag{160}$$

Substituting this into equation (159) gives

$$W_T = A\rho\left[\frac{L}{\sigma f}\right]^{\frac{1}{\sigma-1}} \varphi_T^* \tag{161}$$

Now since $\varphi_A^* < \varphi_T^*$, we have $W_A < W_T$.

Proof of Proposition 6:

Recall the following revenue functions in autarky and trade

$$r_A(\varphi) = (\sigma f\varphi^{\sigma-1})\varphi_A^{*\,1-\sigma} \qquad\qquad r_T(\varphi,d) = (b_x^{d_x}b_m^{d_m})(\sigma f\varphi^{\sigma-1})\varphi_T^{*\,1-\sigma} \tag{162}$$

(i.) $r_T(\varphi,0,0) < r_A(\varphi)$ follows directly from Proposition 1.

Consider two economies with partial trade. First, the open economy considered here with $\zeta = b_m = 1$. This would be an economy with only exports and no imports and we label all variables associated with this economy with an X. This is the analog to the open economy that Melitz studied. Second, consider the open economy considered here with $\zeta = b_x = 1$. This is an economy with only imports and no exports and we label all variables associated with this economy with an M. Define the following functions

$$\gamma_X(\varphi) \equiv \gamma_A(\varphi) + \delta_1(\varphi, \tilde{\varphi}_X, \underline{\epsilon}, \bar{\epsilon}) \tag{163}$$

where $\tilde{\varphi}_X$ equals $\tilde{\varphi}_0$ evaluated at $\zeta = b_m = 1$, and

$$\gamma_M(\varphi) \equiv \gamma_A(\varphi) + \delta_2(\varphi, \tilde{\varphi}_M, \underline{\epsilon}, \bar{\epsilon}) \tag{164}$$

where $\tilde{\varphi}_M$ equals $\tilde{\varphi}_1$ evaluated at $\zeta = b_x = 1$. Now, the equilibrium cutoff productivity for operation in each economy satisfies

$$\gamma_X(\varphi_X^*) = \xi f_e \tag{165}$$

$$\gamma_M(\varphi_M^*) = \xi f_e \tag{166}$$

Let the associated revenue functions in each economy be denoted $r_X(\varphi, d)$ and $r_M(\varphi, d)$. We now derive the following result

$$r_A(\varphi) < r_X(\varphi, 1, 0) < r_T(\varphi, 1, 1) \tag{167}$$

We first note that $lim_{b_x \to 1} r_X(\varphi, 1, 0) = r_A(\varphi)$. Now if we can show that $r_X(\varphi, 1, 0)$ is increasing in b_x, then we have proved the first inequality. Recall that $r_X(\varphi, 1, 0) = b_x \left[\frac{\varphi}{\varphi_X^*}\right]^{\sigma-1} \sigma f$. The elasticity of this function with respect to b_x is given by

$$\left(\frac{\partial r_X(\varphi, 1, 0)}{\partial b_x}\right)\left(\frac{b_x}{r_X(\varphi, 1, 0)}\right) = 1 + (1-\sigma)\left(\frac{\partial \varphi_X^*}{\partial b_x}\right)\left(\frac{b_x}{\varphi_X^*}\right) \tag{168}$$

Differentiating (165) with respect to b_x and rearranging gives

$$\left(\frac{\partial \varphi_X^*}{\partial b_x}\right)\left(\frac{b_x}{\varphi_X^*}\right) = -\frac{\left(\frac{\partial \gamma_X}{\partial b_x}\right)\left(\frac{b_x}{\gamma_X}\right)}{\left(\frac{\partial \gamma_X}{\partial \varphi}\right)\left(\frac{\varphi_X^*}{\gamma_X}\right)} \tag{169}$$

Now if we derive these elasticities and substitute into equation (168), we derive the following

$$Sign\left\{\left(\frac{\partial r_X(\varphi, 1, 0)}{\partial b_x}\right)\left(\frac{b_x}{r_X(\varphi, 1, 0)}\right)\right\} = Sign\left\{\int_{\varphi_X^*}^{\infty} z^{\sigma-1} g(z) dz - \int_{\tilde{\varphi}_X}^{\infty} z^{\sigma-1} g(z) dz\right\}. \tag{170}$$

Now, since $\varphi_X^* < \tilde{\varphi}_X$, this expression must be positive, so $r_X(\varphi, 1, 0)$ is increasing in b_x and $r_A(\varphi) < r_X(\varphi, 1, 0)$ for all φ.

Next, note that $lim_{b_m \to 1} r_T(\varphi, 1, 1) = r_X(\varphi, 1, 0)$. Now if we can show that $r_T(\varphi, 1, 1)$ is increasing in b_m, then we have proved the second inequality. Recall that $r_T(\varphi, 1, 1) = b_x b_I \left[\frac{\varphi}{\varphi_T^*}\right]^{\sigma-1} \sigma f$. The elasticity of this function with respect to b_m is given by

$$\left(\frac{\partial r_T(\varphi, 1, 1)}{\partial b_m}\right)\left(\frac{b_m}{r_T(\varphi, 1, 1)}\right) = 1 + (1-\sigma)\left(\frac{\partial \varphi_T^*}{\partial b_m}\right)\left(\frac{b_m}{\varphi_T^*}\right) \tag{171}$$

Differentiating (144) with respect to b_m and rearranging gives

$$\left(\frac{\partial \varphi_T^*}{\partial b_m}\right)\left(\frac{b_m}{\varphi_T^*}\right) = -\frac{\left(\frac{\partial \gamma_T}{\partial b_m}\right)\left(\frac{b_m}{\gamma_T}\right)}{\left(\frac{\partial \gamma_T}{\partial \varphi}\right)\left(\frac{\varphi_T^*}{\gamma_T}\right)} \tag{172}$$

Now if we derive these elasticities and substitute into equation (173), we derive the following

$$\begin{aligned} Sign\left\{\left(\frac{\partial r_T(\varphi, 1, 1)}{\partial b_m}\right)\left(\frac{b_m}{r_T(\varphi, 1, 1)}\right)\right\} = Sign\Bigg\{ & \int_{\epsilon_1}^{\epsilon_2}\left[\int_{\varphi_T^*}^{\infty} z^{\sigma-1} g(z)dz - \int_{\tilde{\varphi}_1}^{\infty} z^{\sigma-1} g(z)dz\right] h(\epsilon)d\epsilon \\ & + \int_{\epsilon_2}^{\epsilon_3}\left[\int_{\varphi_T^*}^{\infty} z^{\sigma-1} g(z)dz - \int_{\tilde{\varphi}_2}^{\infty} z^{\sigma-1} g(z)dz\right] h(\epsilon)d\epsilon \\ & + \int_{\epsilon_3}^{\epsilon_4}\left[\int_{\varphi_T^*}^{\infty} z^{\sigma-1} g(z)dz - \int_{\tilde{\varphi}_4}^{\infty} z^{\sigma-1} g(z)dz\right] h(\epsilon)d\epsilon \\ & + b_x \int_{\epsilon_3}^{\epsilon_4}\left[\int_{\tilde{\varphi}_4}^{\infty} z^{\sigma-1} g(z)dz - \int_{\tilde{\varphi}_3}^{\infty} z^{\sigma-1} g(z)dz\right] h(\epsilon)d\epsilon \Bigg\} \end{aligned} \tag{173}$$

Now, we know that $\varphi_T^* < \tilde{\varphi}_j$ for $j \in \{1, 2, 4\}$ and $\epsilon > \underline{\epsilon}$. We also have $\tilde{\varphi}_4 < \tilde{\varphi}_3$ for $\epsilon \in \{\epsilon_2, \bar{\epsilon}\}$. These inequalities imply that the elasticity in equation (173) is positive. Thus, we have demonstrated that $r_T(\varphi, 1, 1)$ is increasing in b_m and so $r_X(\varphi, 1, 0) < r_T(\varphi, 1, 1)$. Finally, combining these two results, we have $r_A(\varphi) < r_T(\varphi, 1, 1)$ for all φ.

Using similar methods, we can argue that $\lim_{b_m \to 1} r_M(\varphi, 0, 1) \to r_A(\varphi)$ and that $r_M(\varphi, 0, 1)$ is increasing in b_m. Together, these imply that $r_A(\varphi) < r_M(\varphi, 0, 1)$. (Derivations omitted for now.)

Proof of Proposition 6 (Incomplete)

Proof of Proposition 7 (Incomplete)

Consider the variables on the axes of Figures 1 and 2:

$$\Phi \equiv \left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1}\left(\frac{f}{N}\right) \tag{174}$$

and

$$\theta \equiv f_m + \epsilon. \tag{175}$$

Now, because these variables are monotonically increasing functions of φ and ϵ, respectively, they will have the same densities and cdfs as those variables. Using, Figure 2, we can see that the fraction of firms who export in the open economy equilibrium can be written as

$$\nu_T^X(b_x, b_m) \equiv \nu(\varphi_T^*, 1, 0) + \nu(\varphi_T^*, 1, 1) =$$

$$\int_0^{\theta_2} \left(1 - G(\Phi_x^1)\right) h(\theta) d\theta + \int_{\theta_2}^{\theta_3} \left(1 - G(\Phi_{xm})\right) h(\theta) d\theta + \int_{\theta_3}^{\theta_4} \left(1 - G(\Phi_x^0)\right) h(\theta) d\theta. \tag{176}$$

Note that in this expression that θ_2, θ_3, Φ_x^1, and Φ_{xm} depend upon b_m and so changes in b_m will affect the fraction of exporting firms through its effect on these variables.

We first note that the derivatives of the right-hand side of this expression with respect to θ_2 and with respect to θ_3 equal zero (because $\Phi_x^1 = \Phi_{xm}$ at θ_2 and $\Phi_x^0 = \Phi_{xm}$ at θ_3. Denote the elasticity of a function $f(b_m)$ with respect to b_m as $\hat{f}$. Hence, the elasticity of v_T^X with respect to b_m, is given by

$$\hat{\nu}_T^X = \frac{-\left(\int_0^{\theta_2} g(\Phi_x^1)\Phi_x^1 \hat{\Phi}_x^1 h(\theta)\theta + \int_{\theta_2}^{\theta_3} g(\Phi_{xm})\Phi_{xm}\hat{\Phi}_{xm} h(\theta)\theta\right)}{(1 - G(f/N))\nu_T^X} \tag{177}$$

Using equations (24) and (26), we can derive the following elasticities with respect to b_m:

$$\hat{\Phi}_x^1 = -1 < 0 \qquad\qquad \hat{\Phi}_{xm} = \frac{-b_x b_m}{b_x b_m - 1} < 0. \tag{178}$$

Since these elasticities are both negative, $\hat{v}_T^X$ must be positive. This implies that the fraction of exporting firms falls as b_m falls.

12 Appendix III: Properties of Type I Extreme-Value Distributions

Here, we discuss the properties of the Type I extreme-value distributed random variables. Assume that $\epsilon(0)$ and $\epsilon(1)$ are independently drawn from the identical extreme-value distribution with mean zero and variance normalized to $\frac{\pi^2}{6}$.[19] Let $V(0)$ and $V(1)$ be some real numbers. All we have to know about the properties of the extreme-value distributed random variables is the following two properties.

The first property is:

$$E[\max(V(0)+\epsilon(0), V(1)+\epsilon(1)] = \ln[exp(V(0)) + \exp(V(1))],$$

where the expectation is taken with respect to the distribution of $\epsilon(0)$ and $\epsilon(1)$.

The second property is:

$$P(V(0)+\epsilon(0) > V(1)+\epsilon(1)) = \frac{\exp(V(0))}{\exp(V(0)) + \exp(V(1))}.$$

In multivariate case, when we have $\epsilon(d)$ for $d = 0, 1, 2, ..., J$, the first property is

$$E[\max_{j=0,1,\ldots,J} V(j) + \epsilon(j)] = \ln[\sum_{j=0}^{J} \exp(V(j))].$$

The second property is

$$P[V(d)+\epsilon(d) > V(j)+\epsilon(j) \text{ for all } j \neq d] = \frac{\exp(V(d))}{\sum_{j'=0}^{J} \exp(V(j'))}.$$

One implication is

$$\begin{aligned} E[\max(V(0)+\varrho\epsilon(0), V(1)+\varrho\epsilon(1)] &= \varrho E[\max(V(0)/\varrho+\epsilon(0), V(1)/\varrho+\epsilon(1)] \\ &= \varrho \ln[\exp(V(0)/\varrho) + \exp(V(1)/\varrho)], \end{aligned}$$

which I have used to derive the first equation in (54). Note that by letting $\varrho \to 0$, we have $E[\max(V(0)+\varrho\epsilon(0), V(1)+\varrho\epsilon(1)] \to \max(V(0), V(1))$ so that, if we do not like adding extreme-value distributed random variables into the model, then we may manually set ϱ to be a very small number so that we get a computational advantage of this specification while getting the almost identical result as the case of no extreme-value distributed random variables.

[19] The cumulative distribution function of $\epsilon(d)$ for $d = 0, 1$ is $\exp(-\exp(-(\epsilon(d) - \gamma)))$, where γ is Euler's constant.

Assume that $\epsilon(0)$ and $\epsilon(1)$ are independently drawn from the identical extreme-value distribution with mean zero and variance normalized to $\frac{\pi^2}{6}$. To derive labor demand in the presence of extreme value distributed shocks, we need to know $P(\epsilon(0)|\epsilon(0)+V(0) \geq \epsilon(1)+V(1))$. Below, we prove that $\epsilon(0)$ conditional on $d=0$ is chosen is extreme value distributed with the mean $-\ln P(0)$.

First,

$$P(\epsilon(0)+V(0) \geq \epsilon(1)+V(1)|\epsilon(0)) = \exp(-e^{-(\epsilon(0)-\gamma+V(0)-V(1))}).$$

Then,

$$\begin{aligned} P(\epsilon(0)+V(0) \geq \epsilon(1)+V(1), \epsilon(0)) &= P(\epsilon(0))P(\epsilon(0)+V(0) \geq \epsilon(1)+V(1)|\epsilon(0)) \\ &= -e^{-(\epsilon(0)-\gamma)} \exp(-e^{-(\epsilon(0)-\gamma)}) \exp(-e^{-(\epsilon(0)-\gamma+V(0)-V(1))}). \end{aligned}$$

Using $P(V(0)+\epsilon(0) > V(1)+\epsilon(1)) = \frac{\exp(V(0))}{\exp(V(0))+\exp(V(1))}$,

$$\begin{aligned} P(\epsilon(0)|\epsilon(0)+V(0) \geq \epsilon(1)+V(1), \epsilon(0)) &= \frac{P(\epsilon(0)+V(0) \geq \epsilon(1)+V(1), \epsilon(0))}{P(V(0)+\epsilon(0) > V(1)+\epsilon(1))} \\ &= -e^{-(\epsilon(0)-\gamma+\ln P(0))} \exp(-e^{-(\epsilon(0)-\gamma+\ln P(0))}). \end{aligned}$$

Figure 1: Export and Import Status without Complementarities (ζ=1)

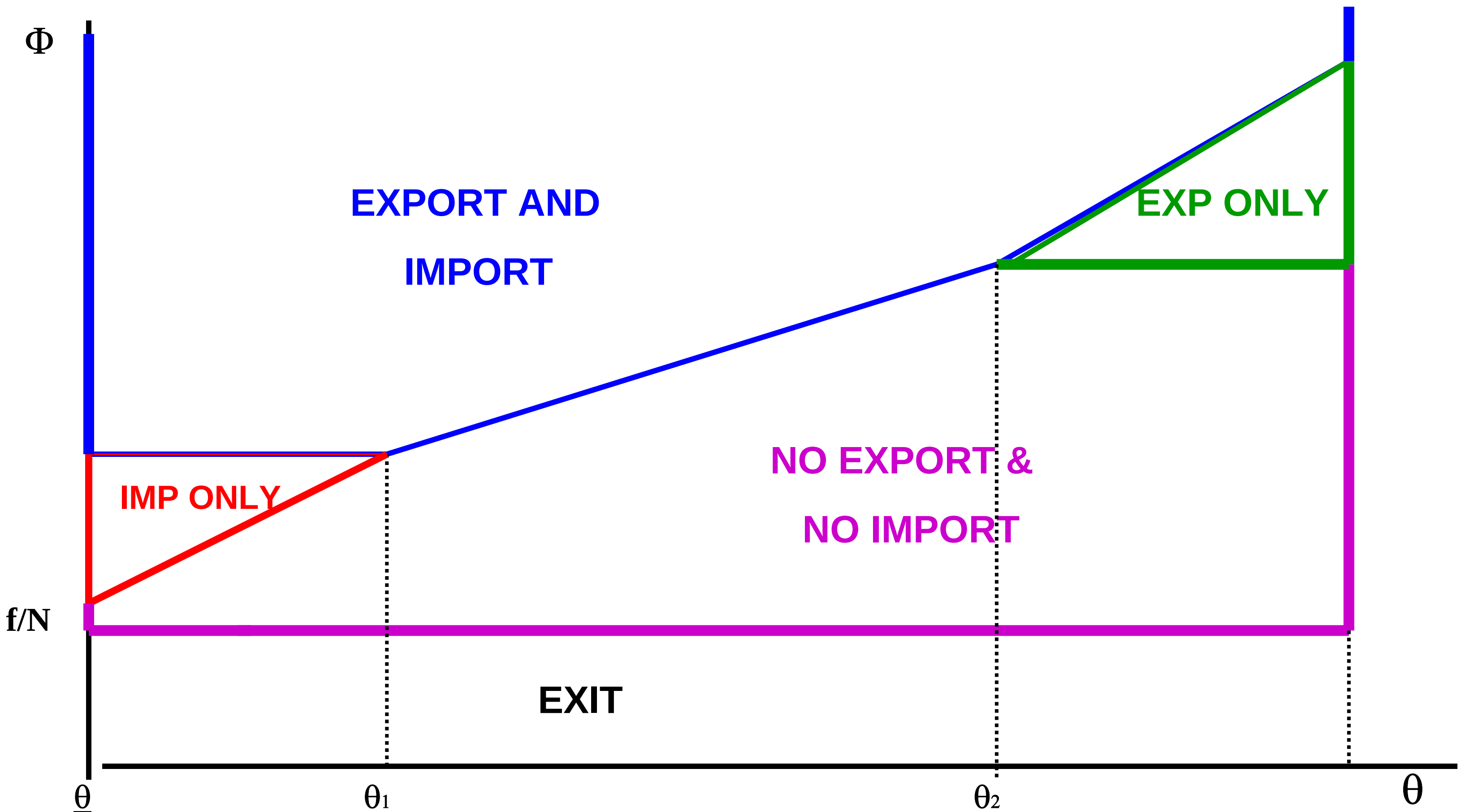

Figure 2: Export and Import Status with Complementarities (ζ<1)

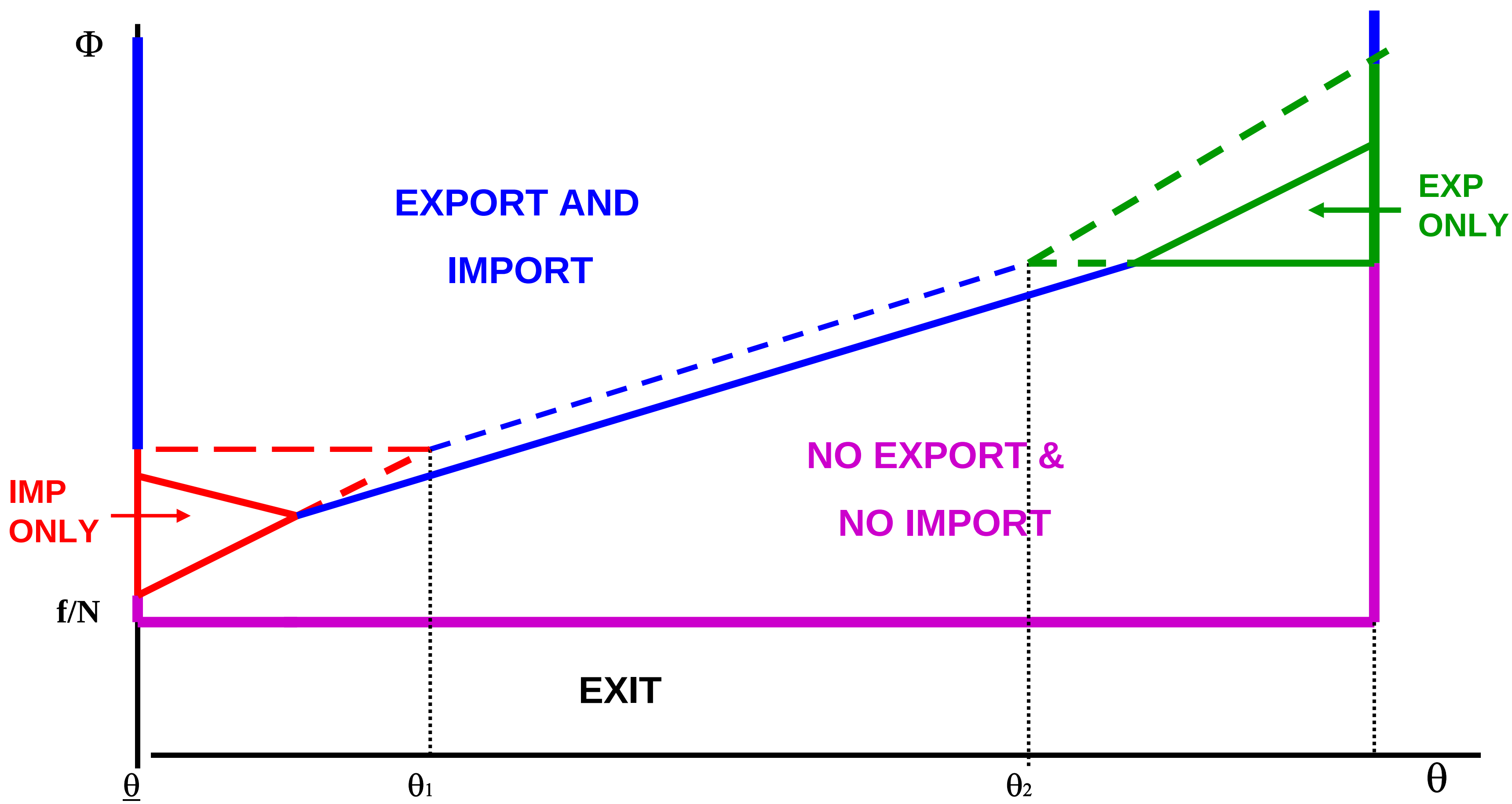

Figure 3: Import Protection as Export Destruction

(ζ=1) ($b_m \to 1$)

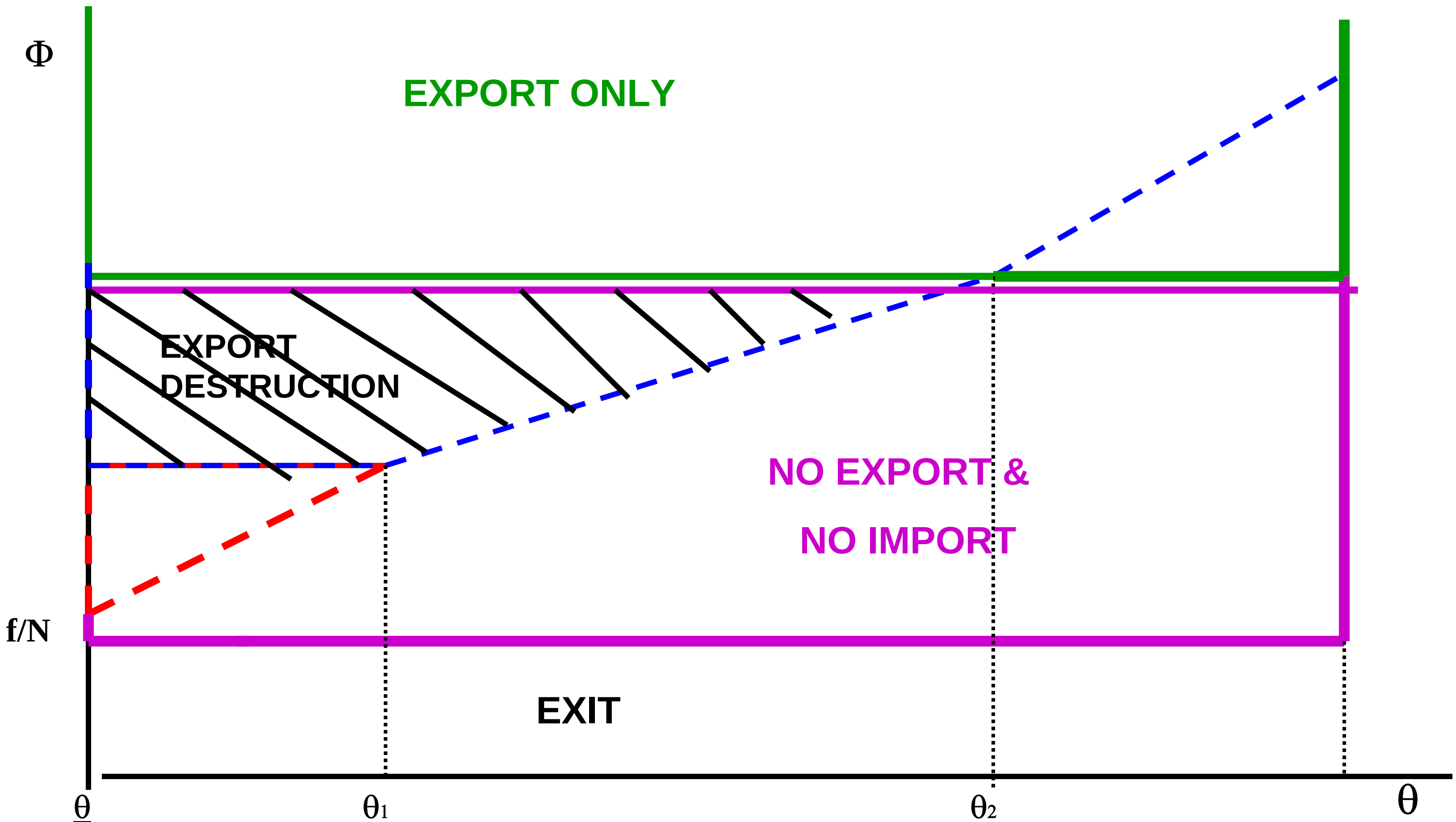

Figure 4: Revenue and Profit Comparisons
(Medium ε)

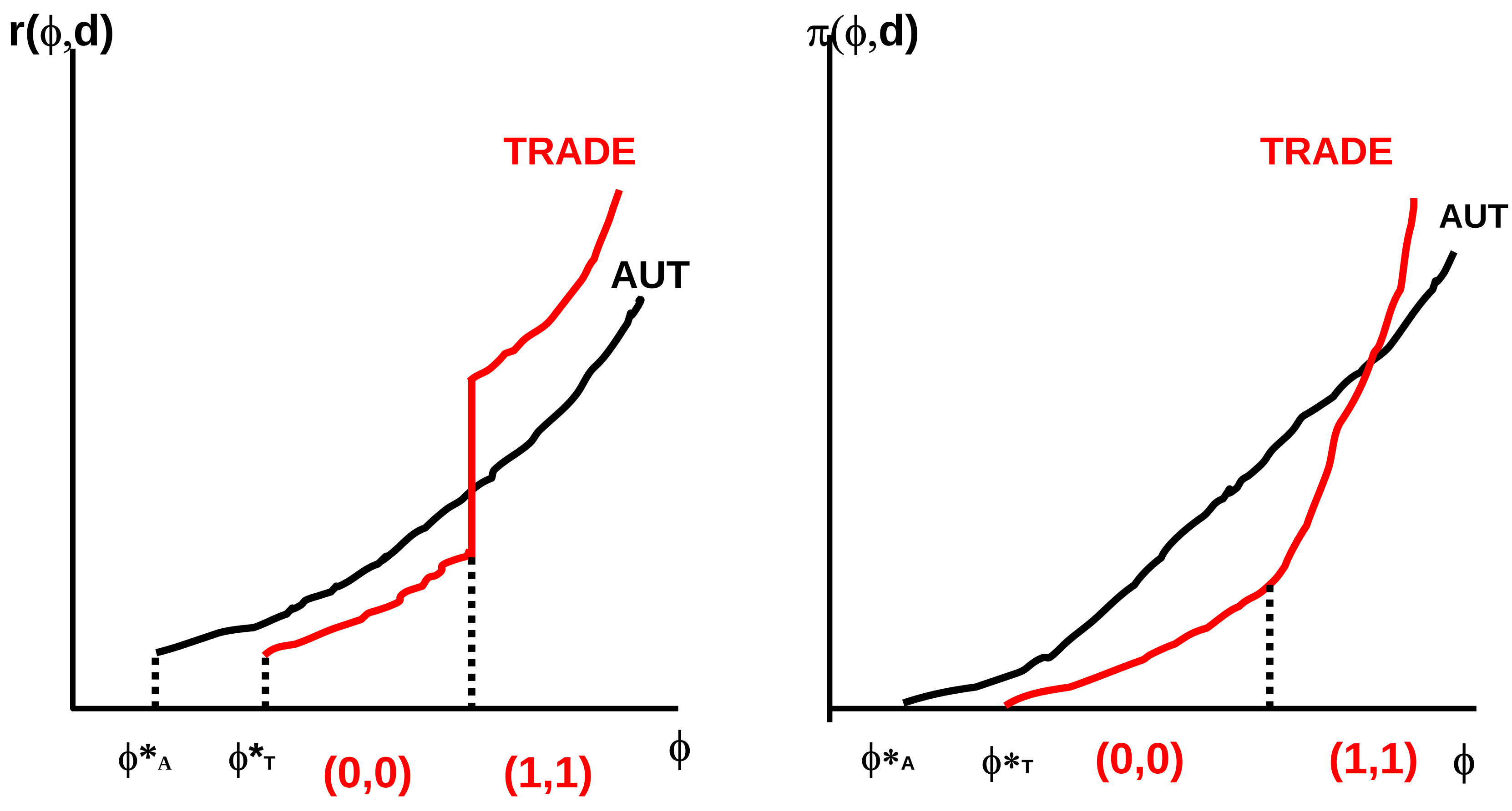

Figure 5: Revenue and Profit Comparisons
(Low ε) (Firms which only import lose market share)

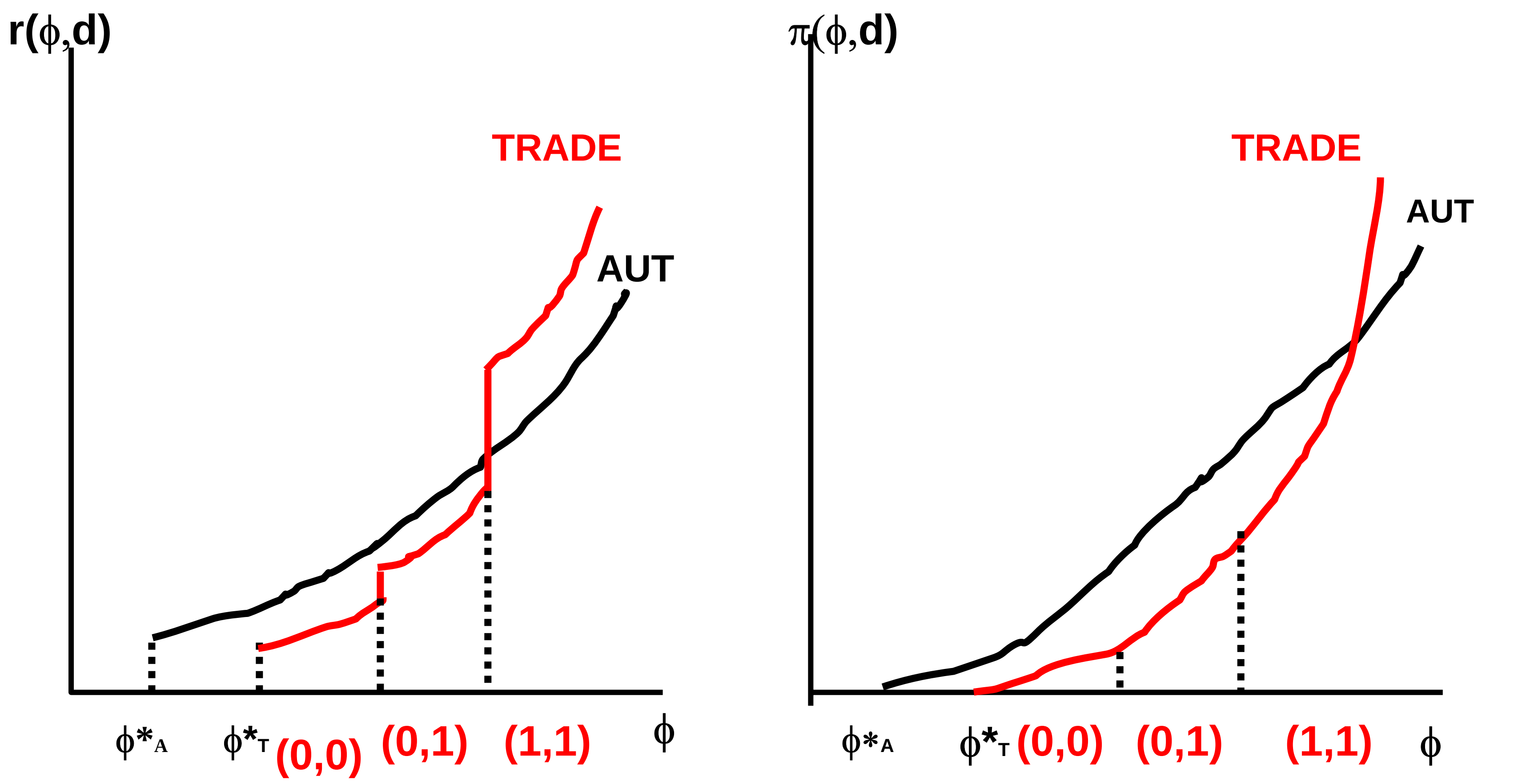

Figure 6: Revenue and Profit Comparisons
(Low ε) (Firms which only import gain market share)