Federal Reserve Bank of Minneapolis
Research Department Staff Report 277

October 2000

Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?

V. V. Chari*

University of Minnesota
and Federal Reserve Bank of Minneapolis

Patrick J. Kehoe*

Federal Reserve Bank of Minneapolis
and University of Minnesota

Ellen R. McGrattan*

Federal Reserve Bank of Minneapolis
and University of Minnesota

ABSTRACT

The central puzzle in international business cycles is that real exchange rates are volatile and persistent. The most popular story for real exchange rate fluctuations is that they are generated by monetary shocks interacting with sticky goods prices. We quantify this story and find that it can account for some of the observed properties of real exchange rates. When prices are held fixed for at least one year, risk aversion is high and preferences are separable in leisure, the model generates real exchange rates that are as volatile as in the data. The model also generates real exchange rates that are persistent, but less so than in the data. If monetary shocks are correlated across countries, then the comovements in aggregates across countries are broadly consistent with those in the data. Making asset markets incomplete or introducing sticky wages does not measurably change the results.

*This paper is a revised version of our 1996 paper entitled "Monetary Shocks and Real Exchange Rates in Sticky Price Models of International Business Cycles." Materials for the paper are available at our website, http://minneapolisfed.org/research/sr/sr277.html. Chari and Kehoe thank the National Science Foundation for support. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

The central puzzle in international business cycles is that real exchange rates are volatile and persistent. Ever since the work of Dornbusch (1976), the most popular story for exchange rate fluctuations is that they result from the interaction of monetary shocks and sticky prices. To date, however, few researchers have attempted to develop quantitative general equilibrium models of this story. In this paper, we do that with some success. We develop a general equilibrium monetary model with sticky prices that builds on the pioneering work of Svensson and van Wijnbergen (1989) and Obstfeld and Rogoff (1995) to investigate the extent to which monetary shocks can account for the observed volatility and persistence of real exchange rates. We show that if risk aversion is high and preferences are separable in leisure, then the model can account for the volatility of real exchange rates. With price-stickiness of one year, the model also produces real exchange rates that are quite persistent, but less so than in the data. If monetary shocks are correlated across countries, then the comovements in aggregates across countries are broadly consistent with those in the data. Minor discrepancies are that the model generates too much volatility in relative price levels and employment and too little in investment.

In constructing our model, we need to choose whether to make real exchange rate fluctuations arise from deviations from the law of one price for traded goods across countries or from fluctuations in the relative prices of nontraded to traded goods across countries or from both. We choose to abstract from nontraded goods; so that, in our model, fluctuations in real exchange rates arise solely from deviations from the law of one price for traded goods. This focus is guided by the data. We present evidence that fluctuations in the relative prices of nontraded to traded goods across countries account for essentially none of the volatility of real exchange rates. Using data for the United States and an aggregate of Europe (and our admittedly imperfect measures), we find that less than 2 percent of the variance of real exchange rates is due to fluctuations in the relative prices of nontraded to traded goods. This evidence is consistent with studies which document that even at a very disaggregated level, the relative price of traded goods has large and persistent fluctuations. (See, for example, the work of Engel (1993, 1999) and Knetter (1993).)

Our model with only traded goods is a version of Svensson and van Wijnbergen's (1989) model modified to allow for price discrimination, staggered price-setting, and capital accumulation. We introduce price-discriminating monopolists in order to get fluctuations in real exchange rates from fluctuations in the relative price of traded goods. (See the work of Dornbusch (1987), Krugman (1987), Knetter (1989), Marston (1990), and Goldberg and Knetter (1997).) We introduce staggered price-setting in order to get persistent real exchange rates. We introduce capital accumulation in order to generate the relative volatility of consumption and output observed in the data. In our model, this relative volatility is closely connected to that for the real exchange rate and output.

In this benchmark model, the real exchange rate is the ratio of the marginal utilities of consumption of households in the two countries. Hence, the volatility of real exchange rates is essentially determined by the risk aversion parameter and the volatility of consumption, while the persistence of real exchange rates is essentially determined by the persistence of consumption. More precisely, we show that the volatility of real exchange rates is approximately equal to the product of the risk aversion parameter and the volatility of relative consumption in the two countries. We show that this calculation implies that a risk aversion parameter of about 6 will produce the real exchange rate volatility in the data.

We also show that the persistence of real exchange rates is approximately the autocorrelation of relative consumption in the two countries. If prices are set for a substantial length of time, then monetary shocks lead to persistent fluctuations in consumption and, hence, in real exchange rates. In our quantitative analysis, we assume that prices are set for one year at a time along the lines of the evidence summarized by Taylor (1999). We find that with this amount of price-stickiness, real exchange rates are persistent in our model, but somewhat less so than in the data.

We investigate two variations of the benchmark model designed to increase the persistence of real exchange rates. One variation is to replace the model's complete international asset markets with incomplete markets that allow for trade only in an uncontingent nominal bond. The idea is that with incomplete markets, monetary shocks can lead to wealth redistributions that increase the persistence of real exchange rates. In the other variation of the benchmark model, markets are complete, but we replace the model's frictionless labor markets with sticky wages. The idea is that with sticky wages, nominal marginal costs respond less to monetary shocks, so prices do too, thereby increasing persistence. While both of these avenues are conceptually promising, both give quantitatively small effects.

In our benchmark model, the volatility of real exchange rates depends importantly on the specification of preferences. Our preferences are additively separable in leisure and a consumption-money aggregate. If ongoing technological progress occurs only in the market sector, then the benchmark preferences are inconsistent with balanced growth. We show that if technological progress occurs in the production of both market goods and leisure services, then the preferences can be consistent with balanced growth.

Many researchers have investigated the economic effects of sticky prices. For some early work in a closed-economy setting, see the studies by Svensson (1986), Blanchard and Kiyotaki (1987), and Ball and Romer (1989). The international literature on sticky prices has three branches. The pioneering work laying out the general theoretical framework is by Svensson and van Wijnbergen (1989) and Obstfeld and Rogoff (1995). (See also the recent work by Corsetti, Pesenti, Roubini, and Tille (1999).) More closely related to our paper are those by Kollmann (1996) and Betts and Devereux (2000), who consider economies with price-discriminating monopolists who set prices as in the work of Calvo (1983). Kollmann considers a semi-small open-economy model without capital in which both prices and wages are sticky; he shows that the model generates volatile exchange rates. Betts and Devereux are primarily interested in replicating the vector autoregression evidence on monetary policy shocks and exchange rates. Finally, for some other work on the implications of sticky prices for monetary policy under fixed exchange rates, see the work of Ohanian and Stockman (1997).

1. DATA

Here we document properties of measures of bilateral exchange rates between the United States and individual European countries and a European aggregate. The series are constructed from raw data for individual countries collected by the International Monetary Fund (IMF) and the Organisation for Economic Co-operation and Development (OECD), much of which are available from the Data Resources, Inc. data base. The raw data are quarterly and cover the period from 1972:1 through 1994:4. The data clearly support the notion that real exchange rates between the United States and Europe are volatile and persistent. We then demonstrate, using disaggregated price data, that very little—less than 2 percent—of the volatility in real exchange rates arises from fluctuations in the relative prices of nontraded to traded goods. This motivates our decision to exclude nontraded goods from the model.

A. *Volatility and persistence of exchange rates*

Our measure of the U.S. nominal exchange rate e_t between the United States and Europe is a trade-weighted average of the bilateral nominal exchange rates with individual European countries.[1] We construct a price index for the European countries, denoted P_t^*, in an analogous way, using each country's consumer price index (CPI). The U.S. real exchange rate with Europe is $q_t = e_t P_t^*/P_t$, where P_t is the price index for the United States.

In Figure 1, we plot the U.S. nominal and real exchange rates with Europe and the ratio of the CPI for Europe to that for the United States. Our aggregate of Europe consists of the nine countries for which we could get complete data: Austria, Finland, France, West Germany, Italy, Norway, Spain, Switzerland, and the United Kingdom. Clearly, both the nominal and real exchange rates are highly volatile, especially when compared to the relative price level. The exchange rates are also highly persistent. (For an earlier analysis emphasizing these features of the data, see Mussa (1986).)

In Table 1, we present some statistics for exchange rates and CPIs for the United States and the European aggregate and for the nine individual European countries. (All the data reported in the table is logged and Hodrick-Prescott (H-P) filtered.) The standard deviation of the real exchange rate between the United States and Europe is 7.81.[2] That is about 4.4 times the volatility of U.S. output (which is only 1.76 percent). Clearly, real exchange rates are very volatile.

We also see in Table 1 that both nominal and real exchange rates between the United States and Europe are highly persistent, with autocorrelations of .86 and .83, respectively, and nominal and real exchange rates are very highly correlated with each other, with a cross-correlation of .99. The data on the individual countries show that these patterns are also evident in bilateral comparisons between each European country and the United States.

B. *Decomposing real exchange rate fluctuations*

In the data, movements in real exchange rates arise from two sources: deviations from the law of one price for traded goods across countries and movements in the relative prices of nontraded to traded goods across countries. To investigate the relative magnitudes of these sources, define the traded goods real exchange rate as $q_T = eP_T^*/P_T$ where P_T and P_T^* are traded goods price indices in the two countries. Let the *nontraded goods relative price* be defined by $p = q/q_T$. To interpret this price, assume the price indices in the two countries are given by $P = (P_T)^{1-\alpha}(P_N)^{\alpha}$ and $P^* = (P_T^*)^{1-\gamma}(P_N^*)^{\gamma}$, where P_N and P_N^* are nontraded goods price indices, and α and γ are the consumption shares of nontraded goods. Then the nontraded goods relative price p is equal to $(P_N^*/P_T^*)^{\gamma}/(P_N/P_T)^{\alpha}$, and it depends on the relative prices of nontraded to traded goods in the two countries. Notice that if the law of one price holds, then q_T is constant and all the variance in q is attributable to the relative prices of nontraded to traded goods. Here, we follow Engel (1999) and use several measures of disaggregated price data to construct this decomposition.

One measure uses disaggregated CPI data. The Organisation for Economic Co-operation and Development (OECD) reports price index data in its *Main Economic Indicators,* where it disaggregates the consumer price index for all items into indices for food, all goods less food, rent, and services less rent. We construct a price index for traded goods as a weighted average of the price indices for food and for all goods less food. Since data on expenditure shares among traded goods by country are not readily available, we use U.S. weights obtained from the U.S. Department of Labor (1992) to construct this price index for each country in Europe which has disaggregated price data. These six countries are Denmark, France, Italy, the Netherlands, Norway, and Switzerland. For the European aggregate, we use the trade-weighting procedure described above.

Figure 2 plots the real exchange rate, q; the traded goods real exchange rate, q_T; and the nontraded goods relative price, p. This figure shows that virtually none of the movement in real exchange rates is due to fluctuations in the relative prices of nontraded to traded goods across countries. The variance of the real exchange rate can be decomposed as $\text{var}(\log q) = \text{var}(\log q_T) + \text{var}(\log p) + 2\text{cov}(\log q_T, \log p)$. In the data, the variance decomposition becomes $(4.29) = (4.89) + (.08) + (-.68)$. Since the covariance between the two components is negative, the maximum portion of the variance of real exchange rates attributable to variability in the nontraded goods relative price is less than 2 percent. (More precisely, the portion is 1.86 percent $= (.08/4.29) \times 100$ percent.)

C. *Alternative decompositions*

Table 2 gives some additional statistics on relative prices and nominal and real exchange rates for individual European countries as well as for the aggregate. Here, although there is some heterogeneity in the individual country statistics, the bilateral comparisons have the same basic patterns as the comparison of aggregates. For our European aggregate, the correlation between the traded goods real exchange rate and the all-goods real exchange rate is .99. In other respects, the statistics in this table are similar to those in Table 1.

These measures provide evidence that the relative price of traded goods varies a great deal across countries. Since these measures are constructed from broad aggregates, the law of one price may hold for each traded good; and the volatility of the traded goods real exchange rate may arise from compositional effects among traded goods. But we doubt that composition effects account for much of the volatility of real exchange rates: European countries have consumption baskets similar to that of the United States, and these consumption baskets do not change much over time.

The OECD also reports nominal and real consumption expenditures for four categories: durable goods, semi-durable goods, nondurable goods, and services. We used these data to construct traded and nontraded goods price indices and found similar results. (For details, see Chari, Kehoe, and McGrattan (1998).)

Our measures of the price of traded goods are clearly imperfect in another way, however. They measure the price paid by the final user of the goods and, hence, incorporate the value of intermediate nontraded services, such as distribution and retailing. Thus, if the value of such nontraded services is volatile, we would expect the real exchange rate for traded goods to be volatile even if the law of one price held for goods net of the value of the nontraded services.

One way to measure the volatility induced by distribution and retailing services is to examine wholesale price indices (WPIs). These data reflect prices received by producers and thus do not include many distribution and retailing costs. These price indices do, however, include the prices of exported goods and exclude the prices of imported goods; thus, they are imperfect measures of the real exchange rate. We report in Table 3 relative prices and exchange rates constructed using WPIs. The procedure we use to construct these indices is the same as that for the measures in Tables 1 and 2, but the number of European countries is different; WPI data are available for the 11 countries listed in Table 3. For the European aggregate relative to the United States, the standard deviation of the real exchange rate constructed using WPIs is 7.61, fairly close to the 7.81 standard deviation found using CPIs (Table 1). The closeness of these measures suggests that volatile distribution costs are unlikely to be a significant source of real exchange rate volatility.

2. THE WORLD ECONOMY

Here we develop a two-country model with infinitely lived consumers that we will use to confront the observations on exchange rates in Europe and the United States. In our model, competitive final good producers in each country purchase intermediate goods from monopolistically competitive intermediate good producers. Each intermediate good producer can price-discriminate across countries and must set prices in the currency of the local market. Once prices are set, each intermediate good producer must satisfy the forthcoming demand. The intermediate good producers set prices in a staggered fashion.

Specifically, consider a two-country world economy consisting of a *home* country and a *foreign* country. Each country is populated by a large number of identical, infinitely lived consumers. In each period of time t, the economy experiences one of finitely many events s_t. We denote by $s^t = (s_0, \ldots, s_t)$ the history of events up through and including period t. The probability, as of period 0, of any particular history s^t is $\pi(s^t)$. The initial realization s_0 is given.

In each period t, the commodities in this economy are labor, a consumption-capital good, money, a continuum of intermediate goods indexed by $i \in [0, 1]$ produced in the home country, and a continuum of intermediate goods indexed by $i \in [0, 1]$ produced in the foreign country. In this economy, the intermediate goods are combined to form final goods which are country-specific. All trade between the countries is in intermediate goods that are produced by monopolists who can charge different prices in the two countries. We assume that all intermediate good producers have the exclusive right to sell their own goods in the two countries. Thus, price differences in intermediate goods cannot be arbitraged away.

In terms of notation, goods produced in the home country are subscripted with an H, while those produced in the foreign country are subscripted with an F. In the home country, final goods are produced from intermediate goods according to a production function that combines features from the industrial organization literature (Dixit and Stiglitz (1977)) and the trade literature (Armington (1969)):

$$y(s^t) = \left[a_1 \left(\int_0^1 y_H(i, s^t)^\theta di\right)^{\rho/\theta} + a_2 \left(\int_0^1 y_F(i, s^t)^\theta di\right)^{\rho/\theta}\right]^{\frac{1}{\rho}}, \tag{1}$$

where $y(s^t)$ is the final good and $y_H(i, s^t)$ and $y_F(i, s^t)$ are intermediate goods produced in the home and foreign countries, respectively. This specification of technology will allow our model to be consistent with three features of the data. The parameter θ will determine the markup of price over marginal cost. The parameter ρ, along with θ, will determine the elasticity of substitution between home and foreign goods. And the parameters a_1 and a_2, together with ρ and θ, will determine the ratio of imports to output.

Final good producers in our economy behave competitively. In the home country, in each period t, producers choose inputs $y_H(i, s^t)$ for $i \in [0, 1]$ and $y_F(i, s^t)$ for $i \in [0, 1]$ and output $y(s^t)$ to maximize profits given by

$$\max\ P(s^t)y(s^t) - \int_0^1 P_H(i, s^{t-1})y_H(i, s^t)\, di - \int_0^1 P_F(i, s^{t-1})y_F(i, s^t)\, di \tag{2}$$

subject to (1), where $P(s^t)$ is the price of the final good in period t, $P_H(i, s^{t-1})$ is the price of the home intermediate good i in period t, and $P_F(i, s^{t-1})$ is the price of the foreign intermediate good i in period t. These prices are in units of the domestic currency. The intermediate goods prices can, at most, depend on s^{t-1} because producers set prices before the realization of the period t shocks. Solving the problem in (2) gives the input demand functions

$$y_H^d(i, s^t) = \frac{[a_1 P(s^t)]^{\frac{1}{1-\rho}}\ \bar{P}_H(s^{t-1})^{\frac{\rho-\theta}{(1-\rho)(\theta-1)}}}{P_H(i, s^{t-1})^{\frac{1}{1-\theta}}} y(s^t) \tag{3}$$

$$y_F^d(i, s^t) = \frac{[a_2 P(s^t)]^{\frac{1}{1-\rho}}\ \bar{P}_F(s^{t-1})^{\frac{\rho-\theta}{(1-\rho)(\theta-1)}}}{P_F(i, s^{t-1})^{\frac{1}{1-\theta}}} y(s^t), \tag{4}$$

where $\bar{P}_H(s^{t-1}) = \left(\int_0^1 P_H(i, s^{t-1})^{\frac{\theta}{\theta-1}} di\right)^{\frac{\theta-1}{\theta}}$ and $\bar{P}_F(s^{t-1}) = \left(\int_0^1 P_F(i, s^{t-1})^{\frac{\theta}{\theta-1}} di\right)^{\frac{\theta-1}{\theta}}$. Using the zero-profit condition, we have

$$P(s^t) = \left(a_1^{\frac{1}{1-\rho}} \bar{P}_H(s^{t-1})^{\frac{\rho}{\rho-1}} + a_2^{\frac{1}{1-\rho}} \bar{P}_F(s^{t-1})^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}}.$$

Thus, in equilibrium, the price of the final good in period t does not depend on the period t shock.

The technology for producing each intermediate good i is a standard constant returns to scale production function

$$y_H(i, s^t) + y_H^*(i, s^t) = F(k(i, s^{t-1}), l(i, s^t)), \tag{5}$$

where $k(i, s^{t-1})$ and $l(i, s^t)$ are the inputs of capital and labor, respectively, and $y_H(i, s^t)$ and $y_H^*(i, s^t)$ are the amounts of this intermediate good used in home and foreign production of the final good, respectively. The capital used in producing good i is augmented by investment of final goods $x(i, s^t)$ and is subject to adjustment costs. The law of motion for such capital is given by

$$k(i, s^t) = (1-\delta)k(i, s^{t-1}) + x(i, s^t) - \phi\left(\frac{x(i, s^t)}{k(i, s^{t-1})}\right) k(i, s^{t-1}), \tag{6}$$

where δ is the depreciation rate and where the adjustment cost function ϕ is convex and satisfies $\phi(\delta) = 0$ and $\phi'(\delta) = 0$.

Intermediate good producers behave as imperfect competitors. They set prices for N periods in a staggered way. In particular, in each period t, a fraction $1/N$ of the home country producers choose a home currency price $P_H(i, s^{t-1})$ for the home market and a foreign currency price $P_H^*(i, s^{t-1})$ for the foreign market before the realization of the event s_t. These prices are set for N periods, so for this group of intermediate good producers, $P_H(i, s^{t+\tau-1}) = P_H(i, s^{t-1})$ and $P_H^*(i, s^{t+\tau-1}) = P_H^*(i, s^{t-1})$ for $\tau = 0, \ldots, N-1$. The intermediate good producers are indexed so that those with $i \in [0, 1/N]$ set new prices in 0, N, $2N$, and so on, while those with $i \in [1/N, 2/N]$ set new prices in 1, $N+1$, $2N+1$, and so on, for the N cohorts of intermediate good producers.

Consider, for example, producers in a particular cohort, namely $i \in [0, 1/N]$. These producers choose prices $P_H(i, s^{t-1}), P_H^*(i, s^{t-1})$, inputs of labor $l(i, s^t)$, capital $k(i, s^t)$, and investment $x(i, s^t)$ to solve

$$\max \sum_{t=0}^{\infty} \sum_{s^t} Q(s^t)[P_H(i, s^{t-1}) y_H(i, s^t) + e(s^t) P_H^*(i, s^{t-1}) y_H^*(i, s^t) \\ - P(s^t) w(s^t) l(i, s^t) - P(s^t) x(i, s^t)] \quad (7)$$

subject to (5), (6), and the constraints that their supplies to the home and foreign markets $y_H(i, s^t)$ and $y_H^*(i, s^t)$ must equal the amount demanded by home and foreign final good producers, $y_H^d(i, s^t)$ from (3) and its analogue. In addition, the constraints that prices are set for N periods are $P_H(i, s^{t-1}) = P_H(i, s^{-1})$ for $t = 0, \ldots, N-1$, and $P_H(i, s^{t-1}) = P_H(i, s^{N-1})$ for $t = N, \ldots, 2N-1$ and so on, with similar constraints for $P_H^*(i, s^{t-1})$. Here $Q(s^t)$ is the price of one unit of home currency in s^t in an abstract unit of account, $e(s^t)$ is the nominal exchange rate, and $w(s^t)$ is the real wage. The initial capital stock $k(i, s^{-1})$ is given and is the same for all producers in this cohort.

The optimal prices for $t = 0,\ N,\ 2N$ are

$$P_H(i, s^{t-1}) = \frac{\sum_{\tau=t}^{t+N-1} \sum_{s^\tau} Q(s^\tau) P(s^\tau) v(i, s^\tau) \Lambda_H(s^\tau)}{\theta \sum_{\tau=t}^{t+N-1} \sum_{s^\tau} Q(s^\tau) \Lambda_H(s^\tau)}$$

$$P_H^*(i, s^{t-1}) = \frac{\sum_{\tau=t}^{t+N-1} \sum_{s^\tau} Q(s^\tau | s^{t-1}) P(s^\tau) v(i, s^\tau) \Lambda_H^*(s^\tau)}{\theta \sum_{\tau=t}^{t+N-1} \sum_{s^\tau} Q(s^\tau | s^{t-1}) \Lambda_H^*(s^\tau)}$$

where $v(i, s^t)$ is the real unit cost which is equal to the wage rate divided by the marginal product of labor, $w(s^t)/F_l(i, s^t)$, $\Lambda_H(s^t) = [a_1 P(s^t)]^{\frac{1}{1-\rho}} \bar{P}_H(s^{t-1})^{\frac{\rho-\theta}{(1-\rho)(\theta-1)}} y(s^t)$, and $\Lambda_H^*(s^t) = [a_2 P^*(s^t)]^{\frac{1}{1-\rho}} \bar{P}_H^*(s^{t-1})^{\frac{\rho-\theta}{(1-\rho)(\theta-1)}} y^*(s^t)$. Here, $F_l(i, s^t)$ denotes the derivative of the production function with respect to l. We use similar notation throughout the paper.

In a symmetric steady state, the real unit costs are equal across firms. Hence, in this steady state, these formulas reduce to $P_H(i) = eP_H^*(i) = Pv/\theta$, so that the law of one price holds for each good and prices are set as a markup $(1/\theta)$ over nominal costs Pv. Thus, in this model, all deviations from the law of one price are due to shocks which keep the economy out of the deterministic steady state.

In this economy, the markets for state-contingent money claims are complete. We represent the asset structure by having complete, contingent, one-period nominal bonds denominated in the home currency. We let $B(s^t, s_{t+1})$ denote the home consumers' holdings of such a bond purchased in period t and state s^t with payoffs contingent on some particular state s_{t+1} at $t+1$. Let $B^*(s^t, s_{t+1})$ denote the foreign consumers' holdings of this bond. One unit of this bond pays one unit of the home currency in period $t+1$ if the particular state s_{t+1} occurs and 0 otherwise. Let $Q(s^{t+1}|s^t)$ denote the price of this bond in units of the home currency in period t and state s^t. Clearly $Q(s^{t+1}|s^t) = Q(s^{t+1})/Q(s^t)$. (Notice that also including bonds denominated in the foreign currency would be redundant.) For notational simplicity, we assume that claims to the ownership of firms in each country are held by the residents of that country and cannot be traded.

In each period $t = 0, 1, \ldots$, consumers choose their period t allocations after the realization of the event s_t. Consumers in the home country face the sequence of budget constraints

$$\begin{aligned} &P(s^t)c(s^t) + M(s^t) + \textstyle\sum_{s_{t+1}} Q(s^{t+1}|s^t)B(s^{t+1}) \\ &\qquad \leq P(s^t)w(s^t)l(s^t) + M(s^{t-1}) + B(s^t) + \Pi(s^t) + T(s^t) \end{aligned} \tag{8}$$

and a borrowing constraint $B(s^{t+1}) \geq -P(s^t)\bar{b}$, where $c(s^t)$, $l(s^t)$, and $M(s^t)$ are consumption, labor, and nominal money balances, respectively, and $s^{t+1} = (s^t, s_{t+1})$. Here $\Pi(s^t)$ is the profits of the home country intermediate good producers, $T(s^t)$ is transfers of home currency, and the positive constant $\bar{b}$ constrains the amount of real borrowing of the consumer. The initial conditions $M(s^{-1})$ and $B(s^0)$ are given.

Home consumers choose consumption, labor, money balances, and bond holdings to maximize their utility:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U\big(c(s^t), l(s^t), M(s^t)/P(s^t)\big) \tag{9}$$

subject to the consumer budget constraints. Here β is the discount factor. The first-order conditions for the consumer can be written as

$$-\frac{U_l(s^t)}{U_c(s^t)} = w(s^t), \tag{10}$$

$$\frac{U_m(s^t)}{P(s^t)} - \frac{U_c(s^t)}{P(s^t)} + \beta \sum_{s_{t+1}} \pi(s^{t+1}|s^t) \frac{U_c(s^{t+1})}{P(s^{t+1})} = 0, \tag{11}$$

$$Q(s^t|s^{t-1}) = \beta\pi(s^t|s^{t-1}) \frac{U_c(s^t)}{U_c(s^{t-1})} \frac{P(s^{t-1})}{P(s^t)}. \tag{12}$$

Here $U_c(s^t)$, $U_l(s^t)$, and $U_m(s^t)$ denote the derivatives of the utility function with respect to its arguments, and $\pi(s^t|s^{t-1}) = \pi(s^t)/\pi(s^{t-1})$ is the conditional probability of s^t given s^{t-1}.

The problems of the final good producers, the intermediate good producers, and the consumers in the foreign country are analogous to these problems. Allocations and prices in the foreign country are denoted with an asterisk.

Now let's develop a relationship between the real exchange rate and marginal utilities of consumption of the consumers in the two countries, which is implied by arbitrage. The budget constraint of a consumer in the foreign country is given by

$$\begin{aligned} &P^*(s^t)c^*(s^t) + M^*(s^t) + \textstyle\sum_{s_{t+1}} Q(s^{t+1}|s^t)B^*(s^{t+1})/e(s^t) \\ &\qquad \leq P^*(s^t)w^*(s^t)l^*(s^t) + M^*(s^{t-1}) + B^*(s^t)/e(s^t) + \Pi^*(s^t) + T^*(s^t), \end{aligned} \tag{13}$$

where $B^*(s^t)$ denotes the foreign consumer's holdings of the home country bonds at s^t. The first-order condition with respect to bond holdings for a foreign consumer is

$$Q(s^t|s^{t-1}) = \beta\pi(s^t|s^{t-1}) \frac{U_c^*(s^t)}{U_c^*(s^{t-1})} \frac{e(s^{t-1})}{e(s^t)} \frac{P^*(s^{t-1})}{P^*(s^t)}.$$

Substituting for the bond price in this equation from (12) and iterating, we obtain

$$\frac{U_c(s^t)}{U_c(s^0)} \frac{P(s^0)}{P(s^t)} = \frac{U_c^*(s^t)}{U_c^*(s^0)} \frac{e(s^0)}{e(s^t)} \frac{P^*(s^0)}{P^*(s^t)}.$$

Defining the real exchange rate as $q(s^t) = e(s^t)P^*(s^t)/P(s^t)$, we obtain

$$q(s^t) = \kappa \frac{U_c^*(s^t)}{U_c(s^t)}, \tag{14}$$

where the constant $\kappa = e(s^0)U_c(s^0)P^*(s^0)/U_c^*(s^0)P(s^0)$. We use this relationship between real exchange rates and marginal rates of substitution in developing intuition for our quantitative results.

The money supply processes in the home and foreign countries are given by $M(s^t) = \mu(s^t)M(s^{t-1})$ and $M^*(s^t) = \mu^*(s^t)M^*(s^{t-1})$, where $\mu(s^t)$ and $\mu^*(s^t)$ are stochastic processes and $M(s^{-1})$ and $M^*(s^{-1})$ are given. New money balances of the home currency are distributed to consumers in the home country in a lump-sum fashion by having transfers satisfy $T(s^t) = M(s^t) - M(s^{t-1})$. Likewise, transfers of foreign currency to foreign consumers satisfy $T^*(s^t) = M^*(s^t) - M^*(s^{t-1})$.

An equilibrium requires several market-clearing conditions. The resource constraint in the home country is given by

$$y(s^t) = c(s^t) + \int_0^1 x(i, s^t)\, di$$

and the labor market-clearing condition is $l(s^t) = \int l(i, s^t)\, di$. Similar conditions hold for the foreign country. The market-clearing condition for contingent bonds is $B(s^t) + B^*(s^t) = 0$.

An *equilibrium* for this economy is a collection of allocations for home consumers $c(s^t)$, $l(s^t)$, $M(s^t)$, $B(s^{t+1})$; allocations for foreign consumers $c^*(s^t)$, $l^*(s^t)$, $M^*(s^t)$, $B^*(s^{t+1})$; allocations and prices for home intermediate good producers $y_H(i, s^t)$, $y_H^*(i, s^t)$, $l(i, s^t)$, $x(i, s^t)$, and $P_H(i, s^{t-1})$, $P_H^*(i, s^{t-1})$ for $i \in [0, 1]$; allocations and prices for foreign intermediate good producers $y_F(i, s^t)$, $y_F^*(i, s^t)$, $l^*(i, s^t)$, $x^*(i, s^t)$, and $P_F(i, s^{t-1})$, $P_F^*(i, s^{t-1})$ for $i \in [0, 1]$; and allocations for home and foreign final good producers $y(s^t), y^*(s^t)$, final good prices $P(s^t)$, $P^*(s^t)$, real wages $w(s^t)$, $w^*(s^t)$, and bond prices $Q(s^{t+1}|s^t)$ that satisfy the following five conditions: (*i*) the consumer allocations solve the consumers' problem; (*ii*) the prices of intermediate good producers solve their maximization problem; (*iii*) the final good producers' allocations solve their problem; (*iv*) the market-clearing conditions hold; and (*v*) the money supply processes and transfers satisfy the specifications above.

We are interested in a stationary equilibrium and thus restrict the stochastic processes for the growth rates of the money supplies to be Markovian. To make the economy stationary, we deflate all nominal variables by the level of the relevant money supply. A stationary equilibrium for this economy consists of stationary decision rules and pricing rules that are functions of the state of the economy. The state of the economy when monopolists make their pricing decisions (that is, before the event s_t is realized) must record the capital stocks for a representative monopolist in each cohort in the two countries, the prices set by the other $N-1$ cohorts in the two countries, and the period $t-1$ monetary shocks. The shocks from period $t-1$ are needed because they help forecast the shocks in period t. The current shocks are also included in the state of the economy when the rest of the decisions are made (that is, after the event s_t is realized). We compute the equilibrium using standard methods to obtain linear decision rules (Chari, Kehoe, and McGrattan (2000)). For the benchmark preferences with one-quarter price-stickiness, we checked the accuracy of the linear decision rules against nonlinear decision rules obtained by the finite element method. (For an introduction to the finite element method, see McGrattan (1996).)

3. CALIBRATION

Now we must choose values for this benchmark model's parameters. We report all our choices in the top panel of Table 4.

We consider a benchmark utility function of the form

$$U(c,l,M/P) = \frac{1}{1-\sigma}\left[\left(\omega c^{\frac{\eta-1}{\eta}} + (1-\omega)\left(\frac{M}{P}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}\right]^{1-\sigma} + \psi\frac{(1-l)^{(1-\gamma)}}{1-\gamma} \qquad (15)$$

and an intermediate good production function of the form $F(k,l) = k^{\alpha}l^{1-\alpha}$. Notice that the utility function is separable between a consumption-money aggregate and leisure.

Consider first the preference parameters. The discount factor β is set so as to give an annual real return to capital of 4 percent. The parameters ψ and γ are set so that the elasticity of labor supply, with marginal utility held constant, is 2 and the time devoted to work is one-quarter of the total time in the steady state. The literature has a wide range of estimates for the curvature parameter σ. We set it to 6 and show later that this value is critical for generating the right volatility in the real exchange rate.

To obtain η and ω, we draw on the money demand literature. Our model can be used to price a variety of assets, including a nominal bond which costs one dollar at s^t and pays $R(s^t)$ dollars in all states s^{t+1}. The first-order condition for this asset can be written as $U_m(s^t) = U_c(s^t)[R(s^t) - 1]/R(s^t)$. When we use our benchmark specification of utility, the first-order condition can be rewritten as

$$\log \frac{M(s^t)}{P(s^t)} = -\eta \log \frac{\omega}{1-\omega} + \log c(s^t) - \eta \log \left(\frac{R(s^t) - 1}{R(s^t)} \right), \tag{16}$$

which has the form of a standard money demand function with consumption and interest rates. To obtain η, we ran a quarterly regression from 1960 to 1995 (inclusive) in which we used M1 for money; the GDP deflator for P; consumption of durables, nondurables, and services for c; and the three-month U.S. Treasury bill rate for R. Our estimate of the interest elasticity is $\eta = .39$, and the implied value for ω is .94.

Consider next the final good technology parameters. In our model, the elasticity of substitution between home goods and foreign goods is $1/(1-\rho)$. Studies have estimated quite a range for this parameter. The most reliable studies seem to indicate that for the United States the elasticity is between 1 and 2, and values in this range are generally used in empirical trade models. (See, for example, the survey by Stern, Francis, and Schumacher (1976).) For an aggregate of Europe, the elasticity seems to be smaller. (See, for example, the discussions of Whalley (1985, Ch. 5) and Deardorff and Stern (1990, Ch. 3).) We follow the work of Backus, Kehoe, and Kydland (1994) and use an elasticity of 1.5. To set a_1 and a_2, note that in a symmetric steady state, $y_H/y_F = [a_1/a_2]^{\frac{1}{1-\rho}}$. In U.S. data, imports from Europe are roughly 1.6 percent of GDP. This implies that $y_H/y_F = .984/.016$. Together with our normalization, this gives the values of a_1 and a_2.

For the intermediate good technology parameters, we set the capital share parameter

$\alpha = 1/3$ and the depreciation rate $\delta = .021$, as is standard in the real business cycle literature. Based on work of Basu and Fernald (1994,1995), Basu and Kimball (1997), and Basu (1996), we chose $\theta = .9$, which implies a markup of 11 percent and an elasticity of demand of 10. We also need to choose the length of price-stickiness N. Taylor (1999) summarizes the empirical studies on price-setting and concludes that the average length of time between price changes is about a year. We set $N = 4$, so that prices are set for four quarters.

We consider an adjustment function of the form $\phi(x/k) = b(x/k - \delta)^2/2$. Notice that with this specification at the steady state, both the total and marginal costs of adjustment are 0. Uncertainty about the size of these adjustment costs is high. In all of our experiments, we choose the parameter b so that the standard deviation of consumption relative to the standard deviation of output is equal to that in the data. One measure of the adjustment costs is the resources used up in adjusting capital relative to investment given by $\phi(x/k)/x$. For our benchmark economy, the resource cost in adjusting capital is .22 percent of investment when averaged across firms, time, and simulations.

The details of the monetary rules followed in the United States and Europe are extensively debated. Here we assume that all the monetary authorities follow a simple rule, namely, that the growth rate of the money stocks for both areas follows a process of the form

$$\begin{aligned} \log \mu_t &= \rho_\mu \log \mu_{t-1} + \varepsilon_{\mu t} \\ \log \mu_t^* &= \rho_\mu \log \mu_{t-1}^* + \varepsilon_{\mu t}^*, \end{aligned} \tag{17}$$

where $(\varepsilon_\mu, \varepsilon_\mu^*)$ is a normally distributed, mean-zero shock. (Notice that each period now has a continuum of states. Our earlier analysis with a finite number of states extends immediately to this case.) Each shock has a standard deviation of σ_μ, and the shocks have a positive cross-correlation. The stochastic process for money in the foreign country is the same. We choose $\rho_\mu = .57$ from the data by running a regression of the form (17) on quarterly U.S. data for M1 from 1973 through 1995 (obtained from Citibase). In our experiments, we choose the standard deviation of these shocks that will give the same volatility for output as in the U.S.

data. We also choose the cross-correlation of these shocks so as to produce a cross-correlation for outputs that is similar to that in the data. We choose the standard deviation and the cross-correlation of these shocks in this way because we want to investigate whether a model in which monetary shocks account for the observed movements in outputs can also account for the observed movements in exchange rates and other macroeconomic variables.

4. FINDINGS

We report on the H-P–filtered statistics for the data, the benchmark economy, and some variations on that economy in Tables 5 and 6. The statistics for the data are all computed with the United States as the home country and the aggregate of Europe as the foreign country. Thus, all the numbers that refer to GDP, consumption, net exports, and so on are from U.S. data, while the price ratio, for example, refers to the price index for the United States relative to that for Europe. Statistics for exchange rates and the price ratio for the data reported in Table 5 are taken from Table 1 (with the standard deviations normalized by the standard deviation of U.S. output). Business cycle statistics reported in Table 6 are based on data reported by the OECD.[3]

Overall, we find that the benchmark model generates nominal and real exchange rates that match the data qualitatively: they are volatile, persistent, and highly cross-correlated. However, quantitatively, along some dimensions, the model does less well: while its volatility of exchange rates is about right, it generates too little persistence in exchange rates, too much volatility in the price ratio and employment, and too little volatility in investment.

In Table 5, we see that in the benchmark model, compared to output, the nominal exchange rate is 4.84 times as variable and the real exchange rate is 4.88 times as variable. These values are close to those in the data (4.74 and 4.43). The benchmark model also produces substantial persistence (autocorrelations) of nominal and real exchange rates (.68 and .63), but this persistence is less than that in the data (.86 and .83).

The high volatility of real exchange rates comes from our choice of a high curvature parameter σ, which corresponds to a choice of high risk aversion. To see the connection between volatility and σ, log-linearize the expression for real exchange rates, (14), to obtain

$$\hat{q} = A(\hat{c} - \hat{c}^*) + B(\hat{m} - \hat{m}^*) + D(\hat{l} - \hat{l}^*), \tag{18}$$

where a caret denotes the deviation from the steady state of the log of the variable and m, m^* denote real balances. The coefficients A, B, and D are given by

$$A = -\frac{cU_{cc}}{U_c},\ B = -\frac{mU_{cm}}{U_c},\ D = -\frac{lU_{cl}}{U_c},$$

evaluated at the steady state. For preferences expressed in the utility function (15), the coefficient of relative risk aversion A is approximately equal to the curvature parameter $\sigma = 6$, B is unimportant, and $D = 0$. (The actual values are $A = 5.94$ and $B = .06$. Notice that A is only approximately equal to σ because of the nonseparability between consumption and money balances.) Thus, for our preferences,

$$\frac{\text{std}(\hat{q})}{\text{std}(\hat{y})} \cong \sigma \frac{\text{std}(\hat{c} - \hat{c}^*)}{\text{std}(\hat{y})}.$$

In Figure 3 we graph the benchmark model's volatility of the real exchange rates against the curvature parameter σ, where this volatility is measured as in Table 5. As we vary σ, we alter the adjustment cost parameter b to keep roughly unchanged the standard deviation of consumption relative to that of output.[4] We see that a curvature parameter of about 6 is needed to reproduce the data's volatility of real exchange rates relative to output (4.43). Note also in Figure 3 that as σ is varied, the autocorrelation of real exchange rates is essentially unchanged.

In terms of the persistence of real exchange rates, for our preferences the autocorrelation of real exchange rates can be written as

$$\text{corr}(\hat{q}, \hat{q}_{-1}) \cong \text{corr}(\hat{c} - \hat{c}^*, \hat{c}_{-1} - \hat{c}^*_{-1}).$$

This expression suggests that the autocorrelation of real exchange rates is essentially determined by the autocorrelation of consumption. In Table 6, we see that the autocorrelation of consumption in the model is high (.63) but less than that in the data (.85), which mirrors the feature (from Table 5) that the autocorrelation of real exchange rates is high in the model but less than that in the data.

Note that without substantial price-stickiness, neither consumption nor real exchange rates would have much persistence. To see this, consider Figure 4 in which we graph the autocorrelation of consumption, the autocorrelations of real and nominal exchange rates, and the volatility of the price ratio relative to that of output against the number of periods that prices are held fixed, N. Notice that the autocorrelations of consumption and the real exchange rate match almost exactly. When $N = 1$, consumption is negatively autocorrelated, as is the real exchange rate. As N increases, so do the autocorrelations of consumption and the real exchange rate. Notice also that as the periods of price-stickiness are increased, the price ratio responds less to monetary shocks; its volatility declines, and the behavior of the real exchange rate comes to mirror that of the nominal exchange rate.

Consider now the rest of the statistics for the benchmark economy in Tables 5 and 6. In Table 5, we see that the price ratio is substantially more volatile in the model (3.15) than in the data (.74) while real and nominal exchange rates are a little less correlated in the model (.79) than in the data (.99). Both of these occur because prices move to offset nominal exchange rate movements more in the model than in the data. We also see that real exchange rates and output are more correlated in the model than in the data (.51 vs. .10), while real exchange rates and net exports are only slightly correlated both in the model and in the data (both .09). It is worth noting that, across countries, there is greater heterogeneity in the correlations between real exchange rates and various aggregates, like output and net exports, than for other statistics, like the volatility and persistence of real exchange rates or the cross-correlation of real and nominal exchange rates.

In Table 6, we see that investment is only about half as volatile in the model as in the data (1.71 vs. 3.28), while employment is nearly twice as volatile in the model as in the data (1.50 vs. .72). Investment is less volatile in the model because when we use the high curvature parameter of $\sigma = 6$, we need to use a relatively high adjustment cost parameter to make consumption have the right volatility, and with that level of adjustment costs, investment is not very volatile.

Employment is more rather than less volatile than output in the model because almost all of the movement in output comes from variations in the labor input. Specifically, note that log-deviations in output can be written as $\hat{y} = \alpha \hat{k} + (1 - \alpha)\hat{l}$. Since investment is only a small percentage of the capital stock, this stock moves only a small amount at business cycle frequencies, and we roughly have that $\text{std}(\hat{y}) \cong (1 - \alpha)\text{std}(\hat{l})$. With $\alpha = 1/3$, this gives $\text{std}(\hat{l})/\text{std}(\hat{y}) \cong 1.5$. So, in a sticky price model like ours, we should expect employment to be much more volatile than output. This feature does not arise in real business cycle models because in them the technology shock accounts for much of the movement in output.[5] (A related feature of sticky price models more generally is that labor productivity is countercyclical in the model but procyclical in the data.)

In Table 6, we also see that in the model, the cross-country correlation of output is similar to that of consumption (.48 vs. .49) while in the data, the cross-correlation of output is much higher than that of consumption (.52 vs. .27). While the cross-correlation of consumption is higher in our model than that in the data, the model does much better on this dimension than does the standard real business cycle model (see Backus, Kehoe and Kydland (1994)). In the real business cycle model the law of one price holds for all traded goods and the real exchange rate does not vary as much as it does in our model. Since an equation like (14) holds in both models, the lower variability of real exchange rates in the real business cycle model leads to a higher correlation of the marginal utilities of consumption and, thus, to a higher cross country correlation of consumption. A minor discrepancy between the benchmark model and the data is that in the data, net exports are somewhat countercyclical ($-.37$) while in the model they are essentially acyclical (.14 with a standard error of .17).

5. SENSITIVITY ANALYSIS

Here we examine the findings of our benchmark model by varying assumptions about three of the model's features: the export share, the specification of shocks, and the monetary rule. The sensitivity analysis determines that the initial findings are fairly robust.

A. *Export share*

We have chosen parameters so that the export share of output is 1.6 percent, which is similar to the share that the United States has in its bilateral trade with Europe. More open economies have much larger shares than this. To see what difference a larger share might make, we consider a variation of the model with an export share of 15 percent (by adjusting a_1 and a_2 accordingly). To put this number in perspective, note that it is similar to the share that the United States has with the rest of the world.

In Tables 5 and 6, the columns labeled "High Exports" list the model's predictions with the 15 percent share. The increase clearly has had little effect on most of the statistics. It decreases the volatility of both nominal and real exchange rates only slightly. It worsens the performance of net exports by making them more procyclical and by slightly lowering their correlation with real exchange rates. But, overall, there is little change.

B. *Real shocks*

So far the only shocks in the model are monetary shocks. Now we add real shocks of two types: shocks to technology and to government consumption. Here we primarily want to examine whether adding these shocks improves the model's performance on business cycle statistics. As noted above, employment is too volatile in our model because variation in labor input is the primary source of variation in output at business cycle frequencies. Adding other shocks will add other sources of variation in output.

We allow for country-specific technology shocks which are common across all intermediate good producers. The technology for producing intermediate goods in the home country and foreign countries is now $F(k_t, A_t l_t)$ and $F(k_t^*, A_t^* l_t^*)$. Here the technology shocks A_t and A_t^* are common across all intermediate goods and follow a stochastic process given by $\log A_{t+1} = \rho_A \log A_t + \varepsilon_{At+1}$ and $\log A_{t+1}^* = \rho_A \log A_t + \varepsilon_{At+1}^*$, where the technology innovations ε_A and ε_A^* have zero means, are serially uncorrelated, and are uncorrelated with shocks to money and government consumption. We follow Kehoe and Perri (2000) and use $\rho_A = .95$, $\text{var}(\varepsilon_A) = \text{var}(\varepsilon_A^*) = (.007)^2$, and $\text{corr}(\varepsilon_A, \varepsilon_A^*) = .25$.

We add government consumption shocks as follows. The final good is now used for government consumption as well as private consumption and investment. The resource constraint for the home country is now

$$y_t = c_t + g_t + \int_0^1 x_t(i)\, di,$$

where home government consumption g_t follows a stochastic process $\log g_{t+1} = (1 - \rho_g)\mu_g + \rho_g \log g_t + \varepsilon_{gt+1}$. To obtain estimates for this autoregressive process, we ran a regression with data on real government purchases for the United States over the period 1947:1 through 1998:4. Our estimates from this regression are as follows: $\mu_g = .13$, $\rho_g = .97$, and $\text{var}(\varepsilon_g) = (.01)^2$. We assume that the shock ε_g is serially uncorrelated and uncorrelated with shocks to money and technology and to the shock to government consumption in the foreign country. We model government consumption in the foreign country symmetrically. In each period, first the technology and government consumption shocks are realized, then prices are set, and then the monetary shock is realized. (Alternative timing assumptions lead to similar results.)

We report the results for this economy in the columns labeled "Real Shocks" in Tables 5 and 6. Again, most of the statistics change little. However, the relative volatility of employment actually increases slightly (from 1.50 in the benchmark model to 1.58 in the model with real shocks). To understand this finding note that here the log-deviations in output are approximately given by $\hat{y} \cong (1-\alpha)(\hat{A}+\hat{l})$ so that

$$\frac{\operatorname{var}\hat{l}}{\operatorname{var}\hat{y}} \cong \frac{1}{(1-\alpha)^2} - \frac{\operatorname{var}\hat{A}}{\operatorname{var}\hat{y}} - \frac{2\operatorname{cov}(\hat{A},\hat{l})}{\operatorname{var}\hat{y}}.$$

From this expression we see that introducing technology shocks can increase the variability of employment if technology shocks and employment are sufficiently negatively correlated. In the model, on impact a positive technology shock leads to a fall in employment since firms can meet the same demand with fewer workers. This feature of the model makes technology shocks and employment negatively correlated enough to raise the relative volatility of employment. Government consumption shocks, meanwhile, have a quantitatively insignificant role.

C. *Taylor rule*

There is a lively debate over the most appropriate way to model monetary policy. A recently popular way to do so has been with an interest rate rule. Here we discuss how our money growth rule can be interpreted as an interest rate rule, and we describe the properties of our model economy under a particular interest rate rule popularized by Taylor (1993).

Logically, any interest rate rule can be interpreted as a money growth rule and vice versa. To see this, posit an interest rate rule and work out the equilibrium of the economy. This equilibrium has a corresponding money growth process associated with it. Clearly, if one views this money growth process as the policy, then the equilibrium for this economy with this money growth is the same as that for an economy with the interest rate rule. Of course, if there are multiple equilibria under the interest rule, then for each equilibrium, there is a different money growth process that implements it. The converse also holds. (Of course, such rules can be represented as either a function of both past endogenous variables and exogenous shocks or as a function of solely the history of exogenous shocks.) Moreover, there is empirical evidence in support of our choice for the money growth rule. In particular, Christiano, Eichenbaum, and Evans (1998) have shown with vector autoregression analysis that a money growth process of the kind considered here is a good approximation to a process that implements their estimated interest rate rule.

As a practical matter, however, some simple interest rate rule might be a better approximation to the policy in the data than is our simple money growth rule. Thus, we consider the implications of replacing our simple rule for money growth rates with an interest rate rule similar to those studied by Taylor (1993) and Clarida, Gali, and Gertler (2000).

In particular, we assume that nominal interest rates r_t are set as a function of lagged nominal rates, expected inflation rates, and output according to

$$r_t = \rho_r r_{t-1} + (1 - \rho_r) \left(\alpha_\pi E_t \pi_{t+1} + \alpha_y [\ln \text{gdp}_t]/4 \right) + \varepsilon_{rt}, \tag{19}$$

where we have dropped the constant and converted units to quarterly rates. In (19) π_{t+1} is the inflation rate from t to $t+1$, gdp_t is real gross domestic product at t, and ε_{rt} is a normally distributed, mean-zero shock. We set $\rho_r = .79$, $\alpha_\pi = 2.15$, and $\alpha_y = .93$. (The numbers are taken from Clarida, Gali, and Gertler (2000), Table II.) We choose the volatility of the shocks to match the volatility of output, and we choose the correlation of the home shock ε_{rt} and the foreign shock to match the cross-correlation of output.

When we use this Taylor rule in our benchmark model, we are unable to generate reasonable business cycle behavior. Briefly, for low values of the adjustment cost parameter the relative volatility of consumption is tiny. For high values of the adjustment cost parameter the relative volatility of consumption increases but the correlation between consumption and output is negative. On closer investigation, we found that these features of the model were driven by the nonseparability of consumption and money balances. Since we do not view this nonseparability as a crucial feature of our model, we investigate a version with the Taylor rule and with preferences of the form

$$\frac{c^{1-\sigma}}{1-\sigma} + \omega \frac{(M/P)^{1-\sigma}}{1-\sigma} + \psi (1-l)^{(1-\gamma)}/(1-\gamma).$$

We set the parameters σ, ψ and γ as before. (The parameter ω is not relevant since money demand is determined residually.) In Tables 5 and 6, we report the results for this exercise in the columns labeled "Taylor Rule." This model moves the volatilities of the price ratio and the exchange rates closer to those in the data. Unfortunately, however, the model's nominal and real exchange rates, with autocorrelations of .45 and .48, are much less persistent than those in either the data or the benchmark model.

We also investigated the properties of the economy using a Taylor rule estimated by Rotemberg and Woodfoord (1997). Their estimated Taylor rule uses three lags of nominal interest rates and inflation together with current output and two of its lags. When we use this rule we obtain essentially the same results as we did with the one estimated by Clarida, Gali, and Gertler (2000). For example, the autocorrelations of nominal and real exchange rates were .40 and .42.

Nominal exchange rates are less persistent in the Taylor rule model than in the benchmark model because the endogenous policy reaction tends to offset the exogenous shocks. For example, a negative shock to interest rates in (19) raises the quantity of money and leads in subsequent periods to a rise in inflation. This rise in inflation leads to an endogenous increase in interest rates that offsets the initial shock. As a result, interest rates are not very persistent and, hence, neither are movements in consumption or real exchange rates. We confirmed this intuition by analyzing the properties of the model for higher values of ρ_r. For example, when we raised ρ_r from .66 to .95, the autocorrelations of nominal and real exchange rates increased from .45 and .48 to .61 and .59, while for $\rho_r = .99$, these autocorrelations increased even further, to .67 and .62.

6. INCREASING PERSISTENCE

The primary discrepancy between the benchmark model and the data is that while the model generates the exchange rate volatility in the data, it does not generate quite the amount of persistence in the data. Two avenues for increasing persistence seem particularly promising: making markets incomplete and adding labor frictions by making wages sticky. However, neither change turns out to lead to a significant improvement in the benchmark model's persistence performance.

A. *Incomplete markets*

Theoretically, with incomplete markets, monetary shocks can lead to wealth redistributions which lead to persistent changes in relative consumption and, hence, to more persistent real exchange rates. Furthermore, the real exchange rate could potentially be volatile even with a much smaller value of the curvature parameter. The idea is that with incomplete markets, the simple static relationship between the real exchange rate and the ratio of the marginal utilities given in (14) is replaced by one that holds only in expectation.

The market incompleteness we introduce into the benchmark model is to replace the complete set of contingent bonds traded across countries by a single uncontingent nominal bond. This bond is denominated in units of the home currency. The home consumer's budget constraint is now

$$\begin{aligned} &P(s^t)c(s^t) + M(s^t) + \bar{Q}(s^t)D(s^t) \\ &\qquad \leq P(s^t)w(s^t)l(s^t) + M(s^{t-1}) + D(s^{t-1}) + \Pi(s^t) + T(s^t), \end{aligned} \tag{20}$$

where D is the consumer's debt. The real value of these bonds $D(s^t)/P(s^t)$ is bounded below. Here each unit of $D(s^t)$ is a claim on one unit of the home currency in all states s^{t+1} that can occur at $t+1$, and $\bar{Q}(s^t)$ is the corresponding price. The foreign consumer's budget constraint is modified similarly.

The first-order condition for bond-holding in the home country is now given by

$$\bar{Q}(s^t) = \sum_{s_{t+1}} \beta\pi(s^{t+1}|s^t)\frac{U_c(s^{t+1})}{U_c(s^t)}\frac{P(s^t)}{P(s^{t+1})}, \tag{21}$$

while that in the foreign country is

$$\bar{Q}(s^t) = \sum_{s_{t+1}} \beta\pi(s^{t+1}|s^t)\frac{U_c^*(s^{t+1})}{U_c^*(s^t)}\frac{e(s^t)}{e(s^{t+1})}\frac{P^*(s^t)}{P^*(s^{t+1})}. \tag{22}$$

Equating (21) and (22) and log-linearizing the resulting equations gives

$$E_t\left[\hat{U}_{ct+1} - \hat{U}_{ct} + \hat{P}_t - \hat{P}_{t+1}\right] = E_t\left[\hat{U}^*_{ct+1} - \hat{U}^*_{ct} + \hat{e}_t + \hat{P}^*_t - \hat{e}_{t+1} - \hat{P}^*_{t+1}\right], \tag{23}$$

where carets denote log-deviations from a steady state with $D = 0$. Noting that $\hat{q}_t = \hat{e}_t + \hat{P}_t^* - \hat{P}_t$, we can rewrite (23) as

$$E_t \left[\hat{q}_{t+1} - \hat{q}_t \right] = E_t \left[(\hat{U}_{ct+1}^* - \hat{U}_{ct+1}) - (\hat{U}_{ct}^* - \hat{U}_{ct}) \right]. \tag{24}$$

Thus, with incomplete markets, the relation between real exchange rates and marginal utilities only holds in expected first-differences.

In Tables 5 and 6, we report statistics for an incomplete market economy which has the same parameters as does the benchmark economy, but has the asset structure just discussed. In the columns labeled "Incomplete Markets," the statistics in both tables are virtually identical to those for the benchmark economy with complete markets. Thus, while adding incomplete markets theoretically could help with both volatility and persistence, quantitatively it does not.

The intuition behind this result is that the wealth effects in our model are extremely small. To see why, consider starting in a steady state in which net claims on foreigners are zero. For the home country to increase its wealth, it must increase its net exports. The home country's net exports are

$$NX_t = e_t P_{Ht}^* y_{Ht}^* - P_{Ft} y_{Ft}, \tag{25}$$

where y_{Ht}^* are the exports of home-produced goods and y_{Ft} are the imports of foreign-produced goods. With sticky prices, a positive monetary shock in the home country leads to a depreciation of the exchange rate e_t, a rise in domestic use of all intermediate goods, including y_{Ft}, and essentially no change in foreign use of intermediate goods, including y_{Ht}^*. Since the prices foreigners pay for home exports are fixed in the foreign currency, the depreciation of the home currency leads to a rise in export earnings. This rise helps pay for the rise in imports that comes from increased home demand for foreign goods. Indeed, for a version of our model, the rise in export earnings turns out to exactly pay for the rise in imports, so that monetary shocks lead to no wealth redistribution at all.

Let's work through the details of such a model. Consider a deterministic version of the benchmark model without capital, with a utility function of the form

$$U\left(c, l, M/P\right) = \log c + \gamma \log(1 - l) + \phi \log M/P$$

and with prices set for one period ($N = 1$). Suppose that, starting from a steady state with zero debt, the home country experiences a onetime unanticipated monetary shock of 1 percent in period 1, and the money supply is constant thereafter. We show that such a shock leads to the following outcomes. In period 1, net exports are zero, home use of both home and foreign intermediate goods rises 1 percent, foreign use of intermediate goods is unaffected, and nominal and real exchange rates depreciate 1 percent. In period 2, then, the economy returns to a new steady state with the same real allocations as in the old steady state. The domestic price level rises 1 percent, and the nominal exchange rate stays at its depreciated level, while the real exchange rate returns to its old steady-state level. In order for these outcomes to constitute an equilibrium, debt at the end of period 1 must be zero or, equivalently, net exports in period 1 must be zero.

We prove this claim by showing that our conjectured outcome satisfies the equilibrium conditions. Note first that if net exports in period 1 are zero, then the economy in the beginning of period 2 is identical in all respects to that in the beginning of period 1 except that the money stock is 1 percent higher. Clearly, there is an equilibrium in which all real variables from period 2 on are identical to those in the original steady state and domestic prices and the exchange rate rise 1 percent. To see that net exports in period 1 are indeed zero, consider the three key equilibrium conditions in period 1: the money demand equation,

$$\frac{M_1}{P_1} = \frac{\phi c_1}{1 - Q_1}; \tag{26}$$

the bond price equation,

$$Q_1 = \beta \frac{P_1 c_1}{P_2 c_2}; \tag{27}$$

and the interest parity equation,

$$Q_1 = Q_1^* e_1 / e_2 \tag{28}$$

(which must hold from period 1 on).

We claim that there is an equilibrium in which, in period 1, domestic consumption rises 1 percent, domestic and foreign bond prices are unchanged, and the nominal exchange rate rises 1 percent. From the money demand equation (26), we can see that if the bond price Q_1 is unchanged and the money supply M_1 rises 1 percent, then consumption c_1 must rise 1 percent, since the price P_1 is preset. Since c_1 and P_2 both rise 1 percent and P_1 and c_2 are unchanged, we know from (27) that Q_1 is also unchanged. Using the analogous equilibrium conditions for the foreign country, we know that Q_1^* is unchanged. From (28), we then see that since e_2 rises 1 percent, e_1 does too. Since intermediate goods prices are preset, a one percent rise in home consumption of the final good leads to a one percent rise in the home use of both home and foreign intermediate goods. A contradiction argument can be used to show that this outcome is the unique equilibrium. (See Chari, Kehoe, and McGrattan (1998).)

B. *Labor market frictions*

So far we have considered frictions in goods markets and asset markets and have let labor markets function perfectly. But we want to try to increase persistence in our benchmark model, and adding labor market frictions in the form of sticky wages might do that. In the benchmark model, after a monetary shock, wages immediately rise. This rise in wages leads intermediate good producers to increase their prices as soon as they can. Thus, the benchmark model generates little endogenous price-stickiness, that is, price-stickiness beyond that exogenously imposed. Preset nominal wages might generate some endogenous price-stickiness and, hence, more persistence in exchange rates.

We extend the benchmark model to include sticky wages by letting labor be differentiated and having monopolistically competitive unions that set wages in a staggered way for

M periods.

The final good producers in the model remain as before, while the problems of the intermediate good producers and the consumers are altered. The only change in technology is that the labor input $l(i, s^t)$ of intermediate good producer i is now a composite of a continuum of differentiated labor inputs $j \in [0, 1]$ and is produced according to

$$l(i, s^t) = \left[\int l(i, j, s^t)^{\vartheta} \, dj \right]^{1/\vartheta}, \tag{29}$$

where $l(i, j, s^t)$ denotes the amount of differentiated labor input j used by intermediate good producer i in date t. The nominal wage for the jth type of labor in date t is denoted $W(j, s^{t-1})$. The problem of the intermediate good producer is the same as before except that now we have a sub-problem of determining the cost-minimizing composition of the different types of labor. The term $w(s^t)l(i, s^t)$ in the intermediate good producer's problem (7) is now replaced by

$$w(s^t)l(i, s^t) = \min_{\{l(i,j,s^t)\}, j \in [0,1]} \int \frac{W(j, s^{t-1})}{P(s^t)} \, l(i, j, s^t) \, dj \tag{30}$$

subject to (29). The solution to this problem is the demand for labor of type j by intermediate good producer i, namely

$$l(i, j, s^t) = \left(\frac{\bar{W}(s^t)}{W(j, s^{t-1})} \right)^{\frac{1}{1-\vartheta}} l(i, s^t),$$

where $\bar{W}(s^t) = \left[\int W(j, s^{t-1})^{\frac{\vartheta}{\vartheta - 1}} \, dj \right]^{\frac{\vartheta - 1}{\vartheta}}$ is the nominal wage index. Substitution of the demand into (30) implies that the real wage index is given by $w(s^t) = \bar{W}(s^t)/P(s^t)$.

The consumer side of the labor market can be thought of as being organized into a continuum of unions indexed by j. Each union consists of all the consumers in the economy with labor of type j. Each union realizes that it faces a downward-sloping demand for its type of labor. The total demand for labor of type j is obtained by integrating across the demand of the intermediate good producers and is given by

$$l^d(j, s^t) = \left(\frac{\bar{W}(s^t)}{W(j, s^{t-1})} \right)^{\frac{1}{1-\vartheta}} \int l(i, s^t) \, di. \tag{31}$$

We assume that a fraction $1/M$ of unions set their wages in a given period and hold them fixed for M subsequent periods. The unions are indexed so that those with $j \in [0, 1/M]$ set new wages in 0, M, $2M$, and so on, while those with $j \in [1/M, 2/M]$ set new wages in 1, $M+1$, $2M+1$, and so on, for the M cohorts of unions. In each period these new wages are set before the realization of the current money shocks. Notice that the wage-setting arrangement is analogous to the price-setting arrangement for intermediate good producers.

The problem of the jth union, for say $j \in [0, 1/M]$, is to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t)\, U\left(c(j, s^t), l^d(j, s^t), M(j, s^t)/P(s^t)\right) \tag{32}$$

subject to the labor demand schedule (31), the budget constraints

$$\begin{aligned} &P(s^t)c(j, s^t) + M(j, s^t) + \textstyle\sum_{s_{t+1}} Q(s^{t+1}|s^t)B(j, s^{t+1}) \\ &\qquad \leq W(j, s^{t-1})l^d(j, s^t) + M(j, s^{t-1}) + B(j, s^t) + \Pi(s^t) + T(s^t), \end{aligned} \tag{33}$$

and the constraints that wages are set for M periods, $W(j, s^{t-1}) = W(j, s^{-1})$ for $t = 0, \ldots, M-1$, and $W(j, s^{t-1}) = W(j, s^{M-1})$ for $t = M, \ldots, 2M-1$ and so on. We choose the initial debt of the unions so that each union has the same present discounted value of income.

In this problem, the union chooses the wage and agrees to supply whatever is demanded at that wage. The first-order conditions are changed from those in the benchmark economy as follows. The condition for the labor choice (10) is replaced by the condition for nominal wages

$$W(j, s^{t-1}) = -\frac{\sum_{\tau=t}^{t+M-1} \sum_{s^\tau} Q(s^\tau)P(s^\tau)U_l(j, s^\tau)/U_c(j, s^\tau)l^d(j, s^\tau)}{\vartheta \sum_{\tau=t}^{t+M-1} \sum_{s^\tau} Q(s^\tau)l^d(j, s^\tau)}. \tag{34}$$

Notice that in a steady state, this condition reduces to $W/P = (1/\vartheta)(-U_l/U_c)$, so that real wages are set as a markup over the marginal rate of substitution between labor and consumption. The conditions (11) and (12) are now indexed by j. These conditions, together with our assumption on initial debt, imply that $U_c(j, s^t)$ and $U_m(j, s^t)$ are equated across unions.

The new parameters in the model are the number of periods of wage-setting M and the markup parameter ϑ. Following Taylor's (1999) discussion of the evidence, we set $M = 4$. We set $\vartheta = .87$ so that the markup is about 15 percent. This markup is consistent with estimates of the markup of union wages over non-union wages. (See Lewis 1986.)

In Tables 5 and 6, in the columns labeled "Sticky Wages," we see that the sticky wage model slightly improves on the benchmark model. The sticky wage model decreases the volatility of the price ratio (from 3.15 to 2.56) and increases the volatility of real exchange rates (from 4.88 to 5.04). (With these changes, the volatility in the data can be matched with a slightly lower value of the curvature parameter σ, if so desired.) The sticky wage model also slightly increases the persistence of real exchange rates (from .63 to .68) and the cross-correlation of real and nominal exchange rates (from .79 to .87). The business cycle statistics remain basically unchanged, except for the correlation of real exchange rates with both GDP and net exports, which worsen.

7. PREFERENCE SPECIFICATION

One problem arises with the benchmark model when productivity grows in the market sector but not in the nonmarket sector: preferences of the form (15) are inconsistent with balanced growth. Here we discuss this problem as well as two potential solutions to it.

To derive our benchmark model's implications for growth paths, we suppress uncertainty and add labor-augmenting technical change z_t, so that the technology for each intermediate good producer is given by $F(k_t, z_t l_t)$, where z_t grows at a constant rate z. We say that an economy is on a *balanced growth path* if output, consumption, real balances, the capital stock, and wages all grow at rate z, while labor and interest rates are constant. Inspecting consumers' first-order conditions in the benchmark model, (10)–(12), we see that for the economy to have a balanced growth path, three homotheticity conditions must be satisfied: U_{lt}/U_{ct} must be homogeneous of degree one in c_t and m_t when l_t is held fixed; U_{mt}/U_{ct} must be homogeneous of degree zero in c_t and m_t when l_t is held fixed; and U_{ct+1}/U_{ct} must be homogeneous of degree zero in c_t and m_t, with $c_{t+1} = (1+z)c_t$ and $m_{t+1} = (1+z)m_t$, when l_t is held fixed.

The preferences in our benchmark model violate the first condition, that U_{lt}/U_{ct} must be homogeneous of degree one (except when $\sigma = 1$). Thus, in the benchmark economy, a balanced growth path does not exist. To get some feel for how far the economy is from a balanced growth path, suppose that consumption, real balances, and wages grow 2 percent a year. Then, with our benchmark parameter values, we easily see that leisure grows 7.5 percent per year. If leisure is initially three-quarters of the time endowment, then within four years, leisure uses up the entire time endowment and labor supply is zero.

This violation may initially seem troublesome because the economy becomes unbalanced even at business cycle frequencies. We may be able to mitigate the effect, however, once we understand where the violation comes from.

A. *Technical progress in nonmarket activities*

The feature of the benchmark model that leads to this violation is that technical progress raises the productivity of time allocated to market activities but not that of time allocated to nonmarket activities. In the spirit of Becker (1993), suppose that technical progress does raise the productivity of time allocated to nonmarket activities, so that an input of $(1 - l_t)$ units of time outside the market produces $z_t(1-l_t)$ units of leisure services. With this formulation, we can easily show that

$$-\frac{U_{lt}}{U_{ct}} = \frac{\psi(1+z)^{(1-\gamma)t}(1-l_t)^{-\gamma}}{\omega c_t^{\frac{-1}{\eta}}\left[\omega c_t^{\frac{\eta-1}{\eta}} + (1-\omega)m^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}(1-\sigma)-1}}.$$

If c_t and m_t grow at the same rate as z_t and if l_t is a constant, then

$$-\frac{U_{lt}}{U_{ct}} = \kappa\frac{(1+z)^{(1-\gamma)t}}{(1+z)^{-\sigma t}},$$

where κ is a constant. Along a balanced growth path, wages grow at the same rate as z_t, so that in order for the economy to have a balanced growth path, it must have $\sigma = \gamma$.

The parameter γ, together with the fraction of time allocated to the market, determines the Frisch elasticity of labor supply. If the fraction of time allocated to the market is one-quarter, then this labor supply elasticity is $3/\gamma$. The balanced growth restriction $\sigma = \gamma$ thus connects the labor supply elasticity to the intertemporal elasticity of substitution in consumption.

In our earlier experiments, when we varied σ, we left γ unchanged. We have also done experiments in which as we vary σ, we change γ, so that the balanced growth restriction holds. We found that imposing this balanced growth restriction makes little difference to our results. Note that with $\sigma = \gamma = 6$, the implied labor supply elasticity is 1/2, which is within the wide range of estimates of the labor supply elasticity.

B. *Nonseparable preferences*

Thus, one way of dealing with the issue of unbalanced growth is to add technical progress in nonmarket activities. Another way, of course, is to use preferences that lead to balanced growth with technical progress only in the market sector. Such preferences, however, do not generate volatile real exchange rates.

A typical specification of such preferences is

$$U(c, l, M/P) = \left[\left(\omega c^{\frac{\eta-1}{\eta}} + (1-\omega)(M/P)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} (1-l)^{\xi} \right]^{1-\sigma} / (1-\sigma).$$

It is easy to verify that these preferences satisfy the three homotheticity conditions for balanced growth. We set the parameters η, σ, and ω as in the benchmark model. We set $\xi = 2.25$, as is typical in the business cycle literature (for example, Chari, Christiano, and Kehoe (1991)), and we display the resulting statistics in the columns labeled "Nonseparable Preferences" in Tables 5 and 6. Unfortunately, these preferences do not generate volatile exchange rates: real exchange rates now vary hardly at all.

We can get some intuition for why these preferences do not generate more volatility by examining the expression for log-linearized real exchange rates (18). With nonseparable preferences, the coefficients are $A = 5.94$, $B = .06$, and $D = -3.78$. Thus, an increase in the money growth rate in the home country increases both consumption and employment in that country. The increase in consumption decreases the marginal utility of home consumption and, hence, leads to a depreciation in the real exchange rate. With nonseparable labor and the curvature parameter $\sigma > 1$, the increase in employment decreases the marginal utility of home consumption and partially offsets the real exchange rate depreciation. This offsetting effect of employment increases with σ; hence, increasing the curvature σ does not substantially increase the volatility of real exchange rates.

8. CONCLUSION

The central puzzle in international business cycles is the large and persistent fluctuations in real exchange rates. In this paper, we have taken a step toward solving that puzzle. We have developed a general equilibrium sticky price model which can generate fluctuations in real exchange rates that are appropriately volatile and quite persistent, though not quite persistent enough. We have found that for monetary shocks to generate these data, the model needs to have separable preferences, high risk aversion, and price-stickiness of at least one year. We have also found that if monetary shocks are correlated across countries, then the comovements in aggregates across countries in the model are broadly consistent with those in the data.

The main failing of our model is that it does not generate enough persistence in real exchange rates. As we have seen, this is primarily because the model does not generate enough persistence in consumption. One avenue to generate more persistence in consumption is to include habit persistence or consumption durability in preferences. Unfortunately, this avenue typically leads to less volatile consumption and, hence, less volatile exchange rates. We need to find ways to increase the persistence of consumption without decreasing its volatility.

We have seen that without substantial price-stickiness, real exchange rates are not persistent. We have assumed that prices are exogenously fixed for one year. While this assumption generates movements in prices that are consistent with the evidence in Taylor (1999, it is somewhat unappealing to simply assume that firms cannot change their prices for a year. A major challenge in this line of research is to find a mechanism that generates substantial amounts of endogenous price-stickiness from small frictions. By this, we mean a mechanism that leads firms to optimally choose not to change prices much even when they can freely do so. To be successful, such a mechanism must be consistent with microeconomic data on firms' pricing decisions and must generate business cycles of the kind seen in the data. One avenue worth exploring is to depart from the simple monopolistic competition setup and allow for richer strategic interactions among firms.

Minor failings of the model are that it produces too much volatility in relative price levels and employment and too little in investment. Modifications to strategic interactions among firms may reduce the volatility of relative price levels. In terms of employment, the labor input may be mismeasured due to considerations like labor-hoarding, so that measured employment fluctuates much less than the true labor input. In terms of investment, recall that we chose the adjustment cost parameter to match the relative volatility of consumption. Adding other sources of variation in consumption might let us simultaneously match the volatilities of consumption and investment in the data.

Acknowledgments. This paper is a revised version of our 1996 paper entitled "Monetary Shocks and Real Exchange Rates in Sticky Price Models of International Business Cycles." Materials for the paper are available at our website, http://minneapolisfed.org/research/sr/sr277.html. Chari and Kehoe thank the National Science Foundation for support. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

Notes

[1]In particular, our constructed index is $e_t = \sum_{i \in \mathcal{I}} \omega_i e_{it}/e_{i0}$, where e_{it} is the exchange rate for country i in period t, e_{i0} is the exchange rate for country i in the first quarter of 1972, and the weight ω_i is the time series average of the ratio of the dollar value of exports plus imports between country i and the United States to the total dollar value of all exports plus imports between the European countries included in set $\mathcal{I}$ and the United States. The countries (trade weights) included in our dataset are: Austria (1.0), Belgium (7.5), Denmark (1.7), Finland (1.0), France (12.4), West Germany (23.3), Italy (10.6), Netherlands (9.7), Norway (2.0), Spain (4.7), Switzerland (5.1), and the United Kingdom (20.9).

[2]Our real exchange rate measure is substantially more volatile than another measure of the real exchange rate between the United States and the rest of the world. The IMF's *International Financial Statistics* reports that the effective real exchange rate for the United States, based on weights derived from the multilateral exchange rate model (MERM), has a standard deviation of 5.43. The autocorrelation of this MERM exchange rate is .85. The MERM measure is less volatile presumably because shocks affecting bilateral exchange rates are not perfectly correlated across countries, and the MERM measure averages across more countries than our measure does.

[3]Our series for the foreign country are aggregates for the nine European countries in Table 1. For each country, we obtain output, consumption, and investment in 1990 local currency units. We convert these series into dollars using the OECD's 1990 purchasing power parity exchange rate and add the results to obtain our aggregates for Europe. Exports and imports are reported in U.S. dollars. Since employment data for Austria are not available, employment for Europe is the sum of the employment in the eight remaining countries.

[4]If we keep the adjustment cost parameter unchanged, then as we increase σ, the relative volatility of consumption and output decreases somewhat. Hence, the volatility of the real exchange rate increases with σ, but at a somewhat slower rate. For example, with b held fixed, the volatility of the real exchange rate at $\sigma = 1.01$ and 10 is 1.20 and 6.58, while when b is adjusted, these volatilities are .82 and 8.42.

[5]One extension that might help sticky price models in this dimension is to have cyclical variations in the intensity that measured capital and labor are worked.

References

ARMINGTON, P. S. (1969), "A Theory of Demand for Products Distinguished by Place of Production", *International Monetary Fund Staff Papers,* **16**, 159–178.

BACKUS, D. K.; KEHOE, P. J.; and KYDLAND, F. E. (1994), "Dynamics of the Trade Balance and the Terms of Trade: The J-Curve?", *American Economic Review,* **84**, 84–103.

BALL, L., and ROMER, D. (1989), "Are Prices Too Sticky?", *Quarterly Journal of Economics,* **104**, 507–524.

BASU, S. (1996), "Procyclical Productivity: Increasing Returns or Cyclical Utilization?" *Quarterly Journal of Economics,* **111**, 719–751.

BASU, S., and FERNALD, J. G. (1994), "Constant Returns and Small Markups in U.S. Manufacturing," (International Finance Discussion Paper 483, Board of Governors of the Federal Reserve System).

__________. (1995), "Are Apparent Productive Spillovers a Figment of Specification Error?" *Journal of Monetary Economics,* **36**, 165–188.

BASU, S., and KIMBALL, M. S. (1997), "Cyclical Productivity With Unobserved Input Variation," (Working Paper 5915, National Bureau of Economic Research).

BECKER, G. S. (1993), *Human Capital: A Theoretical and Empirical Analysis, With Special Reference to Education,* 3rd ed. (Chicago: University of Chicago Press).

BETTS, C., and DEVEREUX, M. B. (2000), "Exchange Rate Dynamics in a Model of Pricing-to-Market", *Journal of International Economics,* **50**, 215–244.

BLANCHARD, O. J., and KIYOTAKI, N. (1987), "Monopolistic Competition and the Effects of Aggregate Demand", *American Economic Review,* **77**, 647–666.

CALVO, G. A. (1983), "Staggered Prices in a Utility-Maximizing Framework", *Journal of Monetary Economics,* **12**, 383–398.

CHARI, V. V.; CHRISTIANO, L. J.; and KEHOE, P. J. (1991), "Optimal Fiscal and Monetary Policy: Some Recent Results", *Journal of Money, Credit, and Banking,* **23**, 519–539.

CHARI, V. V.; KEHOE, P. J.; and MCGRATTAN, E. R. (1998), "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?" (Staff Report 223, Federal Reserve Bank of Minneapolis, http://minneapolisfed.org/research/sr/sr223.html).

CHARI, V. V.; KEHOE, P. J.; and MCGRATTAN, E. R. (2000), "Appendix: 'Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?' " (Federal Reserve Bank of Minneapolis, http://minneapolisfed.org/research/sr/sr277.html).

CHRISTIANO, L. J.; EICHENBAUM, M.; and EVANS, C. L. (1998), "Modeling Money"

(Working Paper 6371, National Bureau of Economic Research).

CLARIDA, R.; GALI, J.; and GERTLER, M. (2000), "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory" *Quarterly Journal of Economics*, **115**, 147-180.

CORSETTI, G.; PESENTI, P.; ROUBINI, N.; and TILLE, C. (1999), "Competitive Devaluations: A Welfare-Based Approach" (Working Paper 6889, National Bureau of Economic Research).

DEARDORFF, A. V., and STERN, R. M. (1990), *Computational Analysis of Global Trading Arrangements* (Ann Arbor: University of Michigan Press).

DIXIT, A. K., and STIGLITZ, J. E. (1977), "Monopolistic Competition and Optimum Product Diversity", *American Economic Review,* **67**, 297–308.

DOMOWITZ, I.; HUBBARD, R. G.; and PETERSEN, B. C. (1986), "Business Cycles and the Relationship Between Concentration and Price-Cost Margins", *Rand Journal of Economics,* **17**, 1–17.

DORNBUSCH, R. (1976), "Expectations and Exchange Rate Dynamics", *Journal of Political Economy,* **84**, 1161–1176.

__________. (1987), "Exchange Rates and Prices", *American Economic Review,* **77**, 93–106.

ENGEL, C. (1993), "Real Exchange Rates and Relative Prices: An Empirical Investigation", *Journal of Monetary Economics,* **32**, 35–50.

__________. (1999), "Accounting for U.S. Real Exchange Rate Changes", *Journal of Political Economy,* **107**, 507–538.

GOLDBERG, P. K., and KNETTER, M. M. (1997), "Goods Prices and Exchange Rates: What Have We Learned?", *Journal of Economic Literature,* **35**, 1243–1272.

KEHOE, P. J., and PERRI, F. (2000), "International Business Cycles With Endogenous Incomplete Markets" (Staff Report 265, Federal Reserve Bank of Minneapolis).

KNETTER, M. M. (1989), "Price Discrimination by U.S. and German Exporters", *American Economic Review,* **79**, 198–210.

__________. (1993), "International Comparisons of Price-to-Market Behavior", *American Economic Review,* **83**, 473–486.

KOLLMANN, R. (1996), "The Exchange Rate in a Dynamic-Optimizing Current Account Model with Nominal Rigidities: A Quantitative Investigation" (Working Paper 319, University of Montreal).

KRUGMAN, P. R. (1987), "Pricing to Market When the Exchange Rate Changes", in S. W. Arndt and J. D. Richardson (eds.), *Real-Financial Linkages Among Open Economies,* pp. 49–70 (Cambridge, Mass.: MIT Press).

LEWIS, H. G. (1986), "Union Relative Wage Effects," in O. Ashenfelter and R. Layard (eds.), *Handbook of Labor Economics*, Vol. II, Chap. 20, pp. 1139-1181 (Amsterdam: Elsevier/North-Holland).

MARSTON, R. C. (1990), "Pricing to Market in Japanese Manufacturing", *Journal of International Economics,* **29**, 217–236.

MCGRATTAN, E. R. (1996), "Solving the Stochastic Growth Model With a Finite Element Method", *Journal of Economic Dynamics and Control,* **20**, 19–42.

MUSSA, M. (1986), "Nominal Exchange Rate Regimes and the Behavior of Real Exchange Rates: Evidence and Implications", *Carnegie-Rochester Conference Series on Public Policy,* **25**, 117–213.

OBSTFELD, M., and ROGOFF, K. (1995), "Exchange Rate Dynamics Redux", *Journal of Political Economy,* **103**, 624–660.

OHANIAN, L. E., and STOCKMAN, A. C. (1997), "Short-Run Independence of Monetary Policy Under Pegged Exchange Rates and Effects of Money on Exchange Rates and Interest Rates", *Journal of Money, Credit, and Banking,* **29**, 783–806.

ROTEMBERG J. J., and WOODFORD, M. (1997), "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy," *NBER Macroeconomics Annual* (Cambridge, Mass.: MIT Press).

STERN, R. M.; FRANCIS, J.; and SCHUMACHER, B. (1976), *Price Elasticities in International Trade: An Annotated Bibliography* (Toronto: Macmillan).

SVENSSON, L. E. O. (1986), "Sticky Goods Prices, Flexible Asset Prices, Monopolistic Competition, and Monetary Policy", *Review of Economic Studies,* **53**, 385–405.

SVENSSON, L. E. O., and VAN WIJNBERGEN, S. (1989), "Excess Capacity, Monopolistic Competition, and International Transmission of Monetary Disturbances", *Economic Journal,* **99**, 785–805.

TAYLOR, J. B. (1993), "Discretion versus Policy Rules in Practice", *Carnegie-Rochester Conference Series on Public Policy,* **39**, 195–214.

__________. (1999), "Staggered Price and Wage Setting in Macroeconomics", in J. B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics,* Vol. 1B, Chap. 15, pp. 1009–1050 (Amsterdam: Elsevier/North-Holland).

U.S. DEPARTMENT OF LABOR, Bureau of Labor Statistics (1992), *BLS Handbook of Methods,* Bulletin 2414, September (Washington, D.C.: U.S. Bureau of Labor Statistics).

WHALLEY, J. (1985), *Trade Liberalization Among Major World Trading Areas* (Cambridge, Mass.: MIT Press).

Table 1

Properties of exchange rates and consumer price indices

Statistic	Country Relative to U.S. Austria	Finland	France	Germany	Italy	Norway	Spain	Switzerland	UK	Europe Relative to U.S.
Standard Deviations										
Price ratio	1.70	1.95	1.29	1.54	1.86	1.94	2.58	1.69	2.08	1.30
Exchange rate										
Nominal	8.52	7.87	8.97	8.75	8.97	6.40	9.43	9.24	8.81	8.34
Real	8.18	7.21	8.28	8.44	8.08	6.21	8.79	8.98	8.41	7.81
Autocorrelations										
Price ratio	.88	.92	.92	.90	.87	.90	.90	.91	.83	.90
Exchange rate										
Nominal	.83	.83	.86	.84	.86	.78	.88	.82	.83	.86
Real	.82	.80	.84	.83	.83	.76	.87	.82	.80	.83
Cross-Correlations										
Real and nominal exchange rates	.98	.97	.99	.98	.98	.95	.96	.98	.97	.99

NOTE: The statistics are based on logged and H-P-filtered quarterly data for the period 1972:1–1994:4. The statistics for Europe are trade-weighted aggregates of countries in the table. (See the text for details on construction of the data for Europe.)

Table 2

Properties of exchange rates and disaggregated consumer price indices

Statistic	Country Relative to U.S.: Denmark	France	Italy	Netherlands	Norway	Switzerland	Europe Relative to U.S.
Standard Deviations							
Price ratio							
All goods	1.33	1.07	1.62	1.48	1.91	1.69	1.26
Traded goods	1.58	1.57	2.12	2.04	2.24	1.56	1.65
Exchange rate							
Nominal	8.37	8.97	8.97	8.60	6.40	9.24	8.50
All goods real	8.05	8.41	8.27	8.26	6.15	9.01	7.95
Traded goods real	8.24	8.18	8.17	8.05	6.31	8.85	7.86
Autocorrelations							
Price ratio							
All goods	.87	.88	.83	.94	.90	.91	.92
Traded goods	.71	.87	.81	.88	.89	.73	.88
Exchange rate							
Nominal	.84	.86	.86	.84	.78	.82	.85
All goods real	.83	.84	.83	.83	.75	.81	.83
Traded goods real	.83	.84	.83	.82	.77	.82	.83
Cross-Correlations of Exchange Rates							
Real and nominal							
All goods	.99	.99	.99	.99	.95	.98	.99
Traded goods	.98	.99	.97	.97	.94	.99	.98
All and traded goods real	.99	1.00	.99	.99	.99	.99	.99

NOTE: The statistics are based on logged and H-P-filtered quarterly data for the period 1972:1–1994:4. The statistics for Europe are trade-weighted aggregates of countries in the table. (See the text for details.)

TABLE 3

Properties of exchange rates and wholesale price indices

Statistics	Austria	Belgium	Denmark	Finland	Germany	Italy	Netherlands	Norway	Spain	Switzerland	UK	Europe Relative to U.S.
	Country Relative to U.S.											
Standard Deviations												
Price ratio	2.62	4.30	2.49	1.93	2.18	3.23	2.84	1.75	3.24	2.07	3.36	2.42
Exchange rate												
Nominal	8.71	9.47	8.55	7.75	8.95	9.16	8.80	6.48	9.57	9.43	9.04	8.57
Real	7.80	6.65	6.80	6.78	8.24	7.79	8.03	6.13	8.13	9.10	8.03	7.61
Autocorrelations												
Price ratio	.74	.92	.86	.81	.88	.84	.91	.79	.87	.84	.89	.90
Exchange rate												
Nominal	.83	.87	.84	.84	.84	.86	.84	.78	.87	.82	.83	.85
Real	.79	.78	.80	.80	.82	.82	.82	.77	.84	.81	.79	.82
Cross-Correlations												
Real and nominal exchange rates	.96	.92	.97	.97	.97	.94	.95	.96	.95	.98	.93	.96

NOTE: The statistics are based on logged and H-P-filtered quarterly data for the period 1972:1–1993:3. The statistics for Europe are trade-weighted aggregates of countries in the table. (See the text for details.)

Table 4

Parameter values

Benchmark Model	
Preferences	$\beta = .99$, $\psi = 50$, $\gamma = 1.5$, $\sigma = 6$, $\eta = .39$, $\omega = .94$
Final good technology	$\rho = 1/3$, $a_1 = .9397$, $a_2 = .0603$
Intermediate good technology	$\alpha = 1/3$, $\delta = .021$, $\theta = .9$, $N = 4$
Money growth process	$\rho_\mu = .57$, $\text{corr}(\varepsilon_\mu, \varepsilon_\mu^*) = .5$
Variations[a]	
High exports	$a_1 = .7607$, $a_2 = .2393$
Real shocks	
Technology	$\rho_A = .95$, $\text{var}(\varepsilon_A) = \text{var}(\varepsilon_A^*) = (.007)^2$, $\text{corr}(\varepsilon_A, \varepsilon_A^*) = .25$
Government consumption	$\mu_g = .13$, $\rho_g = .97$, $\text{var}(\varepsilon_g) = \text{var}(\varepsilon_g^*) = (.01)^2$
Taylor rule	$\rho_r = .66$, $\alpha_\pi = 1.8$, $\alpha_y = .03$, $\text{corr}(\varepsilon_r, \varepsilon_r^*) = .5$
Incomplete markets	No changes
Sticky wages	$\vartheta = .87$, $M = 4$
Nonseparable preferences	$\xi = 2.25$

[a] Other parameters in the variations are the same as in the benchmark model, except for two parameters. The adjustment cost parameter is chosen to keep the relative volatility of consumption and output the same as in the data. The innovations to the monetary policy are chosen to keep the volatility of output the same as in the data.

Table 5

Exchange rates and prices for the models

Statistic	Data[b]	Benchmark Economy	Variations on the Benchmark Economy[a]: High Exports	Real Shocks	Taylor Rule	Incomplete Markets	Sticky Wages	Nonseparable Preferences
Standard Deviations Relative to GDP[c]								
Price ratio	.74	3.15 (.67)	3.51 (.67)	3.15 (.65)	1.36 (.27)	3.15 (.67)	2.56 (.59)	.02 (.00)
Exchange rate								
Nominal	4.74	4.84 (.77)	4.78 (.73)	4.83 (.80)	4.38 (.57)	4.74 (.76)	4.79 (.79)	.06 (.01)
Real	4.43	4.88 (.74)	4.65 (.67)	4.86 (.78)	4.69 (.62)	4.80 (.73)	5.04 (.83)	.05 (.01)
Autocorrelations								
Price ratio	.90	.93 (.02)	.92 (.02)	.93 (.02)	.92 (.02)	.93 (.02)	.95 (.01)	.81 (.05)
Exchange rate								
Nominal	.86	.68 (.08)	.68 (.08)	.68 (.08)	.45 (.09)	.68 (.08)	.69 (.07)	.83 (.04)
Real	.83	.63 (.07)	.58 (.08)	.63 (.07)	.48 (.09)	.63 (.07)	.68 (.07)	.78 (.05)
Cross-Correlations								
Real and nominal exchange rates	.99	.79 (.05)	.72 (.05)	.79 (.05)	.96 (.01)	.78 (.05)	.87 (.04)	.98 (.00)

NOTE: The statistics are based on logged and H-P filtered data. For each economy the standard deviation of monetary shocks are chosen so that the standard deviation of GDP is the same as in the data, namely, 1.76 percent. Numbers in parentheses are standard deviations of the statistic across 100 simulations.

[a] See Table 4 for specifications of the variations of the benchmark economy.

[b] Except for the standard deviations relative to GDP, the statistics in the data column are taken directly from Table 1. The standard deviations relative to GDP are the standard deviations in Table 1 divided by the standard deviation of U.S. GDP, which is 1.76 percent.

[c] The standard deviations of the variables are divided by the standard deviation of GDP.

TABLE 6

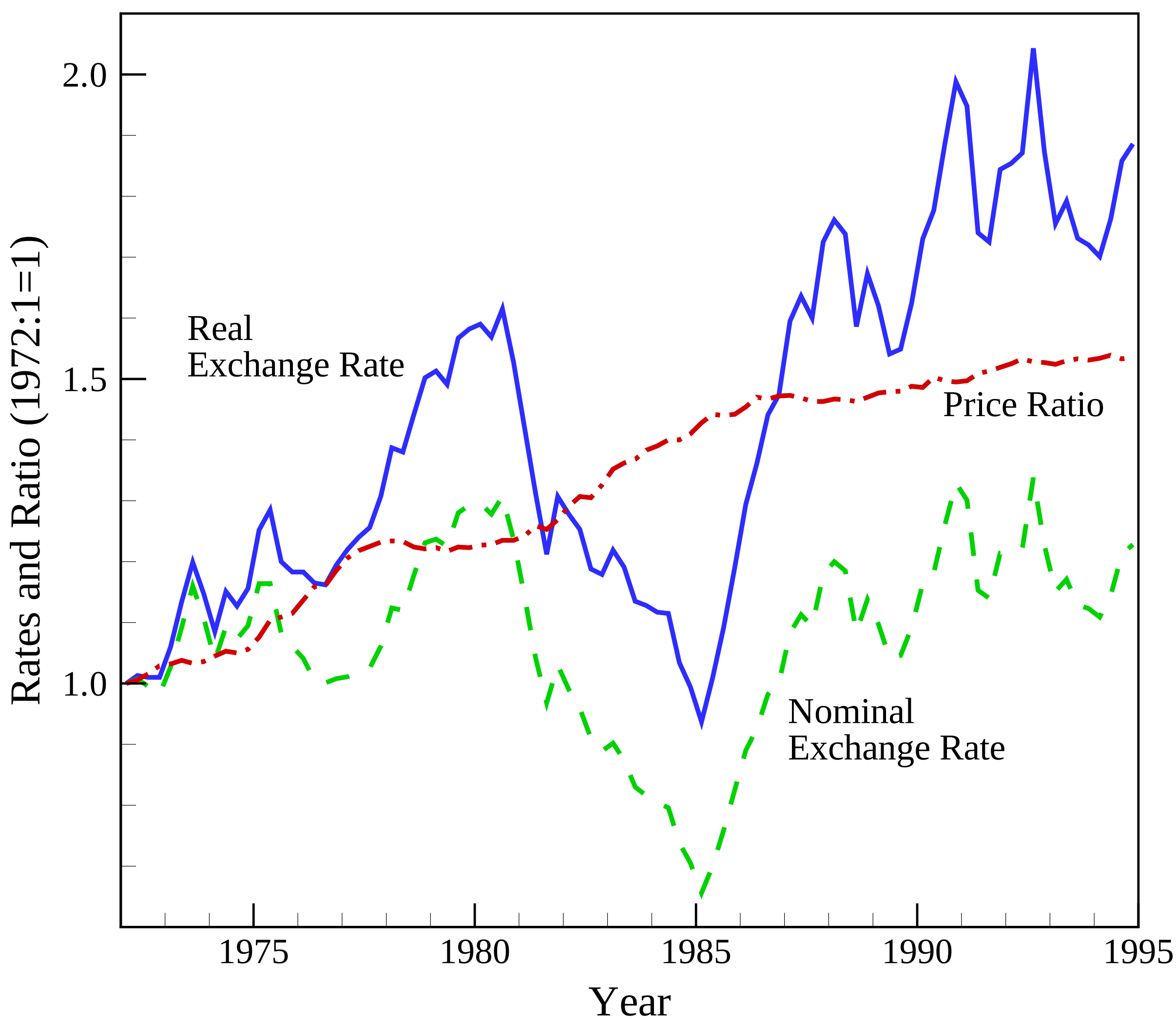
Business cycle statistics for the models

Statistics	Data[a]	Benchmark Economy	High Exports	Real Shocks	Taylor Rule	Incomplete Markets	Sticky Wages	Nonseparable Preferences
			Variations on the Benchmark Economy					
Standard Deviations Relative to GDP								
Consumption	.79	.79	.79	.79	.79	.79	.79	.96
		(.01)	(.07)	(.03)	(.01)	(.01)	(.02)	(.00)
Investment	3.28	1.71	1.32	2.41	1.78	1.71	1.64	1.19
		(.02)	(.12)	(.07)	(.01)	(.02)	(.03)	(.01)
Employment	.72	1.50	1.40	1.58	1.51	1.50	1.50	1.51
		(.01)	(.05)	(.09)	(.00)	(.01)	(.01)	(.00)
Net exports	.04	.13	.94	.14	.08	.12	.19	.04
		(.02)	(.12)	(.02)	(.01)	(.02)	(.03)	(.00)
Autocorrelations								
GDP	.87	.64	.72	.64	.49	.64	.70	.05
		(.08)	(.06)	(.07)	(.09)	(.08)	(.07)	(.09)
Consumption	.85	.63	.62	.63	.48	.63	.67	.05
		(.08)	(.08)	(.08)	(.09)	(.08)	(.07)	(.09)
Investment	.91	.63	.62	.63	.48	.63	.69	.05
		(.08)	(.08)	(.08)	(.09)	(.08)	(.07)	(.09)
Employment	.95	.64	.68	.64	.48	.64	.69	.05
		(.08)	(.07)	(.07)	(.09)	(.08)	(.07)	(.09)
Net exports	.61	.74	.64	.75	.64	.73	.81	.12
		(.05)	(.07)	(.05)	(.07)	(.05)	(.04)	(.10)
Cross-Correlations								
Between foreign and domestic								
GDP	.52	.48	.27	.47	.52	.48	.44	.50
		(.14)	(.20)	(.15)	(.11)	(.14)	(.17)	(.09)
Consumption	.27	.49	.53	.49	.50	.49	.50	.50
		(.13)	(.12)	(.14)	(.11)	(.14)	(.15)	(.09)
Investment	.22	.49	.52	.49	.50	.49	.49	.39
		(.14)	(.13)	(.14)	(.11)	(.14)	(.15)	(.11)
Employment	.40	.48	.46	.45	.52	.48	.45	.50
		(.14)	(.16)	(.14)	(.11)	(.14)	(.16)	(.09)
Between net exports and GDP	−.37	.14	.50	.14	−.07	.15	.27	−.49
		(.17)	(.13)	(.17)	(.16)	(.17)	(.18)	(.09)
Between real exchange rates and								
GDP	.10	.51	.35	.50	.49	.51	.52	.16
		(.13)	(.14)	(.13)	(.11)	(.13)	(.14)	(.10)
Net exports	.09	.09	−.17	.10	−.26	.10	.26	−.25
		(.12)	(.12)	(.13)	(.12)	(.12)	(.12)	(.05)

NOTE: Notes *a* and *c* of Table 5 also apply here. With the exception of net exports, the standard deviation of each variable is divided by the standard deviation of output. Throughout the table we measure net exports as the H-P-filtered ratio of real net exports to real gross domestic product. Thus, the standard deviation of net exports is simply the standard deviation of this ratio.

[a] With the exception of net exports, the standard deviations and autocorrelations in the data column are based on logged and H-P-filtered U.S. quarterly data for the period 1972:1–1994:4. The cross-correlations between domestic and foreign variables are based on the U.S. and an aggregate of Europe.

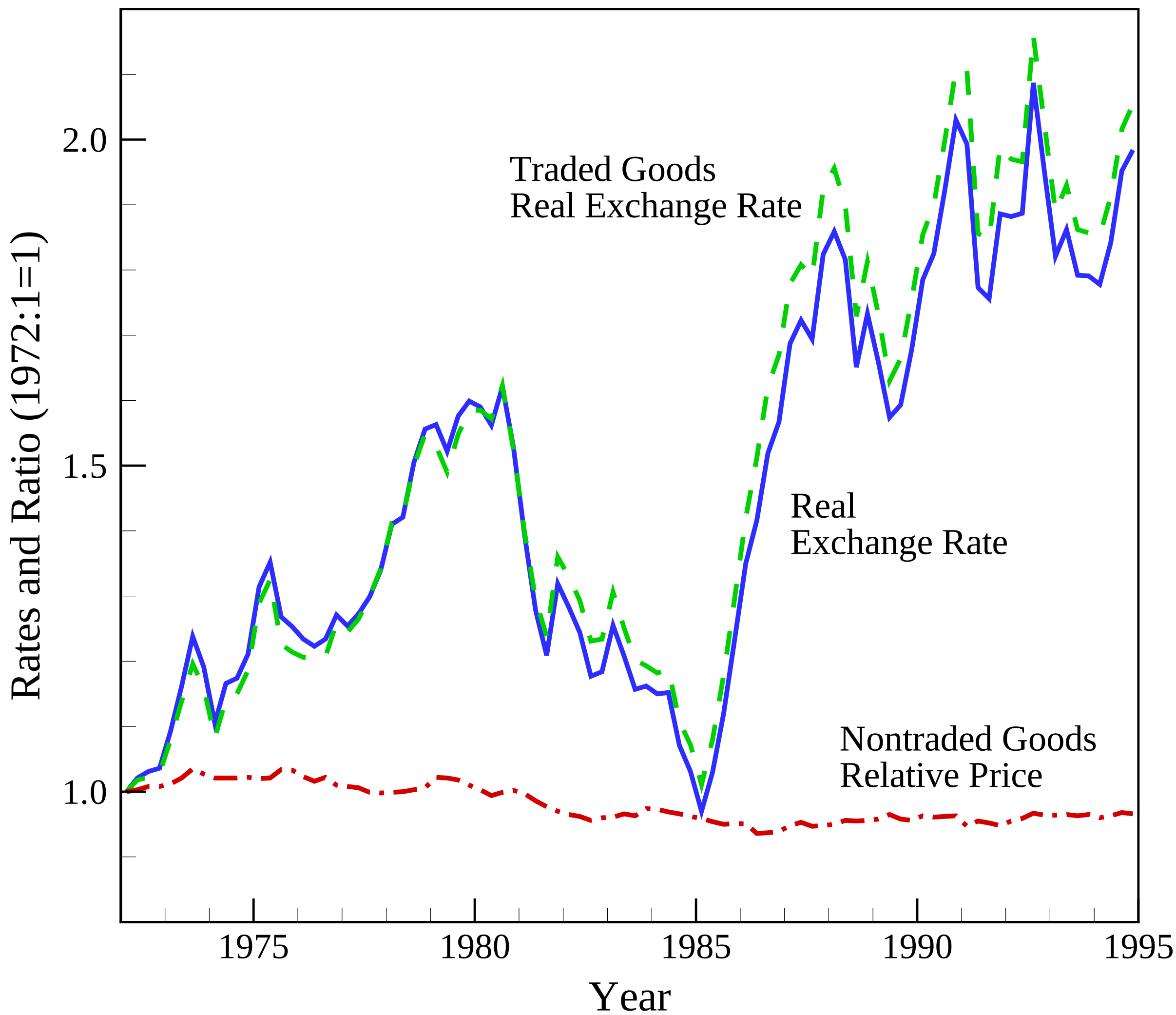
Figure 1
Exchange rates and price ratio
between the United States and Europe



Source of basic data: DRI

NOTE: The real exchange rate is eP^*/P, where the nominal exchange rate e is the U.S. dollar price of a basket of European currencies, P^* is an aggregate of European CPIs, and P is the U.S. CPI. The price ratio is P^*/P.

Figure 2
Decomposing real exchange rates



Source of basic data: OECD and U.S. Department of Labor

Figure 3

The benchmark model's real exchange rate properties
vs. Its curvature parameter

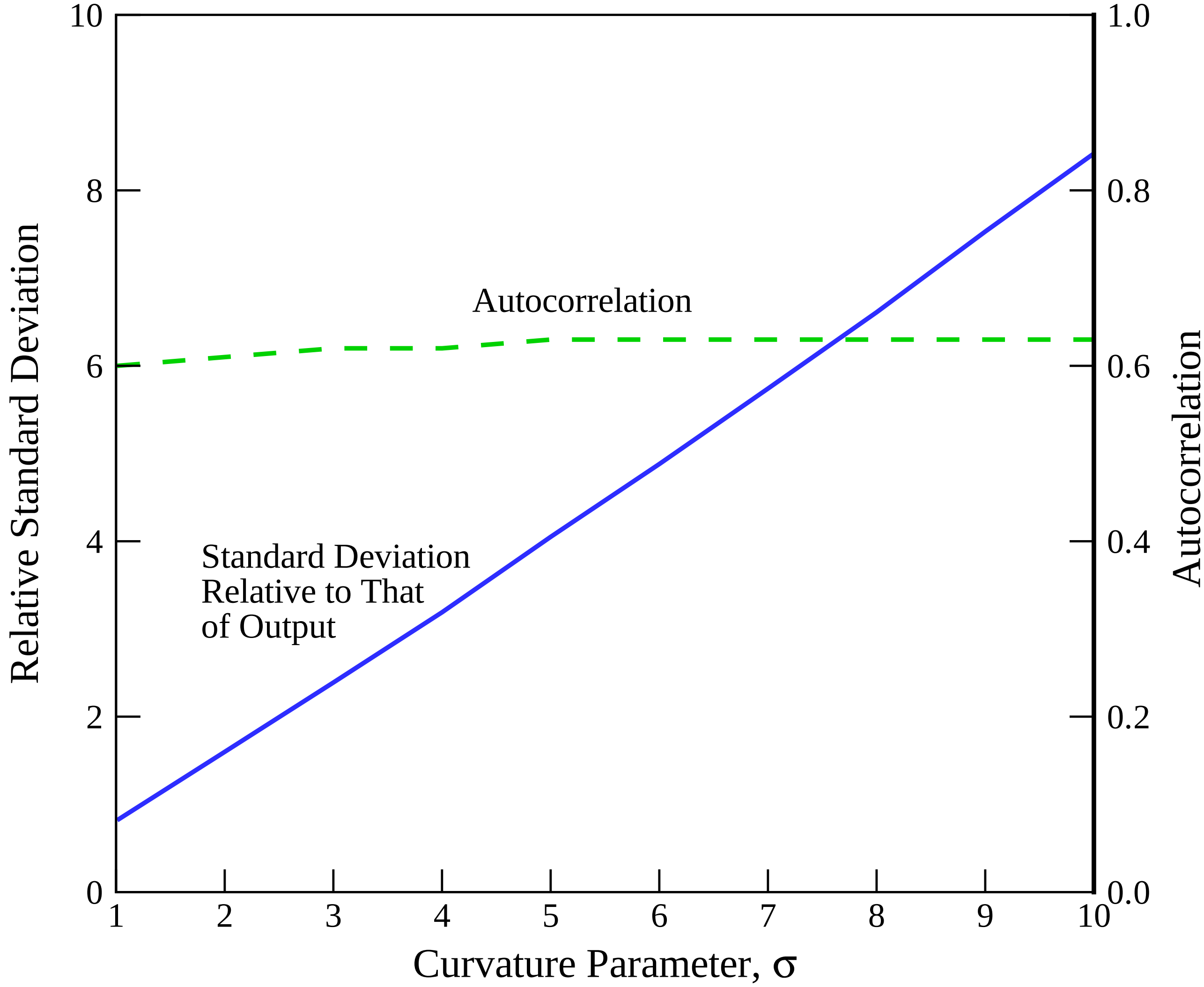


Figure 4
Measures of persistence vs. Price-stickiness