Department of Economics

Queen's University

Econ320: Macroeconomic Theory II

Instructor: Afifa Khazri

Assignment 1

(Due: Wednesday, February 16th, 2005)

Section A: Read the following statements and indicate whether they are **True**, **False** or **Uncertain**. Briefly explain your answer. All questions have equal value. NO MARKS WILL BE GIVEN FOR UNSUPPORTED ANSWERS.

A1: The 1983 *Economic report of the President* contained the following statement: "Devoting a larger share of national output to investment would help restore rapid productivity growth and rising living standards."

A2: In a pure Keynesian model (with fixed prices and interest rates), and increase in saving will reduce aggregate production.

A3: In the augmented Solow model, with Cobb-Douglas technology:

$$Y_t = {K_t}^{\alpha} {H_t}^{\beta} (A_t L_t)^{1-\alpha-\beta}$$

and capital accumulation equations:

$$\dot{K} = s_K Y_t$$

$$\dot{H} = s_H Y_t$$

the economy has a unique balanced growth path with positive growth.

Section B: Answer any 2 of the following 3 Long Questions.

B1: Consider the basic Solow growth model with production technology

$$Y_t = {K_t}^{\alpha}(A_t L_t)^{1-\alpha}$$

The savings rate is s, the rate of depreciation is δ, the population growth rate is n and the rate of technical change is g. Assume output and factor markets are competitive. Suppose the economy starts on a Balance Growth Path and then the savings rate increases permanently.

(a) What is the impact on the long-run values of the capital stock and output per unit of effective labor, and the long run level and growth rate of per capita output ?

(b) Illustrate how these variables evolve towards their long-run values following the increase in s.

(c) How does consumption per unit of effective labor, c, evolve after the increase in s. Does the long run value of c rise or fall relative to its initial value ? Explain.

(d) What savings rate would ensure that along the BGP the economy is dynamically efficient ?

(e) The Solow model implicitly assumes that technical change in consumption and investment goods producing sectors is equal in the long run. What consequences would relaxing this assumption have for the aggregate resource constraint and the user-cost of capital ?

B2: In the Cobb-Douglas version of the Solow model, suppose that output depends upon natural resources as well as capital and labor, so that:

$$Y_t = {K_t}^{\alpha}{T_t}^{\beta}(A_t L_t)^{1-\alpha-\beta}$$

where T_t represents the level of natural resources used in period t. Suppose that the "user cost" of a unit of natural resources is v_t, the user cost of capital is q_t and the wage per worker is w_t.

(a) Derive the intensive form production function. Assuming competitive factor markets, derive expressions for factor prices as a function of capital, k_t, and natural resources, τ_t, per effective worker.

(b) If Z_t represents the supply of resources, write down the equilibrium condition which determines v_t.

(c) Under what conditions will the intensive form production function have the same general form as that without natural resources (i.e. where output per effective worker can be expressed as a constant function of capital per effective worker).

(d) If the growth in natural resources is in fact given by $\frac{\dot{Z}_t}{Z_t} = -\eta < 0$, characterize the long-run growth path of the economy.

B3: Consider a simple economy inhabited by N identical households. Each household has a one period planning horizon, is endowed with one unit of time which it can allocate between work at wage w_t or leisure, and is endowed at time t with a stock of assets k_t which earns a return q_t. Assume the household's preferences over consumption, c_t, savings, s_t, and fraction of hours worked, l_t, are given by:

$$u(c_t, s_t, h_t) = lnc_t + \beta lns_t + \gamma ln(1 - l_t)$$

The household faces a time t budget constraint given by:

$$c_t + s_t = (1 - \tau_t)w_t l_t + q_t k_t + T_t = y_t$$

where y_t is total household income, τ is a proportional wage tax and T_t is a lump sum transfer from the government. Assume that the government balances its budget each period, so that:

$$\tau w_t l_t = T_t$$

but that the household takes the transfer T_t as given when making its decisions.

(a) Given the one-period planning horizon, show that consumption and savings are proportional to total income, y_t.

(b) Show that optimal household labor supply can be expressed as:

$$l_t = 1 - \frac{a y_t}{(1-\tau) w_t}$$

where a is constant term. What is the impact of an increase in τ on household labor supply?

Now suppose that we append this household model to a Solow growth model (with no technical change and no population growth). There are N identical households and the aggregate production function is given by:

$$Y_t = {K_t}^{\alpha} {L_t}^{1-\alpha}$$

where L_t denotes aggregate hours worked and the aggregate capital stock evolves according to:

$$K_{t+1} = (1-\delta) K_t + I_t$$

where I_t denotes aggregate investment. Factor markets are competitive and clear every period.

(c) Derive expressions for the wage, w_t, and the user cost of capital, q_t, as functions of the capital stock per hour worked. Show that the economy's aggregate resource constraint (in per household terms) is equivalent to the household budget constraint.

(d) Show that household labor supply is constant in general equilibrium. Why do you think the equilibrium labor supply is unrelated to the wage? (Hint: think about the income and substitution effects on labor supply for this example).

(e) What is the capital stock per household and income per household along the balanced growth path? What is the impact of an increase in the tax rate τ on income per household along the balanced growth path? Using a diagram, characterize the transitional dynamics associated with this increase.