Introduction

- Economic growth has been one of the most hotly debated topics in the economic theory history.
- The importance of studying economic growth is due to the vast differences in standards of living over time and across countries.
- The goal of research on economic growth is to determine whether there are possibilities for rising overall growth or bringing standards of living in poor countries closer to those in the world leaders.

Kaldor Facts in industrialized countries

- Output per worker grows over time
- Capital per worker grows over time
- Capital-output ratio does not change much over time
- Rate of return to capital(interest rate) is steady
- Constant shares of labor and capital expenditures in National Income
- Wide disparities in growth rates across countries

Growth Facts

- Variability of growth rates (over time and across countries)
- Average real income today is between 10 to 30 times larger than a century ago.
- Decline in productivity growth (overage annual growth in output per person)in industrialized countries since 1970.
- Enormous differences in standards of living countries.

Questions

- Are there any policies that the government of poor countries could have implemented to grow as fast as Japan?
- If so, which ones?
- If not, what was so special about the nature of Japan?

Solow's Model

(Modeling economic growth)

•Solow model is the starting point for almost all analyses on long-run growth and first-order cross country differences.

•The principal conclusion of the Solow model is that ***the accumulation of physical capital cannot account for either the vast growth over time in output per person or the vast geographic differences in output per person***.

Assumptions of the model

- The evolution of labor and knowledge is exogenous.
- Population equals labor force
- Labor and Knowledge grow at constant rates

$$?L_t = n L_t$$

$$?A_t = g A_t$$

- Effective labor grows at the rate (g +n), the combined rates of technical change and population growth.

A. Producers

•The economy has some amounts of capital (K), labour (L) and knowledge (A), and these are combined to produce output (Y)

$$Y_t = F(K_t, A_t L_t)$$

•The production function has constant returns to scale in its two arguments, capital and effective labour

$$F(\lambda K_t, \lambda A_t L_t) = \lambda F(K_t, A_t L_t)$$

- The initial level of K, L and A are taken as given.

- The intensive form of the production function: the amount of output per unit of effective labour depends only on the quantity of capital per unit of effective labour.

Y/AL = F(K/AL,1)

y = f(k)

- Where f'(.) > 0 and f''(.) < 0
- The change in capital stock equals aggregate investment less depreciation.

? K_t =I_t - dK_t

EXAMPLE: $Y_t = K_t^{\theta}(A_t L_t)^{(1-\theta)}$

- Solving the firm profit maximizing problem, we have,

$$\max_{K,L} \{(A_t L_t)^{(1-\theta)} K_t^{\theta} - w_t L_t - r_t K_t\}$$

$F.O.C.$

$w_t = (1-\theta)(A_t)^{(1-\theta)} L_t^{-\theta} K_t^{\theta}$ and $r_t = \theta_t (A_t L_t)^{(1-\theta)} K_t^{\theta-1}$

we can then compute the shares,

$\frac{w_t L_t}{Y_t} = 1-\theta$, and $\frac{r_t K_t}{Y_t} = \theta$ which are constant.

- In "per worker" terms: Output per effective worker

(Y_t/$A_t L_t$)= ($K_t^?$/$A_t L_t$) ($A_t L_t$)$^{(1-?)}$

=(K_t/$A_t L_t$)$^?$

y_t = $k_t^?$

B. Consumer

- Assume <u>fixed saving rate</u>: S =*sY*

(Consumers have already made the consumption-saving decision)

- Supply of saving will determine the level of investment (Say's low)
- Given an income of Y
  - consumption: C = *Y* - *S* = (1 - *s*) ·*Y*
  - saving (invest): I = S =sY

The Dynamics of the model
1. The dynamic of k

? k_t/k_t = (? K_t/K_t) - (? A_t/A_t) – (? L_t/L_t)

? k_t = (? K_t/K_t) k_t – (g + n) k_t

? k_t = (? K_t/K_t) (K_t/$A_t L_t$) – (g + n) k_t

= (? K_t/$A_t L_t$) – (g + n) k_t

= (sY_t - dK_t)/($A_t L_t$) – (g + n) k_t

= (sy_t $A_t L_t$- dk_t $A_t L_t$)/($A_t L_t$) – (g + n)k_t

= sy_t - dk_t – (g + n) k_t

= sy_t -(g + n + d) k_t

- The first term in right hand of this equation, is saving per worker witch is equal to gross investment per worker. The second term, is the break-even investment, the amount of investment that must be done to keep capital stock per effective worker constant. Reasons:
  - To replace capital witch depreciate (dk)
  - To produce capital to new employees in the market (nk)
  - To produce capital to more productive employees (gk)
- Thus:
  - If Δ*k* > 0: economy accumulates capital per worker
  - If Δ*k* < 0: economy reduces capital per worker
  - If Δ*k* = 0: constant capital per worker: **steady state**

2.Computing the steady state

Steady state: situation in which the economy replicates itself over time

- In steady state, $\Delta k = 0$

$$\Delta k = s \cdot f(k) - (n+g+\delta)k$$
$$0 = s \cdot f(\bar{k}) - (n+g+\delta)\bar{k}$$
$$s \cdot f(\bar{k}) = (n+g+\delta)\bar{k}$$
$$\frac{s}{n+g+\delta} = \frac{\bar{k}}{f(\bar{k})}$$

- Along the balanced growth path, the output and capital stock grow at the same rate as that of the effective labour, which is independent of the saving rate. The saving rate only affects the level of steady state, not the long-run growth.

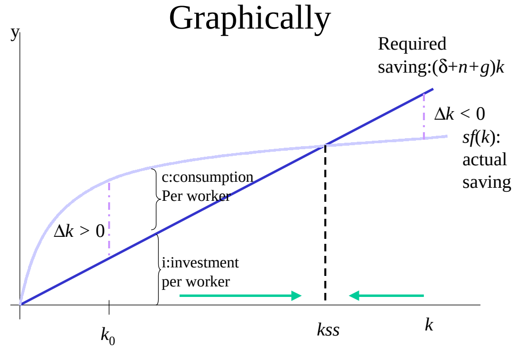
$$K_t = \bar{k} A_t L_t \Rightarrow \frac{\Delta K}{K} = n + g$$
$$Y_t = \bar{y} A_t L_t \Rightarrow \frac{\Delta Y}{Y} = n + g$$

- Along the BGP,the capital stock grows at the same rate as income, so that K/Y is constant:

$$\frac{K_t}{Y_t} = \frac{\bar{k} A_t L_t}{\bar{y} A_t L_t} = \frac{\bar{k}}{\bar{y}} = \frac{s}{n+g+\delta}$$

- Along the BGP, the rate of growth of income per worker depends only on the rate of technological progress, not on the rates of saving and population growth.

Graphically



Implications of the model

- The economy converges, over time, to its steady state.
  - If the economy starts BELOW the steady state, it accumulates capital until it reaches the steady state.
  - If the economy starts ABOVE the steady state, it reduces capital until it reaches the steady state.

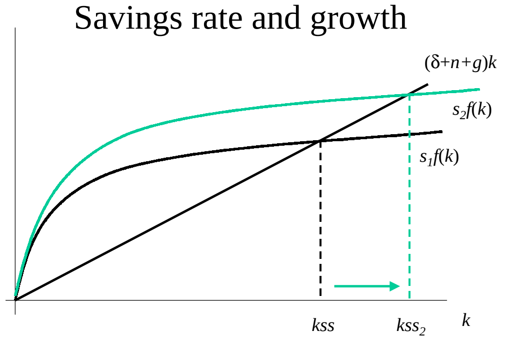
Comparative statistics

Parameters of the model: s, n,g, δ

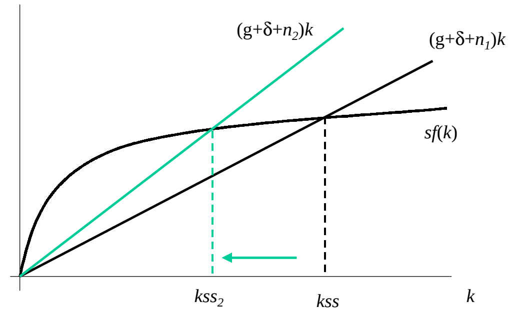
Predictions of the model:

- In steady state:
  - Higher savings rate implies higher income per worker
  - Higher population growth implies lower income per worker
  - Growth rates decrease as the country approaches the steady state
  - Countries that start further away from the steady state grow faster

Savings rate and growth



Population growth rate and growth



Long-run effect of a rise in saving on output

$$\frac{\partial y^*}{\partial s} = f'(k^*)\frac{\partial k^*(s,n,g,\delta)}{\partial s}$$

$$\frac{\partial k^*}{\partial s} = ?$$

$$sf(k^*(s,n,g,\delta)) = (n+g+\delta)k^*(s,n,g,\delta)$$

$$\Rightarrow sf'(k^*(s,n,g,\delta))\frac{\partial k^*}{\partial s} + f(k^*) = (n+g+\delta)\frac{\partial k^*}{\partial s}$$

$$\Rightarrow \frac{\partial k^*}{\partial s} = \frac{f(k^*)}{(n+g+\delta) - sf'(k^*)}$$

$$\Rightarrow \frac{\partial y^*}{\partial s} = f'(k^*)\frac{f(k^*)}{(n+g+\delta) - sf'(k^*)}$$

$$\frac{\partial y^*}{\partial s}\frac{s}{y^*} = \frac{s}{f(k^*)} f'(k^*)\frac{f(k^*)}{(n+g+\delta) - sf'(k^*)}$$

$$= \frac{f'(k^*)}{f(k^*)}\frac{sf(k^*)}{(n+g+\delta) - sf'(k^*)}$$

- $sf(k^*) = (n+g+\delta)k^*$

$$\Rightarrow \frac{\partial y^*}{\partial s}\frac{s}{y^*} = \frac{f'(k^*)(n+g+\delta)k^*}{f(k^*)\left[(n+g+\delta) - (n+g+\delta)k^*\frac{f'(k^*)}{f(k^*)}\right]}$$

$$= \frac{k^* f'(f^*)/f(k^*)}{1 - k^* f'(f^*)/f(k^*)}$$

$$\equiv \frac{\theta(k^*)}{1-\theta(k^*)}$$

•In competitive market, capital earn its marginal product. In most country, the share of income paid to capital is about 1/3. Thus the elasticity of output with respect to the saving rate in the long-run is about ½.

•Significant changes in saving have moderate effects on the level of output on the balanced growth path.

Convergence

- There are three reasons for convergence:
- Countries converge to their BGP
- The rate of return on capital is lower in countries with more capital per worker. Capitals flow from rich to poor countries.
- Diffusion of knowledge converges the effectiveness of labour.

Growth accounting

$$dY = \frac{\partial Y}{\partial K}dK + \frac{\partial Y}{\partial L}dL + \frac{\partial Y}{\partial A}dA$$

$$\frac{dY}{Y} = \frac{K}{Y}\frac{\partial Y}{\partial K}\frac{dK}{K} + \frac{L}{Y}\frac{\partial Y}{\partial L}\frac{dL}{L} + \frac{A}{Y}\frac{\partial Y}{\partial A}\frac{dA}{A}$$

$$\frac{dY}{Y} = \theta\frac{dK}{K} + (1-\theta)\frac{dL}{L} + R$$

•The first term, in right is the fraction of output growth attributable of the capital. The second term is the fraction of output growth due to growth of labor force. The third term is the fraction of output due to productivity growth which is called the ***Solow residual***.

•Solow residual reflects all sources of growth other than the contribution of capital and labour growth.

•The Solow residual is used by Real Business Cycles Models

Central Questions of the model

- How explain the differences in output per worker across time and space?
- Two sources of variation in output per worker in the Solow model: capital per worker (K/L) and effectiveness of labour (A)
- The output per worker in industrialized countries is about 10 times larger than poor countries:

$$\frac{y_1}{y_2} = 10$$

$$\Rightarrow \frac{k_1^{\theta}}{k_2^{\theta}} = 10$$

$$\Rightarrow \frac{k_1}{k_2} = 10^{\theta} = 10^3 = 100($$

- There is no evidence of such differences in capital stocks. Differences in physical capital per worker cannot account for the differences in output per worker in space and time.
- Conclusion: ***differences in standards of living is attributed to differences in the effectiveness of labour.***
- Solow model successfully explained the stylized facts of Kaldor. In this model capital accumulation does not account for a large part of long-run growth. In fact, if we don't consider population and technology growth in our model, there is no long run growth.

- In addition, this model attributes most of long run growth to the growth in effectiveness labor, whose exact meaning is vague and whose behavior is taken as exogenous.
- Hence the model could not provide us with satisfying answer to our central question about economic growth: What is a basic driving force of the economic growth?
- Development of endogenous growth theory (these models are beyond the scope of this course.)
- Solow model suggest that, if there are no barriers to capital mobility, a country with high saving does not mean a country with high investment.
- Feldstein and Horika (1980)found that there is strong relation between saving and investment.