

Rational Expectations And The Lucas Critique

According to Phillips curve, one could achieve and maintain a permanently low level of unemployment merely by tolerating a permanently high level of inflation. As you can recall from Phillips curve, the cost of reducing inflation is often measured by the sacrifice ratio, which is the number of percentage points of GDP that must be forgone to reduce inflation by 1 percentage point. Because these estimates of the sacrifice ratio are often large, they lead some economists to argue that policymakers should learn to live with inflation, rather than incurring the large cost of reducing it. According to advocates of the rational expectations approach, however, these estimates of the sacrifice ratio are unreliable because they are based on adaptive expectations, so they are subject to the Lucas critique.

Adaptive expectations imply systematic errors in forecasting and do not take account of other relevant information. In the early 1970s, Robert E. Lucas Jr, developed an alternative theory of the Phillips curve and the money-driven business cycle, under the assumption of rational expectations. He showed that a positive relationship between output and inflation could arise because of imperfect information regarding the aggregate price level. The Lucas supply function with rational expectations implies that only unanticipated changes in the money supply affect real output. Anticipated changes in the money supply affect only the price level leaving real output equal to potential. Two crucial assumptions underlie the policy ineffectiveness result: Prices and wages are perfectly flexible, and expectations are rational. If prices are sticky, anticipated changes in the money supply have an effect on real output, even if expectations are rational.

Lucas Imperfect Information Model

I) Assumptions:

1. Microeconomic foundations:

This model attempts to explain the departures of actual GDP from potential GDP using the microeconomic theory of firm's supply. Price and wages are assumed to be flexible in this model. According to microeconomic theory, a firm produces up to the point where its price equals marginal cost. Marginal cost depends on the price of the firm's inputs to production. The firm will produce more only if the price of its output rises relative to some other prices in the economy, in particular, its input prices.

When the general price level rises, as it does in booms, there is not necessarily any change in relative prices. Hence, the firm's output price will not rise relative to its input price. So how do we explain the boom that usually occurs when the general level price rises?

To generate the price output relation, we have to add another assumption about the information available to different firms.

2. **Imperfect information:**

Firms are assumed to have difficulty getting information about prices in the economy other than their own output price. There are temporary information barriers through which firms cannot see what is going on in other markets. They therefore tend to confuse movements in the overall price level with the movements in relative prices. An unanticipated inflation leads individual to infer that the relative prices of the goods they produce are temporarily high, which induces them to increase the quantity supplied. This story thus implies that output depends on the deviation of inflation from expected inflation. In this way, the assumption of imperfect information was used to generate the expectations-augmented Phillips curve of Friedman and Phelps.

3. **Rational expectations**

Lucas has emphasised the issue of how people form expectations of the future. Expectations play a crucial role in the economy because they influence all sorts of economic behaviour. Households decide how much to consume based on expectations of future income, and firms decide how much to invest based on expectations of future probability. These expectations depend on many things, including the economic policies being pursued by the government. Lucas has argued that traditional methods of policy evaluation do not adequately take into account this impact of policy on expectations and, thereby on behaviour.

4. **The model has two types of shocks:**

- A random shifts in preferences that change the relative demand for different goods. These shocks lead to changes in relative prices and in the relative production of different goods.
- Disturbances to the money supply. When these shocks are observed, they change only the aggregate price level and have no real effects. But when they are unobserved, they change both the price level and aggregate output.

II) The model

In what follows I will use lower case letters to denote the logarithm of the associated aggregate.

1. Aggregate supply

Suppliers are located in a large number of competitive markets. Producers do not observe the aggregate price level (P_t); as a result, they make their production decisions without full knowledge of the relative prices they will receive for their goods.

The main idea of the Lucas model is that when a producer observes a change in the price of his product, he does not know whether it reflects a change in the good's relative price or a change in the aggregate price level. A change in the relative price alters the optimal amount to produce.

Quantity supplied in each market will be viewed as the product of a normal component common to all market and a cyclical component, which varies from market to market. The secular component, reflects capital accumulation and population change.

$$Y_t(Z) = Y_{nt} + Y_{ct}(Z)$$

$$Y_{ct}(Z) = \gamma(P_t(Z) - E(P_t / I_t(Z))) \quad (1)$$

Where E is the statistical expectation of P_t , which may be based on both past observations of P_t , current observations of other relevant data and information about the structure of the economy. The symbol / is shorthand for "conditional upon". Thus this should be read as the expectation of P_t conditional upon the information available at time t.

$$I_t(Z) = \{I_{t-1}, P_t(Z)\} \quad (2)$$

$P_t(Z)$ is the price on market Z. It depends on two variables:

- P_t : an increase in the general level of price due to an increase in aggregate demand
- Z_t : random shock specific to market Z, an increase in demand on the market Z.

$$P_t(Z) = P_t + Z_t \quad (3)$$

Where: $Z_t \rightarrow N(0, \tau^2)$

$$P_t = E(P_t / I_{t-1}) + \varepsilon_t \quad (4)$$

$$\varepsilon_t \rightarrow N(0, \sigma^2)$$

When the price of the producer's good increases, there is some chance that the increase reflects a rise in the price level, and some chance that it reflects a rise in the good's relative price. The rational response for the producer is to attribute part of the change to an increase in the price level and part to an increase in the relative price, and therefore to increase output somewhat:

$$E(P_t / I_t(Z)) = \theta E(P_t / I_{t-1}) + (1 - \theta) P_t(Z) \quad (5)$$

Where: $\theta = \frac{\tau^2}{\sigma^2 + \tau^2}$

Substituting (5) into (1)

$$Y_{ct}(Z) = \gamma\theta(P_t(Z) - E(P_t / I_{t-1})) \quad (6)$$

Consider the case where all prices in the economy rise by the same amount. Each firm will observe only the increase in price of its own product, and will have to make a guess about all the other prices as summarized in the index P. If the firm does not adjust its guess of P, then it will clearly produce more. The firm thinks that its own relative price has increased. If the other firms in the economy behave the same way, then they could all produce more as a result of the general increase in prices. With all firms producing more than their potential, output in the economy, as a whole will surely be above potential. Hence we have explanation of a departure of real GDP from potential GDP, based on the confusion within individual firms about what is going on elsewhere in the economy.

The **aggregate supply function** for the entire economy is obtained by adding up the entire representative firms' supply functions. The aggregate price level P is simply the sum of the individual firms' prices divided by N, the number of firms.

$$Y_t^s = \sum_{Z=1}^N Y_t(Z) = Y_n + \sum_{Z=1}^N Y_{ct}(Z)$$

$$Y_t^s = Y_n + \gamma\theta(P_t - E(P_t / I_{t-1})) \quad (7)$$

This implies that the aggregate supply curve slopes up: when the aggregate price level rises, all producers see increases in the prices of their goods, and (not knowing that the increases reflect a rise in the price level) thus raise their output.

The slope of the aggregate supply function varies with the fraction θ of total individual price variance, $\sigma^2 + \tau^2$, which is due to relative price variation.

Remarks:

- When $\tau^2 \rightarrow 0 \Rightarrow \theta \rightarrow 0 \Rightarrow$ so those individual price changes are virtually certain to reflect general price changes, the supply curve is nearly vertical. At the other extreme when are stable (σ^2 is relatively small) the slope of the supply curve approaches the limiting value of γ .
- This Lucas aggregate supply curve is a version of **the augmented Phillips curve**:

$$\begin{aligned}
 Y_t &= Y_n + \gamma\theta(P_t - E(P_t / I_{t-1})) \\
 \Rightarrow P_t &= \frac{1}{\gamma\theta}(Y_t - Y_n) + E(P_t / I_{t-1}) \\
 \Rightarrow P_t - P_{t-1} &= \frac{1}{\gamma\theta}(Y_t - Y_n) + [E(P_t / I_{t-1}) - P_{t-1}] \\
 \Rightarrow (P_t - P_{t-1}) - [E(P_t / I_{t-1}) - P_{t-1}] &= \frac{1}{\gamma\theta}(Y_t - Y_n) \\
 \Rightarrow \gamma\theta(\pi_t - \pi_t^e) &= (Y_t - Y_n)
 \end{aligned}$$

Thus, the model predicts that inflation and the output gap are positively correlated over time.

2. Aggregate demand

The author suggests a demand function for goods of the form:

$$Y_t^d = x_t - P_t,$$

Where x_t is an exogenous shift variable equal to the observable log of nominal GNP. An empirical representation of x_t is

$$\begin{aligned}
 x_t &= x_{t-1} + \delta + \mu_t \quad (8) \\
 \mu_t &\rightarrow N(0, \sigma_\mu^2)
 \end{aligned}$$

In equilibrium:

$$\begin{aligned}
 Y_t^d &= Y_t^s = Y_t \\
 \left\{ \begin{aligned} P_t &= -Y_t + x_{t-1} + \mu_t + \delta \Rightarrow E(P_t / I_{t-1}) = x_{t-1} + \delta - E(Y_t / I_{t-1}) \\ Y_t &= \gamma\theta[P_t - E(P_t)] \Rightarrow E(Y_t / I_{t-1}) = 0 \end{aligned} \right. \\
 \Rightarrow Y_t &= \gamma\theta[-Y_t + x_{t-1} + \delta + \mu_t - E(P_t)] \\
 \Rightarrow (1 + \gamma\theta)Y_t &= \gamma\theta[x_{t-1} + \delta + \mu_t - E(P_t)] \\
 \Rightarrow Y_t &= \frac{\gamma\theta}{(1 + \gamma\theta)}\mu_t \quad (9)
 \end{aligned}$$

$$\Rightarrow P_t = x_{t-1} + \delta + \frac{1}{(1 + \gamma\theta)}\mu_t \quad (10)$$

$$\sigma_P = \left(\frac{1}{1 + \gamma\theta} \right)^2 \sigma_\mu^2 \quad (11)$$

III) Conclusion:

- From equation (9), only non-anticipated aggregate demand shocks can affect real output. However, anticipated and non-anticipated aggregate demand shocks affect the price level.
 - Variance of price level depends on the variance of aggregate demand.
 - For example, deviations in output from its natural rate, depend only on unanticipated movements in the money supply. Anticipated movements affect only the price level. Anticipated monetary policy has an effect on price because, with rational expectation, individuals take into account this policy.
 - The slope of Lucas aggregate supply is flexible, it depends on the behavior of individuals, on government policies.
 - The model's parameters depend on the individual behavior: **Structural parameters**
- Thus the main implication of the Lucas Critique is that dynamic macroeconomic should be based on microeconomic foundations. Consumer and firm behavior must be derived from dynamic microeconomic models in which their response to changes in policy and other phenomena is determined by policy-invariant structural parameters.
- Policy evaluation must take the reaction of private sector into account.
 - It shows how a positive statistical relationship (Phillips curve) can arise between output and inflation even though prices adjust instantaneously and markets clear. This positive correlation between inflation and output, is not due to sluggish price adjustment, but to the effects of unanticipated movements in the money supply which initially affect output temporarily, but eventually only cause inflation.