Investment

Introduction: Capital versus Investment

Investment is the most volatile component in GDP. When aggregate demand falls in recession, it is essentially due to a fall in investment expenditure. Total investment is equal to the sum of fixed investment, residential investment and variation of inventories.

What is investment? Strictly speaking, investment is the *change* in capital stock during a period. Consequently, unlike capital, investment is a *flow* term and not a *stock* term. This means that while capital is measured at a *point in time*, while investment can only be measured *over a period* of time.

How is the theory of investment different from the theory of capital? If all capital is circulating capital, so that it is completely used up within a period, then no capital built up during the previous period can be brought over into next period. In this special case, the theory of capital and the theory of investment become one and the same thing. With fixed capital, two decisions must be addressed: the amount of capital and the amount of investment. These are different decisions. One is about the desired level of capital stock. The other is about the desired rate of investment flow. The decisions governing one will inevitably affect the other, but it is not necessarily the case that one is reducible to the other.

In this course, I will investigate the demand for investment. I will emphasise on the standard aspects of the theory.

In this chapter, you will learn:

- Leading theories to explain each type of investment.
- Why the relationship between investment and interest rate is negative?
- Why investment increases in expansion and decrease in recession?
- What lead investment function to shift?

There are three types of investment:

1. **Business fixed investment**: businesses'spending on equipment and structures for use in production.
2. **Residential investment**: purchases of new housing units
3. **Inventory investment**: the value of change in inventories of finished goods, materials and supplies, and work in progress.

I. Understanding business fixed investment:

A) Investment and the cost of capital: Neo-classical theory

Suppose there are two types of firms:

- Production firms rent the capital they use to produce goods and services
- Rental firms own capital, rent it out to production firms.

In this context, investment is the rental firms' spending on new capital goods.

The firm's profits function:

$$\pi(K, X_1, X_2, ..., X_n) - rK,$$

It is firm's income (net of other production cost) minus capital cost. X's are exogenous, include for example, the price of firm's product and the costs of other inputs. We suppose that income function is concave over the capital input

$$\pi_k > 0, \quad \pi_{kk} < 0.$$

The profit maximising choice of capital implies that firm rents capital up to the point where its marginal revenue product equals its rental price:

$$\pi_k(K, X_1, X_2, ..., X_n) = r$$

If we differentiate this FOC, we will have:

$$\pi_{kk}(K, X_1, X_2, ..., X_n)dK = dr$$

$$\rightarrow \frac{dK}{dr} = \frac{1}{\pi_{kk}} < 0.$$

For the Cobb-Douglas production function $Y = AK^{\alpha}L^{1-\alpha}$, the marginal productivity of capital and the equilibrium is: $r = \alpha A\left(\frac{L}{K}\right)^{1-\alpha}$

The real rental price r would increase if:

- K decreases (due for example to earthquake or war.)
- L increases (due for example to population growth or immigration)
- A increases (technological progress)

When rent capital price increase, firms use less capital. The problem here is that most capital is not rented but is owned by the firms that use it. Thus there is no clear empirical counterpart of r_k. What is the *user cost of capital* for firms which own its proper capital? It is an *opportunity cost* because firm can sell its capital.

Keeping the capital has three costs to the firm:

1. **iP_k:** the nominal interest that the firm would receive if it sold the capital at price p_k and saved the proceeds. If firms borrow in the loanable funds market to finance their purchases of capital, then they incur interest. But even if firms use their own funds, they incur an opportunity cost equal to the interest they could have earned if they purchased P_k bonds instead of spending P_k to buy a piece of capital.
2. **δP_k**: depreciation cost
3. Δp_k : the price of the capital may be changing. This increase the cost of using the capital if the price is falling (since the firm obtains less if it waits to sell the capital) and decrease the cost if the price is rising.

The total cost of capital is the sum of these three parts:

$$r_k = iP_k + \delta P_k - \Delta P_k$$

$$r_k = \left[i + \delta - \frac{\Delta p_k}{p_k} \right] p_k$$

The investment tax credit reduces a firm's taxes by a certain amount for each dollar it spends on capital. Hence, the investment tax credit effectively reduces P_k.

If we take on account the tax treatment of investment, the user cost of capital is $(1-\tau)\ P_k$ in place of P_k, we have

$$r_k = \left[i + \delta - \frac{\Delta p_k}{p_k} \right] (1-\tau) p_k$$

Thus the investment tax credit reduces the user cost of capital; and increases firm's desired capital stocks.

We said that the profit maximising choice of capital implies that firm rents capital up to the point where its marginal revenue product (MRP) equals its rental price:

$$MRP = r_k = \left[i + \delta - \frac{\Delta p_k}{p_k} \right] (1-\tau) p_k$$

Profits are positive when MRP is greater than capital costs, and it is negative when MRP is less than capital costs:

$$\Delta K = MRP - \left[\mathrm{i(t)} + \delta - \frac{\Delta \mathrm{p_k}}{\mathrm{p_k}} \right](1-\tau)p_k$$

Hence:

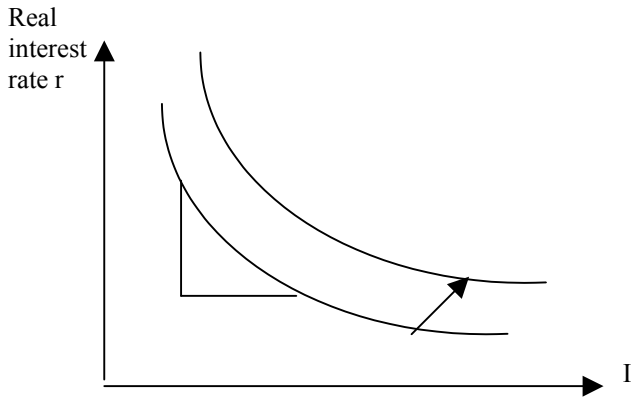
$$I = \Delta K + \delta K$$

$$I = \left\{ MRP - \left[\mathrm{i} + \delta - \frac{\Delta \mathrm{p_k}}{\mathrm{p_k}} \right](1-\tau)p_k \right\} + \delta K$$

$$I = \{MRP - [\mathrm{i} + \delta - \pi](1-\tau)p_k\} + \delta K$$

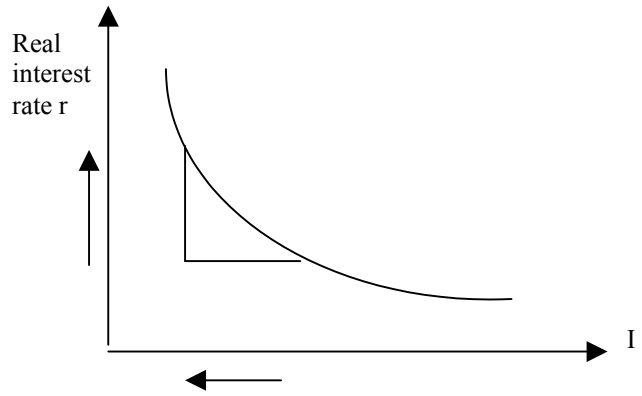
$$I = \{MRP - [\mathrm{r} + \delta](1-\tau)p_k\} + \delta K$$

Fixed investment of firms is function of marginal productivity of capital, of capital cost and depreciation rate. You can see that when interest rate increases, capital cost increases and profit of holding capital decreases so the accumulation of capital decreases and investment decreases too. We can suppose that the growth rate of price is the same that the inflation in the economy so:



- Any factor that increases marginal productivity of capital will increase investment demand and shifts the investment function to the right.

- An increase in real interest rate, r, raises the cost of capital, reduces the profit rate and reduces investment.



- The depreciation rate appears in two different places in the equation:

- It appears in the expression for the profit rate, when δ increases, raises the cost of capital and hence reduce the profit rate and the incentive to invest. This would tend to shift the curve left.
- δ appears as a coefficient on K, an increase in the depreciation rate means that more investment is needed to replace depreciating capital and keep the total capital stock at its optimal level. This effect would shift the curve right.

The net impact of the two opposing forces is ambiguous, without knowing the specific form of the investment function.

- Two of the most important taxes affecting investment are:

- Corporate income tax: A tax on profits. Corporate income tax discourages investment.
- Investment tax credit (ITC): The ITC reduces a firm's taxes by a certain amount for each dollar it spends on capital. The ITC reduces the price of capital P_k , which increases the profit rate and the incentive to invest.

Difficulties with the Baseline Model

The cost of capital theory has at least two major failings as a description of the real evolution of investment**.** A discrete change in user cost of capital leads to discrete change in the desired capital stock. This requires an infinity rate of investment. For the economy as a whole, however, investment is limited by the economy's output; thus aggregate investment cannot be infinite.

The model ignores the role of expectation on the investment demand. In practice, expectation about demand and cost are central to investment decisions. Firms expand their capital stocks when they expect their sales to be growing and the cost of capital to be low, and they contract

them when they expect their sales to be falling and the cost of capital to be high. Thus we need to modify the model.

B) Tobin's q

$$q = \frac{\text{Market value of installed capital}}{\text{remplacement cost of installed capital}}$$

If q>1, assets market evaluate the stock of capital greater than its replacement cost. Hence increases in capital stock increase the value of the firm. If q < 1, assets market evaluate that the replacement cost of installed capital is greater than its actual value. In this situation, firms should not replace their capital, so the demand for capital decreases.[1]

C) A model of investment with Adjustment Costs

Consider an industry with N identical firms. Firm's profits are proportional to its capital stock, and decreasing in the industry-wide capital stock. The production function has constant returns to scale, output markets are competitive, and the supply of all factors other than capital is perfectly elastic. The assumption that profits are decreasing in the industry's capital stock is appropriate if the demand curve for the industry's product is downward-slopping.
The firm maximise the present value of its profits:

$$\prod = \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} [\pi(K_t)\kappa_t - I_t - C(I_t)]$$

Where K_t is the capital stock of the industry, κ_t is the capital stock of the firm, I_t is investment of the firm, and C(I) reflects capital adjustment cost.

Notes:

- We make a distinction between capital stock of the industry and the capital stock of the firm.
- The firm's profits are proportional to its capital, so $\pi_{kk}=0$
- C(.) is a convex function, where C(0) = 0, C'(0) = 0, and C''(.) > 0. these adjustment cost can be due to "intrinsic" factors(i.e. costs of installation) or "extrinsic" factors (rising supply price)

[1] To have more information about the relation between neo-classical model and Tobin'q, see Fumio Hayash,"Tobin's Marginal q and Average q: A neoclassical Approch", Econometrica 50, January 1982.

- The purchase price of capital goods is constant and equal to 1; thus there are only internal adjustment costs.
- The depreciation rate is equal 0. Hence, the capital stock evolves according to

$$\kappa_{t+1} = \kappa_t + I_t$$

The Lagrangian for the firm's maximisation problem is

$$L = \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \left[\pi(K_t)\kappa_t - I_t - C(I_t) + q_t\left(\kappa_t + I_t - \kappa_{t+1}\right)\right]$$

- The Lagrangian multiplier, q_t, is the marginal value of an additional unit of capital at time t+1 in time-t dollars.
- The capital stock κ_t is fixed in the beginning of the period t. The firm choose κ_{t+1} in period t.

The first-order condition for the firm's investment in period t is :

$$-1 - C'(I_t) + q_t = 0$$
$$\Leftrightarrow q_t = 1 + C'(I_t)$$

the cost of acquiring a unit of capital equals the purchase price(equal to1) plus the marginal adjustment cost. The firm invests to the point where the cost of acquiring capital equals the value of the capital.

The first-order condition for capital in period t:

$$\frac{1}{1+r}\left[\pi(K_t) + q_t\right] = q_{t-1}$$
$$\Leftrightarrow \pi(K_t) = rq_t - \Delta q_t - r\Delta q_t$$

On the left–hand side we have the marginal revenue of capital. And the right-hand side is the opportunity cost of one unit of capital.

If the last period of activities of the firm is the period T, to be optimising, the firm cannot have capital holdings at T that have a strictly positive present value. The present value of the firm's capital at T is $(\frac{1}{1+r})^T q_T \kappa_T$, this means that this expression cannot be strictly positive, nor strictly negative either. So

$$(\tfrac{1}{1+r})^T q_T \kappa_T = 0$$

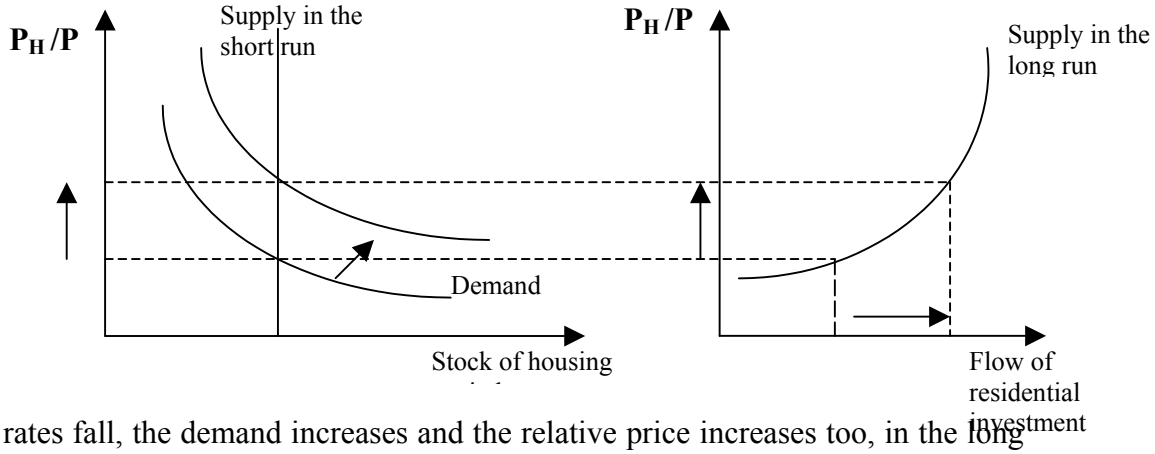
When the horizon is infinite:

$$\lim_{t\to\infty}(\tfrac{1}{1+r})^t q_t \kappa_t = 0$$

This is the *transversality condition*. It implies that the value of the stock of the capital must approach to zero. If this condition fails, then the firm is holding valuable capital forever, and so it can increase the present value of its profits by reducing its capital stock.
This reflects the fact that, if the firm has a finite horizon, it will reduce the present capital value to zero before the end of its activities.

II. Residential investment

- The flow of new residential investment, I_H , depends on the relative price of housing, (P_H /P).
- (P_H /P) is determined by supply and demand in the market for existing houses



When interest rates fall, the demand increases and the relative price increases too, in the long run the supply of new housing increases.

III. Inventory investment

Inventories consist of raw materials, goods in the process of production, and completed goods held by firms in anticipation of the products'sale.
There are four motives for holding inventories:

1) Production smoothing: Sales fluctuate, but many firms find it cheaper to produce at a steady rate.

- When sales < production, inventories rise.
- When sales > production, inventories fall.

2) Inventories as a factor of production: inventories allow some firms to operate more efficiently.

- Samples for retail sales purposes
- Spare parts for when machines break down.

3) Stock-out avoidance: to prevent lost sale in the event that demand is higher than expected.
4) Work in process: Goods not yet completed are counted as part of inventory.

The theory that explains the behaviour of inventory is **the accelerator model.** Inventory investment is proportional to the change in output. When output is raising, firms increase their inventories. When output is falling, firms allow their inventories to run down.

$$I = \alpha(Y - Y_{-1})$$

The effect of the interest rate:

The opportunity cost of holding goods in inventory is the interest rate that could been earned on the revenue from selling those goods.

Hence, inventory investment depends on the real interest rate.