Consumption, Saving and Investment

In these notes, I will investigate households' consumption choices and firms' investment choices. Consumption and investment represent the two major components of the aggregate demand for goods and services, so changes in consumption and investment decisions have important effect on the behaviour of the economy. They are important to both growth and fluctuation. The decision of how much to spend or save and invest depends on the economy's future. There is a trade off between the present and the future.

I) Keynes

A) Assumptions

1) Marginal propensity to consume (MPC) is between 0 and 1. MPC plays a central role in politics proposed by Keynes to decline unemployment (across th multiplier)
2) Average propensity to consume (APC) falls when income rises. Rich people save more than poor people.
3) Income is the most important component that affects consumption, interest rate plays a marginal role.

B) Empirically

On the basis of household data, the Keynesian theory was verified empirically in the short-run. However in the long run the negative relationship between APC and income (the second assumption) isn't realistic. In fact, at the end of the second war world II, income was higher than at the beginning of the war. Economists were afraid that the economy would be in secular stagnation. They expected that when income rises after the war consumption would fall this lead to a decrease in demand for goods and services. Fortunately for the economic activity and not for Keynes, saving rate didn't rise hence this assumption is unvalid.
Using data on consumption and income for long period, since 1869 until 1940, Simon Kuznets found that consumption was stable in spite of rising in income in these periods. APC is relatively constant in long periods.
The question is: why in short-run, the Keynesian theory is approved and in log term it isn't?

II) Irving Fisher and intertemporal choice

Irving Fisher creates the model that will be used by economists to analyse how rational consumers expect future and make their intertemporal choices of consumption and saving.

Intertemporal budgetary constraint

Suppose that household lives two periods: young and old corresponding to his working years and his retirement years. He has an income Y_1 and consume C_1 when he works, and Y_2 and C_2 when he is old. He can borrow and save in each period with the same real interest rate r.

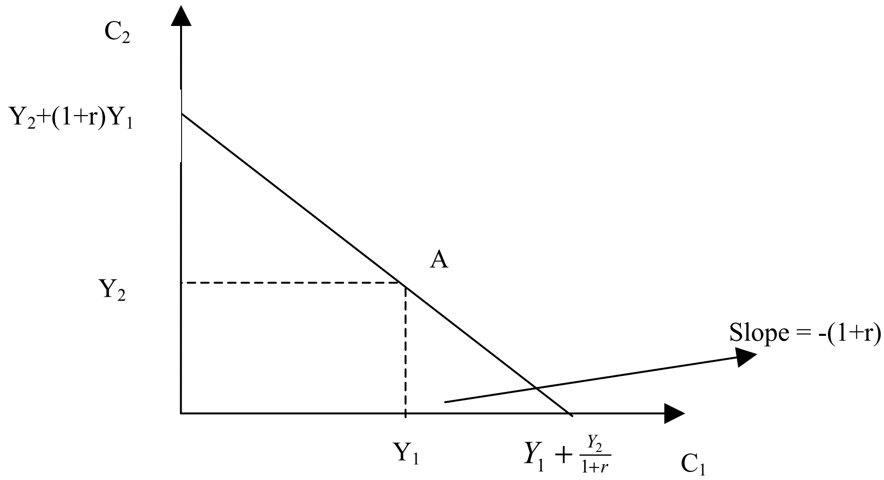
In period 1: $S = Y_1 - C_1$

$$C_2 = (1+r)S + Y_2$$

In period 2: $\Rightarrow C_2 = (1+r)(Y_1 - C_1) + Y_2$

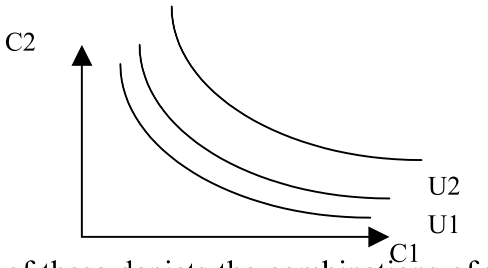
$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

This constraint means that the household's present discounted value lifetime consumption must equal its present discounted value of lifetime income.



Consumer preferences

How the consumer will allocate its consumption across the two periods in a way which achieves the highest level of utility and satisfying his budget constraint.



Each one of these depicts the combinations of C1 and C2 that yield the same level of utility U.

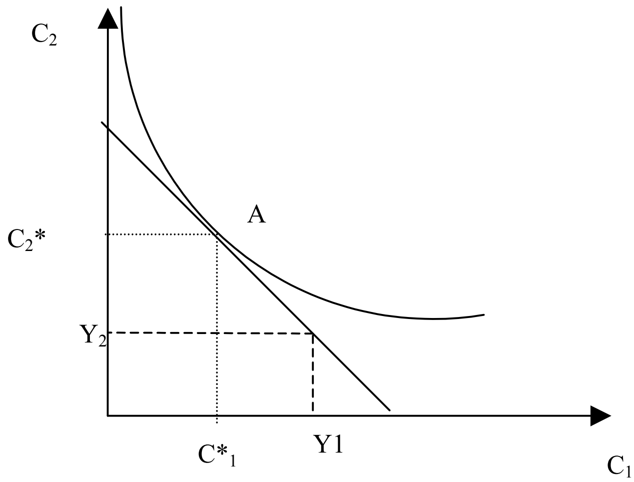
$$\overline{U} = U(C_1) + \beta U(C_2)$$
$$0 = U'(C_1)dC_1 + U'(C_2)dC_2$$
$$\left.\frac{dC_2}{dC_1}\right|_{\overline{U}} = \frac{-U'(C_1)}{\beta U'(C_2)}$$

The slope is the marginal rate of intertemporal substitution.

Optimisation: Consumer determines his consumption in each period The optimal of current consumption and saving is represented graphically by the point at which consumer's budget line is tangent to an indifference curve (point A)

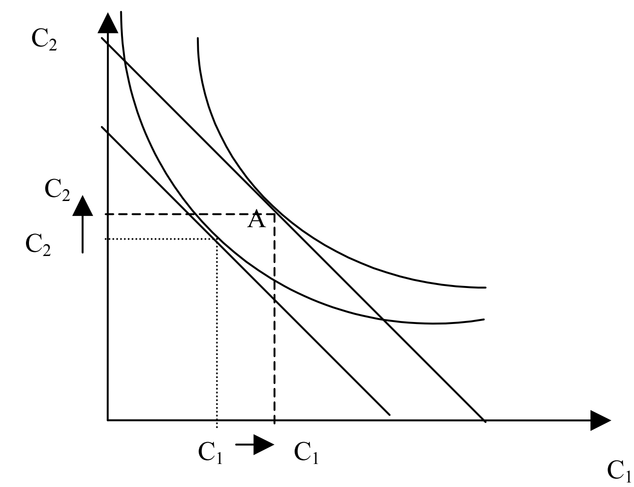
$$\frac{U'(C_1)}{\beta U'(C_2)} = (1+r)$$
$$U'(C_1) = \beta(1+r)U'(C_2)$$

Euler equation: This equation says that the marginal cost of saving an extra \$1 in period 1 and carry it over into period 2, should be equal the marginal benefit. In fact, by saving an extra \$1, the household loses \$1 of consumption in period 1, the resulting loss in utility is therefore $U'(c_1)$. In period 2, the household will have \$(1+r) extra, which it will spend on consumption. Valued in period 2, the additional utility would be (1+r) $U'(c_2)$.
Values of c_1 and c_2 that satisfy this condition and the intertemporal budget constraint constitute a solution to the household's consumption-savings decision.



In this case the household saves in the first period and consume more than its income in the second period.

Impact of an increase in income: On the contrary of the Keynesian's consumption function, consumption in Fisher's model depends on current and escompted income over life. An increase in current income or future income will increase current and future consumption (consumption smoothing motives).



III) Franco Modigliani:[1] Life-Cycle assumption

Modigliani tries to resolve the contradiction between Keynesian consumption function and the relation observed in data. He uses Fisher's model. He added the assumption of life cycle. Income changes in each period and saving helps households to carry out a part of their income from period where income is high to periods where it is low.
The origin of the assumption of life cycle is retirement. Suppose a household with an initial wealth W, he expected that he will live T periods again. He expects that he will receive an income Y before his retirement in R periods.

Resources = W+ RY
We suppose that real interest rate is zero. The consumer wants to smooth his consumption along his lifetime.
Each year he will consume:

$$C = \frac{W + RY}{T} = \frac{W}{T} + \frac{R}{T}Y$$

For example, if the consumer expect that he will live 50 years again, to work 30 years. His consumption function will be: $C = \frac{1}{50}W + \frac{30}{50}Y$

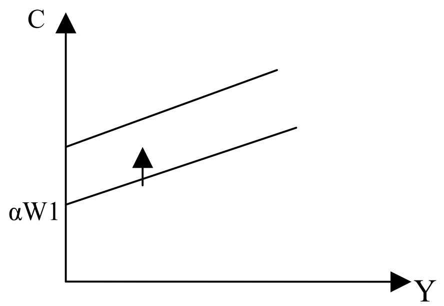
$$C = 0.02\,W + 0.6Y$$

Consumption is function of wealth and income. An increase of 1$ in his wealth will increase his consumption about 0.02 and an increase of 1$ in his income will increase his consumption about 0.6.

If all consumers have the same behaviour, the aggregate consumption function is :

$$C = \alpha W + \gamma Y$$

$$\frac{C}{Y} = \alpha \frac{W}{Y} + \gamma$$



In the short-run, when wealth is constant, the consumption function of life cycle looks like Keynesian consumption function. In the long run, wealth rises and the consumption function shifts upward. The average propensity to consume doesn't fall when income rises.

[1] Franco Modigliani ,"Life cycle, Individual thrift, and wealth of Nations", AE.R 76, Jun 86.

A Formal Model of Life-Cycle Consumption and Saving

1. Assumptions:

- The household lives for t discrete time periods.
- The household's optimal labour supply choice results in levels of real income equal to Y_1 in the first period, Y_2 in the second period and Y_T in the period T.
- The individual's wealth is such that consumption is always in the range where marginal utility is positive.
- The household can borrow and lend in unlimited amounts at the prevailing rate of interest, r.
- The individual must pay off any standing debts at the end of life.

2. The Model

The consumer maximises:

$$U = E_t \sum_{i=0}^{\infty} (1+\rho)^{-i} U(c_{t+i})$$

where:

- U(): one period utility function, separable in time, and strictly concave;
- $U'(.) > 0$ and $U'' < 0$
- E_t : mathematical expectation conditional on all information available in t;
- ρ: household's rate of time preference. The household's subjective discount factor β measures the extent to which the household values future utility relative to current utility

$$\rho \to \infty \Rightarrow \beta = \frac{1}{1+\rho} \to 0 \Leftrightarrow \text{the individual is impatient}$$

$$\rho \to 0 \Rightarrow \beta = \frac{1}{1+\rho} \to 1 \Leftrightarrow \text{the individual is patient}$$

- C_t : non durable real consumption goods in period t
- ∞: length of economic life

Subject to the T dynamics budget constraint:

$$Y_t + (1+r_t)A_t = A_{t+1} + c_t$$
$$\text{for all } t \in \{1, \ldots, \infty\}$$

r_t: real interest rate;
Y_t : earnings, are stochastic and are the only source of uncertainty.

A_t : assets apart from human capital.

The Lagrangian maximisation problem is therefore:

$$L = E_t\{U(c_t) + \lambda_t[Y_t + (1+r_t)A_t - A_{t+1} - c_t] + \beta U(c_{t+1}) + \beta\lambda_{t+1}[Y_{t+1} + (1+r_{t+1})A_{t+1} - A_{t+2} - c_{t+1}] + ...\}$$

First Order Conditions (FOC):

C_t :

$$\frac{\partial U}{\partial C_t} - \lambda_t = 0$$

A^i_{t+1}:

$$-\lambda_t + \beta E_t\{\lambda_{t+1}(1 + r^i_{t+1})\} = 0$$

λ_t :

$$Y_t + (1 + r_t)A_t = A_{t+1} + c_t$$

Euler condition:

$$\boxed{E_t\left[\frac{(1 + r_{t+1})}{(1 + \rho)}\frac{U'(c_{t+1})}{U'(c_t)} - 1\right] = 0}$$

$\Leftrightarrow$

$$E_t\left[\frac{U'(c_{t+1})}{U'(c_t)}\right] = E_t\left[\frac{(1+\rho)}{(1+r_{t+1})}\right]$$

If $r_{t+1} \uparrow \Leftrightarrow \frac{1}{1+r_{t+1}} \downarrow \Leftrightarrow \frac{U'(c_{t+1})}{(1+\rho)U'(c_t)} \downarrow$

$\Leftrightarrow U'(c_{t+1}) \downarrow$ and $U'(c_t) \uparrow$

$\Leftrightarrow c_{t+1} \uparrow$ and $c_t \downarrow$

The optimal allocation of consumption across time should satisfy the Euler equation:

IV) Milton Friedman and the assumption of disposable permanent-income[2]

As Modigliani, Friedman uses the Fisher assumption, that consumption does not depend only on the current income. However, on the contrary of the life cycle assumption that income have a regular path along the life of a person, the assumption of permanent income emphasise on the fact that income changes from year to another due to temporary and random shocks.

Assumptions

- Suppose that $r = \rho = 0$;
- Under certainty;
- Economic life length is T

$$\frac{U'(c_t)}{U'(c_{t+1})} = 1$$

$\forall$ t=1,2,3….T

$$U'(c_t) = U'(c_{t+1})$$

$$c_1 = c_2 = c_3 = \ldots = c_T = c^*$$

the intertemporal budget constraint:

$$\sum_{t=1}^{T} C_t = A_0 + \sum_{t=1}^{T} Y_t$$

$$TC_t = A_0 + \sum_{t=1}^{T} Y_t$$

$$C_t = \frac{1}{T}\left[A_0 + \sum_{t=1}^{T} Y_t\right]$$

The representative household divides his lifetime resources equally among each period of life. In the terminology of Friedman (1957), the right hand side of the equation is permanent income, and the difference between current (Y) and permanent income (Y^P) is transitory income (Y^T). Consumption is positively correlated to permanent income.

Saving is :

$$S_t = Y_t - C_t$$

$$S_t = Y_t - \left[\frac{1}{T}A_0 + \frac{1}{T}\sum_{t=1}^{T} Y_t\right]$$

$$S_t = Y_t - Y^P = Y^T$$

- When $Y_t > Y^p$, transitory income is high and saving is high too.
- When $Y_t < Y^p$, saving is negative.

---

[2] A theory of the Consumption Function , Princeton University, Press 57

Thus the individual uses saving and borrowing to smooth the path of consumption. It is the permanent income and not the current income that can explain consumption function.

$$Y = Y^P + Y^T$$

$$C = \alpha Y^P$$

$$\frac{C}{Y} = \alpha \frac{Y^P}{Y}$$

- The APC depends on the ratio of permanent income to current income.
- If changes in current income are due to changes in permanent income, the APC remains constant.
- If an increase in current income is due to an increase in transitory income, consumers does not rise their consumption. That's why, economists found that household with high income have an APC low.
- In the short-run, it is the transitory income, which changes so the average of consumption falls when income rises.
- In long run, changes in current income are attributable to changes in permanent income so the APC will remain constant.

<u>Empirical application</u>: Does consumption positively correlated to disposable current income (Keynes) or to permanent income (Friedman)?

To estimate consumption, we use the relationship:

$$C_i = a + bY_i + \varepsilon_i;$$

The estimated slop coefficient is:

$$\hat{b} = \frac{Cov\ (Y, C)}{Var\ (c)};$$

The estimated intercept is:

$$\hat{a} = \overline{C} - \hat{b}\overline{Y}$$

Empirical researches did not demonstrate a consistent stable relationship between consumption and current income. (Fig 7.1 Romer). However, the Friedman's permanent income hypothesis provides a straightforward explanation of all these findings. In short run, variations are transitory so the slope is less than one and the estimated intercept is positive (fig 7.1. a). In long run, almost all the variations reflects permanent increase in the economy's resources (fig 7.1. b).

With permanent income hypothesis, an increase in current income is associated with a proportional increase in consumption only if the variation of the permanent increase is greater than the variation of the transitory income. If an increase of the current income is attributable to an increase of the transitory income, consumption rises little with current income. For policy analysis, the pure life cycle-permanent income hypothesis supports the view that only unexpected changes in policy affect consumption, everything known about future changes in policy is already incorporated in present consumption. Further, unexpected changes in policy affect consumption only to the extend that they affect permanent income, and their effects are expected to be permanent. Policies that have a transitory effect on income are incapable of having a transitory effect on consumption.

IV) The Random-Walk hypothesis (Hall 1978)

- If the permanent income is true and if consumers have rational expectations, it is not possible to predict the variation of consumption in time. Consumption is a random-walk.
- Implication: with rational expectations, only non expected policies can have an effect ion consumption

Example:

- Suppose that r = ρ =0;
- Suppose that the instantaneous utility function is quadratic:

$$U(c_t) = C_t + \tfrac{a}{2} C_t^2$$

$$\frac{U'(c_t)}{E_t U'(c_{t+1})} = \frac{(1-ac_t)}{E_t(1-ac_{t+1})} = 1$$

- Suppose that the individual has chosen first period consumption optimally, given the information available in period 1, $E_1(.)$, and suppose he choose consumption in each period optimally given the information then available, $E_1(C_t)$, ∀ t=2,3,…,T.

$$(1-ac_t) = E_t(1-ac_{t+1})$$
$$E_t c_{t+1} = c_t$$

No information available in period t apart from the level of consumption**,** c_t **,** helps predict consumption c_{t+1} . These results are independent of the relative magnitudes of $Y_1, Y_2, \ldots, Y_T$

The simplest testable implication of the pure life cycle-permanent income hypothesis is that only the first lagged value of consumption helps predict current consumption.

$$\boxed{c_{t+1} = c_t + \varepsilon_{t+1}}$$

This is the famous result of Hall (1978), Consumption is a random walk. We can test this hypothesis by regressing c_t on c_{t-1} and verifying if other known variables at t-1 have a strong predictive power on consumption. Empirical researches rejected this theory. In fact, other variables can predict variations in consumption.