

ADF Test for Mean Stationarity:

$$\Delta y_t = \alpha_0 + \alpha_1 y_{t-1} + \sum_{j=1}^n \alpha_j \Delta y_{t-j} + \varepsilon_t$$

Where: y = Canadian real GNP per capita; t = 1870-2005; α = parameters to be estimated with OLS; ε = iid error term $\sim N(0, \sigma)$.

Unit Root Test:

$$H_0: \alpha_1 = 0$$

$$H_1: \alpha_1 < 0$$

\therefore if we can reject the null of a unit root (p value ≤ 0.10), then y is said to be *mean stationary*, which implies that all shocks or deviations from mean are transitory. Note that standard "t test" statistics are invalid, so ADF critical values must be derived.

ADF Test for Trend Stationarity:

$$\text{Linear: } \Delta y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 t + \sum_{j=1}^n \alpha_j \Delta y_{t-j} + \varepsilon_t$$

$$\text{Quadratic: } \Delta y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 t + \alpha_3 t^2 + \sum_{j=1}^n \alpha_j \Delta y_{t-j} + \varepsilon_t$$

ADF Test for Difference Stationarity:

$$\Delta D(1) y_t = \alpha_0 + \alpha_1 D(1) y_{t-1} + \sum_{j=1}^n \alpha_j \Delta D(1) y_{t-j} + \varepsilon_t$$

Where: $D(1)$ = difference operator = $y_t - y_{t-1}$.

ADF Test for Segmented Trend Stationarity:

$$\Delta y_t = \alpha_0 + \sum_{i=1}^7 \beta_i \text{dum}_i + \alpha_1 y_{t-1} + \alpha_2 t + \sum_{i=1}^7 \gamma_i \text{dum}_i \times t + \sum_{j=1}^n \alpha_j \Delta y_{t-j} + \varepsilon_t$$

Where: dum_i = seven dummy variables to segment time series (1870-96, 1897-1921, 1922-29, 1930-45, 1946-72, 1973-82, 1983-88, 1989-2005).

Note that standard ADF critical values are invalid with segmented trends, so ADF critical values must be derived for every trend break test. In general, the critical values for the unit root tests will be dependent on both the number and timing of the segments.

ADF Unit Root Tests on Real Canadian GNP per Capita, 1870-2005

	Unit Root Test Statistic	Approximate P Value
Mean Stationary:	2.730	0.999
Linear Trend Stationary:	-0.168	0.992
Quadratic Trend Stationary:	-3.060***	0.000
Difference Stationary:	-5.218***	0.000
Segmented Trend Stationary:		
All Breaks	-5.860***	0.000
1896	-0.690	0.495
1921	-2.310***	0.021
1930	-3.700***	0.000
1945	-3.780***	0.000
1972	-0.131	0.192
1982	-0.480	0.631
1988	-0.240	0.814
1896, 1921	-2.230**	0.027
1896, 1921, 1945	-3.840***	0.000

Note: Number of lags (n=2) should be determined on basis of SC or AIC information criteria testing.

* indicates statistical significance with at least 90% confidence.

** indicates statistical significance with at least 95% confidence.

*** indicates statistical significance with at least 99% confidence.