

Midterm Exam

Consider a model with two countries, H and F, two goods, X and Y, and one factor of production, labour (L). Labour is mobile between industry X and Y within a country but is not mobile across countries. Labour in each country is fully employed.

The technologies for producing the goods in country H are given by

$$X^H = a_x^H L_x^H \quad \text{and} \quad Y^H = a_y^H L_y^H.$$

The technologies for producing the goods in country F are given by

$$X^F = a_x^F L_x^F \quad \text{and} \quad Y^F = a_y^F L_y^F.$$

The price of X is normalized to 1. Denote the prices of Y (relative to X) in country H and country F by p^H and p^F , respectively. Consumers in both countries have the identical utility function given by

$$U(X, Y) = X^\beta Y^{1-\beta}.$$

In each country, there is a single consumer with the labour supply \bar{L} . The representative consumer maximizes her utility given her budget constraint,

$$X + pY \leq I,$$

where w is the wage rate (relative to the price of X) and I is income which is treated as given ($I = w\bar{L}$).

The values of productivity parameters $(a_x^H, a_y^H, a_x^F, a_y^F)$ satisfy the following relationship:

$$\begin{aligned} a_x^H &> a_x^F > 0 \\ a_y^H &> a_y^F > 0 \\ \frac{a_x^H}{a_y^H} &< \frac{a_x^F}{a_y^F} \end{aligned}$$

1. (8 points) Determine each country's autarkic equilibrium goods price ratio in terms of the model's parameters.

2. (8 points) Determine the pattern of comparative advantage between these two countries.
3. (8 points) Determine the free trade equilibrium price ratio in an equilibrium in which each country completely specialized in its good of comparative advantage.
4. (8 points) Show that there are aggregate gains from trade for each country if both countries are completely specialized by comparing each consumer's utility level in free trade with her utility level in autarky.
5. (8 points) Suppose that the consumers in both Country H and Country F supplies $2\bar{L}$ units of labour rather than \bar{L} . Determine the new free trade equilibrium price ratio in which both countries are completely specialized and explain how/why an increase in the size (labour supply) of both countries changes the equilibrium price ratio intuitively.
6. (10 points) Draw the World Production Possibility Frontier.
7. (10 points) The equilibria can be classified into three cases: (Case 1) Country H is completely specialized but Country F produces both goods, (Case 2) Both countries are completely specialized, and (Case 3) Country F is completely specialized but Country H produces both goods. *For each case*, draw graphs of the World Production Possibility Frontier and the production possibility frontiers of both countries together with indifference curves and export-import triangles.
8. (10 points) Derive the inequality conditions in terms of parameters for the case in which Country H is completely specialized but Country F produced both goods (i.e., Case 1 in Question 6).
9. (10 points) Show whether there are gains from trade for Country F and Country H when Country H is completely specialized but Country F produced both goods (i.e., Case 1 in Question 6). [Use the graphs of the production possibility frontiers for both countries].
10. Suppose that Country H is completely specialized but Country F produced both goods initially. Now suppose that Country H's labor supply increases from \bar{L} to $2\bar{L}$. (i) (10 points) Is it *possible* for Country F to become completely specialized after this change in the size of country H? (ii) (10 points) Suppose, it turns out, that Country F still produces both goods after the increase in the size of Country H. Did Country F's welfare increase due to the increase in the size of Country H?