Homework 4

(Due: April 1 before the class begins)

Consider the following monopolistic competition model. There are two countries: home country and foreign country. Both countries are identical in their preference and technology but are different in their sizes, where L denotes the home country's size and L^* denotes the foreign country's size.

All individuals have the same utility function,

$$\begin{aligned} U &= \sum_{i=1}^{n} c_{h,i}^{\alpha} + \sum_{i=1}^{n^*} c_{h,i}^{*\alpha}, \\ U^* &= \sum_{i=1}^{n} c_{f,i}^{\alpha} + \sum_{i=1}^{n^*} c_{f,i}^{*\alpha}, \end{aligned}$$

where n and n^* represent the number of goods produced in home country and foreign country, respectively. $c_{h,i}$ is the consumption of the ith home good in home country and $c_{h,i}^*$ is the consumption of the ith foreign good in home country. $c_{f,i}$ is the consumption of the ith home good in foreign country and $c_{f,i}^*$ is the consumption of the ith foreign good in foreign country.

All goods will be produced with the same cost function:

$$l_i = a + bx_i, \qquad l_i^* = a + bx_i^*,$$

where $a, b > 0$ and l_i is labor used in producing the ith good and x_i is output of that good.

Since output must be equal to consumption (i.e., supply = demand), $x_i = c_{h,i}L + c_{f,i}L^*$ and $x_i^* = c_{h,i}^*L + c_{f,i}^*L^*$. Labor market clearing implies that $L = \sum_{i=1}^{n} a + bx_i$ and $L^* = \sum_{i=1}^{n^*} a + bx_i^*$.

Initially, both home country and foreign country impose import tariffs at the rate of τ and τ^*, respectively. The consumer receives tariff revenue so that a budget constraint for home country's consumer is given by:

$$\sum_{i=1}^{n} p_i c_{h,i} + \sum_{i=1}^{n^*} (1+\tau) p_i^* c_{h,i}^* \leq I \equiv w + T$$

, where I is individual income which consists of wage, denoted by w, and lump-sum tariff transfer from the government $T = \sum_{i=1}^{n^*} \tau p_i^* c_{h,i}^*$. Importantly, each individual treats her income as given.

1. (10 points) Solve the utility maximization problem for home country:

$$\max_{c_{h,i}, c_{h,i}^*} \sum_{i=1}^{n} c_{h,i}^{\alpha} + \sum_{i=1}^{n^*} c_{h,i}^{*\alpha}$$

   subject to

$$\sum_{i=1}^{n} p_i c_{h,i} + \sum_{i=1}^{n^*} (1+\tau) p_i^* c_{h,i}^* \leq I,$$

   and show that home country's demand functions for home goods and foreign goods, $c_{h,i}$ and $c_{h,i}^*$, are:

$$\begin{aligned} c_{h,i} &= \frac{I}{p_{h,i}^{\sigma} P^{1-\sigma}} \\ c_{h,i}^* &= \frac{I}{[(1+\tau)p_{h,i}]^{*\sigma} P^{1-\sigma}}, \end{aligned}$$

   respectively, where $\sigma = \frac{1}{1-\alpha}$ and

$$P = [\sum_{i=1}^{n} p_{h,i}^{1-\sigma} + \sum_{i=1}^{n^*} ((1+\tau)p_{h,i}^*)^{1-\sigma}]^{\frac{1}{1-\sigma}} \tag{1}$$

   is the aggregate price index. Note: $\alpha = \frac{\sigma-1}{\sigma}$. Similarly, show that

$$\begin{aligned} c_{f,i} &= \frac{I^*}{[(1+\tau^*)p_{f,i}]^{\sigma} P^{*1-\sigma}} \\ c_{f,i}^* &= \frac{I}{p_{f,i}^{*\sigma} P^{*1-\sigma}}, \end{aligned}$$

   where $P^* = [\sum_{i=1}^{n} ((1+\tau)p_{f,i})^{1-\sigma} + \sum_{i=1}^{n^*} p_{f,i}^*)^{1-\sigma}]^{\frac{1}{1-\sigma}}$.

2. (10 points) Show that the aggregate price index of the equation (1) is given by

$$P = \min_{c_{h,i}, c_{h,i}^*} \sum_{i=1}^{n} p_{h,i} c_{h,i} + \sum_{i=1}^{n^*} (1+\tau) p_{h,i}^* c_{h,i}^*$$

   subject to

$$\sum_{i=1}^{n} c_{h,i}^{\alpha} + \sum_{i=1}^{n^*} c_{h,i}^{*\alpha} = 1.$$

   That is, P is the unit price of "aggregate consumption goods": $C \equiv \sum_{i=1}^{n} c_{h,i}^{\alpha} + \sum_{i=1}^{n^*} c_{h,i}^{*\alpha}$.

3. (10 points) Solve the profit maximization problem for the ith producer in home country:

$$\max_{c_{h,i},c_{f,i}} p_{h,i}(c_{h,i})c_{h,i}L + p_{f,i}(c_{f,i})c_{f,i}L^* - [a + b(c_{h,i}L + c_{f,i}L^*)],$$

where $p_{h,i}(c_{h,i})$ and $p_{f,i}(c_{f,i})$ are inverse demand functions and show that a firm sets the same price across countries as:

$$p_{h,i} = p_{f,i} = \frac{b}{\alpha} \equiv p_i.$$

4. (10 points) Assuming free entry, and hence zero profit, show that, in equilibrium,

$$x_i = c_{h,i}L + c_{f,i}L^* = \frac{\alpha a}{(1-\alpha)b}.$$

5. (10 points) Using the labor market clearing condition, $L = \sum_{i=1}^{n} a + bx_i$, show

$$n = \frac{(1-\alpha)L}{a}.$$

6. (10 points) Show that $c_{h,i} = \frac{w}{(n+n^*(1+\tau)^{-\sigma})p_i}$, $c^*_{h,i} = \frac{(1+\tau)^{-\sigma}w}{(n+n^*(1+\tau)^{-\sigma})p_i}$, $c_{f,i} = \frac{(1+\tau^*)^{-\sigma}w^*}{(n^*+n(1+\tau^*)^{-\sigma})p^*_i}$, and $c^*_{f,i} = \frac{w^*}{(n^*+n(1+\tau^*)^{-\sigma})p^*_i}$.

7. (10 points) Derive the ratio of home country's wage to foreign country's wage $\frac{w}{w^*}$ in terms of τ, τ^*, σ, L, and L^*. How does $\frac{w}{w^*}$ change as τ increases?

8. (10 points) How does a decrease in the import tariff rate of home country affect the home country's welfare? [Derive the expression for the utility level in terms of tariff rates and analyze how a decrease in tariff rates affect the utility level.]

9. (10 points) Suppose that foreign country's population increased from L^* to L^{**}. How does this increase in foreign country's population affect the home country's welfare? Explain.

10. (10 points) Suppose that, due to the technological progress in foreign countries, the fixed cost of production decreased from a to a' in foreign country. On the other hand, the fixed cost a in home country remains the same. How does this change in foreign country affect the *home* country's welfare? Explain your intuition briefly.