Answer Key to HW 2 (Econ 425) ①

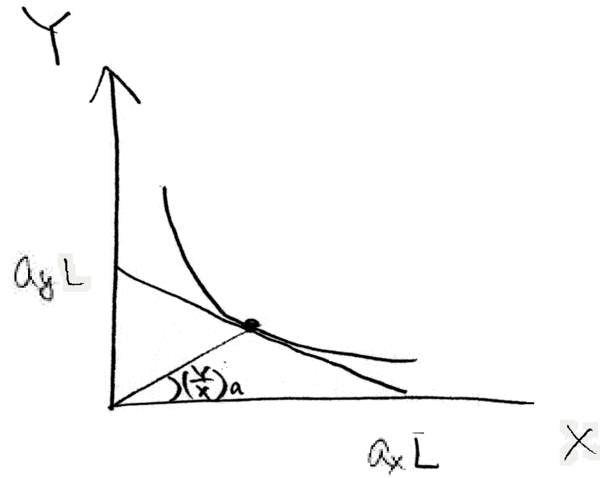
1)

* Country H

$X \quad a_x^H L_x^H$

$Y \quad a_y^H L_y^H$

$L \quad + L_y^H$



(Autarky)

Profit Maximization

$$\max_{L^H} \quad a_x^H L_x \quad w^H L_x^H$$

$$\Rightarrow \boxed{a_x^H = w^H} \qquad ⓐ$$

$$\max_{L_y^H} \quad p\, a_y^H L_y^H \quad w L_y^H$$

$$\Rightarrow \boxed{p\, a_y^H = w^H} \qquad ⓑ$$

Utility Maximization

$$\max_{X\,Y} \quad X^{\beta} Y^{1-\beta}$$

$$\text{s.t} \quad X + pY \le w^H L^H$$

$$\rightarrow \boxed{\begin{aligned} X &= \beta\, w^H \bar{L}^H \\ Y &= (1-\beta)\, \frac{w^H \bar{L}^H}{p} \end{aligned}} \qquad ⓒ$$

From a & b ,

$$a_x^H = w^H = p\, a_y^H$$

$$\Rightarrow \boxed{p_a^H = \frac{a_x^H}{a_y^H}}$$

autarky price

$$\boxed{\left(\frac{X}{Y}\right)_a^H = \frac{\beta}{1-\beta}\, p_a^H = \frac{\beta}{1-\beta}\, \frac{a_x^H}{a_y^H}}$$

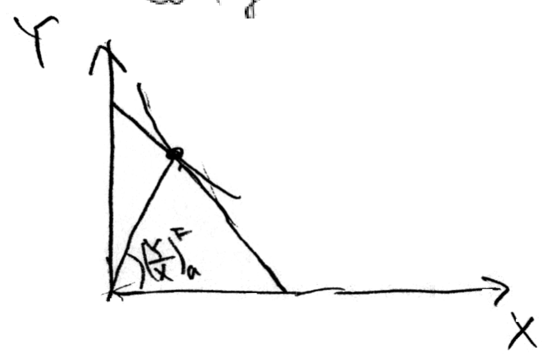
autarky production & consumption

Similarly for country F

$$\boxed{p_a^F = \frac{a_x^F}{a_y^F}}$$

$$\left(\frac{X}{Y}\right)_a^F \quad \frac{\beta}{1-\beta} \quad \frac{a_x^F}{a_y^F}$$

Country F

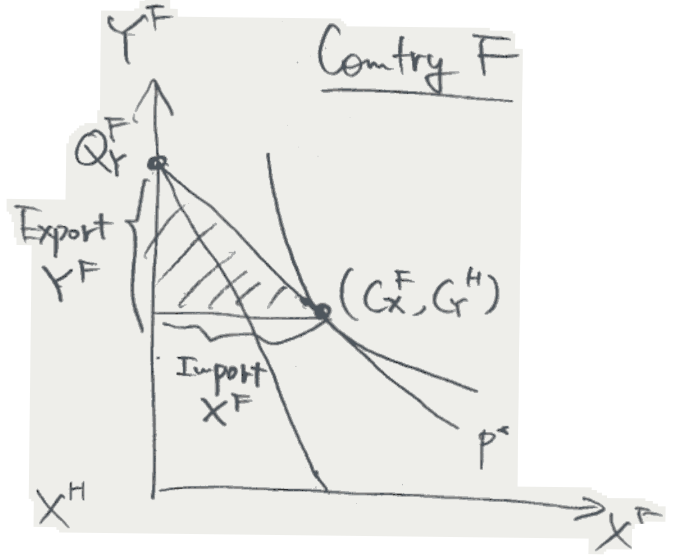


Since $\underbrace{p_a^H}_{}$ $\frac{a_x^H}{a_y^H} > \frac{a_x^F}{a_y^F}$ a

relative prices of Y to X

Country H has a comparative advantage in producing X.

(b) Y^H Country H



Country H Profix max

$$\max_{L_x} \; a_x L_x^H \quad w^H L_x^H$$

$$\boxed{a_x^H = w^H}$$

$$\max_{L} \; p^* a_y^H L_y^H \quad w^H L_y^H$$

$$a_y^H < w^H \quad (\text{negative profi})$$

$$\Rightarrow \boxed{L_y^H = 0} \; \& \; L_x^H = \bar{L}^H \quad (\text{no production of } Y)$$

$$\boxed{Q_x^H = a_x^H \bar{L}^H}$$

Util max

$$C_x^H \quad \beta w^H L \quad \beta a_x^H L^H$$

$$C_y^H = (1-\beta)\frac{w^H \bar{L}^H}{p^*} = (1-\beta)\frac{a_x^H \bar{L}}{p^*}$$

$$\boxed{\text{Export } X^H = Q_x^H - C_x^H = a_x^H \bar{L}^H - \beta a_x^H \bar{L}^H = (1-\beta) a_x^H \bar{L}^H}$$

$$\boxed{\text{Import } Y^H = C_Y^H = (1-\beta)\frac{a_x^H \bar{L}^H}{P^*}}$$

Country F

Profit max

$$\left\{ \begin{array}{l} a_x^F < w^F \Rightarrow L_x^F \quad 0 \quad \$ \quad L_y^F \quad L \\ P \quad a_y^F \quad w^F \end{array} \right.$$

$$Q_Y^F = a_y^F L$$

Util. max

$$C_X^F \quad \beta w^F \bar{L} \quad \beta \quad a_y^F L$$

$$C_Y^F \quad \frac{(1-\beta) w^F \bar{L}}{P^*} \quad (1-\beta) a_y^F L$$

Export Y^F $\quad Q_Y^H - C_Y^H = a_y^F \bar{L} - (1-\beta) a_y^F L$

$$\Rightarrow \boxed{\text{Export } Y^F = \beta a_y^F \bar{L}}$$

Import X^F $\quad C_X^F \quad \beta P a_y^F L$

$$\Rightarrow \boxed{\text{Import } X^F = \beta P^* a_y^F \bar{L}}$$

Equilibrium

Export X^H Import X^F

$$\Rightarrow \boxed{(1-\beta) a_x^H \bar{L}^H = \beta P^* a_y^F \bar{L}^F}$$

Export Y^F Import Y^H

$$\Rightarrow \boxed{\beta a_y^F \bar{L}^F = \frac{(1-\beta) a_x^H \bar{L}^H}{P^*}}$$

Note both equations are the same

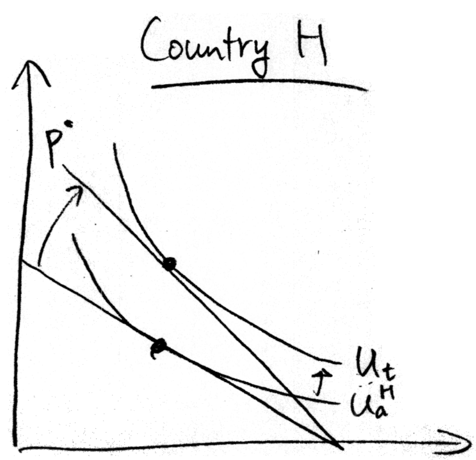
i.e. Clearing Market X automatically

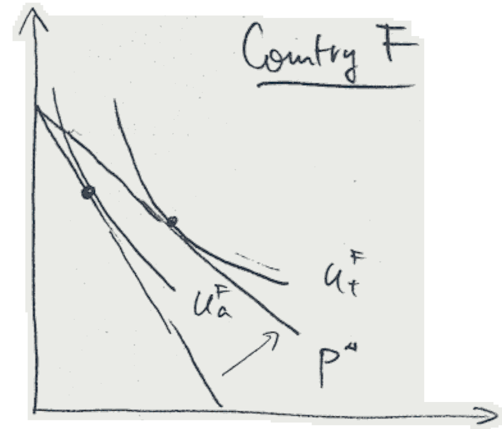
clears Market Y or vice versa

$$\boxed{P^* = \frac{1-\beta}{\beta} \frac{a_x^H}{a_y^F}}$$

since $\bar{L}^H$ $\bar{L}^F$ L

(c)





In both countries autarky → free trade relaxes its budget constraint leading to an increase welfare

(d)

(1) $p^* = \frac{1-\beta}{\beta} \frac{a_x^H}{a_y^F} \frac{L^H}{L^F}$ $\qquad \frac{1-\beta}{\beta} \frac{a_x^H}{a_y^F} \; 2$

f both countries specialized

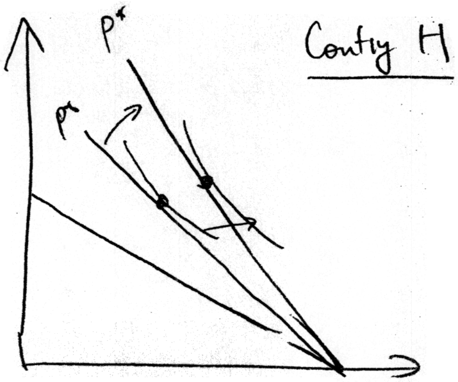
e $\underbrace{P_a^F}_{} < P^* < \underbrace{P_a^H}_{}$

$\frac{a_x^F}{a_y^F} \qquad \frac{a_x^H}{a_y^H}$

An increase in the size f country F

⇒ Export Supply of Y^F ↑
Import Demand of X^F ↑ } ⇒ P $\quad P_y$ ↓ $\;P_x^*$ ↑ ↓

(d) (ii)



$p^* \quad \frac{1-\beta}{\beta} \frac{a_x}{a_y}$

$p^* = \frac{1}{2} \frac{1-\beta}{\beta} \frac{a_x^H}{a_y^F}$

An increase in the size of country F improves the terms of trade for country H and thus by relaxing its budget constraint increases the utility level in Country H

(e) **Country H**

Profit max

$a_x^H \quad w^H$

$L_y^H = 0$ & $L_x^H \quad \bar{L}^H$

$Q_x^H \quad a_x^H \bar{L}^H$

Util max

$C_x^H \quad \beta a_x^H \bar{L}^H$

$C_y^H \quad \frac{(1-\beta) a_x^H \bar{L}^H}{(1+t) p^*}$

Import Y^H $\quad C_y^H \quad \dfrac{(1-\beta)\, a_x^H\, \bar{L}^H}{(1+t)\, P^*}$ $\quad$ (Due to the tariffs demand for Y^H less

Export Y^F $\quad \beta\, a_y^F\, L^F$

$$\Rightarrow \quad P^*(t) \quad \frac{1-\beta}{\beta\,(1+t)} \; \frac{a_x^H \; \bar{L}^H}{a_y^F \; \bar{L}^F}$$

world price

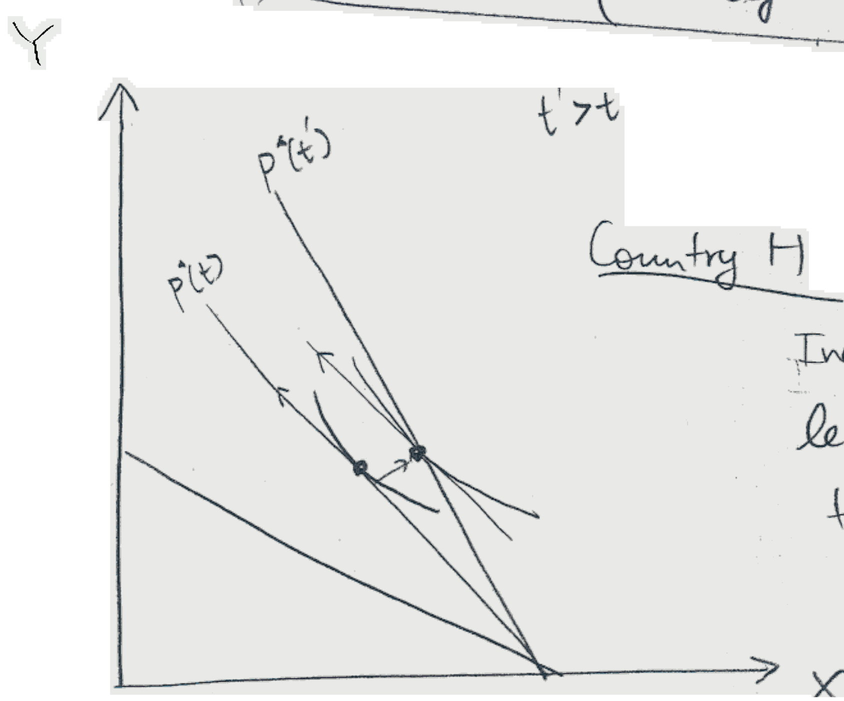
An increase in tariffs

$\Rightarrow$ demand for $Y \downarrow$

$\Rightarrow$ Relative Prices of $Y \downarrow$

Domestic Price for Y in Country H is

$$\boxed{(1+t)P^* = \frac{1-\beta}{\beta}\,\frac{a_x^H}{a_y^F}\,\frac{\bar{L}^H}{\bar{L}^F} = \frac{1-\beta}{\beta}\,\frac{a_x^H}{a_y^F}.}$$



Country H

Increases its welfare level by improving its terms of trade, which offsets the distortionary effect of tariffs

2 (a) Profit Maximization

$$\Rightarrow \quad K_x \quad a_x L_x$$

$$K_y \quad a_y L_y$$

Labor Market & Capital Market Clear

$$\Rightarrow \quad \bar{K} \quad K_x + K_y \quad a_x L_x + a_y L_y$$

$$L \quad L_x + L_y$$

$$\Rightarrow \quad K \quad a_x (\bar{L} \quad L_y) + a_y L_y$$

$$\Rightarrow \left\{ \begin{array}{ll} \boxed{L_y = \dfrac{\bar{K} - a_x \bar{L}}{a_y - a_x}} & \\ K_y & \dfrac{a_y (K \quad a_x L)}{a_y - a_x} \\ L_x & \dfrac{\bar{K} - a_y \bar{L}}{a_x - a_y} \\ K_x & \dfrac{a_x (\bar{K} - a_y \bar{L})}{a_x - a_y} \end{array} \right.$$

Utility Max

$$\rightarrow \quad \frac{X}{Y} \qquad \frac{\beta}{1-\beta} p_a$$

Equilibrium

Since $\quad X \quad \dfrac{\bar{K} - a_y \bar{L}}{a_x - a_y}$

$\dfrac{\bar{K} - a_x \bar{L}}{a_y - a_x}$,

This can be only possible if

$\downarrow \boxed{a_y < \bar{k} < a_x}$

Assume this

$$\boxed{p_a^* = \frac{1-\beta}{\beta} \frac{\bar{K} - a_y \bar{L}}{a_x \bar{L} - \bar{K}} = \frac{1-\beta}{\beta} \frac{\bar{k} - a_y}{a_x - \bar{k}}}$$

Country H $\qquad p^H \quad \dfrac{1-\beta}{\beta} \dfrac{\bar{k}^H - a_y}{a_x \quad \bar{k}^H}$

Country F $\qquad {}_a^F \quad \dfrac{1-\beta}{\beta} \dfrac{\bar{k}^F - a_y}{a_x - \bar{k}^F}$

We assume $\qquad a_y < \bar{k}^F, \bar{k}^H < a_x$

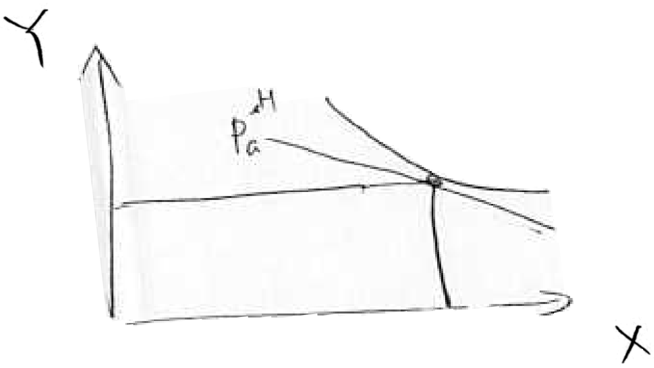
so that full employments of capital & labor are possible

Since $\quad \bar{k}^H > \bar{k}^F$

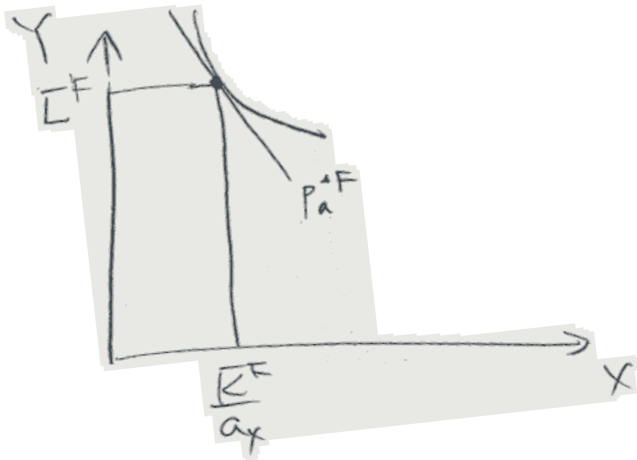
$p_a^H > p^F$

Therefore Country H has comparative advantage producing Good X

Country H



Country F



2(b) Since aggregate income equals national products

$wK + rL \qquad X + pY$

Aggregate Demand for X in Country H is

$X_D^H \quad \beta(X_S^H + pY_S^H)$

where $X_S^i = \dfrac{\bar{K}^i - a_z \bar{L}^i}{a_z - a_y}$

$Y_S = \dfrac{\bar{K}^i - a_x \bar{L}}{a_y - a_x}$

Similarly $X_D^F \quad \beta(X_S^F + pY_S^F)$

The excess supply for X^H is (Export)

Export = X^H $\quad X_S^H \quad X_D^H \qquad X_S^H \quad \beta(X_S^H + pY_S^H)$

Import X^F $\; - \; X_D^F - X_S^F = \beta(X_S^F + pY_S^F) \quad X_S^F$

In equilibrium $\quad$ Export X^H $\quad$ Import X^F

Thus $X_s^H \quad \beta(X_s^H + p Y_s^H)$

$$\beta(X_s^F + p Y_s^F) \quad X_s^F$$

r

$$\beta(Y_s^F + Y_s^H)p \quad X_s^H + X_s^F \quad \beta(X_s^H + X_s^F)$$

$$\boxed{p^* = \frac{(1-\beta)(X_s^H + X_s^F)}{\beta(Y_s^F + Y_s^H)}}$$

where

$$X_s^H \quad \frac{\bar{K}^H - a_y \bar{L}^H}{a_x \quad a_y}$$

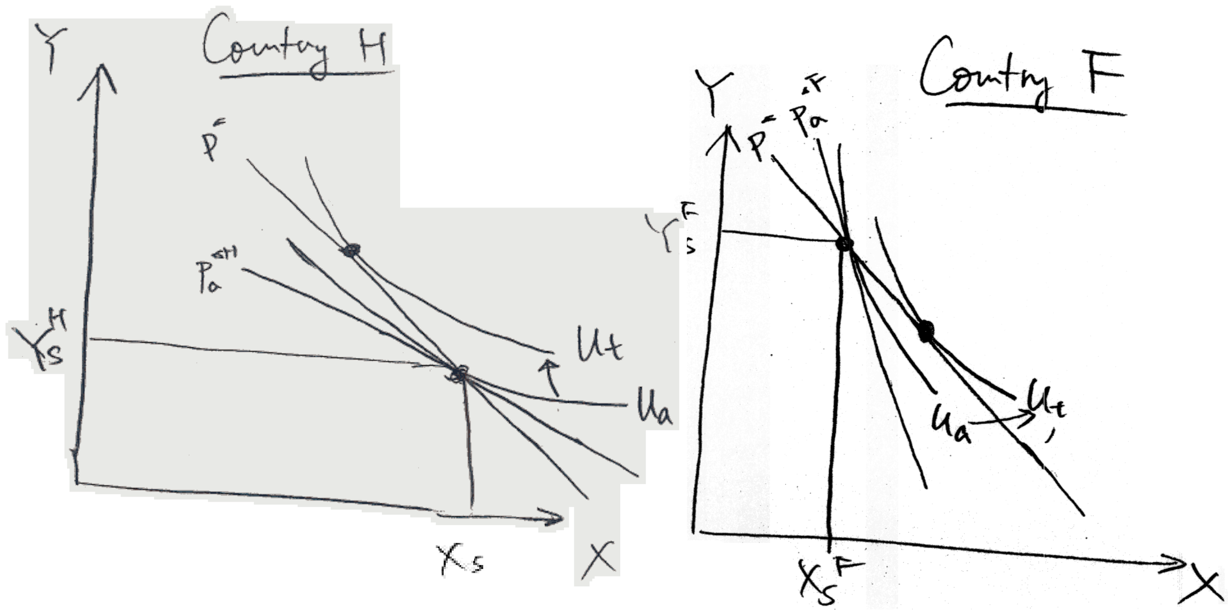
$$Y_s^H \quad \frac{K^H \quad a_x L^H}{a_y \quad a_x} \qquad \frac{a_x \bar{L}^H - K^H}{a_x \quad a_y}$$

$$X_s^F \quad \frac{\bar{K}^F - a_y \bar{L}^F}{a_x \quad a_y}$$

$$Y_s^F \quad \frac{K^F \quad a_x L^F}{a_y \quad a_x} \qquad \frac{a_x L^F \quad K^F}{a_x - a_y}$$

2(c) is not difficult to check

$$P_a^H > P > P_a^F$$



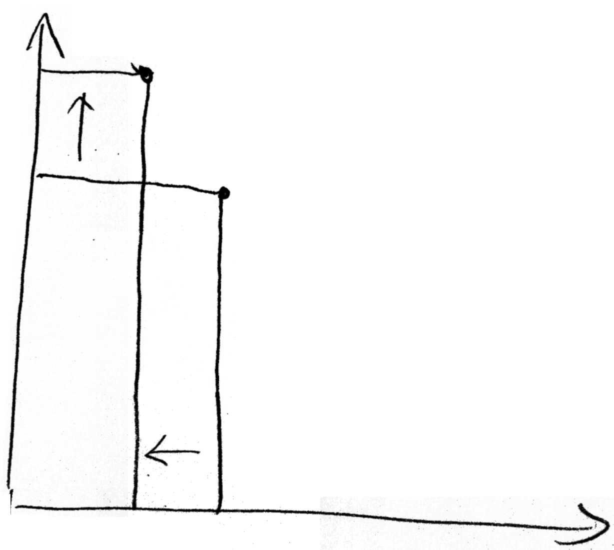
There are aggregate gains from trade as shown in the above figures

2. (d).

$$\,^{F} \downarrow \quad \text{from} \quad \frac{\bar{K}^F - a_y \bar{L}^F}{a_x - a_y} \quad \text{to} \quad \frac{\bar{K}^F - 2a_y \bar{L}^F}{a_x - a_y}$$ ⑭

$$Y_s^F \uparrow \quad \text{from} \quad \frac{\bar{K}^F - a_x \bar{L}^F}{a_y - a_x} \quad \text{to} \quad \frac{2a_x \bar{L}^F - \bar{K}^F}{a_x - a_y}$$

Country F's Production Possibility Frontier.



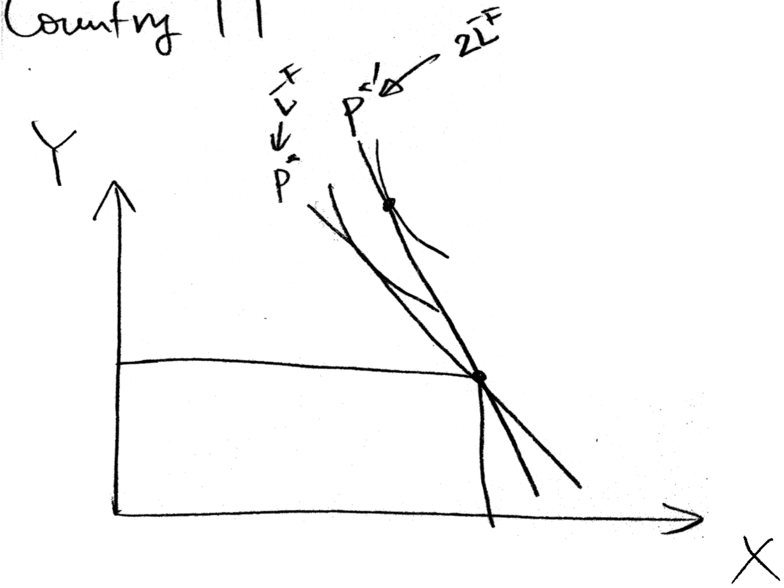
d)
$$p^* = \frac{1-\beta}{\beta} \cdot \frac{(X_s^H + X_s^F \downarrow}{Y_s^F \uparrow + Y_s^H} \quad \downarrow$$

Relative Prices of Y to X falls

An increase in the labour supply of country F reduces the production of X (capital-intensive goods)
& increases " Y (labour ")

Therefore, $p^* = \frac{P_Y^* \downarrow}{P_X^* \uparrow} \downarrow$

(ii) Country H



A decrease in relative prices of Y

$\Rightarrow$ improves the terms of trade in Country H

$\Rightarrow$ increases Country H's utility level

2. (e)

(1)

Since Aggregate income equals national products

$(w\bar{K}+r\bar{L})$ $\quad$ $(X^H + (1+t)pY^H)$

$(X^F + pY^F)$

$$X_D^H = \beta\left(X_S^H + (1+t)pY_S^H\right)$$

$$X_D^F = \beta\left(X_S^F + pY_S^F\right)$$

Export $X^H = X_S^H - X_D^H = (1-\beta)X_S^H - \beta(1+t)pY_S^H$

Import $X^F = X_D^F - X_S^F = -(1-\beta)X_S^F + \beta pY_S^F$

In equilibrium, Export X^H = Import X^F

$$\Rightarrow \quad p^* = \frac{(1-\beta)(X_S^H + X_S^F)}{\beta\left(Y_S^F + (1+t)Y_S^H\right)}$$

An increase in t

$\Rightarrow$ World Demand for $Y \downarrow$

$\Rightarrow$ Relative prices of $Y \downarrow$

Price distortion.
$\Rightarrow$ worsen welfare

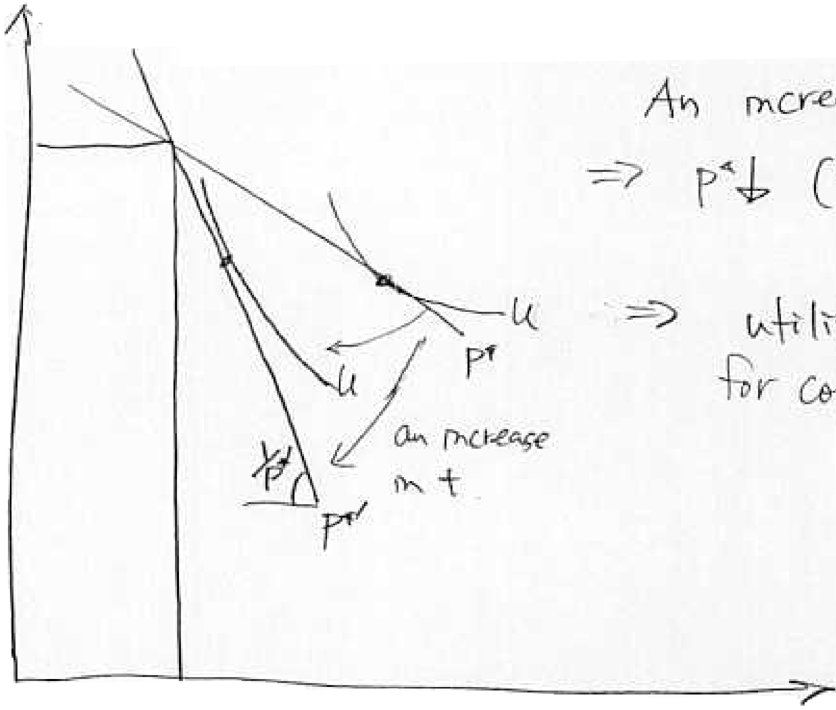
The domestic relative prices of $Y \uparrow$ as $t \uparrow$.

Domestic price after tariff is

$$(1+t)p^* = \frac{(1+t)(1-\beta)(X_S^H + X_S^F)}{\beta\left(Y_S^F + (1+t)Y_S^F\right)} = \frac{(1-\beta)(X_S^H + X_S^F)}{\beta\left(\frac{Y_S^F}{(1+t)} + Y_S^F\right)}$$

Country F

$p^* \downarrow$



An increase in tariffs

$\Rightarrow$ $p^* \downarrow$ (the terms of trade for country F $\downarrow$)

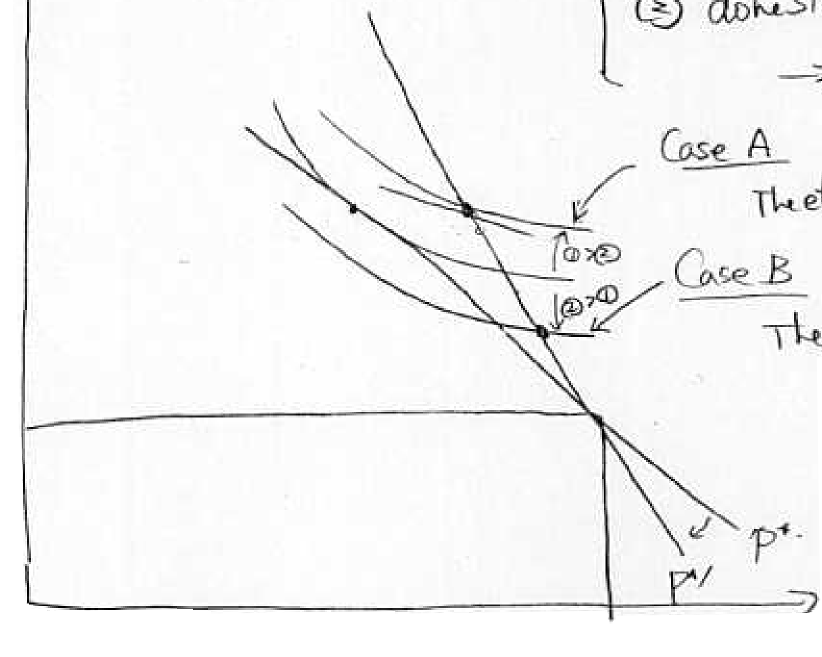
$\Rightarrow$ utility level $\downarrow$ for country F

Country H

There are two factors.

① the terms of trade $\uparrow$
→ improve welfare

② domestic price distortion
→ worsen welfare



Case A
The effect ① > The effect ②

Case B
The effect ② > The effect ①