

Homework 2

(Due: February 9 before the class begins)

1. Consider a model with two countries, H and F, two goods, X and Y, and one factor of production, labour (L). Labour is mobile between industry X and Y within a country but is not mobile across countries. Labour in each country is fully employed.

The technologies for producing the goods in country H are given by

$$X^H = a_x^H L_x^H \quad \text{and} \quad Y^H = a_y^H L_y^H.$$

The technologies for producing the goods in country F are given by

$$X^F = a_x^F L_x^F \quad \text{and} \quad Y^F = a_y^F L_y^F.$$

The price of X is normalized to 1. Denote the prices of Y (relative to X) in country H and country F by p^H and p^F , respectively. Consumers in both countries have the identical utility function given by

$$U(X, Y) = X^\beta Y^{1-\beta}.$$

In each country, there is a single consumer with the labour supply \bar{L} . The representative consumer maximizes her utility given her budget constraint,

$$X + pY \leq w\bar{L},$$

where w is the wage rate (relative to the price of X).

The values of productivity parameters ($a_x^H, a_y^H, a_x^F, a_y^F$) satisfy the following relationship:

$$\begin{aligned} a_x^H &> a_x^F > 0 \\ a_y^H &> a_y^F > 0 \\ \frac{a_x^H}{a_y^H} &> \frac{a_x^F}{a_y^F} \end{aligned}$$

- (a) (10 points) Determine each country's autarkic equilibrium goods price ratio in terms of the model's parameters and then determine the pattern of comparative advantage between these two countries.
- (b) (10 points) Determine the free trade equilibrium price ratio in an equilibrium in which each country specialized in its good of comparative advantage.

- (c) (10 points) Show that there are aggregate gains from trade for each country by comparing each consumer's utility level in free trade with her utility level in autarky.
- (d) Suppose that the consumer in country F supplies $2\bar{L}$ units of labour rather than \bar{L} .
- (5 points) Determine the new free trade equilibrium price ratio and explain how/why an increase in the size (labour supply) of country F changes the equilibrium price ratio intuitively.
 - (5 points) Explain how an increase in the size of country F affects the utility level in country H.
- (e) Suppose that the government in country H places an ad valorem tax of t on each unit of Y imported into the country so that the representative consumer's budget constraint is now given by

$$X^H + (1+t)p^HY^H \leq w^H\bar{L}.$$

- (5 points) Determine the equilibrium domestic price in country H, p^H .
 - (5 points) Explain how an increase in this import tariffs t imposed by the government of country H affects the utility levels of country H and country F.
2. Consider an economy with two factors, two goods, and two countries (H and F). Both countries use capital and labour to produce two types of goods: agriculture and automobiles. Two countries are different in their endowments of labour and capital. Country H has \bar{L}^H units of labour and \bar{K}^H units of capital to supply to production. Country F has \bar{L}^F units of labour and \bar{K}^F units of capital. We assume that

$$\begin{aligned} \bar{K}^F &> \bar{K}^H > 0, \\ \bar{L}^F &> \bar{L}^H > 0, \\ \frac{\bar{K}^H}{\bar{L}^H} &> \frac{\bar{K}^F}{\bar{L}^F}. \end{aligned}$$

Countries are identical in their technologies:

$$X = \min \left\{ \frac{K_x}{a_x}, L_x \right\}, \quad Y = \min \left\{ \frac{K_y}{a_y}, L_y \right\}.$$

We assume that $a_x > a_y$. The price of agriculture products is normalized to 1. Denote the prices of automobiles (relative to agriculture products) in country H and country F by p^H and p^F , respectively. Consumers in both countries have the identical utility function given by

$$U(X, Y) = X^\beta Y^{1-\beta}.$$

Representative consumer maximizes her utility given her budget constraint,

$$\begin{aligned} X^H + p^HY^H &\leq w^H\bar{L}^H + r^H\bar{K}^H, \\ X^F + p^FY^F &\leq w^F\bar{L}^F + r^F\bar{K}^F, \end{aligned}$$

where w^i and r^i are the wage rate and the rental price of capital (relative to the price of agriculture products) of country i for $i = H, F$.

- (a) (10 points) Determine each country's autarkic equilibrium goods price ratio in terms of the model's parameters and then determine the pattern of comparative advantage between these two countries.
- (b) (10 points) Determine the free trade equilibrium price ratio in an equilibrium in which each country specialized in its good of comparative advantage.
- (c) (10 points) Show that there are aggregate gains from trade for each country by comparing each consumer's utility level in free trade with her utility level in autarky.
- (d) Suppose that the consumer in country F supplies $2\bar{L}^F$ units of labour rather than \bar{L}^F . Factor endowments of capital in both countries and factor endowment of labor in country H remain the same.
- i. (5 points) Determine the new free trade equilibrium price ratio and explain how/why an increase in the size (labour supply) of country F changes the equilibrium price ratio intuitively.
 - ii. (5 points) Explain how an increase in the size of country F affects the utility level in country H.
- (e) Suppose that the government in country H places an ad valorem tax of t on each unit of Y imported into the country so that the representative consumer's budget constraint is now given by

$$X^H + (1+t)p^HY^H \leq w^H\bar{L}^H + r^H\bar{K}^H.$$

- i. (5 points) Determine the equilibrium domestic price in country H, p^H .
- ii. (5 points) Explain how an increase in this import tariffs t imposed by the government of country H affects the utility levels of country H and country F.