Economics 425 **Winter 2004**
International Trade Theory and Policy **Hiroyuki Kasahara**

Homework 1

(Due: January 26 before the class begins)

1. Consider the industry which uses capital and labour with the following Cobb-Douglas production function:

$$X = K^{\alpha} L^{1-\alpha},$$

where $0 < \alpha < 1$ is the share of capital. Denote the rental price of capital by r and the wage by w.

   (a) (10 points) Determine the capital to labour *ratio* $\left(\frac{K^*}{L^*}\right)$ which minimizes the cost of producing a fixed amount of output $\bar{X}$. Explain why this optimal ratio does or does not depend on $\bar{X}$.

   (b) (10 points) Determine the *levels* of capital K^* and labour L^* which minimize the cost of producing $\bar{X}$ as a function of the rental price of capital, the wage, and $\bar{X}$.

2. Suppose there are two consumers in a country: consumer 1 and consumer 2. Two consumers have the following Cobb-Douglas utility function defined over consumption of goods X and Y:

$$U(X, Y) = X^{\beta} Y^{1-\beta}, \tag{1}$$

where $1 > \beta > 0$. Consumers are different in their incomes. Consumer 1 has exogenous income equal to I_1, and consumer 2 has exogenous income equal to I_2. Denote aggregate income by $I \equiv I_1 + I_2$.

   (a) (10 points) Derive each consumer's individual Marchallian demands and then derive aggregate Marshallian demands to show that aggregate demand for the goods depends only on aggregate income and not the distribution of income.

   (b) (10 points) Suppose now that, while consumer 1's utility function is given by (1), consumer 2's utility function is different from consumer 1 and given by:

$$U(X, Y) = X^{\gamma} Y^{1-\gamma}. \tag{2}$$

   where $1 > \gamma > \beta > 0$. How does a redistribution of income from consumer 1 to consumer 2 affect aggregate Marshallian demands for goods X? [No point will be assigned to unsupported answer].

3. Consider a country that uses labour to produce two types of goods: apple and orange. The country has $\bar{L}$ units of labour.

   The technology for apple is given by the following linear production function:

   $$X = a_x L_x,$$

   where $a_x > 0$. The technology for orange is given by the following linear production function:

   $$Y = a_y L_y,$$

   where $a_y > 0$. The price of apple is normalized to 1. Denote the price of orange (relative to apple) by p. Consumer's utility function is given by

   $$U(X, Y) = X^{\beta} Y^{1-\beta}.$$

   The representative consumer maximizes her utility given her budget constraint,

   $$X + pY \leq I,$$

   where w is the wage rate (relative to the price of apple) and I is income which is treated as given ($I = w\bar{L}$).

   (a) (5 points) Draw a figure of the production possibility frontier (corresponding to Figure 2.7 of the textbook).

   (b) (5 points) What is the equilibrium price ratio of orange to apple p^*?

   (c) (5 points) Determine the equilibrium allocation of labour (L_x^* and L_y^*).

   (d) (5 points) Determine the equilibrium quantity of apples (X^*) and oranges (Y^*).

   (e) Suppose that the government places an ad valorem tax of t on each unit of orange so that the representative consumer's budget constraint is now given by

   $$X + (1 + t)pY \leq I.$$

   The government redistributes what it collects as a tax so that $I = w\bar{L} + tpY$. Note, however, that each consumer treats her income I as given. Determine (i) (5 points) the equilibrium price rate p^* and (ii) (5 points) the equilibrium quantity of apples and oranges (X^* and Y^*).

4. Consider a country that uses capital and labour to produce two types of goods: agriculture and automobiles. The country has $\bar{L}$ units of labour and $\bar{K}$ units of capital to supply to production.

   The technology for agriculture is given by the following Cobb-Douglas production function:

   $$X = K_x^{\alpha_x} L_x^{1-\alpha_x},$$

The technology for automobiles is given by the following Cobb-Douglas production function:

$$Y = K_y^{\alpha_y} L_y^{1-\alpha_y}.$$

We assume that $\alpha_x > \alpha_y$. The price of agriculture products is normalized to 1. Denote the price of automobiles (relative to agriculture products) by p. Consumer's utility function is given by

$$U(X, Y) = X^{\beta} Y^{1-\beta}.$$

Representative consumer maximizes her utility given her budget constraint,

$$X + pY \leq I,$$

where w and r are the wage rate and the rental price of capital (relative to the price of agriculture products), respectively, and $I = w\bar{L} + r\bar{K}$.

(a) (5 points) Derive the capital to labour ratio which minimizes cost for each industry. Is it true that automobile uses labour relatively intensively and agriculture production uses capital relatively intensively *for any pair of wage and rental price of capital*?

(b) (5 points) Assuming that capital and labour are fully employed, determine the equilibrium qunatity of agriculture products (X^*) and automobiles (Y^*).

(c) (5 points) Determine the equilibrium allocation of labour between agriculture and automobiles (L_x^* and L_y^*) and the equilibrium allocation of capital (K_x^* and K_y^*).

(d) (5 points) Determine the equilibrium ratio of wage to rental price $\frac{w^*}{r^*}$ and the equilibrium price ratio of automobile to agriculture p^*.

(e) Suppose that the government places an ad valorem tax of t on each unit of automobiles so that the representative consumer's budget constraint is now given by

$$X + (1 + t)pY \leq I$$

The government redistributes what it collects as a tax so that $I = w\bar{L} + r\bar{K} + tpY$. Note, however, that each consumer treats her income I as given. Determine (i) (5 points) the equilibrium price rate p^* and (ii) (5 points) the equilibrium quantity of apples and oranges (X^* and Y^*).