Estimation of Dynamic Hedonic Housing Models with Unobserved Heterogeneity*

Susumu Imai, Queen's University

Sidibe Modibo, University of Lyon, GATE LSE

October 26, 2011

*Very preliminary. Comments are very welcome.

Abstract

In this paper, we estimate the dynamic hedonic model with unobserved housing specific characteristics. Our approach allows for the unobserved housing characteristics to be correlated with the observed ones. The estimation only requires single market housing data. We use the mobility choice from the housing tenure data and the observed hedonic housing characteristics as the main source of identification. The traditional hedonic approach only uses the latter for estimation. The estimation problem turns out to be much simpler than the location choice based approach common in the literature, where the potential number of choices are large. We recover for each housing unit one house specific unobserved characteristics as well as individual specific utility shocks.

We use mobility data on rental apartments from the French Housing Survey. We exploit the strong rent control regulation in France, which makes the rent effectively invariant to changes to local economic conditions. Therefore, we do not need to worry about the endogeneity of rent in the mobility decision due to housing specific unobserved heterogeneity.

Keywords: Hedonic analysis, Identification

JEL Codes: R21, C31, C11.

1 Introduction

We propose a new methodology to estimate a housing demand model with unobserved housing characteristics, which also includes unobserved neighborhood characteristics. Most of the literature on location choice, such as **?** or **?**, analyzes individuals who choose which house among all housing units to live or, in a more simplified setting, where to live among all neighborhoods. The difficulty of the approach is that the choice probability of each housing unit is a function of unobserved housing characteristics of all available housing units. Even in the simpler setup of neighborhood choice, the choice probability is a function of unobserved neighborhood characteristics in all neighborhoods plus other structural parameters. Estimation of those models are extremely complex due to the large number of choices, since the number of housing units or, even in a simplified model the number of neighborhoods are usually large, thus causing finite sample difficulties. Often strong assumptions on the error term of the utility function need to be imposed.

To overcome this, we follow **?** and estimate a dynamic model of housing demand. Their framework, inspired from **?**, separates the individual's problem into two distinct choices: an optimal stopping problem in the decision to stay or move; and conditional on this decision, a static discrete choice over where to move, which follows the static location choice model of **?**. In contrast to **?** and other literature, we use the following two sets of data as the main sources of identification. First, we use the conditional choice probability of whether to stay or move as well as housing tenure information. Since the only housing characteristics that should influence the mobility decision of individuals is the one in where she is living, the estimation problem becomes much simpler. It consists in recovering for each housing unit one housing specific characteristics parameter based on one conditional choice probability of moving out. The dynamic choice problem of staying or moving out does not increase its complexity with the number of housing units or neighborhoods at all. This greatly helps identification and estimation. This is in contrast to the conventional location model where choices are over all housing units. There, choice probability of each house is a function of unobserved housing characteristics of all housing units.

Second, instead of the housing location information, we use the observed housing characteristics. That is, we combine the static hedonic literature and the dynamic mobility choice literature. The conventional static hedonic approach by **?**, **?**, **?** and others only use data on the choice of observed housing characteristics, and thus have difficulties in dealing with unobserved housing characteristics. They have to assume that unobserved housing characteristics are independent to the observed ones. By using both tenure and mobility choice data and the housing characteristics data, we allow for correlation between observed and unobserved housing characteristics. We also recover flexibly specified hedonic per period utility function and the individual specific utility shocks as well, where conventionally separability between housing and individual characteristics are assumed. Otherwise, the literature requires multimarket data and the assumption that the distribution of observed individual and housing characteristics vary across market but the unobserved characteristics do not. In our analysis, we only need data on one housing market.

The two separate data sources are complementary in identifying the model. Mobility and tenure information helps identify hedonic utility function of housing units because individuals whose deterministic utility of housing consumption is high stay at the housing unit longer. If we just used the discrete mobility data, then we would only be able to identify the finite mixture distribution or parameterized distribution of unobserved heterogeneity of each housing unit. But by additionally using the housing characteristics data, which comes from the continuous choice of individuals, we would be able to recover the unobserved characteristics of each housing unit, and the individual taste shocks as well.

We use data on rental apartments from the French Housing Survey. We exploit the strong rent control regulation in France, which makes the rent effectively invariant to changes to local economic conditions. Therefore, we do not need to worry about the endogeneity of rent in the mobility decision due to housing specific unobserved heterogeneity. Therefore, we avoid the need for an instrument for price of a housing consumption, which has always been a difficult issue in the housing literature.

2 The Model

2.1 The Hedonic Model of Housing

We first briefly review the literature on the residential mobility using hedonic framework. The analysis of hedonic models was pioneered by **?**. In this paper, Rosen suggests a two-step estimation to recover preferences. In the first step, the marginal price function is recovered from the regression of price on attributes. Then, the first order conditions are used to estimate the utility function. However as noted by **?**, the second stage suffers from a simultaneity issue. Hence, **?** argue that using a linear approximation of the first order conditions, relying on multi-markets data, and assuming that agent's preferences parameters are common across market while individual heterogeneity vary across markets, identification can be achieved. Subsequently, **?** and **?** suggest that the model was still not identified because unobserved taste affect both the quantity of an amenity consumed by an individual and its price. Hence suggested to use instrumental variables.

The current literature departs from the preceding ones in two directions: by using nonparametric methods instead of linear approximation of the first order conditions **?**, **?**, and by allowing for unobserved heterogeneity (**?**).

Let's introduce some notation based on **?**'s model to make clear the identification issues. The individual's utility function of a house with observed characteristics z and utility shock ϵ is

$$U(z, x, c, \epsilon)$$

where x is the vector of individual observed characteristics, and c is the non-housing consumption. An individual maximizes her utility subject to the following budget constraint:

$$c + P(z) \leq y$$

where the price of non-housing consumption c is normalized to one. Then, the first order condition that allows to recover housing demand is

$$P_z(z) = h(x, z, \epsilon) \equiv \frac{U_z(z, x, y - P(z), \epsilon)}{U_c(z, x, y - P(z), \epsilon)}$$

? show that hedonic models with additively separable utility function are nonparametrically identified with single market data and present two methods for recovering the structural functions in such models. **?** relax the additivity assumptions, and show that only multimarket data identifies all the parameters of the model. There, additional assumptions need to be imposed: the observed individual or housing characteristics need to be differently distributed across markets but the unobserved individual or housing characteristics need to remain the same. The implicit assumption in **?** and **?** literature is that all the housing characteristics z are observable. It is important to note that ϵ cannot be interpreted as the unobserved housing characteristics since it is not included in the price function. If the individual knows that the price of unobserved characteristics is zero, she would consume it at the bliss point and thus the distribution of ϵ would be degenerate. If we include ϵ in the price equation, then the price equation becomes

$$P(z, \epsilon) = \delta_z z + \delta_\epsilon \epsilon.$$

This is the price equation analyzed in **?**. Applications of this model include **?**, who use nonparametric methods to estimate the coefficients δ_z and interpret the residuals as the unobserved housing characteristics. Once unobserved characteristics are known, then one could just proceed to estimate the parameters in a standard way discussed above. However, consistently estimating the coefficients of the price equation would require dealing with the correlation of z and ϵ. **?**, **?** assume that they are not correlated with each other.

To allow for the correlation between z and ϵ requires an instrument. Finding a variable that is correlated with the observed housing characteristics z but uncorrelated with the unobserved housing characteristics ϵ would be an extremely difficult task.

Another strand of the literature tries to estimate the unobserved housing characteristics directly from the housing choice of the individuals, which was pioneered by **?**. Denote the observable housing characteristics of a housing unit j to be z_j and the unobservable characteristics to be η_j. Because it is not observable, they follow the convention of the discrete choice model and specify the housing specific unobserved heterogeneity η_j.

Then, the optimal housing choice of the individual i can be expressed as follows.

$$k = \underset{j}{argmax}\{U(z_j, x_i, y_i - P(z_j, \eta_j, w_j), \eta_j, \epsilon_{ij})\}$$

where ϵ_{ij} is the individual i specific utility shock for housing unit j, which is assumed to be i.i.d. extreme value distributed. Now, the price is a function of both the observed characteristics z_j and the unobserved characteristics η_j and the price shock w_j. Furthermore, they assume that

$$U(z_j, x_i, p_j, \eta_j) = V(z_j, x_i, p_j) + \eta_j + \epsilon_{ij}$$

where $p_j = P(z_j, \eta_j, w_j)$. Then, the household i's choice probability of a housing unit k equals

$$p(i,k) = \frac{exp\left[V(z_k, x_i, p_k, \eta_k)\right]}{\sum_l exp\left[V(z_l, x_i, p_l, \eta_l)\right]}$$

The ML estimation is then based on the above household choice probability over all the housing units in the data. Notice that the number of housing specific heterogeneity term η_j equals the total sample size of houses, which makes the estimation exercise subject to the finite sample problem.[1]

In a more simpler setting, one could modify the specification of the housing specific unobserved heterogeneity as follows.

$$\eta_j = \xi_n$$

where ξ_n is the neighborhood specific unobserved heterogeneity. Even then, the estimation problem could be subject to the finite sample problem if the number of observations per neighborhood is small, which is typical in a disaggregated neighborhood level data. Furthermore, it is known that the logit choice framework imposes strong functional form (I.I.A.) on the utility function, which may distort welfare calculation (see **?** for more details.).

Another issue in the estimation of the above location mode is the endogeneity of the housing characteristics z_k, i.e. their potential correlation with the housing specific unobserved heterogeneity. A proposed solution for location-related characteristics rely on quasi-random variation like geographical boundaries (**??**). However, it is in general difficult to find an appropriate instrument for all the other housing characteristics, such as number of rooms, since one has to find an instrument that is correlated with the observed characteristics but uncorrelated with the unobserved characteristics[2].

[1] **?**, after implementing the above estimation algorithm, hint that a potential approach that is not subject to the finite sample bias would be to assume that η_j follows a distribution that is nonparametrically estimated, perhaps with finite mixture. This would certainly avoid the finite sample problem mentioned above, but it would also create an additional issue that $V(z_j, x_i, p_j) + \eta_j$ can no longer be obtained a la BLP style in the first stage using the contraction mapping, which may add complexity in the estimation.

[2] For example, **?** uses prices of far away neighborhoods as instruments, where the implicit assumption is that the unobserved characteristics of the far away neighborhoods are not correlated.

The literature that is closest to our paper is **?**. of a neighborhood. This approach is based on the sorting model of **?** which predicts the positive correlation between neighborhood quality and average permanent income of its residence. The issue there is that the permanent income is not observed, and additional instruments are required since **?** predicts negative within neighborhood correlation between observed and unobserved permanent income. **?** estimates a dynamic model of location choice with moving cost. The estimation proceeds in two steps. In the first, a dynamic model of migration is estimated and a location fixed effect is recovered. Then, the location fixed effect is decomposed between observed and unobserved determinants. The main limitation of this paper is that the second step may suffer from an endogeneity bias due to individual sorting.

2.2 The Dynamic Model

Instead of identifying the structural parameters solely from the residential choice of the individuals, we additionally use the dynamic mobility choice, i.e. whether to stay in the same residential unit or move. Let τ be the length of stay in the neighborhood, and Z_i and w_i be the vector of the observed and unobserved characteristics of a housing unit i. We assume that the relative rent of the apartment i depends on the length of stay τ, housing observed attributes and an unobserved component v_i :

$$r_i = r_0(\tau, Z_i, w_i)$$

We assume that the relative rent of the housing unit is determined as follows

$$r_0(\tau, Z_i, w_i) = r_0(\tau, Z_i, w_i)g(\tau) = r_0(Z_i, w_i)g(\tau)$$

where $g(\tau)$ represents the over time change in relative rent, which is entirely determined by the regulated uniform growth rate of rent and the average rent of the rest of France at time $t_0 + \tau$, where t_0 is the initial period. We let the period zero rent equation to be linearly separable between observed and unobserved heterogeneity, i.e.

$$r_0(Z, w) = h(Z) + v(w)$$

Then, the individual in each period makes the choice between staying and moving out of the house. Let the per period utility of staying in a house be specified as.

$$\gamma_{r1} r_i(\tau,) + \frac{1}{2}\gamma_{r2} r_i(\tau)^2 + \mu_{i\tau} + \epsilon_{0\tau i}$$

where the first two terms represent the negative utility from paying the rent r_i, and the fourth term represents the per period utility shock. The third term is a function of observable and unobservable characteristics:

$$\mu_{i\tau} = [b(Z_i, X_{i,\tau}, u_Z) + u(w_i)]\, c\,(X_{i,\tau})\, u_i$$

where $b()$ is the utility component for observed attributes, which is a function of housing observed characteristics Z_i and individual observed characteristics $X_{i,\tau}$, u_i is the shock for the marginal utility of housing characteristics, $c()$ is a function of individual observed characteristics $X_{i,\tau}$. As we discussed before, we do not observe each component of the vector w_i. Given parameters of the rent equation $h()$, we can only recover $v(w_i)$. We thus approximate the utility component of unobservable characteristics as follows.

$$u(w_i) \approx u_{v.i} v(w_i)$$

From now on, we drop the term w_i without loss of generality. This is a setup that is similar to the one where the utility function is linearly separable in other goods $x = y - r_i$ and a function of housing characteristics Z.

We denote the tenure invariant component of the rent as $r_i(0)$. Since the rent of all subsequent periods are a deterministic function of the initial rent, all the subsequent mobility choice is a function of $r_i(0)$. The choice specific value of staying is

$$V_0(r_i(0), X_i, \tau, \mu_i, \epsilon_{0\tau i}) = \gamma_{1r} r(\tau, Z_i, v_i) + \frac{1}{2}\gamma_{2r} r(\tau, Z_i.v_i)^2 + \mu_{i0} + \epsilon_{0\tau i} + \beta EV(r_i(0), \tau+1, X_i', \mu_i)$$

The value of moving out is

$$V_1(a_i, \epsilon_{1\tau i}) = \varphi(a_i) + \epsilon_{1\tau i}.$$

where a_i is the age of the individual i, φ captures the life-cycle effect and $\epsilon_{1\tau}$ is per period shock to the utility of moving out. We assume that since the individual is moving out of the current housing unit, the utility of moving out is not a function of its characteristics. Both $\epsilon_{0\tau i}$ and $\epsilon_{1\tau i}$ are assumed to be i.i.d. extreme value distributed.

Now, denote $\bar{V}_\tau$ to be the deterministic value of staying. That is,

$$\bar{V}_{0\tau}(r_i(), Z_i, \tau, X_i, v_i) = \gamma_{1r} r_i(0) g(\tau) + \frac{1}{2}\gamma_{2r}\left[r_i(0)g(\tau)\right]^2 + \mu_{i\tau} + \beta EV(r_i(), Z_i, \tau+1, X_i', v_i).$$

Similarly, let $\bar{V_{1i}}$ be the deterministic value of moving out, i.e.

$$\bar{V}_1(a_i) = \varphi(a_i).$$

Then, the probability of leaving the house is

$$p(r_i(0), Z_i, \tau, a_i, \mu_i) = \frac{exp(\bar{V}_1(a_i))}{exp\left[\bar{V}_{0i}(r_i(0), \mu_{i0}, \tau, a_i)\right] + exp(\bar{V}_1(a_i))} \tag{1}$$

During the period when the individual is looking for a house, she tries to find a housing unit that maximizes the following value function with respect to observed housing characteristics Z_i and unobserved housing characteristics v_i.

$$EV(a_i, \xi_i, u_{Z,i}) = argmax_{\{Z_i, v_i\}} EV(r_i, Z_i, u_{Z,i}, v_i, \xi_i, a_i, \theta) \tag{2}$$

Here, we impose a restriction that there is only one unobserved characteristics v_i. That is, if there are several unobserved characteristics of the rental unit, individuals do not choose optimally the quantity of each of them, and thus they can be considered as an index. Finally, we assume that the individual's utility shocks ξ_i, $u_{Z,i}$ only get realized after she decides to move. We then specify the deterministic value of moving out as the difference between the expected value of a new house and the mobility cost, i.e.

$$\varphi(a_i) = E_{\xi_i, u_{Z,i}}\left[EV(a_i, \xi_i, u_{Z,i})\right] - C(a_i)$$

3 Data

We use two waves of the French Housing Survey 2002, and 2006. The French Housing Survey is a 4 years cross-section data that aims to describe the housing conditions of the population using a representative sample. Hence it describes the characteristics of the houses (*size, area, tenure*, . . .) and the characteristics of the individual living in those houses (*income, labor market outcome, children*, ...) including their former mobility (*length of stay, former tenure*, ...).

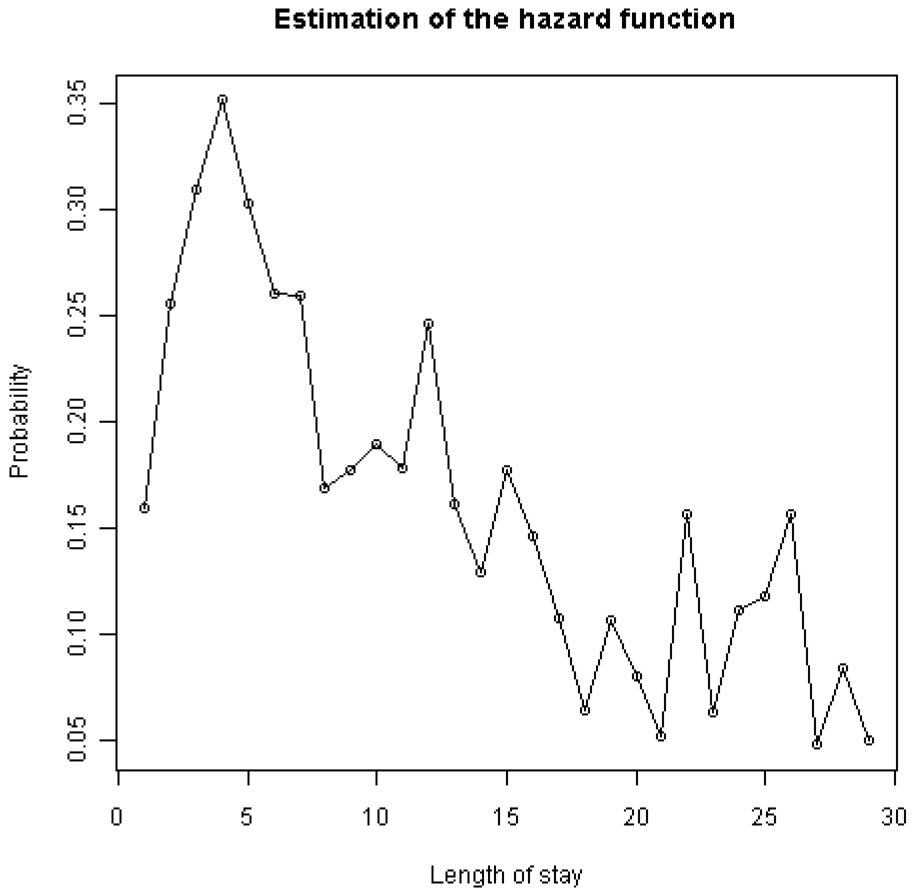
The main advantage of this data source is that it provides a large number of households (close to 80,000) and their location at the finest geographical level: ILOTS or statistical block. However this geographical level does not allow to identify several households per neighborhood. Our final sample is composed of 9,888 individuals whose selection is explained in the first section of the appendix.

Higher fraction of the mobile households have male household heads. Lower fraction of households who have plans to move are either married or in cohabitations, which could mean that their mobility costs are higher. The number of children are similar for mobile and nonmobile households. The ratio of immigrants are slightly higher for mobile households, and mobile households tend to be younger. Mobile households tend to be more educated. Mobile households tend to have smaller housing units than not mobile ones suggesting a search for more space.

Table 1: **Description of the sample**

Variables		Not Mobile Households	Mobile Households	All Households
Female		34.5	33.9	34.4
Married/cohab		47.5	50.6	48.3
Number of children	0	63.1	61.6	62.7
	1	16.7	19.1	17.3
	2	12.8	12.4	12.7
	3 and more	7.4	6.2	7.3
Citizenship	Native	88.2	87.1	87.9
	Immigrant	11.8	12.9	12.1
Age	Less than 30	32.1	42.2	34.6
	30 to 39 years	26.4	31.9	27.8
	40 to 49 years	20.9	15.9	19.6
	50 to 64 years	20.1	9.6	17.4
	65	0.6	0.4	0.6
Education	Intermediate	48.2	42.6	46.7
	Higher	17.6	19.1	17.9
	University	34.2	38.9	35.4
Number of rooms	1	16.7	23.7	18.4
	2	24.9	28.0	25.7
	3	28.2	25.3	27.5
	4	19.1	15.1	18.1
	5	7.9	5.9	7.4
	6	2.3	1.6	2.1
	7	0.9	0.4	0.7
Mean size per room m^2		23.88	23.80	23.86
Mean rent / 100		5.37	5.29	5.35
Number of observations		7,427	2,461	9,888

In the figure , we present the mobility information. The hazard rate turns out to decrease with tenure, indicating either unobserved heterogeneity or the negative effect of decreasing rent over time on mobility.



4 Identification

4.1 The French renting market

The French renting market is characterized by a large number of regulations. As noted by **?**, these regulations are biased towards tenants. The major regulation takes a form of a winter recess during which time landlords have to cope with unpaid bills. The second major characteristic, which is more of interest for our paper, is related to the presence of a rent evolution regulation. Landlords have the power to set the initial rent of their dwelling. After the initial period, the rent evolution is given by a rent index calculated by the French Statistical Institute (INSEE). This rent index is based on the consumer price index for all goods except tobacco and shelter. This regulation has many two consequences. First, it reduces significantly the degree of uncertainty on future rents, and hence operates as an insurance against the shock of rents. A potential implication is to decrease the value of homeownership compared to renting. The homeownership rate in 2006 in France is close to 56 % lower than most of the developed countries.[3] Second, the rent regulation creates an exogenous variation paths for rent that is independent of market conditions. This is very useful in order to circumvent the endogeneity of rent.

4.2 Identification of the model

The identification is based on the optimal housing choice of an individual. That is, at the beginning of the period, individual chooses housing units which maximizes his expected utility. Given the parameters, we can recover v_i from the initial rent equation as follows.

$$v_i = r_i(0) - h(Z_i)$$

We then use the following F.O.C. of the optimal choice of housing characteristics,

$$\frac{\partial EV(r_i(0), Z_i, v_i, u_{zi}, u_{vi}, u_i, X_i, \theta)}{\partial v_i} = -\frac{\partial EV(r_i(0), Z_i, v_i, u_{zi}, u_{vi}, u_i, X_i, \theta)}{\partial r_i(0)} \frac{\partial r_i(0)}{\partial v_i} \quad (3)$$

$$\frac{\partial EV(r_i(0), Z_i, v_i, u_{zi}, u_{vi}, u_i, X_i, \theta)}{\partial Z_i} = -\frac{\partial EV(r_i(0), Z_i, v_i, u_{zi}, u_{vi}, u_i, X_i, \theta)}{\partial r_i(0)} \frac{\partial r_i(0)}{\partial Z_i} \quad (4)$$

Now, notice that

$$\begin{aligned}
\frac{\partial EV(r_i(0), Z_i, v_i, u_{zi}, u_{vi}, u_i, X_i, \theta)}{\partial \mu_{i\tau}} &= \beta^\tau \frac{\partial EV(r_i(\tau), Z_i, v_i, u_{zi}, u_{vi}, u_i, X_{i\tau}, \theta)}{\partial \mu_{i\tau}} \times P(s \geq \tau, r_i, Z_i, v_i, u_{Zi}, u_{vi}, u_i, X_i) \\
&= \beta^\tau \frac{exp(\overline{V}_{0t})}{exp(\overline{V}_{0t}) + exp(\overline{V}_{1t})} \times P(s \geq \tau, r_i, Z_i, v_i, u_{Zi}, u_{vi}, u_i, X_i) \\
&= \beta^\tau P(s > \tau, r_i, Z_i, v_i, u_{Zi}, u_{vi}, u_i, X_i)
\end{aligned}$$

Hence,

$$\frac{\partial EV(r_i(0), Z_i, v_i, u_{zi}, u_{vi}, u_i, X_i, \theta)}{\partial v_i} = \sum_{\tau=1}^{T} \beta^\tau P(s > \tau, r_i, Z_i, v_i, u_{Zi}, u_{vi}, u_i, X_i) c\left(X_{i,\tau}\right) \times u_{vi} u_i$$

[3] The French homeownership rate is higher than very specific housing market Germany (price fallout after reunification boom), Netherlands (extremely high proportion of Public housing), Denmark (....)

Furthermore, notice that

$$\begin{aligned}
&\frac{\partial EV(r_i(0), Z_i, v_i, u_{zi}, u_{vi}, u_i, X_i, \theta)}{\partial Z_i} \\
= & \sum_{\tau=1}^{T}\left[\beta^{\tau} P(s>\tau, r_i, Z_i, v_i, u_{zi}, u_{vi}, u_i, X_i) \times \frac{\partial b\left(Z_i, X_{i,\tau}, u_{zi}\right)}{\partial Z_i}\right] c\left(X_{i,\tau}\right) u_i
\end{aligned}$$

Therefore, we obtain for each element j of vector Z

$$\frac{\sum_{\tau=1}^{T}\left[\beta^{\tau} P(s>\tau,) \times b_{Zj}\left(Z_i, X_{i\tau}, u_{zi}\right) c\left(X_{i,\tau}\right)\right]}{\sum_{\tau=1}^{T} \beta^{\tau} P(s>\tau,) c\left(X_{i,\tau}\right) u_{vi}} = h_{Z_j}(Z_i)$$

Notice that there are $J+1$ unknown variables, u_{vi}, u_{zi} with only J equations. Hence, without loss of generality, we normalize u_{z0} to be zero. Then,

$$u_{v,i} = \frac{\sum_{\tau=1}^{T} \beta^{\tau} P(s>\tau,) \times b_{Z_j}\left(Z_i, X_{i\tau}, u_{zi}\right) c\left(X_{i,\tau}\right)}{\sum_{\tau=1}^{T} \beta^{\tau} P(s>\tau,) c\left(X_{i,\tau}\right) h_{Z_j}(Z_i)} \tag{5}$$

Lastly, we need to derive the utility shock parameter u_i. For that, we use the following relationship.

$$u_i = -\frac{h'(Z) \sum_{l=1}^{T} \beta^{l} P(s>l,)\left[\gamma_{1r} + \gamma_{2r} r_i(0) g(l)\right] g(l)}{\sum_{l=1}^{T} \beta^{l} P(s>l,) \times b_{Z}\left(Z_i, X_{i\tau}, u_{zi}\right) c\left(X_{i,l}\right)} \tag{6}$$

That is, substituting u_{vi} and u_i, we get

$$\begin{aligned}
\mu_{i\tau} &= \left[b\left(Z_i, X_{i\tau}, u_{zi}\right) + u_{v,i}\left(r_i - h(Z_i)\right)\right] c\left(X_{i,\tau}\right) u_i \\
&= h_{Z_j}(Z_i) b\left(Z_i, X_{i\tau}, u_{zi}\right) c\left(X_{i,\tau}\right) \times \frac{\sum_{l=1}^{T} \beta^{l} P(s>l,)\left(-\gamma_{1r} - \gamma_{2r} r_i(0) g(l)\right] g(l)}{\sum_{l=1}^{T} \beta^{\tau} P(s>l,) b_{Z_j}\left(Z_i, X_{i,l}, u_{zi}\right) c\left(X_{i,l}\right)} \\
&\quad + \left[r_i - h(Z_i)\right] c\left(X_{i,\tau}\right) \times \frac{\sum_{l=1}^{T} \beta^{l} P(s>l,)\left[-\gamma_{1r} - \gamma_{2r} r_i(0) g(l)\right] g(l)}{\sum_{l=1}^{T} \beta^{l} P(s>l,) c\left(X_{i,l}\right)}
\end{aligned} \tag{7}$$

Given $\mu_{i,\tau}$, we then can derive the likelihood of the dynamic discrete choice model where the probability of moving out after tenure τ is

$$p(r_i(), Z_i, a_i, \tau, \mu_i) = \frac{exp(\bar{V}_1(a_i))}{exp\left[\bar{V}_{0i}(r_i(0), Z_i, a_i, \tau, \mu_i)\right] + exp(\bar{V}_1(a_i))}$$

Let us make several remarkes about the above identification strategy. First, it is the predicted over time change that identifies the rent coefficient γ_r. With over time variation in rent, γ_{r1} and γ_{r2} can be separately identified as long as the variation of $g(\tau)$ over tenure is independent of the variation of Z_i, $X_{i,\tau}$. Next, consider the terms that includes observed characteristics Z_i.

We now show that $h()$ is identified if $A_j(Z_i, X, u_z)$ varies with X.

Proposition

Suppose that $h(Z)$ is the price equation term for Z and $b_{zj}(Z_i, X, u_z)$ defined as above is a nonconstant function of X and suppose that $u_z(Z, X)$ is a continuously differentiable function of Z and X. Then, among the class of functions $\tilde{b}_j(Z_i, X, \tilde{u}_z)$ with the same property, h is identified and b is identified up to a multiplicative constant.

Proof of Proposition

Without loss of generality, assume that $h(0) = 0$, i.e. the function h does not include a constant. Consider the part of $\mu_{i\tau}$ which is a function of Z_i. Then, it is

$$\overline{\mu}_{i\tau} = \left[\sum_{l=1}^{T} \beta^l P(s > l,) \left[-\gamma_{1r} - \gamma_{2r} r_i(0)\right] g(l)\right] c(X_{i\tau}) \left[\frac{b(Z_i, X_{i\tau}, u_{zi}) h_{zj}(Z_j)}{\sum_{l=1}^{T} \beta^\tau P(s > l,) b_{zj}(Z_i, X_{il}, u_{zi}) c(X_{il})} - \frac{h(Z_j)}{\sum \beta^l P(s > l,) c(X_{il})}\right]$$

Since on the RHS, b and b_z appear in the numerator and the denominator, b is at best identified up to a multiplicative constant. Suppose there exists $\tilde{h}_j() \neq h_j()$, b_j,$\tilde{b}_j()$,u_{zi}, and $\tilde{u}_{zi}$, which gives the same $\overline{\mu}_{i\tau}$, i.e. such that

$$\frac{b(Z_i, X_{i\tau}, u_{zi}) h_{zj}(Z_j)}{\sum_{l=1}^{T} W(l) b_{zj}(Z_i, X_{il}, u_{zi})} - \frac{h(Z_j)}{\sum_{l=1}^{T} W(l)} = \frac{\tilde{b}(Z_i, X_{i\tau}, \tilde{u}_{zi}) \tilde{h}_{zj}(Z_j)}{\sum_{l=1}^{T} W(l) \tilde{b}_{zj}(Z_i, X_{il}, \tilde{u}_{zi})} - \frac{\tilde{h}(Z_j)}{\sum_{l=1}^{T} W(l)}$$

where $W(l) \equiv \beta^l P(s > l,) c(X_{il})$. Thus,

$$\frac{h(Z_j) - \tilde{h}(Z_j)}{\sum_{l=1}^{T} W(l)} = \frac{b(Z_i, X_{i\tau}, u_{zi}) h_{zj}(Z_j)}{\sum_{l=1}^{T} W(l) b_{zj}(Z_i, X_{il}, u_{zi})} - \frac{\tilde{b}(Z_i, X_{i\tau}, \tilde{u}_{zi}) \tilde{h}_{zj}(Z_j)}{\sum_{l=1}^{T} W(l) \tilde{b}_{zj}(Z_i, X_{il}, \tilde{u}_{zi})}$$

Then,

$$\frac{h(Z_j) - \tilde{h}(Z_j)}{\sum_{l=1}^{T} W(l)} = \frac{\sum_{l=1}^{T} W(l) \left[\tilde{b}_{zj}(Z_i, X_{il}, \tilde{u}_{zi}) b(Z_i, X_{i\tau}, \tilde{u}_{zi}) h_{zj}(Z_i) - b_{zj}(Z_i, X_{il}, \tilde{u}_{zi}) \tilde{b}(Z_i, X_{i\tau}, \tilde{u}_{zi}) \tilde{h}_{zj}(Z_i)\right]}{\left[\sum_{l=1}^{T} W(l) b_{zj}(Z_i, X_{il}, u_{zi})\right] \left[\sum_{l=1}^{T} W(l) \tilde{b}_{zj}(Z_i, X_{il}, \tilde{u}_{zi})\right]}$$

Thus,

$$\left[\sum_{l=1}^{T} W(l)\right] \left\{\sum_{l=1}^{T} W(l) \left[\tilde{b}_{zj}(l) b(\tau) h_{zj} - b_{zj}(l) \tilde{b}(\tau) \tilde{h}_{zj}\right]\right\} = \left[\sum_{l=1}^{T} W(l) b_{zj}(l)\right] \left[\sum_{l=1}^{T} W(l) \tilde{b}_{zj}(l)\right] \left[h(Z_j) - \tilde{h}(Z_j)\right]$$

Now, let X_{i1},...,X_{iT} be such that

$$u_{zi}\left(Z_i, X_i\right) = \overline{u}_{zi},\ \widetilde{u}_{zi}\left(Z_i, X_i\right) = \overline{\widetilde{u}}_{zi}$$

That is, X_i can vary given that u_{zi}, $\widetilde{u}_{zi}$ remains constant. Now, if we compare each terms involving $X_{il}, X_{ik}, l \neq k$, $l \neq \tau$, $k \neq \tau$, then

$$W_l W_k \left[\tilde{b}_{zj}(l) b(\tau) h_{zj} - b_{zj}(l)\tilde{b}(\tau)\tilde{h}_{zj} + \tilde{b}_{zj}(k) b(\tau) h_{zj} - b_{zj}(k)\tilde{b}(\tau)\tilde{h}_{zj}\right] = 2W_l W_k b_{zj}(l)\tilde{b}_{zj}(k)\left[h(Z_j) - \tilde{h}(Z_j)\right]$$

Since the LHS term contains $b_{zj}(\tau)$, which is a function of $X_{i\tau}$ whereas the RHS term does not, and the RHS contains the cross term $b_{zj}(k)b_{zj}(l)$ but the LHS does not. In order for the above equality to hold for any X_i given z, b_{zj} cannot be a function of X, which contradicts the assumption.

That is, for the identification to work, we need the elasticity of substitution between observable and unobservable housing characteristics to be a function of observable individual characteristics. This is similar to the standard hedonic literature, which assumes that the marginal rate of substitution between observed housing characteristics Z_i and other consumption goods C_i is a function of individual observed characteristics $X_{i\tau}$. The restriction used in the above identification strategy is that the rental market is competitive. That is, individual observed characteristics enter in the marginal rate of substitution between Z_i and v_i, but not in the rent equation.

The parameters are estimated on the mobility data. That is, parameters are chosen so that the generated unobserved heterogeneity results in per period utility that explains the pattern of mobility best. That is, parameters should be chosen such that individuals who stayed longer in an apartment have higher per period utility. If we can assume that the unobserved characteristics v_i is orthogonal to the observed one, Z_i, then, similarly to the standard hedonic literature, we can identify all the parameters of the hedonic model without duration data, using the static hedonic model. To see this, first consider the rent equation. With the additional orthogonality condition, we can obtain the function $h(Z)$ of the rent equation just by OLS. Then, if we modify equation 6 and add another term $b_x X_i$, it becomes

$$r_v b_Z(Z_i, X_i) u_{zi} + r_v b_x X_i = h'(Z_i) \tag{8}$$

This setup is very similar to that of **?** and **?** and other hedonic literature, where they show identification of the parameters of the function $r_v b_Z$ given that the utility shock u_{zi} is independent of X_i. It is of some interest that in static hedonic model can only identify the utility function parameters well if the marginal utility function is fully flexible. One common restriction is that it is linearly separable in Z_i and X_i. However, we have shown that in dynamic hedonic models, the utility function component $b(Z_i, X_{i,\tau})$ can be made more flexible. In sum, both the hedonic optimal choice part and the dynamic discrete mobility choice part complement each other in identifying the structural parameters and the unobserved heterogeneity of each individual.

5 Estimation

We estimate the model by maximum likelihood. The likelihood function is set up as follows. The likelihood increment for individual i who has been living in an apartment for τ years and whose mobility decision is $I_{m,i}$, is

$$L_i = \left[\prod_{l=1}^{\tau} (1 - P_m(l, X_i, Z_i))\right] [I_{m,i} P_m(\tau+1, X_i, Z_i) + (1 - I_{m,i})(1 - P_m(\tau+1, X_i, Z_i))]$$

where the mobility probability is defined as in equation 2. The likelihood is the product of individual likelihood increments, i.e.

$$L = \prod_{i=1}^{N} L_i$$

We set the parameters of the rental price equation as follows:

$$h(Z_i) = r_{z1} Z_i$$

and the parameters of the components of the per period utility function for staying in the apartment to be:

$$b(Z, X) = b_1(Z_i + u_{zi}) + \frac{1}{2} Z' b_2 Z + X b_X Z$$

and we normalize one element of the vector b_{11} to be 1, and, as discussed before, u_{z1} to be zero. Furthermore, we specify

$$c(X) = c_1 + c_x X$$

where we also normalize c_1 to be 1. The utility of moving out is specified as

$$\varphi_0 + \varphi_1 log(a_i)$$

The derivation of the likelihood increment for each individual i requires the computation of individual specific unobserved utility components u_v, $u_{z,i}$ and u_i. Equations 6 and 7, which show the relationship between v_v, $u_{z,i}$, u_i and the mobility probability and other variables and parameters also describe the fixed point that u_v, $u_{z,i}$ and u_i need to satisfy. Hence, we compute the unobserved utility components using the following iterative algorithm.

Step 1 Given u_v, $u_{z,i}$, u_i, derive the choice probability, and thus $P(s > l)$ for each l.

Step 2 Given $P(s > l)$, derive the next iteration u'_v,$u'_{z,i}$, u'_i using equations 6 and 7.

We stop the above iteration if

$$\left\| \begin{matrix} u_v - u'_v \\ u_{z,i} - u'_{z,i} \\ u_i - u'_i \end{matrix} \right\| < \delta$$

for a small $\delta > 0$

6 Preliminary Estimation Results

Below, we report the Maximum likelihood estimation results of the basic model. All the parameters have the expected sign. The price coefficients γ_{r1}, γ_{r2} for the per period utility of staying are both negative but insignificant. This could be due to the high correlation between the rent and rent squared. The coefficient of the squared term of the number of rooms, b_2 is negative but insignificant. Thus, there is no evidence for convexity of the per period utility function with respect to number of rooms. The coefficient that measures the interaction between age and number or rooms in the utility function is positive but insignificant, implying that marginal utility of number of rooms not decreasing with age. The coefficients of the utility of moving out are both significant. The coefficient of log age is negative, which reflects the data where older people move less often. Finally, the coefficient of the number of rooms in the rent equation is positive and significant. We also report the standard regression results where the dependent variable is the initial rent and the RHS variable is the number of rooms. This is a very simple version of the typical price equation estimated in the hedonic literature. We can see that the OLS estimated coefficient for number of rooms is 0.78, and significant, about half of the value of the coefficient estimated by ML. The ML and OLS results imply that the unobserved housing specific heterogeneity is negatively correlated with the number of rooms. This is sensible because in France, apartments in expensive neighborhoods tend to have lower number of rooms for the same rent.

Table 2: Parameter Estimates and Standard Errors

Parameters	Parameters	S.E.
b_1	1.0	
c_1	1.0	
r_v	1.0	
b_2	−0.2609	0.1091
b_x	1.2236	0.2365
c_x	−0.2339	0.0028
r_z	5.320	0.3186
γ_{r1}	−0.0325	0.01166
γ_{r2}	−0.1292	0.0635
φ_0	3.5588	0.2374
φ_1	−1.6679	0.073

Table 3: OLS

Parameters	Parameters	S.E.
$Const$	2.5748	0.122
$\#room$	0.8017	0.019
$size\ per\ room$	0.0452	0.003

7 Conclusion

In this paper, we propose to estimate the parameters of the price equation and the structural hedonic model by utilizing the tenure and mobility data of rental apartments in France. We utilized the French data on rental apartments, because it has the detailed information on tenure, i.e. length of stay in an apartment and mobility, in addition to the detailed information on the characteristics of the apartment and the renter. The additional benefit of the data on French apartments are that in France, the rent is regulated to grow at the low mandated rate, which is not related to the local housing market. Therefore, we do not need to worry about the endogeneity of the price of rent when we use the overtime variation of rent for identification of the price coefficient. The literature has estimated dynamic models of housing choice, and also estimated the duration models on the tenure choice, but so far it has not explicitly used the duration data to identify the parameters of the hedonic model.

It turns out that the coefficient of the observed housing characteristics are identified without the restriction that the observed and unobserved characteristics are orthogonal, the assumption that is often used in estimating the hedonic model. Furthermore, marginal utility of observed characteristics can be a flexible function of observed housing and individual characteristics. This is in contrast to the conventional identification and estimation strategy of hedonic models, where marginal utility needs to be a separable function of observed housing characteristics and the characteristics of the renter.

The estimation results demonstrate that the observed and unobserved housing characteristics are negatively correlated, which is reasonable, since what we in general see in France is that in expensive neighborhoods, renters live in apartments with lower observed quality to make up for the high unobserved qualities of the neighborhood. Failure to take into account the endogeneity bias could underestimate the true value of the observed characteristics of housing.

An interesting direction for future research would be to extend the analysis of hedonic dynamic model to explicitly include neighborhood effects. There has been much interest in estimating the neighborhood effects in the static and dynamic hedonic literature. However, since neighborhood unobserved heterogeneity is also part of unobserved heterogeneity of a housing unit, we believe that the proper identification of it can be only done as an extension of the identification and estimation of the individual unobserved housing characteristics. This extension is left for future research.

References

A Data selection and variable definition

We start with the 2002 and 2006 French Housing Surveys. We restricted our sample to tenants from private sector and for whom the location variable is available. This yields a sample of 12,084 households. The initial sample selection is the following: we dispose of individuals who have a family relationship with their landlord (216) , individuals older than 65 years (1,413), whose who have been in the same dwelling for more than 30 years (123), and individuals who have been in their house before emancipation age (123).

Finally, to reduce the level of heterogeneity between markets, we dispose of all individuals living in rural areas (XXX), and individuals in IRIS of less than XXX inhabitants.[4]

Our final sample is composed of XXXX individuals. The main advantage of this data source is to locate individuals at the finest geographical levels. However, the neighborhood attributes we rely on are compute from the census, and the finest geographical level available for census data is IRIS.[5] Therefore, a neighborhood is defined as an IRIS.

We measure mobility using a prospective variable, those who said that they are planning to move. When using this variable, the mobility rate is close to 60%. Instead, we restrict our attention to those who said having a precise mobility project within one year. This yields a mobility rate of 24.8 %. This rate is more consistent with the reality. Indeed, in the general population the mobility rate is about 10%. However, private tenants account for approximately two third of mobile households. Considering the fact that private sector accounts for 37 % of the total housing stock, the mobility rate of private tenants is close to 25 %, indicating that our mobility variable does not overestimate the real mobility, put is really close to reality.

[4]Neighborhood attributes are not reliable for them

[5]IRIS are the basic units for the dissemination of local data. Municipalities of at least 10,000 inhabitants and most municipalities of between 5,000 and 10,000 inhabitants are divided up to neighborhoods with a population close to 2000 inhabitants