

Structural Equations, Treatment Effects, and Econometric Policy Evaluation.

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Treatment Model

$$Y = \begin{cases} Y_0 & \text{if } D = 0 \\ Y_1 & \text{if } D = 1 \end{cases}$$

or

$$Y = (1 - D)Y_0 + DY_1$$

D : enrollment in military service.

Y_j : earnings.

Econometric Specification

$$Y_0 = \mu_0(X, U_0)$$

$$Y_1 = \mu_1(X, U_1)$$

The individual treatment effect:

$$\Delta = Y_1 - Y_0$$

The decision rule of program participation:

$$D^* = \mu_D(Z) - U_D$$

$$D = \begin{cases} 1 & \text{if } D^* \geq 0 \\ 0 & \text{if } D^* < 0 \end{cases}$$

Without loss of generality, let U_D be uniformly distributed, as long as $\mu_D(\cdot)$ is a flexible function of Z . Then,

$$\mu_D(Z) = P(D = 1)$$

Assumptions

Assumption 1

$\mu_D(Z)$ is a nondegenerate random variable conditional on X .
There is at least one element of Z that does not belong in X .

Assumption 2

U_1, U_D and U_0, U_D are independent of Z conditional on X .
The instrument is independent of the potential outcome Y_0, Y_1 given X .

Assumption 3

$$1 > Pr(D = 1|X) > 0$$

Assumption 4

$X_0 = X_1$ almost everywhere.

$P(Z)$: probability of receiving the treatment given Z

$$P(Z) = \Pr(D = 1|Z) = \Pr(\mu_D(Z) - U_D \geq 0) = F_{U_D|X}(\mu_D(Z))$$

Index Sufficiency

Notice that those assumptions imply

$$\Pr(Y_j \in A|X, Z, D = j) = \Pr(Y_j \in A|X, P(Z), D = j)$$

That is, $P(Z)$ works as an instrument the same way as Z . To see why, for $j = 0$

$$\begin{aligned} & \Pr(Y_0 \in A|X, Z, U_D : D = 0) \\ &= \Pr(Y_0 \in A|X, Z, U_D : \mu_D(Z) - U_D < 0) \\ &= \Pr(Y_0 \in A|X, Z, U_D : P(Z) - U_D < 0) \\ &= \Pr(Y_0 \in A|X, Z, P(Z), D = 0) \end{aligned}$$

Similarly for $j = 1$

Treatment Effects

Average Treatment Effect

$$\Delta^{ATE}(x) = E[Y_1 - Y_0 | X = x]$$

Treatment Effect of the Treated

$$\Delta^{TT}(x) = E[Y_1 - Y_0 | X = x, D = 1]$$

Treatment Effect of the Treated conditional on $P(Z)$

$$\Delta^{TT}(x, p) = E[Y_1 - Y_0 | X = x, P(Z) = p, D = 1]$$

Local Average Treatment Effect

$$\Delta^{LATE}(x, z, z') = E[Y_1 - Y_0 | X = x, D_z = 1, D_{z'} = 0]$$

Marginal Treatment Effect

$$\Delta^{MTE}(x, u_D) = E[Y_1 - Y_0 | X = x, U_D = u_D]$$

All the other treatment effects are weighted integrated value of the marginal treatment effect.

Average Treatment Effect

$$\Delta^{ATE}(x) = E[Y_1 - Y_0 | X = x] = \int_0^1 \Delta^{MTE}(x, u_D) du_D$$

Treatment of the Treated

$$\begin{aligned} & \Delta^{TT}(x) \\ = & \int_0^1 \Delta^{MTE}(x, u_D) \frac{E_Z(D(Z, u_D) | X = x, U_D = u_D)}{E_{u_D} [E_Z(D(Z, u_D) | X = x, U_D = u_D, P(Z))]} du_D \end{aligned}$$

In order to derive the Treatment of the Treated, given the unobserved term u_D , we look at the fraction of people with value of instruments Z that makes them participate the program, i.e.,

$$u_D \leq \mu_D(Z) = P(Z)$$

Notice that usually, in order to derive the choice probability, we integrate over the unobserved term U_D , i.e.

$$Pr(D = 1|X = x, Z = z) = E_{U_D} [D(z, U_D)|X = x, Z = z]$$

Instead, we integrate over the instrument Z . That is,

$$Pr(D = 1|X = x, U_D = u_D) = E_Z [D(Z, u_D)|X = x, U_D = u_D]$$

In other words,

Treatment of the Treated

$$\begin{aligned} & \Delta^{TT}(x) \\ = & \int_0^1 \Delta^{MTE}(x, u_D) \\ & \times \frac{\Pr(Z : u_D \leq \mu_D(Z)) | X = x, U_D = u_D}{E_{u_D} [\Pr(Z : u_D \leq \mu_D(Z)) | X = x, U_D = u_D, P(Z)]} du_D \\ = & \int_0^1 \Delta^{MTE}(x, u_D) \\ & \times \frac{\Pr(Z : u_D \leq P(Z)) | X = x, U_D = u_D}{E_{u_D} [\Pr(Z : u_D \leq P(Z)) | X = x, U_D = u_D]} du_D \end{aligned}$$

Local Average Treatment Effect

$$\begin{aligned}\Delta^{LATE}(x, z, z') &= E[Y_1 - Y_0 | X = x, D_z = 1, D_{z'} = 0] \\ &= E_{U_D} [Y_1 - Y_0 | X = x, P(z') \leq U_D \leq P(z)] \\ &= \frac{1}{P(z) - P(z')} \int_{P(z')}^{P(z)} \Delta^{MTE}(z, u) du\end{aligned}$$

Hence,

$$\begin{aligned}& \lim_{\delta \searrow 0} \Delta^{LATE}(x, z + \delta, z) \\ &= E[Y_1 - Y_0 | X = x, D_{z+\delta} = 1, D_z = 0] \\ &= \lim_{\delta \searrow 0} \frac{1}{P(z + \delta) - P(z)} \int_{P(z)}^{P(z+\delta)} \Delta^{MTE}(z, u) du = \Delta^{MTE}(x, P(z))\end{aligned}$$

That is, the Marginal treatment effect is the Local Average Treatment Effect of a marginal change in the instrument.

Policy Evaluation

MTE: gross gain from a marginal expansion of the program. Gain can be measured in terms of social welfare function V .

$$\Delta_V^{MTE}(x, u_D) = E [V(Y_{1,a}) - V(Y_{0,a}) | U_{D,a} = u_D, X_a = x]$$

The Policy Relevant Treatment Effect

Assume that the policy change from a' to a only affects the program participation, but not the outcome conditional on the participation status. Or more generally

Assumption 4

The distribution of $(Y_{0,a}, Y_{1,a}, U_{D,a})$ conditional on $X_a = x$ and $(Y_{0,a'}, Y_{1,a'}, U_{D,a'})$ conditional on $X_{a'} = x$ are the same.

$$\begin{aligned}
& E [V(DY_{1,a} + (1 - D)Y_{0,a})|X = x] \\
& - E [V(DY_{1,a'} + (1 - D)Y_{0,a})|X = x] \\
= & \int_0^1 \Delta_V^{MTE}(x, u_D) \\
\times & [Pr(Z_a : u_D \leq P_a(Z))|X = x) - Pr(Z_{a'} : u_D \leq P_{a'}(Z))|X = x)] du_D
\end{aligned}$$

Estimation of MTE: Local Instrumental Variable.

Marginal Treatment Effect can be estimated by using the Local Instrumental Variable.

$$\Delta^{LIV}(x, p) \equiv \frac{\partial E(Y|X = x, P(Z) = p)}{\partial p}$$

Now,

$$\begin{aligned} & E(Y|X = x, P(Z) = p) \\ = & E(Y_0|X = x) + E[Y_1 - Y_0|X = x, U_D \leq p] p \end{aligned}$$

Hence, if we assume additive separability, i.e.

$$\begin{aligned} Y_0 &= \mu_0(X) + U_0 \\ Y_1 &= \mu_1(X) + U_1 \end{aligned}$$

For small δ ,

$$\begin{aligned} & E(Y|X = x, P(Z) = p + \delta) - E(Y|X = x, P(Z) = p) \\ = & E[Y_1 - Y_0|X = x, U_D \leq p + \delta] (p + \delta) \\ & - E[Y_1 - Y_0|X = x, U_D \leq p] p \\ = & \delta [\mu_1(X) - \mu_0(X)] \\ & + \int_{U_1 - U_0, U_D \leq p + \delta} [U_1 - U_0] f(U_1 - U_0, U_D) d(U_1 - U_0) dU_D \\ & + \int_{U_1 - U_0, U_D \leq p} [U_1 - U_0] f(U_1 - U_0, U_D) d(U_1 - U_0) dU_D \\ = & \delta [\mu_1(X) - \mu_0(X)] \\ & + \int_{U_1 - U_0, p < U_D \leq p + \delta} [U_1 - U_0] f(U_1 - U_0, U_D) d(U_1 - U_0) dU_D \\ \approx & \delta [\mu_1(X) - \mu_0(X)] + E[U_1 - U_0|U_D = p] \delta \\ = & E[Y_1 - Y_0|X = x, u_D = p] \delta = \Delta^{MTE}(x, p) \delta \end{aligned}$$