Identification and Estimation of Local Average Treatment Effect

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Local Average Treatment Effect(LATE)

Example: Vietnam Lottery.

Treatment Model

$$Y_i = Y_i(W_i) = \begin{cases} Y_i(1) & if \quad W_i = 1 \\ Y_i(0) & if \quad W_i = 0 \end{cases}$$

W_i: enrollment in military service.
$Y_i(W_i)$: earnings.
Earnings and military service may be correlated, resulting in endogeneity bias.

Instrumental Variables Estimation

Instrument: Outcome of the Draft Lottery Z_i

$$W_i = W_i(Z_i) = \begin{cases} W_i(1) & \textit{if} \quad Z_i = 1 \\ W_i(0) & \textit{if} \quad Z_i = 0 \end{cases}$$

- It is not always true that $W_i(1) = 1$. Some individuals who are drafted $Z_i = 1$ do not serve in the military $W_i(1) = 0$.
- Neither is always true that $W_i(0) = 0$. Some individuals who are not drafted $Z_i = 0$ serve in the military voluntarily $W_i(0) = 1$

Condition for the IV Estimation

Assumption 1 (Independence)

Z_i is orthogonal to $(Y_i(0), Y_i(1), W_i(0), W_i(1))$, or

$$Z_i \perp (Y_i(0), Y_i(1), W_i(0), W_i(1))$$

Assumption 2 (Monotonicity)

$$W_i(1) \geq W_i(0)$$

No individual will decide to join the military if not drafted and refuse to join if drafted. $(W_i(1) < W_i(0))$

Assumption 3 (Correlation)

$$E[W_i|Z_i = 1] = Pr(W_i = 1|Z_i = 1)$$
$$> E[W_i|Z_i = 0] = Pr(W_i = 1|Z_i = 0)$$

Then, the instrumental variable estimator equals the local average treatment effect of the compliers.

$$
\begin{aligned}
\beta_{IV} &= \frac{E\left[Y_i|Z_i=1\right] - E\left[Y_i|Z_i=0\right]}{E\left[W_i|Z_i=1\right] - E\left[W_i|Z_i=0\right]} \\
&= E\left[Y_i(1) - Y_i(0)|complier\right] \\
&= E\left[Y_i(1) - Y_i(0)|W_i(1)=1, W_i(0)=0\right]
\end{aligned}
$$

That is, we can only measure the treatment effect on those who responded to the instrument. That is, who would not serve when not drafted and who would serve when drafted.

There are 4 types of indivdiuals.

- Never Taker: $W_i(0) = 0$, $W_i(1) = 0$. Never serve regardless of whether drafted or not.
- Always Taker: $W_i(0) = 1$, $W_i(1) = 1$. Always serve regardless of whether drafted or not
- Defier: $W_i(0) = 1$, $W_i(1) = 0$. Refuse to serve when drafted. Serve when not drafted.
- Complier: $W_i(0) = 0$, $W_i(1) = 1$: Do not serve when not drafted. Serve when drafted.

Monotonicity Assumption rules out defiers.
Since draft lottery outcome does not change military service status of never takers and always takers, local average treatment effect does not include their treatment effect.

Remarks

Randomness and Exclusion Restriction

Randomness of instrument does not guarantee Assumption 1. Even if Z_i is randomly determined, one needs to have the outcomes $Y_i(0)$, $Y_i(1)$ to be orthogonal to Z_i (exclusion restriction).

Monotonicity

Monotonicity is necessary for the IV estimator to be the local average treatment effect.

Suppose $Z_i = 1$ with probability 0.5 and $Z_i = 0$ with probability 0.5. Suppose the treatment effect β_i is different among individuals. Suppose defiers exist with probabilty 0.4, and $\beta_i = 1$ for them, and compliers exist with probability 0.2 and $\beta_i = 2$ for them. Then, when Z changes from 0 to 1

- W_i moves from 1 to 0, and the outcome from $Y(1)$ to $Y(0)$
- W_i moves from 0 to 1, and the outcome from $Y(0)$ to $Y(1)$

The IV estimate is

$$\begin{aligned}\beta^{IV} &= \frac{E\left[Y_i|Z_i=1\right]-E\left[Y_i|Z_i=0\right]}{P(W_i=1|Z_i=1)-P(W_i=1|Z_i=0)} \\ &= \frac{0.2\times E\left[Y_i(0)-Y_i(1)|def\right]+0.1\times E\left[Y_i(1)-Y_i(0)|compl\right]}{0.2-0.1} \\ &= \frac{0.2\times -1+0.1\times 0.2}{0.1}=0\end{aligned}$$

Even though the treatment effect of military service is positive for both defiers and compliers.

Another assumption which makes a consistent estimation of local average treatment effect is no heterogeneity in treatment. If $\beta_i = 2$ for everybody, then

$$\beta^{IV} = \frac{E\left[Y_i | Z_i = 1\right] - E\left[Y_i | Z_i = 0\right]}{P(1) - P(0)} = \frac{-0.1 \times 2 + 0.2 \times 2}{0.1} = 2$$

Instruments with Multiple Values

Suppose Z is the instrument, taking two values z_0, z_1. That is, it is independent with the error term of the outcome equation, correlated with D_i. Then, Local Average Treatment Effect is

$$\alpha_{z_1,z_0} \equiv E\left[Y_i(1) - Y_i(0) | W_i(z_1) = 1, W_i(z_0) = 0\right]$$

Conditions for the Estimation of LATE.

Assumption 1 (Independence)

Z_i is jointly independent to $(Y_i(0), Y_i(1), W_i(z))$

Assumption 2 (Monotonicity)

For all $z_0 \neq z_1$, either $W_i(z_0) \geq W_i(z_1)$ for all i or $W_i(z_0) \leq W_i(z_1)$ for all i.

Assumption 3 (Correlation)

For $z_0 \neq z_1$,

$$E[W_i|Z_i = z_0] \neq E[W_i|Z_i = z_1]$$

If Assumptions 1, 2 and 3 are satisfied, then the LATE can be estimated as an IV estimator as follows:

$$\begin{aligned}\alpha_{z_1,z_0} &\equiv E\left[Y_i(1) - Y_i(0) | W_i(z_1) = 1, W_i(z_0) = 0\right] \\ &= \frac{E\left[Y_i | Z_i = z_1\right] - E\left[Y_i | Z_i = z_0\right]}{E\left[W_i | Z_i = z_1\right] - E\left[W_i | Z_i = z_0\right]}\end{aligned}$$

Suppose now, we have an instrument Z_i which takes z_0, z_1, ..., z_k. Also, suppose Assumptions 1, 2, and 3 holds for all pairs of z_i, z_j. And suppose the above values are ordered such that

$$p_W(z_{k-1}) \equiv E\left[W_i | Z_i = z_{k-1}\right] \leq E\left[W_i | Z_i = z_k\right] \equiv p_W(z_k)$$

Then, the instrumental variable estimator is the weighted average of the LATE $\alpha_{z_k, z_{k-1}}$

$$\tau = \frac{Cov(Y_i, Z_i)}{Cov(W_i, Z_i)} = \sum_{k=1}^{K} \lambda_k \alpha_{z_k, z_{k-1}}$$

where

$$\lambda_k = \frac{\left(p_W(z_k) - p_W(z_{k-1})\right) \sum_{l=k}^{K} \left(p(z_l) - E\left[Z_i\right]\right)}{\sum_{m=1}^{K} \left(p_W(z_m) - p_W(z_{m-1})\right) \sum_{l=m}^{K} \left(p(z_l) - E\left[Z_i\right]\right)}$$

Proof

$$
\begin{aligned}
& E\left[Y_i | Z_i = z_k\right] - E\left[Y_i | Z_i = z_0\right] \\
= & \sum_{l=1}^{k} \left\{E\left[Y_i | Z_i = z_l\right] - E\left[Y_i | Z_i = z_{l-1}\right]\right\} \\
= & \sum_{l=1}^{k} \left[p_W(z_l) - p_W(z_{l-1})\right] \alpha_{z_l, z_{l-1}}
\end{aligned}
$$

Hence,

$$\begin{aligned}
& E\left[Y\left(Z_i - E[Z]\right)\right] \\
= & \sum_{l=0}^{K} p(z_l) E\left[Y|Z = z_l\right]\left(z_l - E[Z]\right) \\
= & p(z_0) E\left[Y|Z = z_0\right]\left(z_0 - E[Z]\right) \\
& + \sum_{l=1}^{K} p(z_l) \sum_{k=1}^{l} \left[p_W(z_k) - p_W(z_{k-1})\right] \alpha_{z_k, z_{k-1}} \left(z_l - E[Z]\right) \\
= & \sum_{k=1}^{K} \alpha_{z_k, z_{k-1}} \left[p_W(z_k) - p_W(z_{k-1})\right] \sum_{l=k}^{K} p(z_l)\left(z_l - E[Z]\right)
\end{aligned}$$

Similarly,

$$
\begin{aligned}
& E\left[W\left(Z_i - E[Z]\right)\right] \\
= \; & \sum_{l=0}^{K} p(z_l) p_W(z_l) \left(z_l - E[Z]\right) \\
= \; & p(z_0) p_W(z_0) \left(z_0 - E[Z]\right) \\
& + \sum_{l=1}^{K} p(z_l) \sum_{k=1}^{l} \left[p_W(z_k) - p_W(z_{k-1})\right] \left(z_l - E[Z]\right) \\
= \; & \sum_{k=1}^{K} \left[p_W(z_k) - p_W(z_{k-1})\right] \sum_{l=k}^{K} p(z_l) \left(z_l - E[Z]\right)
\end{aligned}
$$