Minimum Wage Effects on Labor Market Outcome Under Search, Matching and Endogenous Contact Rates

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- Card and Krueger (1995) Empirically assess the effect of minimum wage using difference in difference. Comparing between regions that changed the minimum wage and those that did not. Minimum wage increases employment
- Minimum wage increase increasing employment: Monopsony power is needed. Equilibrium search and matchign model creates such a monopsony. Then, increasing minimum wage could increase employment by increasing labor supply.
- In equilibrium search and matching model, changing minimum wage also changes the entire wage distribution. This is due to the change in bargaining power of the workers.

Labor Market Decisions without Minimum Wage

Value of Employment

Employed individual's optimal decisions from period t to period $t + \epsilon$

$$V_e(w) = \frac{w\epsilon + \eta\epsilon V_n + (1 - \eta\epsilon V_e(w)) + o(\epsilon)}{1 + \rho\epsilon}$$

η: job separation rate.
ρ: discount rate
Collecting terms,

$$V_e(w)\left[\rho + \eta\right]\epsilon = \left[w + \eta V_n\right]\epsilon + o(\epsilon)$$

Hence,

$$V_e(w) = \frac{w + \eta V_n}{\rho + \eta}$$

as $\epsilon \to 0$

Value of Nonemployment

$$V_n = \frac{b\epsilon + \lambda\epsilon \int Max\,[V_n, V_e(w(\theta, V_n))]\, dG(\theta) + (1 - \lambda\epsilon)V_n + o(\epsilon)}{1 + \rho\epsilon}$$

Rearrange the terms to get

$$\begin{aligned} & V_n(\rho + \gamma)\epsilon \\ = \quad & \left\{ b\epsilon + \lambda\epsilon V_n + \lambda\epsilon \int Max\,[V_e(w(\theta, V_n)) - V_n, 0]\, dG(\theta) + o(\epsilon) \right\} \end{aligned}$$

Hence,

$$\rho V_n = \left\{ b + \lambda \int Max\,[V_e(w(\theta, V_n)) - V_n, 0]\, dG(\theta) \right\}$$

Wage Determination

Firm's Value Function

$$W_e(w) = \frac{(\theta - w)\epsilon + \eta\epsilon W_e(w) + (1 - \eta\epsilon)0 + o(\epsilon)}{1 + \rho\epsilon}$$

where θ is the total per period output. Correcting terms, we get

$$W_e(w) = \frac{\theta - w}{\rho + \eta}$$

Nash Bargaining Between a Worker and a Firm

- Outside option: workers: V_n, firms: 0
- Worker's surplus: $V_e(w) - V_n$
- Firm's surplus: $W_e(w)$

Nash Bargaining Wage Determination

$$w(\theta, V_n) = argmax\, [V_e(w) - V_n]^\alpha \left[\frac{\theta - w}{\rho + \eta}\right]^{1-\alpha}$$

α: measure of relative bargaining power of the worker.

$$V_e(w) - V_n = \frac{w + \eta V_n}{\rho + \eta} - V_n = \frac{w - \rho V_n}{\rho + \eta}$$

That is, the total surplus is

$$\theta - \rho V_n$$

and worker and firm divide the surplus. From the F.O.C. we derive

$$w(\theta, V_n) = \alpha\theta + (1 - \alpha)\rho V_n$$

Substitute the wage in to derive the worker's surplus.

$$V_e(w) - V_n = \alpha \frac{\theta - \rho V_n}{\rho + \eta}$$

Hence, the value of being unemployed is

$$\rho V_n = b + \frac{\alpha \lambda}{\rho + \eta} \int_{\rho V_n} [\theta - \rho V_n] \, dG(\theta)$$

Notice that the only source of wage heterogeneity comes from the heterogeneity in firm productivity.

Unemployment

The Reservation Match Value

The match value that makes the value of employment equal to the value of unemployment.

$$\theta^* = \rho V_n$$

The reservation wage is

$$w^* = \theta^*$$

That is, firms with productivity higher or equal to θ will be matched, upon meeting with the worker.

The Rate of Leaving Unemployment

$$\lambda Pr(\theta > \theta^*) = \lambda \left[1 - G(\theta)\right]$$

Higher worker bargaining power α implies higher ρV_n, hence higher θ^*. Hence, higher reservation wage and thus higher unemployment. But the worker has higher welfare.

Wage Density

Because

$$w(\theta, V_n) = \alpha\theta + (1-\alpha)\theta^*$$

$$\theta = \frac{w - (1-\alpha)\rho V_n}{\alpha}$$

$$h(w) = \begin{cases} \frac{g(\theta(w.V_n))}{G(\theta^*)} & \text{if } w \geq \theta^* \\ 0 & \text{if } w < \theta^* \end{cases}$$

Endogenous Contact Rates

Constant Returs to Scale matching technology.

$$M(u, v) = vq(\kappa)$$

$$\kappa = \frac{u}{v}$$

Now, let ϕ be the vacancy cost, V_v be the expected value of a vacancy, J be the expected value of a filled vacancy. Then,

$$\rho V_v = -\phi + q(\kappa)\left[1 - G(r)\right](J - V_v)$$

where

$$r = Max\left\{\rho V_n(m), m\right\}$$

With free entry,

$$V_v = 0$$

Hence,

$$J = \frac{\phi}{q(\kappa)\left[1 - G(r)\right]}$$

Now, suppose that the number of labor force participation is l. Then, transition into unemployment is

$$\eta(l - u)$$

and transition out of unemployment

$$\left[1 - G(r)\right] q(\kappa) v.$$

From

$$\eta(l - u) = [1 - G(r)]\, q(\kappa) v$$

we get

$$u = \frac{\eta l}{\eta + [1 - G(r)]\, q(\kappa)/\kappa}$$

Model with Minimum Wage m

Binding Minimum Wage: $m > \theta^*$

Firms with productivity lower than m cannot survive. Hence, lower employment rate given the arrival rate of jobs λ.
The bargaining problem then becomes

$$w(\theta, V_n) = argmax_{w \geq m} [V_e(w) - V_n]^{\alpha} \left[\frac{\theta - w}{\rho + \eta}\right]^{1-\alpha}$$

If the constraint is not binding, then

$$w(\theta, V_n) = \alpha\theta + (1-\alpha)\rho V_n$$

The constraint is barely not binding when

$$m = \alpha\theta + (1-\alpha)\rho V_n$$

where the unconstrained wage equals the minimum wage. The productivity value is

$$\hat{\theta} = \frac{m - (1-\alpha)\rho V_n}{\alpha}$$

Therefore,
$\theta \in [0, m)$: The firm cannot survive.
$\theta \in \left[m, \hat{\theta}\right)$: The firm pays the minimum wage
$\theta \in \left[\hat{\theta}, \infty\right)$: The firm pays the unconstrained Nash bargaining wage.
The model creates a wage distribution with a mass point at the miniumu wage m, which is consistent with the data.

Value of Being Unemployed

Based on the above wage distribution with minimum wage, we get

$$\rho V_n(m) = b + \frac{\lambda}{\rho+\eta}\left\{\int_m^{\hat{\theta}} [m - \rho V_n(m)]\, dG(\theta) + \alpha \int_{\hat{\theta}} [\theta - \rho V_n(m)]\, dG(\theta)\right\}$$

Notice that with binding minimum wage, $m > \rho V_n$.

Direct Effect of Minimum wage

For workers matched with firms with productivity $\theta \in \left[m, \hat{\theta}\right)$, without minimum wage, the worker would have received

$$\alpha\theta + (1-\alpha)\rho V_n < m = \alpha\hat{\theta} + (1-\alpha)\rho V_n$$

Indirect Effect of Minimum Wage

- Minimum wage increase increases the reservation wage, through an increase in the value of being unemployed ρV_n, which comes from an increase in wage if the productivity of the matched firm is $\theta \in \left[m, \hat{\theta}\right)$.
- This increase in value of being unemployed increases the outside option of the worker, thus increases their bargaining power.
- Because of an increase in value of being unemployed, with λ beign constant, minimum wage increases unemployment.

Equilibrium Wage Distribution

$$p(w;m) = \begin{cases} \frac{g(\theta(w,V_n(m)))}{\alpha[1-G(m)]} & \textit{if} \quad w > m \\ \frac{1-G(m)-[1-G(\theta(w,V_n(m)))]}{1-G(m)} & \textit{if} \quad w = m \\ 0 & \textit{if} \quad w < m \end{cases}$$

where

$$\theta(m, V_n(m)) = \frac{m - (1-\alpha)\rho V_n(m)}{\alpha}$$

An increase in minimum wage increases the wage distribution above the minimum wage as well.
This is because of an increase in the value of begin unemployed, which increases the bargaining power of the workers.

The Effect of Minimum Wage on Unemployment

Fixed Contact and Participation Rate

Assume fixed contact and participaton rate. Then,

$$[1 - G(r)]\, q(\kappa)/\kappa = \lambda$$

Then, with $l = 1$

$$u(m) = \frac{\eta l}{\eta + [1 - G(r)]\, q(\kappa)/\kappa} = \frac{\eta}{\eta + \lambda}$$

and

$$u'(m) = \frac{\eta \lambda g(m)}{[\eta + \lambda (1 - G(m))]^2} > 0$$

Minimum wage raises the reservation wage and increases unemployment.

Fixed Contact and Endogenous Participation Rate

- Minimum wage increase raises the reservation wage, i.e. increases the value of unemployment rate. Therefore, in addition it increases the labor force participation rate.
- Minimum wage increases the overall unemployment rate.

Fixed Participation and Endogenous Contact Rate

$$
\begin{aligned}
u(m) &= \frac{\eta}{\eta + \lambda(m)\left[1 - G(m)\right]} \\
u'(m) &= \frac{u(m)}{\eta + \lambda(m)\left[1 - G(m)\right]}\left[\lambda(m)g(m) - \lambda'(m)\left(1 - G(m)\right)\right]
\end{aligned}
$$

Minimum wage decreases unemployment if and only if

$$\frac{\lambda'(m)}{\lambda(m)} > \frac{g(m)}{1 - G(m)}$$

- That is, an increase in minimum wage should increase the contact rate in percentage more than the percentage increase in unemployment given the contact rate being fixed.
- It turns out that this is impossible given the Cobb-Doublass matching technology.
- So, in order the increase in minimum wage to decrease unemployment, both contact rate and participation rate need to change.

Construction of the Likelihood

The likelihood has 3 components.

The Likelihood Increment for the Unemployed

The density of an unemployment spell

$$f_u(t|u) = \lambda \left[1 - G(m)\right] exp \left[\lambda \left(1 - G(m)\right)\right]$$

Then,

$$log \left[f_u(t|u)\right] = log \left[\lambda \left(1 - G(m)\right)\right] - \left[\lambda \left(1 - G(m)\right)\right]$$

The exit rate from unemployment is

$$\frac{dlog \left[f_u(t|u)\right]}{dt} = -\lambda \left(1 - G(m)\right)$$

The probability of being unemployed:

$$p(u) = \frac{\eta}{\eta + \lambda \left[1 - G(m)\right]}$$

The joint probability of unemployment and unemployment spell.

$$f(t, u) = f(t|u)p(u)$$

The Likelihood Increment of Minimum Wage Worker

The proability of minimum wage conditional on employment

$$p(w = m|e) = \frac{1 - G(m) - \left[1 - G(\hat{\theta})\right]}{1 - G(m)}$$

The steady state employment rate at minimum wage

$$\frac{\lambda [1 - G(m)]}{\eta + \lambda [1 - G(m)]}$$

Joint probability

$$p(w = m, e) = p(w = m|e)p(e)$$

The Likelihood Increment of Above Minimum Wage Worker

The wage density:

$$f(w|w > m, e) = \frac{\alpha^{-1} g(\theta(w, V_n))}{1 - G(\theta(\hat{m}, V_n))}$$

where

$$\theta(w, V_n) = \frac{w - (1 - \alpha)\rho V_n(m)}{\alpha}$$

The conditional probability of employment above the minimum wage

$$f(w > m|e) = \frac{1 - G(\hat{\theta})}{1 - G(m)}$$

The steady state employment rate:

$$\frac{\lambda\,[1 - G(m)]}{\eta + \lambda\,[1 - G(m)]}$$

Joint probability

$$f(w, w > m, e) = frac\alpha^{-1}\lambda g(\theta)\eta + \lambda\left[1 - G(m)\right]$$

Identification, Without Minimum Wage

Flinn and Heckman (1982): Let F be the wage distribution

- Reservation wage w^*: minimum observed wage.
- $\lambda F(w)$. Exit rate out of unemployment
- $\frac{f(w)}{1-F(w^*)}$ wage distribution conditional on employment

We cannot separately identify the arrival rate λ and wage offer distribution F, without assuming the functional form on F.

Suppose we somehow get the wage distribution F. Then, the relationship between the wage distribution and the productivity distribution is

$$\frac{f(w)}{1 - F(w^*)} = \frac{\alpha^{-1} g(\frac{w-(1-\alpha)\rho V_n}{\alpha})}{1 - G(\frac{w^*-(1-\alpha)\rho V_n}{\alpha})}$$

One cannot separately identify G and α. Given wages, we do not know whether productivity is high and workers only get a small share of it or productivity is low, and workers get a high share of it.

Binding Minimum Wage

- Lowest observed wage: minimum wage
- One cannot identify the reservation value $\rho V_n(m)$ because this does not equal to the minimum wage.

Use of Labor Share of Earnings

Labor share of earnings

$$\pi_W(\alpha, m) = \frac{mp(w = m) + \int_{\hat{\theta}}^{\infty} [\alpha\theta + (1-\alpha)\rho V_n(m)]\, g(\theta)d\theta / [1 - G(\hat{m}}{\int_m^{\infty} \theta g(\theta)d\theta / [1 - G(\hat{m})]}$$

Then, the labor share of earnings is a nondecreasing function of α

$$\begin{aligned} lim_{\alpha \to 1} \pi_W(\alpha, m) &= 1 \\ lim_{\alpha \to 0} \pi_W(\alpha, m) &= \frac{m}{E[\theta | \theta \geq m]} \end{aligned}$$

Hence, one can estimate the bargaining power from the wage share data.

Table 3
Model Estimates with Profit Information
September 1996 CPS-ORG

	Demographic Group				
Parameter	All	Males	Females	White Non-H	Black or H
			September 1996		
λ	0.309	0.278	0.356	0.328	0.300
	(0.023)	(0.027)	(0.039)	(0.030)	(0.040)
η	0.031	0.030	0.034	0.026	0.055
	(0.003)	(0.004)	(0.005)	(0.003)	(0.010)
μ	2.301	2.342	2.273	2.331	2.150
	(0.036)	(0.044)	(0.057)	(0.040)	(0.092)
σ	0.528	0.554	0.479	0.504	0.627
	(0.020)	(0.026)	(0.028)	(0.021)	(0.053)
$\rho V_n(m)$	3.093	3.313	2.798	2.901	3.572
	(0.146)	(0.173)	(0.273)	(0.202)	(0.165)
α	0.424	0.427	0.429	0.437	0.383
	(0.007)	(0.006)	(0.013)	(0.0122)	0.106
N	2022	1049	973	1612	410
$\ln L$	-5065.676	-2739.338	-2582.245	-3972.758	-1072.138
			August 1997		
λ	0.447	0.397	0.516	0.554	0.341
	(0.036)	(0.043)	(0.063)	(0.056)	(0.046)
η	0.034	0.032	0.037	0.033	0.047
	(0.004)	(0.005)	(0.006)	(0.005)	(0.008)
μ	2.218	2.318	2.104	2.252	2.167
	(0.043)	(0.047)	(0.078)	(0.047)	(0.089)
σ	0.622	0.594	0.646	0.617	0.581
	(0.024)	(0.028)	(0.040)	(0.026)	(0.048)
$\rho V_n(m)$	3.944	3.981	3.960	3.857	4.11

Parameter Estimates

- Average length of period between wage offer

$$\frac{1}{0.309} = 3.23 \ \textit{months}$$

- Average duration of employment

$$\frac{1}{0.031} = 32.3 \ \textit{months}$$

- Reservation wage is lower than the minimum wage.
- Worker's bargaining power: $\alpha = 0.424$

Policy Experiments: Change Minimum Wage

Exogenous Contact Rate

- Increase up to $8 Increases both employment and unemployment, decreases out of labor force.
- Increase after $8 Increases both unemployment and out of labor force, decreases employment
- Aggregate welfare maximizing minimum wage: $8.66

Endogenous Contact Rate

- Minimum wage increase: decreases employment, unemployment, increases out of labor force.
- Aggregate welfare maximizing minimum wage: $3.36

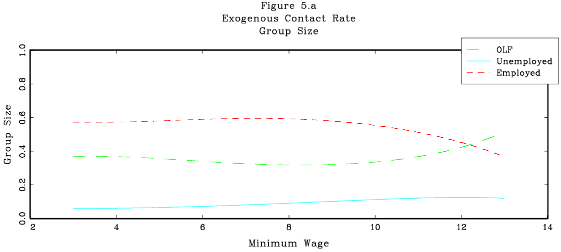
Figure 5.a
Exogenous Contact Rate
Group Size
OLF
Unemployed
Employed
Group Size
0.0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
12
14
Minimum Wage

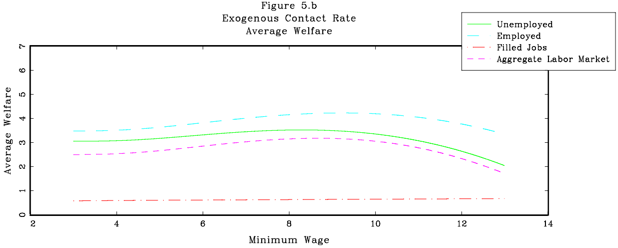
Figure 5.b
Exogenous Contact Rate
Average Welfare
Unemployed
Employed
Filled Jobs
Aggregate Labor Market
Average Welfare
0
1
2
3
4
5
6
7
2
4
6
8
10
12
14
Minimum Wage

Figure 6.a
Endogenous Contact Rate
Group Size

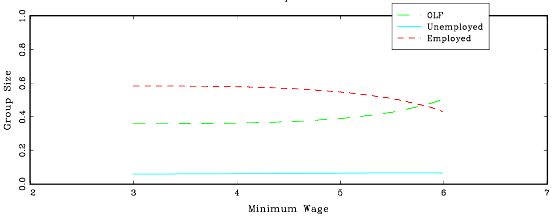

Figure 6.b
Endogenous Contact Rate
Average Welfare