An Equilibrium Search Model of the Labor Market

van den Berg and Ridder

1998

This paper derives the equilibrium wage distribution, that is endogenous, and estimates the empirical equilibrium wage offer distribution from the actual wage distribution.

- Equilibrium wage offer distribution is not observed. Use the dynamic search model to recover it from the actual wage distribution.
- Suppose that only unemployed workers search. Suppose that workers are uniform. Then, suppose firms know the unemployed workers' reservation wage R. Then, profit maximizing firm would only offer reservation wage R. Then, wage distribution is degenerate, which is not consistent with the data.

- Eckstein and Wolpin (1990) addressed this issue by introducing two sources of wage variation. First is the heterogeneity of value of leisure of unemployed workers. Firms with different wage offers attract different types of workers for employment. Second is the measurement error in wage.
- Burdett and Mortensen (1998) introduce a different type of heterogeneity to derive the nondegenerate equilibrium wage offer distribution. Heterogeneity comes from on the job search. employed workers have different reservation wage because they have different wages in their current job.
- This paper uses the Burdett and Mortensen (1998) model to estimate the structural parameters of the Burdett and Mortensen model.

The Equilibrium Search Model

Assumptions

- A1 : Continuous workers and firms with measure m and 1.
- A2 : Workers receive job offers at rate λ_0 if unemployed and λ_1 if employed. Let $F(w)$ be the wage offer distribution. Wage is constant during tenure.
- A3 : An employment match breaks up at rate δ, utility of being unemployed is b
- A4 : Linear production function with marginal revenue pruduct being p. A firm pays all its workers the same wage w.

Optimal Search of Employed and Unemployed Workers

$$
\begin{aligned}
rV_0 &= b + \lambda_0 \left[\int Max\{V_0, V_1(x)\} dF(x) - V_0 \right] \\
rV_1(w) &= w + \lambda_1 \left[\int Max\{V_1(w), V_1(x)\} - V_1(w) \right] dF(x) \\
&\quad + \delta \left[V_0 - V_1(w) \right] \qquad (1)
\end{aligned}
$$

Now, let the reservation wage R be such that

$$V_1(R) = V_0$$

Then,

$$rV_0 = b + \lambda_0 \int_R^\infty \left[V_1(x) - V_0 \right] dF(x)$$

$$rV_1(R) = R + \lambda_1 \int_R^\infty \left[V_1(x) - V_0 \right] dF(x)$$

Together, we obtain

$$R - b = (\lambda_0 - \lambda_1) \int_R^\infty [V_1(x) - V_0] \, dF(x)$$

From Integration by Parts, i.e., from

$$\{[V_1(x) - V_0][1 - F(x)]\}' = V_1'(x)[1 - F(x)] - [V_1(x) - V_0] F'(x)$$

$$R - b = (\lambda_0 - \lambda_1) \int_R^\infty V_1'(x)[1 - F(x)] \, dF(x) \tag{2}$$

Now, taking derivative of equation 1 with respect to w, we get

$$rV_1'(w)dw = dw - \lambda_1 \int_w^\infty V_1'(w)dF(x)dw - \delta V_1'(w)dw$$

$$V_1'(w)[r + \lambda_1 (1 - F) + \delta] = 1 \tag{3}$$

Substituting (3) into (2), we get

$$R - b = (\lambda_0 - \lambda_1) \int_R^\infty \frac{1 - F(x)}{r + \delta + \lambda_1 (1 - F(x))} dF(x)$$

If we set $r = 0$, we get

$$R = b + (\lambda_0 - \lambda_1) \int_R^\infty \frac{1 - F(x)}{\delta + \lambda_1 (1 - F(x))} dF(x)$$

Notice that the reservation wage R does not necessary equal the unemployment benefit b. R can be lower than the unemployment benefit if the arrival rate of jobs λ_1 when employed is higher than when unemployed λ_0. Then, workers want to take jobs just to better search on the job.

Wage Offer Distribution and Wage Distribution

Next, we derive the relationship between wage offer distribution and wage distribution. Denote the wage distribution of $m - u$ employed workers to be $G(w)$. $(m - u)G(w)$ workers receive wages less than w. Then, they either switch into jobs with wages above w, and their total is

$$\lambda_1 [1 - F(w)] G(w)(m - u)$$

or they switch into unemployment, and their total is

$$\delta(m - u)G(w)$$

and the flow of unemployed workers accepting jobs with wages below w is

$$\lambda_0 \left[F(w) - F(R)\right] u$$

where $F(R) = 0$ because no firm will offer wages below the reservation wage.
Because outflow needs to equal inflow,

$$\lambda_1 \left[1 - F(w)\right] G(w)(m - u) + \delta(m - u)G(w) = \lambda_0 F(w) u$$

Hence,

$$G(w) = \frac{F(w)}{\delta + \lambda_1 \left[1 - F(w)\right]} \times \frac{\lambda_0 u}{m - u}$$

set $w = \infty$. Then,

$$1 = \frac{1}{\delta} \times \frac{\lambda_0 u}{m - u}$$

$$\frac{u}{m} = \frac{\delta}{\delta + \lambda_0}$$

$$G(w) = \frac{\delta F(w)}{\delta + \lambda_1 \left[1 - F(w)\right]}$$

Equilibrium Wage Distribution

Firm Profit:

$$(p - w) l(w; R, F)$$

The equilibrium wage distribution has support $[\underline{w}, \overline{w}]$ and because the density of individual wage is $g(w)(m-u)dw$ and the density of firm's wage offer $f(w)dw$ and in equilibrium,

$$f(w) l(w; R, F) dw = g(w)(m-u)$$

Hence, labor demand equals

$$l(w; R, F) = \frac{g(w)dw}{f(w)dw}(m-u) = \frac{m\lambda_0 \delta(\delta + \lambda_1)}{(\delta + \lambda_0)\left[\delta + \lambda_1 (1 - F(w))\right]^2}$$

Equilibrium Wage Offer Distribution

The firm must have the same profit for any wage offer in $[\underline{w}, \overline{w}]$. Thus, the profit for any $w \in [\underline{w}, \overline{w}]$ equals

$$\begin{aligned}\pi &= (p-\underline{w})l(\underline{w}; R, F) = (p-\underline{w})\frac{A}{[\delta+\lambda_1]^2} \\ &= (p-w)l(w; R, F) = (p-w)\frac{A}{[\delta+\lambda_1(1-F(w))]^2}\end{aligned}$$

Hence,

$$F(w) = \frac{\delta+\lambda_1}{\lambda_1}\left[1-\sqrt{\frac{p-w}{p-\underline{w}}}\right]$$

The density function is

$$f(w) = \frac{\delta+\lambda_1}{2\lambda_1}\frac{1}{\sqrt{(p-\underline{w})(p-w)}}$$

Equilibrium Earnings Density

The Equilibrium Earnings Density is

$$g(w) = \frac{\delta\sqrt{p - \underline{w}}}{2\lambda_1} \frac{1}{(p - w)^{3/2}}$$

on $w \in [\underline{w}, \overline{w}]$ Notice that the earnings density is increasing in w but in the data it is not.

The duration of a job with wage w

$$\frac{1}{\delta + \lambda_1 (1 - F(w))}$$

The exit rate (hazard rate) to unemployment

$$\frac{\delta}{\delta + \lambda_1 (1 - F(w))}$$

The exit rate to another job

$$\frac{\lambda_1 (1 - F(w))}{\delta + \lambda_1 (1 - F(w))}$$

Characteristics of the Equilibrium Model

- Without heterogeneity in b for unemployed, the acceptance probability of unemployed is 1.
- Change in b changes the equilibrium wage, leaving the unemployment rate the same.
- The reservation wage can be smaller than the benefit level b, which is observed in the data.
- In this model, firms set the wage. In other models firms and workers bargain over wage.
- In the absence of friction ($\lambda_0 \to \infty$, $\lambda_1 \to \infty$), wages converge to p. Without on the job search ($lambda_1 = 0$), wages converge to b

Data

Dutch OSA Labor Supply Panel Survey

- 4 waves (1985, 1986, 1988, 1990)
- 15-61 year old indivdiuals who are not full time students.
- Labor market history: labor market status including information on job to job transition, spells, wages and occupation, indivdiual characteristics.
- In this analysis, self employed, part time, labor market nonparticipants are excluded. Only labor market status used are employment and unemployment.

Descriptive Analysis:Proportional Hazard Model

Proportional Hazard Model of Unemployment Duration

- Unemployment duration decreases with education, increases with age.
- Evidence of heterogeneity of unemployment spells, but not duration dependence.

Proportional Hazard Model of Employment Duration

- Job spells decrease with education, increase with age.
- Job spells increase with wage.
- Evidence of unobserved heterogeneity. No evidence of duration dependence.

TABLE III

PROPORTIONAL HAZARD MODELS FOR UNEMPLOYMENT SPELLS (STANDARD ERRORS IN PARENTHESES)

	Exponential Model		Two-point Unobs. Heterogeneity		Unobs. Heterogeneity, Weibull Duration Dep.	
Education						
Primary/lower sec.						
Intermediate	−.21	(.20)	−.31	(.24)	−.39	(.30)
Higher	.097	(.30)	.045	(.26)	.0086	(.20)
University	.57	(.56)	.47	(.55)	.49	(.65)
Age						
−29						
30–38	−.46	(.21)	−.49	(.28)	−.71	(.44)
39–	−.65	(.19)	−.63	(.23)	−.80	(.35)
Occupation level						
Un/semi-skilled						
Skilled	.081	(.20)	.11	(.21)	.087	(.28)
Semi-specialized	−.79	(.41)	−.77	(.52)	−.95	(.80)
Specialized	−.12	(.47)	−.031	(.20)	−.0005	(.093)
Constant	−3.58	(.18)	−2.56	(.46)	−3.59	(1.13)
Unobserved heterogeneity						
v_1			.28		.13	
v_2			1.63	(.32)	1.55	(.18)
$\Pr(v = v_2)$			.53	(2.7)	.61	(.18)
Duration dependence						
α					1.39	(3.8)
Log likelihood	−785.56		−780.42		−779.94	

sample is a stock sample,[11] and it is well-known that empirical hazard rates obtained from a stock sample are biased estimates of the underlying cohort hazard rates. Moreover, we want to know how the unemployment and job hazard rates vary over the sample, and we are interested in the relation between

TABLE IV

PROPORTIONAL HAZARD MODELS FOR JOB SPELLS
(STANDARD ERRORS IN PARENTHESES)

	Exponential Model		Two-point Unobs. Heterogeneity		Unobs. Heterogeneity, Weibull Duration Dep.	
Education						
Primary/lower sec.						
Intermediate	.14	(.086)	.15	(.10)	.11	(.087)
Higher	.42	(.12)	.45	(.15)	.32	(.17)
University	.70	(.19)	.80	(.21)	.55	(.27)
Age						
−22						
23–29	−.25	(.12)	−.24	(.16)	−.18	(.13)
30–38	−1.02	(.14)	−1.07	(.17)	−.78	(.33)
39−	−1.62	(.16)	−1.72	(.19)	−1.26	(.51)
Occupation level						
Un/semi-skilled						
Skilled	.045	(.064)	.029	(.11)	.020	(.080)
Semi-specialized	.13	(.13)	.11	(.16)	.096	(.12)
Specialized	.20	(.13)	.14	(.16)	.13	(.12)
Wage/1000	−.21	(.084)	−.21	(.086)	−.14	(.086)
Constant	−4.16	(.16)	−3.52	(.23)	−2.25	(.80)
Unobserved heterogeneity						
v_1			.32		.68	
v_2			1.40	(.23)	1.18	(2.84)
$\Pr(v = v_2)$			.63	(.15)	.64	(4.49)
Duration dependence						
α					.72	(.29)
Log likelihood	−4683.8		−4668.3		−4667.1	

estimates are obtained by maximum likelihood, and the likelihood function deals appropriately with the length bias induced by the stock sample. The estimates for the unemployment spells reported in Table III indicate that unemployment

Wage Distributions

- Accepted wage offer distribution out of unemployment has lower mean than the wage offer distribution of others. Accepted wage offer distribution is stochastically dominated by the other wage distribution.
- Some wages are below the minimum wage: measurement error?
- Zero correlation between accepted wage and unemployment duration.

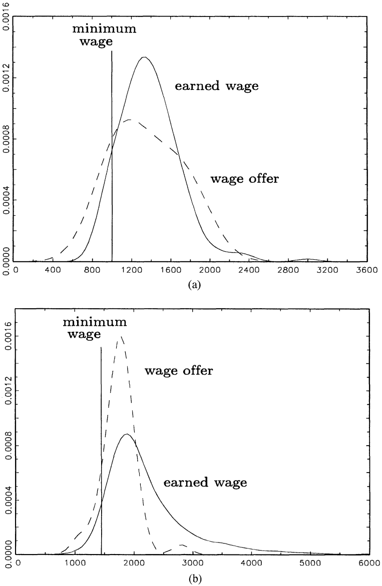

FIGURE 1.—Kernel estimates of wage offer and earnings densities for age −22 and 23− (standard normal kernel and bandwidth $1.06 s_w n^{-1/5}$ with s_w the standard deviation of w). (a) Age −22

Estimation

Identification

- λ_0: identified from the duration of unemployment. Probability of unemployed worker accepting a job:

$$\lambda_0 \int_R^\infty dF(w) = \lambda_0$$

  Duration of unemployment: $\frac{1}{\lambda_0}$
- δ: identified from the duration of jobs that results in unemployment.

To estimate those parameters one does not need any wage data.

Identifying other parameters

- λ_1: from the duration of job with lowest wages

$$\frac{1}{\delta + \lambda_1}$$

- p from the mean of the wage distribution accepted by unemployed F. One needs to know $\underline{w}$, $\overline{w}$

$$
\begin{aligned}
& \int_{\underline{w}}^{\overline{w}} wF'(w)dw \\
= & \int_{\underline{w}}^{\overline{w}} [1 - F(w)]\, dw - \overline{w} \\
= & [\overline{w} - 2\underline{w}] - \frac{\delta + \lambda_1}{\lambda_1} [\overline{w} - \underline{w}] - \frac{1}{2} \frac{\delta + \lambda_1}{\lambda_1} (p - \underline{w})
\end{aligned}
$$

- Assume wage measurement error to be normally distributed with mean zero variance σ^2, independent of the wage.
- Functional form assumption of the measurement error separates the observed wage distribution into measurement error and the true offer wage distribution $F(w)$ with $\overline{w}$ and $\underline{w}$

Heterogeneity

Separate labor markets by education, age, occupation level, by putting those characteristics into the structural parameters and estimate the model for all markets simultaneously.

$$
\begin{aligned}
p &= exp\left[\beta_1' x\right] \\
\lambda_0 &= exp\left[\beta_2' x\right] \\
\lambda_1 &= exp\left[\beta_3' x\right] \\
\delta &= exp\left[\beta_4' x\right]
\end{aligned}
$$

x: age, education, occupation dummies.
Specification including unobserved heterogeneity:

$$p = v \times exp\left[\beta_1' x\right]$$

v: three points, $v_1 < v_2 < v_3$

Results

- $\lambda_0, \lambda_1, \delta$ are monotonically decreasing in age.
- $\delta < \lambda_0, \lambda_1$, δ is much smaller than λ_1. Number of offers during employment is large, and workers quickly climb the wage ladder when young. The wage distribution quickly converges to p with time.
- $\lambda_0 \approx \lambda_1$

TABLE IX

ESTIMATES EQUILIBRIUM SEARCH MODEL: OBSERVED AND UNOBSERVED HETEROGENEITY (STANDARD ERRORS IN PARENTHESES)

Parameter	p	λ_0	λ_1	δ
Constant	0 (−)	−2.92 (0.13)	−2.46 (0.37)	−4.21 (0.14)
Intermediate educ.	0.06 (0.02)	0.31 (0.10)	0.28 (0.22)	−0.02 (0.11)
Higher education	0.14 (0.02)	0.51 (0.15)	0.49 (0.28)	0.21 (0.15)
University	0.24 (0.03)	0.53 (0.20)		
Age category 23–29	0.25 (0.03)	−0.40 (0.16)	−0.40 (0.37)	−0.78 (0.14)
Age category 30–38	0.42 (0.03)	−0.95 (0.16)	−1.20 (0.36)	−1.52 (0.15)
Age category 39–70	0.47 (0.03)	−1.53 (0.15)	−1.75 (0.36)	−2.11 (0.14)
Skilled	0.06 (0.02)	0.05 (0.13)	−0.46 (0.26)	−0.00 (0.11)
Semi-specialized	0.11 (0.02)	−0.19 (0.18)	−0.27 (0.29)	−0.01 (0.15)
Specialized	0.26 (0.02)	0.13 (0.18)		
Log v_1	7.10 (0.03)		q_1	3.35 (0.30)
Log v_2	7.41 (0.04)		q_2	1.72 (0.31)
Log v_3	7.77 (0.05)		q_3	0 (−)
σ^2	0.0220 (0.0013)			
Log likelihood = −26306				

accepted wages from unemployment, then we would expect changes in the estimates. However, the estimates (not reported) do not differ much from those reported. The latter conclusion also holds for estimation results obtained for the

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Table VIII decomposes the total variation in observable wage offers into four omponents: (i) the variation due to wage measurement errors, (ii) the variation ue to observed between-market heterogeneity in p, λ_0, λ_1, δ, b, and $\underline{w}_L$,

TABLE X

CHARACTERISTICS OF THE EQUILIBRIUM: OBSERVED AND UNOBSERVED HETEROGENEITY

Age Category:	16–22	23–29	30–38	39–70	Average
p (Productivity)	1446	1951	2392	2509	2216
λ_0 (Arrival rate in unemployment)	0.066	0.048	0.029	0.016	0.033
λ_1 (Arrival rate in employment)	0.075	0.055	0.025	0.015	0.035
δ (Separation rate)	0.015	0.007	0.003	0.002	0.005
r (Reservation wage)	743	1022	1352	1400	1218
$\overline{w}$ (Highest wage)	1433	1944	2379	2495	2204
$E_F(w)$ (Mean wage offer)	1268	1762	2046	2130	1917
$\underline{w}$ (Lowest wage)	999	1450	1475	1502	1426
b (Benefits level)	807	1120	1248	1320	1192
μ (Monopsony power)	0.13	0.10	0.16	0.17	0.14
u/m (Unemployment)	0.19	0.13	0.11	0.11	0.12
# Observations	212	494	595	648	1949

Decomposition of Total Variation of Wage Offers

ϵ	23 %
x	33 %
v in p	21 %
search friction	22 %

Effect of a Change in the Minimum Wage

An increase in minimum wage: Firms with low levels of p will not be able to profitably hire workers if $w_L > p$. Reduction in employment.

Model	Observed Heterogeneity	Observed and unobserved Heterogeneity
10% increase		
Average	0 %	3 %
16-22	0 %	0 %
23-29	0 %	10 %
30-38	0 %	0 %
39-70	0 %	0 %
25% increase		
Average	11 %	16 %
16-22	0 %	15 %
23-29	42 %	56 %
30-38	0 %	0 %
39-70	0 %	0 %