Ex Ante Evaluation of Social Programs

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- Suppose we want to evaluate ex ande policies, such as tuition subsidy or wage subsidy.
- One could impose parametric functional forms to solve and estimate the dynamic model and then change the policy parameter to simulate the policy.
- Ichimura and Taber (2005) have shown that under some restriction on the model and the type of policy considered, ex ante policy evaluation can be done without any parametric functional forms.

A Multiplicative Wage Subsidy Program

Model without Wage Subsidy

$$Max_h U(c, 1-h)$$

$$s.t.\ c = hw + A$$

Optimal hours: function of wages and nonlabor income.

$$h^* = \phi(w, A)$$

Model with wage subsidy

$$Max_h U(c, 1-h)$$

$$s.t.\ c = hw\tau + A$$

Then, optimal hours with wage subsidy is

$$h^{**} = \phi(w\tau, A)$$

Matching Estimator of the Average Program Effect

Notice that we evaluate wage subsidy program ex ante. The program has not been implemented.
If we compare the outcome without wage subsidy Y_{0i} and with wage subsidy, Y_{1j}

$$Y_{0i}(w\tau, A) = Y_{1j}(w, A)$$

- That is, individual with wage w and gross wage subsidy τ has the same effective wage than the indivdiual with wage $w\tau$ who does not have wage subsidy.
- We can use the hours of individuals with wage $w\tau$ as the policy outcome for individuals with wage w. We do not need to actually observe the policy.

To evaluate the program effect of individual j with wage w_j take the hours data of individual i whose wage w_i equals $w_j\tau$ and compare to that of individual j with wage w_j, controling for assets A as well.
Matching estimator:

$$\frac{1}{\sum_{j,i\in S_p,i=1}^{n}}\sum_{j,i\in S_p,i=1}^{n}[Y_{0i}(w_i = w_j\tau, A_i = A_j) - Y_{0j}(w_j, A_j)]$$

Or Kernel weighted matching estimator:

$$\sum_{i\neq j}[Y_{0i}(w_i = w_j\tau, A_i = A_j) - Y_{0j}(w_j, A_j)]\frac{K(\frac{w_i - w_j\tau}{h})K(\frac{A_i - A_j}{h})}{\sum_{k\neq j}K(\frac{w_k - w_j\tau}{h})K(\frac{A_k - A_j}{h})}$$

No functional form assumption on the utility function is needed to evalute the effect of wage subsidy ex ante.

Allow for unobserved heterogeneity. μ

$$Max_h U(c, 1-h, \mu)$$

$$s.t.\ c = hw + A$$

Optimal labor supply: $h^* = \phi(w, A, \mu)$
The effect of wage subsidy for individual with unobserved heterogeneity μ:

$$\phi(w\tau, A, \mu) - \phi(w, A, \mu)$$

Average effect:

$$\Delta = \int_{w \in S_p} \{\phi(w\tau, A, \mu) - \phi(w, A, \mu)\} f(\mu | A, w) d\mu$$

We can estimate the average effect under the assumption that the distribution of unobserved heterogneity does not depend on wages. That is,

$$f(\mu|w, A) = f(\mu|A)$$

In other words, wage cannot depend on ability. Then, the average subsidy effect is:

$$\frac{1}{\sum_{j,i \in S_p, i=1}^{n}} \sum_{j,i \in S_p, i=1}^{n} [Y_{0i}(w_i = w_j \tau, A_i = A_j) - Y_{0j}(w_j, A_j)]$$

$$\sum_{i \neq j} [Y_{0i}(w_i = w_j \tau, A_i = A_j) - Y_{0j}(w_j, A_j)] \frac{K(\frac{w_i - w_j \tau}{h}) K(\frac{A_i - A_j}{h})}{\sum_{k \neq j} K(\frac{w_k - w_j \tau}{h}) K(\frac{A_k - A_j}{h})}$$

The same as before.

Linear Wage Subsidy and Income Transfer

Original:

$$Max_h U(c, 1-h, \mu)$$

$$s.t.\ c = hw + A$$

Policy:

$$s.t.\ c = h(w+\tau_1) + (A+\tau_2)$$

Program effect:

$$\frac{1}{\sum_{j,i \in S_p, i=1}^{n}} \sum_{j,i \in S_p, j=1}^{n} [Y_{0i}(w_i = w_j + \tau_1, A_i = A_j + \tau_2) - Y_{0j}(w_j, A_j)]$$

Similarly for the Kernel weighted matching estimation for the program effect.

School Attendance Subsidy with Observable Child Wage

s : school attendance indicator.

$$Max_s U(c, s)$$

$$s.t.\ c = y + w(1 - s)$$

with school subsidy τ:

$$s.t.\ c = y + w(1 - s) + \tau s$$

or

$$s.t.\ c = y + \tau + (w - \tau)(1 - s)$$

Use the schooling attendance of individuals with nonlabor income $y + \tau$ and wage $w - \tau$ as a proxy for outcome under policy.

Program effect

$$\frac{1}{\sum_{j,i\in S_p,j=1}^{n}}\sum_{j,i\in S_p,j=1}^{n}[Pr(s=1|w_i=w_j-\tau,Y_i=Y_j+\tau)$$
$$-I(s=1|w_j,A_j)]$$

where $Pr(s=1|w_j,A_j)$ is the school attendance probability estimated by logit or nonparametric regression.

Kernel weighted program effect

$$\sum_{i\neq j}[Pr(s=1|w_i=w_j-\tau,Y_i=Y_j+\tau)-I(s=1|w_j,A_j)]$$
$$\frac{K(\frac{w_i-w_j+\tau}{h})K(\frac{Y_i-Y_j-\tau}{h})}{\sum_{k\neq j}K(\frac{w_k-w_j+\tau}{h})K(\frac{Y_k-Y_j-\tau}{h})}$$

Model of School Subsidy with Leisure

$$Max_{s,l} U(c, l, s)$$

$$s.t.\ c = y + w(1 - l - s) + \tau s$$

or

$$c = y + \tau + (w - \tau)(1 - s) - (w - \tau)l - \tau l$$

The optimal allocation is different from that of individuals with nonlabor income $y + \tau$ and wage $w - \tau$. Hence, it is impossible to use matching methods to estimate the policy effect.

School attendance policy when only accepted child wage offer are observed

$$Max_s U(c, s)$$

$$s.t.\ c = y + w(1 - s) - \delta(k)s$$

$$lnw = \mu_w + \epsilon$$

k: distance to the nearest school.
$\delta(k)$: distance cost of going to school.

We only observe wages of children who work.
Let $\eta(y,k)$ be the reservation wage that determines whether or not to work, i.e. not go to school. Then,

$$E\left[\left(\epsilon | s=0\right)|y,k\right] = \left[E\left(\epsilon|\epsilon > \eta(y,k)\right)|y,k\right] \neq 0$$

For example, if the distance to the next city is large, only workers with high ϵ will work.
Hence, observed wage is a function of y, k. That is,

$$lnw = \mu_w + E\left(\epsilon|\epsilon > \eta(y,k)\right) + u$$

Heckman 2 Step Estimation

We can recover the reservation wage $\eta(y,k)$ from the work choice probability $P(s=0|y,k)$ obtained from the data.

$$P(s=0|y,k) = P\left(\epsilon > \eta(y,k)\right) = 1 - \Phi\left(\frac{\eta(y,k)}{\sigma_\epsilon}\right)$$

Together, the wage equation for workers only can be expressed as

$$lnw = \mu_w + \sigma_\epsilon \frac{\phi\left(\frac{\eta(y,k)}{\sigma_\epsilon}\right)}{1-\Phi\left(\frac{\eta(y,k)}{\sigma_\epsilon}\right)} + u$$

We can then estimate the above equation and recover the parameters of the wage equation μ_w and σ_ϵ.

Since we obtained the parameters of the wage equation, we can generate wages for each individual.
If the person is working, then her wage is the observed wage.
If the person is not working, then draw her wage conditional on the restriction $w_j < \eta(y,k)$

Program effect:

$$\frac{1}{\sum_{j,i \in S_p, j=1}^{n}} \sum_{j,i \in S_p, j=1}^{n} [Pr(s = 1 | w_i = w_j - \tau, Y_i = Y_j + \tau, k_i = k_j)$$
$$- I(s_j = 1 | w_j, y_j, k_j)]$$

Similarly for the Kernel weighted policy effect.

Empirical Application: School Subsidy Program

Conduct ex ante estimation.
Use the model of school attendance when wages of all children are observed. That is, the estimated policy effect is

$$\frac{1}{\sum_{j,i\in S_p,j=1}^{n}} \sum_{j,i\in S_p,j=1}^{n} [Pr(s=1|w_i=w_j-\tau, Y_i=Y_j+\tau)$$
$$-I(s=1|w_j, A_j)]$$

Boys			Girls	
Ages	Experiment	Predicted	Experiment	Predicted
12-13	4.9	0.0	7.7	6.8
14-15	1.6	5.7	14.8	9.9
12-15	2.8	2.1	11.3	8.9