Estimating Production Function Using Inputs to Control for Unobservables:

James levinsohn
Amil Petrin

Bias:

Production function

\[ y_{it} = \beta_0 + \beta_l l_{it} + \beta_k k_{it} + \varepsilon_{it} \]

De-mean:

\[ \Delta y_{it} = \beta_l \Delta l_{it} + \beta_k \Delta k_{it} + \Delta \varepsilon_{it} \]

OLS

\[
\begin{bmatrix}
\hat{\beta}_l \\
\hat{\beta}_k
\end{bmatrix} = \left[X' X\right]^{-1} X' Y = \beta + \left[X' X\right]^{-1} X' E
\]

\[
= \beta + \begin{bmatrix}
\hat{\sigma}_{ll} & \hat{\sigma}_{lk} \\
\hat{\sigma}_{kl} & \hat{\sigma}_{kk}
\end{bmatrix}^{-1} \begin{bmatrix}
\sigma_{l\varepsilon} \\
\sigma_{k\varepsilon}
\end{bmatrix}
\]
\[
\beta_l + \frac{\hat{\sigma}_{k,k} \hat{\sigma}_{l,\varepsilon} - \hat{\sigma}_{l,k} \hat{\sigma}_{k,\varepsilon}}{\hat{\sigma}_{l,l} \hat{\sigma}_{k,k} - \sigma_{l,k}^2} \\
= \beta_k + \frac{\hat{\sigma}_{l,l} \hat{\sigma}_{k,k} - \sigma_{l,k}^2}{\hat{\sigma}_{l,l} \hat{\sigma}_{k,k} - \sigma_{l,\varepsilon}^2}
\]

Likely scenario of bias: \( \hat{\sigma}_{k,\varepsilon} > 0, \hat{\sigma}_{l,\varepsilon} > 0 \)
\( \sigma_{l,\varepsilon} \gg \sigma_{k,\varepsilon} \). Then,

\( \hat{\beta}_l > \beta_l, \hat{\beta}_k < \beta_k \).

Investment proxy:

\[ y_t = \beta_0 + l_t \beta_l + k_t \beta_k + \omega_t + \eta_t \]

\( \omega_t \): productivity.

First stage: investment as productivity proxy:

\[ y_t = l_t \beta_l + \phi_t(i_t, k_t) + \eta_t \]  
\( \phi_t(i_t, k_t) = \beta_0 + k_t \beta_k + \omega_t(i_t, k_t) \)  

Series estimation on (3)
Identify the variable input coefficients using only variation unrelated to $i_t, k_t$.

Or

$$E[y_t | i_t, k_t] = E[l_t | i_t, k_t] \beta_l + \phi_t(i_t, k_t)$$

$$y_t - E[y_t | i_t, k_t] = (l_t - E[l_t | i_t, k_t]) \beta_l + \eta_t$$

Second Stage: obtain the capital coefficient

Using the fact that capital is state variable: slow to adjust.

$k_t$: predetermined at period $t$

Also use the assumption that productivity is Markov

$$\omega_t = E[\omega_t | \omega_{t-1}] + \xi_t$$

Then, $k_t$ does not respond to $\xi_t$

$$y_t^* = y_t - l_t \beta_l = k_t \beta_k + [\beta_0 + E(\omega_t | \omega_{t-1})] + [\xi_t + \eta_t]$$
Problems with the investment proxy:

What if the productivity shock does not have a Markov process?

Unobserved heterogeneity.
i.i.d. component.

Intermediate Inputs as Proxies:

Condition 1 (Monotonicity condition):
Conditional on capital, intermediate input use increases in productivity \( \omega \).

Condition 2 (perfect competition)

Condition 3: separability of the production technology in the intermediate goods.

Production function:

\[
y_t = \beta_0 + \beta_l l_t + \beta_k k_t + \beta_i i_t + \omega_t + \eta_t
\]
Intermediate input demand:

\[ i_t = i_t(\omega_t, k_t) \]

Notice that prices are assumed to be the same across firms.

Intermediate input demand function is monotonic. Invert the intermediate input demand equation:

\[ \omega_t = \omega_t(i_t, k_t) \]

\[ y_t = \beta_0 + \beta_l l_t + \beta_k k_t + \beta_i i_t + \omega_t(i_t, k_t) + \eta_t \]

Then,

\[ y_t = \beta_0 + \beta_l l_t + \phi_t(i_t, k_t) + \eta_t \]

\[ \phi_t(i_t, k_t) = \beta_0 + \beta_k k_t + \beta_i i_t + \omega_t(i_t, k_t). \]
\[
E[y_t | \ell_t, k_t] = E[l_t | \ell_t, k_t] \beta_t + \phi_t(\ell_t, k_t)
\]

\[
y_t - E[y_t | \ell_t, k_t] = (l_t - E[l_t | \ell_t, k_t]) \beta_t + \eta_t
\]