The Dynamics of Productivity in the Telecommunication Equipment Industry

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Potential sources of bias in productivity estimation:

Selection problem: relationship between the unobservable productivity variable and the shutdown (exit) decision.

Simultaneity problem: relationship between productivity and input demands.

Measuring productivity:
There is only one unobserved state variable, which is productivity.

Investment is increasing in productivity.

Use investment as a proxy for productivity.

The Behavioral Framework:

The production function:

\[ y_{it} = \beta_0 + \beta_{a} a_{it} + \beta_{k} k_{it} + \beta_{l} l_{it} + \omega_{it} + \eta_{it} \]
$y_{it}$: log output.

$a_{it}$: age of the firm (plant)

$k_{it}$: log capital input.

$l_{it}$: log labor input.

$\omega_{it}$: state variable: productivity.

Potential bias:

Endogeneity of inputs:
$\omega_{it}$ and inputs $k_{it}, l_{it}$ are correlated.

Selection bias:

$$
\chi_{it} = \begin{cases} 
1 & \text{if stay } \omega_{it} \geq \omega_{i}(a_{it}, k_{it}) \\
0 & \text{if otherwise} 
\end{cases}
$$

$E[\omega_t \mid a_t, k_t, \omega_{t-1}, \chi_t = 1] \neq 0$ is likely to decrease in $k_t$ because with high capital, even unproductive firms stay, hence less firms exit. Downward bias in capital coefficients.
Other assumptions:

Investment and capital accumulation:

\[ k_{t+1} = (1 - \delta)k_t + i_t \]

age:

\[ a_{t+1} = a_t + 1 \]

Productivity follows Markov Process.

\[ \omega_{t+1} \sim F(\cdot, \omega_t) \]

It has been shown that firms that profit maximizes in each period and also determines optimal exit decision and investment decision in each period have satisfy the following properties.
Given state variables $a_t, k_t$, investment is a monotone function of productivity $\omega_t$

$$i_t = i_t(\omega_t, a_t, k_t)$$

There is a threshold in productivity above which the firm stays in and below which the firm exits.

$$\chi_t = 1 \text{ iff. } \omega_t > \omega_t(a_t, k_t)$$

Estimation algorithm

Step 1: Estimate the labor coefficient $\beta_l$

Inversion of the investment function. $i_t = i_t(\omega_t, a_t, k_t)$

$$\omega_t = h_t(i_t, a_t, k_t)$$

productivity can be expressed as a function of observables (investment, age, capital).

$$y_{it} = \beta_l l_{it} + \beta_0 + \beta_a a_{it} + \beta_k k_{it} + h_t(i_{it}, a_{it}, k_{it})$$

which is $\beta_l l_{it}$ plus some unknown function of investment $i_{it}$, age $a_{it}$ and capital $k_{it}$. That is,
\[ y_{it} = \beta l_{it} + \beta_0 + \beta_a a_{it} + \beta_k k_{it} + h_t(i_{it}, a_{it}, k_{it}) \]
\[ y_{it} = \beta l_{it} + \phi_t(i_{it}, a_{it}, k_{it}) + \eta_{it} \]

\[ \phi_t(i_{it}, a_{it}, k_{it}) = \beta_0 + \beta_a a_{it} + \beta_k k_{it} + h_t(i_{it}, a_{it}, k_{it}) \]

Estimate \( \beta_l \) using nonlinear least squares.

\[ y_{it} = \beta l_{it} + \sum_{j_i,j_a,j_k} \gamma_{ji,ja,jk} \ i_{it}^{ji} a_{it}^{ja} k_{it}^{jk} + \eta_{it} \]

Get \( b_l \): nonlinear least squares estimation.

\[ \phi_t(i_{it}, a_{it}, k_{it}) = \sum_{j_i,j_a,j_k} \gamma_{ji,ja,jk} \ i_{it}^{ji} a_{it}^{ja} k_{it}^{jk} \]

How do you separate \( \beta_a, \beta_k \)?

\[ E[y_{t+1} - b_l l_{t+1} \mid a_{t+1}, k_{t+1}, \chi_{t+1} = 1] \]
\[ = \beta_0 + \beta_a a_{t+1} + \beta_k k_{t+1} + E[\omega_{t+1} \mid \omega_t, \chi_{t+1} = 1] \]
\[ = \beta_a a_{t+1} + \beta_k k_{t+1} + g(\omega_{t+1}, \omega_t) \]
\[ g(\omega_{t+1}, \omega_t) = \int_{\omega_{t+1}} \omega_{t+1} \frac{F(d\omega_{t+1} \mid \omega_t)}{\int_{\omega_{t+1}} F(d\omega_{t+1} \mid \omega_t)} \]

What we need to know is \( \omega_t \) and \( \omega_{t+1} \). Use survival probability.

\[ P(stay) = P(\chi_{t+1} = 1 \mid \omega_{t+1}(k_{t+1}, a_{t+1})) = \Pr(\omega_{t+1} \geq \omega_{t+1}(k_{t+1}, a_{t+1}) \mid \omega_{t+1}(k_{t+1}, a_{t+1}), \omega_t) \]

Given the probability of staying \( P(stay) \) from the data, and \( \omega_t \), one can invert to obtain \( \omega_{t+1} \). Hence, \( g \) can also be expressed as a function of \( P(stay) \) and \( \omega_t \). Notice also that

\[ \omega_t = \phi_t - \beta_a a_t - \beta_k k_t \]

Together, we get

\[ y_{t+1} - \beta_t l_{t+1} = \beta_a a_{t+1} + \beta_k k_{t+1} + g(P(stay), \omega_t) + \zeta_{t+1} + \eta_{t+1} \]

\[ = \beta_a a_{t+1} + \beta_k k_{t+1} + g(P(stay), \phi_t - \beta_a a_t - \beta_k k_t) + \zeta_{t+1} + \eta_{t+1} \]

\[ \zeta_{t+1} = \omega_{t+1} - E[\omega_{t+1} \mid \omega_t, \chi_{t+1} = 1] \]
Estimation equation:

\[ y_{t+1} - \beta_l l_{t+1} = \beta_a a_{t+1} + \beta_k k_{t+1} + \sum_{j=0}^{4-m} \sum_{m=1}^{4} \beta_{mj} \hat{\omega}_t^m \hat{P}(stay)_t^j + e_{t+1} \]

Estimation algorithm:

Start with a guess \( \beta_a, \beta_k \)

Regression: OLS

Dependent variable: \( y_{t+1} - b_l l_{t+1} - \beta_a a_{t+1} - \beta_k k_{t+1} \)

Independent variables:
\( \hat{P}(stay) \)
\( \hat{\omega}_t = \hat{\phi}_t - \beta_a a_t - \beta_k k_t \)

Derive sum of squared errors.

Pick \( \beta_a, \beta_k \) which minimizes the sum of squared errors.
Estimation Results

Production function

<table>
<thead>
<tr>
<th></th>
<th>Balanced</th>
<th>Olley-Pakes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>Within</td>
</tr>
<tr>
<td>Labor</td>
<td>0.851 (0.039)</td>
<td>0.728 (0.049)</td>
</tr>
<tr>
<td>Capital</td>
<td>0.173 (0.034)</td>
<td>0.067 (0.049)</td>
</tr>
<tr>
<td>Age</td>
<td>0.002 (0.003)</td>
<td>-0.006 (0.016)</td>
</tr>
<tr>
<td>Time</td>
<td>0.024 (0.006)</td>
<td>0.042 (0.017)</td>
</tr>
<tr>
<td>Other</td>
<td></td>
<td>Polynomial in $P,h$</td>
</tr>
<tr>
<td># Obs</td>
<td>896</td>
<td>896</td>
</tr>
</tbody>
</table>

Increase in capital coefficient, decrease in labor coefficient:

Adding plants that were active at some point in the sample periods.
Sample selection and endogeneity bias leads to high labor coefficients and low capital coefficients.

The Implications for Productivity

Construct plant level productivity between 1974-1987

\[ p_{it} = \exp(y_{it} - b_l l_{it} - b_k k_{it} - b_a a_{it}) \]

Annual productivity growth rate:

<table>
<thead>
<tr>
<th>Time period</th>
<th>Full sample</th>
<th>Balanced panel</th>
</tr>
</thead>
<tbody>
<tr>
<td>74-75</td>
<td>-0.279</td>
<td>-0.174</td>
</tr>
<tr>
<td>75-77</td>
<td>0.020</td>
<td>-0.015</td>
</tr>
<tr>
<td>78-80</td>
<td>0.146</td>
<td>0.102</td>
</tr>
<tr>
<td>81-83</td>
<td>-0.087</td>
<td>-0.038</td>
</tr>
<tr>
<td>84-87</td>
<td>0.041</td>
<td>0.069</td>
</tr>
<tr>
<td>74-87</td>
<td>0.008</td>
<td>0.020</td>
</tr>
<tr>
<td>75-87</td>
<td>0.032</td>
<td>0.036</td>
</tr>
<tr>
<td>78-87</td>
<td>0.034</td>
<td>0.047</td>
</tr>
</tbody>
</table>

The correlation between the productivity change of other industries was zero: most of the productivity is industry specific.

1977-78: registration and certification program
1982: consent degree announcing divestiture: reorganizing and restructuring from 81-83
1984: divestiture
Using only the balanced panel: upward bias in productivity.