Practice Question for the Final Exam

Question 1

Consider the model of bus engine replacement problem of Rust (1987).

1. Carefully write down the Bellman equation of the model, together with the transition probabilities of the state variables.

2. Given you have estimated the parameters of the model, carefully explain how you would obtain the steady state replacement probability.

Question 2

Suppose there are N participants of the sealed bid first price auction. Each bidder's private valuation v is independently and identically distributed with the distribution function being $F(v)$.

1. What is the probability that player i wins the auction when her bid is v_i? Express it in terms of $F(v)$. Write down the profit function of the bidder i.

2. Derive the formula for player i's value maximizing bid given his private value being v_i as a function of $F(v)$.

3. Suppose $F(v)$ is uniformly distributed with support $[0, 1]$. What is the optimal bid for player i if his valuation is v_i? Can you say something about the change in otimal bids and his profit when the number of players increase to infinity?

4. Given the bid of player i being b_i and the distribution of bids being $G(b)$, derive the private value of player i, v_i.