Practice Question for the Midterm Exam

Question 1

Consider the following Cobb-Douglass production function:

$$Y = AL^{\alpha}K^{\beta}\epsilon$$

Suppose you have the data on output, labor cost, capital cost, wage, and rental rate of capital.

1. Derive the First order condition of cost minimization with the Lagrangian coefficient.
Lagrangian:

$$wL + rK + \lambda\left[Y - AL^{\alpha}K^{\beta}e^{\epsilon}\right]$$

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$$\begin{aligned} wL &= \lambda\alpha Y \\ rK &= \lambda\beta Y \\ L &= \frac{\alpha r}{\beta w}K \end{aligned}$$

2. Based on that, derive the cost function.

$$K = \left\{\left[A\left(\frac{\alpha r}{\beta w}\right)^{\alpha}e^{\epsilon}\right]^{-1}Y\right\}^{\frac{1}{\alpha+\beta}}$$

$$L = \left\{\left[A\left(\frac{\beta w}{\alpha r}\right)^{\beta}e^{\epsilon}\right]^{-1}Y\right\}^{\frac{1}{\alpha+\beta}}$$

$$wL + rK = \left[\frac{\alpha+\beta}{\beta}\right]rK = (\alpha+\beta)\left[\left(\frac{w}{\alpha}\right)^{\alpha}\left(\frac{r}{\beta}\right)^{\beta}(Ae^{\epsilon})^{-1}Y\right]^{\frac{1}{\alpha+\beta}}$$

Question 2

Suppose the cost function is as follows.

$$\begin{aligned} lnC &= \beta_0 + \sum_i \beta_i lnp_i + \beta_Q lnQ + \frac{1}{2}\sum_i\sum_j \gamma_{ij} lnp_i lnp_j + \sum_i \gamma_{iQ} lnp_i lnQ \\ &\quad + \frac{1}{2}\gamma_{QQ}\left(lnQ\right)^2 \end{aligned}$$

1. Derive the restrictions that you want to impose on the parameters of the cost function.

2. Derive the equations that you can use to estimate the cost function parameters.

For answers, see the lecture note 8.