Practice Question for the Midterm Exam, No. 2

Question 1

Suppose that the market size is S, inverse demand curve is $P = 3 - \frac{Q}{S}$, where P is the market price and Q is the total quantity. If there are n firms, then the output per firm is $q = \frac{Q}{n}$. The total cost function of a firm is $C(Q) = 1 + q$ where 1 is the fixed cost.

1. Write down the profit function of a firm.

$$\pi = \left(3 - \frac{(n-1)\overline{q}}{S} + \frac{q}{S}\right) q - (1+q)$$

2. Derive the equilibrium price, quantity and profit of the monopolist.

$$q = \frac{(3-1)S}{n+1} = S,\ \ P = 2,\ \ \pi = Pq - (1+q) = S - 1$$

3. Given there are n firms, derive the Nash equilibrium price, quantity and profit of each firm.

The first order condition with respect to q is:

$$\pi' = \left(3 - \frac{(n-1)\overline{q}}{S} + \frac{q}{S}\right) - \frac{q}{S} - 1 = 0$$

In equilibrium, because all firms are the same, $\overline{q} = q$. Hence,

$$\frac{(n+1)q}{S} = 2, q = \frac{2S}{n+1}$$

$$p = 3 - \frac{Q}{S} = \frac{3}{n+1} + \frac{n}{n+1}$$

4. Calculate the market size S which gives zero profit for n firms.

$$\pi = \left(\frac{2}{n+1}\right)^2 S - 1$$

Question 2

Suppose you have the circular city model with the circumference 1 and density D. Suppose that the transportation cost of the consumer is t per unit distance and the firm entry cost is f. Consumer only buys one unit of the good.

1. Derive the equilibrium price and quantity and the equilibrium profit given the number of firms being n.

$$2tx = p - p_i + \frac{St}{n}$$

Demand of firm i is

$$D_i(p_i, p) = 2Dx = D\frac{p - p_i + St/n}{t}$$

$$\pi_i = (p_i - c)\, D_i(p_i, p) - f = (p_i - c)\, D\frac{p - p_i + St/n}{t} - f$$

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$$\frac{p - p_i + St/n}{t} - \frac{p_i - c}{t} = 0$$

$$p_i = \frac{p + St/n + c}{2}$$

$$p = c + St/n$$

$$\pi = S^2 \frac{t}{n}\frac{D}{n} - f$$

2. Derive the zero profit number of firms, equilibrium price and quantity.

$$n = S\sqrt{\frac{Dt}{f}}$$

$$p = c + \sqrt{\frac{tf}{D}}$$

$$q = 2Dx = \frac{DS}{n} = \sqrt{\frac{Df}{t}}$$

3. Derive the total welfare (total consumer unility minus the total production cost minus total travel cost). Suppose that per unit utility of consumer is u. Then, total utility from consumption is

$$SDu$$

and the total travel cost per firm is the integral of all travel costs of its consumers, whose travel cost is 0 for the closest consumer and x for the furthest consumer.

$$2D\int_0^x t dt = 2D\frac{x^2}{2} = Dx^2 = D\left(\frac{q}{2D}\right)^2 = D\left(\frac{Df}{t4D^2}\right) = \frac{f}{4t}$$

Total travel cost is the travel cost per firm times number of firms.

$$TC = \frac{f}{4t}n = \frac{S}{2}\sqrt{\frac{fD}{t}}$$

Total welfare: total consumer utility minus the total production cost minus the total travel cost

$$SD(u-c) - \frac{S}{2}\sqrt{\frac{fD}{t}}$$

4. Suppose the circular city has circumference S and density D. Derive the zero profit number of firms, equilibrium price and quantity. Discuss the difference between the market size effect S and the market density effect D.

From the above eqations, we can see that the market size effect does not change the price and the quantity. That is, an increase in the market size S increases the number of entrants, and thus prices and quantity per firm remains the same. As we can see from the equilibrium number of firm equation, the number of firm per market size S remains the same. Whereas an increase in the market density D reduces the price and increases the quantity. market density