Practice Question No. 1

Question 1

1. Suppose that we have a cross sectional data on markets, with equilibrium prices and quantities and observable demand and supply shifters as data
Consider the following system:

- Demand:$Q_m = \alpha_1 p_m + \alpha_2 x_m + \epsilon_m$
- Supply:$p_m = \beta_1 Q_m + \beta_2 w_m + v_m$

Suppose you estimated the following linear reduced form model of price and quantity.

- $p_m = a_1 x_m + a_2 w_m + \eta_{1,m}$
- $Q_m = b_1 x_m + b_2 w_m + \eta_{2,m}$

Based on the estimation results of the above linear reduced form a_1, a_2, b_1, b_2, recover the structural parameters $\alpha_1, \alpha_2, \beta_1, \beta_2$. Include any assumptions that is necessary to recover the structural parameters from the reduced form regression.

Solving the equilibrium of the model, we get.

$$p_m = \frac{\beta_1 \alpha_2}{1 - \beta_1 \alpha_1} x_m + \frac{\beta_2}{1 - \beta_1 \alpha_1} w_m + \frac{\beta_1}{1 - \beta_1 \alpha_1} \epsilon_m + \frac{1}{1 - \beta_1 \alpha_1} v_m$$

$$Q_m = \frac{\alpha_2}{1 - \beta_1 \alpha_1} x_m + \frac{\alpha_1 \beta_2}{1 - \beta_1 \alpha_1} w_m + \frac{1}{1 - \beta_1 \alpha_1} \epsilon_m + \frac{\alpha_1}{1 - \beta_1 \alpha_1} v_m$$

If we denote $\hat{a}_1$, $\hat{a}_2$, $\hat{b}_1$, $\hat{b}_2$ as the parameter estimates, then $\alpha_1 = \frac{\hat{b}_2}{\hat{a}_2}$, $\beta_1 = \frac{\hat{a}_1}{\hat{b}_1}$. Hence, α_2 can be derived from $\frac{\beta_1 \alpha_2}{1-\beta_1 \alpha_1} = \hat{a}_1$ and β_2 can be derived from $\frac{\alpha_1 \beta_2}{1-\beta_1 \alpha_1} = \hat{b}_2$.

2. Can you also recover $Var(\epsilon)$, $Var(v)$ and $Cov(\epsilon, v)$ from the estimation of the linear reduced form equations?

From 1. we derived the coefficients α_1, α_2, β_1, β_2. Hence, if the sample size is large,

$$\begin{aligned} & Var(p_m \mid x_m, w_m) \\ \approx \; & \left[\frac{\beta_1}{1 - \beta_1 \alpha_1}\right]^2 Var(\epsilon_m) + \left[\frac{1}{1 - \beta_1 \alpha_1}\right]^2 Var(v_m) + 2\frac{\beta_1}{[1 - \beta_1 \alpha_1]^2} Cov(\epsilon_m, v_m) \end{aligned}$$

$$\begin{aligned} & Var(Q_m \mid x_m, w_m) \\ \approx \; & \left[\frac{1}{1 - \beta_1 \alpha_1}\right]^2 Var(\epsilon_m) + \left[\frac{\alpha_1}{1 - \beta_1 \alpha_1}\right]^2 Var(v_m) + 2\frac{\alpha_1}{[1 - \beta_1 \alpha_1]^2} Cov(\epsilon_m, v_m) \end{aligned}$$

$$\begin{aligned} & Cov(P_m, Q_m \mid x_m, w_m) \\ \approx\ & \frac{\beta_1}{[1-\beta_1\alpha_1]^2} Var(\epsilon_m) + \frac{\alpha_1}{[1-\beta_1\alpha_1]^2} Var(v_m) + \frac{1+\alpha_1\beta_1}{[1-\beta_1\alpha_1]^2} Cov(\epsilon_m, v_m) \end{aligned}$$

Question 2

Consider a static oligopoly model with n firms producing homogenous product. Firm i has the following profit function.

$$\pi_i = h(Q)q_i - C_i(q_i)$$

where $p = h(Q)$ is the inverse demand curve and $Q = \sum_{j=1}^{n} q_j$ is the total market output.

1. Write down the first order condition of the profit maximization.

$$h'(Q)\frac{\partial Q}{\partial q_i} q_i + h(Q) - \frac{\partial C_i}{\partial q_i} = 0$$

2. Let $\theta = \frac{1}{n}\sum_{i=1}^{n} \frac{\theta_i q_i}{Q}$ where $\theta_i = 1 + \sum_{j \neq i} \frac{\partial q_j}{\partial q_i}$. Explain how you would specify and estimate the parameters of the inverse demand function, marginal cost function and θ. In particular, discuss what you need to assume to be able to separately estimate θ.

$$p + \theta_i q_i p'(Q) = MC_i$$

$$p + p'(Q)\frac{1}{n}\sum_{i=1}^{n} \frac{\theta_i q_i}{Q} Q = \frac{1}{n}\sum_{i=1}^{n} MC_i$$

Hence,

$$p + p'(Q)\theta Q = \frac{1}{n}\sum_{i=1}^{n} MC_i$$

Specify the aggreate marginal cost as

$$MC = \frac{1}{n}\sum_{i=1}^{n} MC_i = C_0 + C_1 Q + C_2 W$$

and aggregate demand curve as

$$Q = \alpha_0 + \alpha_1 p$$

Then,

$$p + \left[\frac{\theta}{\alpha_1} - C_1\right] Q - C_0 - C_2 W = 0$$

Then, unless additional assumptions are imposed, one cannot separately estimate $\frac{\theta}{\alpha_1}$ and C_1 from the coefficient on Q alone. Therefore, even if we can estimate α_1 from the demand curve using supply shifters as instruments, To estimate θ some additional assumptions are needed. There are two possibilities: 1) Assume $C_1 = 0$, or estimate marginal cost parameter directly given the availability of the cost function or similar information on cost. 2) Assume that demand curve is

$$Q = \alpha_0 + (\alpha_1 + \alpha_2 Z)p$$

Then, the equation becomes

$$p + \left[\frac{\theta}{\alpha_1 + \alpha_2 Z} - C_1\right] Q - C_0 - C_2 W = 0$$

where Z is the demand function slope shifter, and then, given the estimated value of α_1, α_2, in $\frac{\theta}{\alpha_1+\alpha_2 Z} - C_1$, θ and C_1 can be separately estimated.