The Growth and Failure of U.S. Manufacturing Plants

Dunn, Roberts and Samuelson

1989

A 2 Period Dynamic Model of Firm Growth and Exit

Last period:
Production function:

$$Y = PL^{\alpha}A_2$$

where A_2 is the productivity in the second period. Profit function

$$\pi_2 = PL^{\alpha}A_2 - WL - f$$

f: fixed cost of production.
F.O.C of profit maximization with respect to labor input L

$$\alpha PL^{\alpha-1}A_2 = W$$

$$L = \left[\frac{\alpha PA_2}{W}\right]^{\frac{1}{1-\alpha}}$$

Then, the profit is

$$\begin{aligned}
\pi_2 &= P\left[\frac{\alpha P A_2}{W}\right]^{\frac{\alpha}{1-\alpha}} A_2 - W\left[\frac{\alpha P A_2}{W}\right]^{\frac{1}{1-\alpha}} - f \\
&= W^{\frac{-\alpha}{1-\alpha}} P^{\frac{1}{1-\alpha}} A_2^{\frac{1}{1-\alpha}} \left[\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right] - f \\
&= BE - f
\end{aligned}$$

where

$$B = W^{\frac{-\alpha}{1-\alpha}} P^{\frac{1}{1-\alpha}} \left[\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right]$$

$$E_2 = A_2^{\frac{1}{1-\alpha}}$$

Learning about Productivity

Suppose productivity of firm i follow

$$E_{it} = E_i + v_{it}$$

where v_{it} follows i.i.d. normal $N(0, \sigma_v^2)$, and E_i follows $N(0, \sigma_E^2)$.
When making the decision, the firm does not know the productvity.
Instead, it uses the expected productivity $\overline{E}_t$ to make the decision.
Since v_{it} is independent over time, this means

$$\overline{E}_t = E_t [E_i]$$

The firm does not know the constant mean productivity term E_i.
Can only observe the mixture of constant mean productivity term plus the per period shock.
Then, in the first period,

$$\overline{E}_{1i} \sim N(0, \sigma_E^2)$$

In the second, period, after observing E_{i1}, $\overline{E}_{i2}$ given first observation E_{i1} is

$$\overline{E}_{i2}|E_{i1} = E_i|E_{i1} \sim N(\mu_{i2}, \sigma^2_{i2})$$

where

$$\mu_{i2} = \frac{\frac{0}{\sigma^2_E} + \frac{E_{i1}}{\sigma^2_v}}{\frac{1}{\sigma^2_E} + \frac{1}{\sigma^2_v}}$$

and

$$\sigma^2_{i2} = \left[\frac{1}{\sigma^2_E} + \frac{1}{\sigma^2_v}\right]^{-1} < \sigma^2_E$$

Then, the firm produces if

$$\pi_2 = B\overline{E}_2 - f \geq 0$$

and exits if

$$B\overline{E}_2 - f < 0$$

Then,

$$\pi_2 = 0$$

First Period

Profit

$$\begin{aligned} \pi_1 &= B\overline{E}_1 + E\left[Max(B\overline{E}_2 - f, 0)\right] - f \\ &= B\overline{E}_1 + E\left[\int_{\frac{f}{B}}^{\infty} \left(B\overline{E}_2 - f\right) dg(\overline{E}_2|E_1)\right] - f \end{aligned}$$

Then, 2nd period expected productivity which makes exit and production indifferent:

$$\overline{E}_2^* = \frac{f}{B}$$

If $\overline{E}_2 > \overline{E}_2^*$, then $\pi_2 > 0$ and hence the firm produces and if $\overline{E}_2 \leq \overline{E}_2^*$, then $\pi_2 \leq 0$ and the firm exits.
First period breakeven expected productivity

$$\overline{E}_1^* = \frac{1}{B}\left[f - E\left[\int_{\frac{f}{B}}^{\infty}\left(B\overline{E}_2 - f\right) dg(\overline{E}_2|E_1)\right]\right] \leq \overline{E}_2^*$$

If $\overline{E}_1 > \overline{E}_1^*$, then $\pi_1 > 0$ and hence the firm produces and if $\overline{E}_1 \leq \overline{E}_1^*$, then $\pi_1 \leq 0$ and the firm exits.

Without exit, conditional productivity distribution after t period.

$$E_i | \left(E_{it-1}, ..., E_{i1}\right) \sim N(\mu_{it}, \sigma_{it}^2)$$

$$\sigma_{it}^2 = \left[\frac{1}{\sigma_E^2} + \frac{t-1}{\sigma_v^2}\right]^{-1}$$

$$\mu_{it} = \frac{\frac{0}{\sigma_E^2} + \frac{\sum_{s=1}^{t-1} E_{is}}{\sigma_v^2}}{\frac{1}{\sigma_E^2} + \frac{t-1}{\sigma_v^2}}$$

Even without exit, after many periods of observing productivity, variance of $\overline{E}_{it}$ goes down to zero as firms get to learn their permanent productivity more accurately.

The lower the variance of $\overline{E}_{it}$, the lower the value

$$\int_{\frac{f}{B}}^{\infty} \left(B\overline{E}_{it} - f\right) dg_t(\overline{E}_{it})$$

Hence, the higher the threshold value

$$\tilde{E}_{i,t-1} = \frac{1}{B}\left[f - \int_{\frac{f}{B}}^{\infty} \left(B\overline{E}_{it} - f\right) dg_t(\overline{E}_{it})\right]$$

where

$$g_t(\overline{E}_{it}) \approx N(\mu_{it}, \sigma_{it}^2)$$

Theoretical Results of the Learning Model

- Because plant threshold productivity level $\overline{E}_{it}$ increases with age on average, average productivity increases with age.
- Larger plants should have lower failure rate. This is larger plants have higher perceived productivity.
- Plant failure rate decreases with age. This is because over time low perceived productivity firms exit.
- After some size, the mean growth rate decreases with size. This is because the remaining firms, with high perceived productivity firms do not have much room for further positive surprises about their permanent productivity.
- Variance of output growth rate decreases with age. This is because of the variance of perceived productivity decreases with age.

The Growth and Failure of U.S. Manufacturing Plants. Dunne, Roberts and Samuelson

- This paper looks at the dynamics of plant entry, growth and exit of U.S. manufacturing from year 1967 to 1982, explicitly taking into account differences in plant observed characteristics, such as plant size, age, etc.
- The empirical results is compared to the theoretical results by Jovanovic (1982) of firm productivity, learning and exit.

Data: Census of Manufacturers

- All firms in all manufacturing industries, even small firms.
- Each plant has identification number. Hence, researcher can separate between ownership changes (merger, etc) and true entry and exit.
- However, census only collects data every five years.

- Three age categories: 1-5, 6-10, 11-15.
- Five size classes: 5-19, 20-49, 50-99, 100-249, 250-
- 22 digit SIC manufacturing industries.
- 2 Ownership categories: single unit plants (SU), multiunit plants (MU)
- 3 Initial size class: less than, equal to, greater than current size class.

Growth Rate

- Growth rate: $g_t = (q_{t+1} - q_t)/q_t$
- Assume the if the growth rate falls below a critical level g^*, then the firm exits. $(q_{t+1} = 0)$
- If $g'_{t+1} \leq g^*$ then firm exits , and $g_{t+1} = -1$.
  If $g'_{t+1} > g^*$ then $g_{t+1} = g'_{t+1}$
- Overall growth rate and the growth rate of surviving plants are different.

Growth rate of all surviving plants:

$$\mu_h(x) \equiv \frac{\int_{-1}^{\infty} gh(g|x)dg}{\int_{-1}^{\infty} h(g|x)dg}$$

Overall growth rate:

$$\mu_f(x) \equiv \int_{-1}^{\infty} gf(g|x)dg = \mu_h(x)P_s(x) - (1 - P_s(x))$$

where

$$P_s(x) = \int_{-1}^{\infty} h(g|x)dg$$

TABLE I
PLANT GROWTH AND EXIT RATES

Age (years)	Size (number of employees) 5–19	20–49	50–99	100–249	>250	Total
a. Mean employment growth rate of successful plants						
1–5	0.606	0.299	0.187	0.132	0.067	0.446
6–10	0.338	0.136	0.066	0.011	−0.011	0.202
11–15	0.310	0.055	−0.006	−0.015	−0.018	0.153
Total	0.519	0.226	0.130	0.077	0.026	0.353
b. Plant exit rates						
1–5	0.412	0.396	0.390	0.327	0.229	0.397
6–10	0.347	0.268	0.281	0.245	0.158	0.303
11–15	0.304	0.206	0.234	0.212	0.131	0.255
Total	0.391	0.347	0.346	0.291	0.191	0.363
c. Mean employment growth rate of all plants						
1–5	−0.056	−0.216	−0.276	−0.238	−0.178	−0.129
6–10	−0.127	−0.169	−0.234	−0.236	−0.167	−0.162
11–15	−0.089	−0.163	−0.239	−0.224	−0.147	−0.141
Total	−0.074	−0.199	−0.261	−0.236	−0.170	−0.138
d. Number of plant-year observations on successful plants/failing plants						
1–5	75,959/53,325	29,938/19,649	13,758/8,794	9,472/4,601	3,281/977	132,408/87,346
6–10	27,409/14,569	15,268/5,584	7,577/2,961	5,829/1,889	2,630/494	58,713/25,947
11–15	7,773/3,400	4,675/1,216	2,198/673	1,568/421	911/137	17,125/5,847
Total	111,141/71,294	49,881/26,449	23,533/12,428	16,869/6,911	6,822/1,608	208,246/118,690

- Mean growth rate of all successful plants by age and size: Mostly positive. Growth rate declines with age given size. It also decreases with size given age.
  Size and failure probability are negatively correlated. Small firms fail more. Age and failure probability are negatively correlated as well. Younger firms fail more.
- Mean growth rate of all plants: Mostly negative. This is because exit rates lower growth rate.
  The relationship between growth rate and size is U-shaped. Growth rate initially declines with size and then increases. Later increase in growth rate comes from the decrease in failure rate.
  Growth increases with age for most size groups. This increase comes from the decrease in failure rate.

Empirical Model

$$Y_{ct} = \sum_{i=1}^{60} \alpha_i D_i + \sum_{k=1}^{18} \beta_k D_k^c + \epsilon_{ct}$$

D_i: industry time dummies.

D_k^c: size age dummies.

TABLE II
REGRESSION COEFFICIENTS FOR PLANT FAILURE RATES: 1967, 1972, 1977 ENTRANTS
(STANDARD ERRORS IN PARENTHESES)

		Single-plant	Multiplant
Intercept[a]		0.426	0.471
Age/initial-size (IS) versus current-size (CS) category			
2	IS > CS	0.084 (0.012)*	0.104 (0.010)*
2	IS < CS	0.008 (0.010)	0.001 (0.007)
3	IS > CS	0.043 (0.020)	0.065 (0.021)
3	IS < CS	0.002 (0.016)	−0.013 (0.012)
Age/current-size category			
1	2	−0.035 (0.005)*	−0.064 (0.006)*
1	3	−0.002 (0.009)	−0.116 (0.006)*
1	4	0.055 (0.014)	−0.206 (0.007)*
1	5	−0.039 (0.056)	−0.306 (0.008)*
2	1	−0.061 (0.005)*	−0.070 (0.009)*
2	2	−0.166 (0.008)*	−0.158 (0.008)*
2	3	−0.151 (0.012)*	−0.206 (0.008)*
2	4	−0.095 (0.019)*	−0.271 (0.008)*
2	5	−0.174 (0.097)	−0.356 (0.008)*
3	1	−0.087 (0.008)*	−0.150 (0.018)*
3	2	−0.199 (0.013)*	−0.238 (0.015)*
3	3	−0.180 (0.021)*	−0.250 (0.015)*
3	4	−0.099 (0.037)	−0.315 (0.014)*
3	5	−0.193 (0.179)	−0.386 (0.014)*

Age categories: 1 = 1–5 years; 2 = 6–10 years; 3 = 11–15 years.
Size categories: 1 = 5–19 employees; 2 = 20–49 employees; 3 = 50–99 employees; 4 = 100–249 employees; 5 = ≥250 employees.
a. The reported intercept is the average value of the 60 estimated industry-time intercepts.
*Significant at the 0.05 level using Leamer's]1978] correction for sample size.

The pattern of age and size effects on failure rates can be examined more closely by testing several restricted versions of the

- Failure rates are lower for older plants.
- Failure rates are lower for (initially) larger plants, especially for multi-unit plants.

TABLE IV

REGRESSION COEFFICIENTS FOR SUCCESSFUL PLANTS: 1967, 1972, 1977 ENTRANTS (STANDARD ERRORS IN PARENTHESES)

		Mean growth rate		Variance of growth rate	
		Single-plant	Multiplant	Single-plant	Multiplant
Intercept[a]		0.492	0.709	1.650	2.446
Age–initial-size (IS) versus current-size (CS) category					
2	IS > CS	−0.038 (0.025)	0.010 (0.018)	−0.103 (0.014)*	−0.052 (0.010)*
2	IS < CS	0.061 (0.014)	0.026 (0.011)	0.080 (0.012)*	0.015 (0.008)
3	IS > CS	−0.023 (0.037)	0.009 (0.029)	−0.078 (0.020)	−0.056 (0.012)
3	IS < CS	0.045 (0.022)	0.004 (0.017)	0.057 (0.015)	0.004 (0.007)
Age–current-size category					
1	2	−0.292 (0.013)*	−0.360 (0.034)*	−1.166 (0.080)*	−1.812 (0.287)*
1	3	−0.448 (0.016)*	−0.504 (0.032)*	−1.383 (0.078)*	−1.939 (0.284)*
1	4	−0.551 (0.023)*	−0.598 (0.032)*	−1.488 (0.078)*	−2.072 (0.284)*
1	5	−0.664 (0.054)*	−0.683 (0.033)*	−1.651 (0.079)*	−2.167 (0.284)*
2	1	−0.229 (0.014)*	−0.432 (0.039)*	−1.110 (0.083)*	−2.037 (0.285)*
2	2	−0.450 (0.015)*	−0.575 (0.032)*	−1.465 (0.078)*	−2.117 (0.284)*
2	3	−0.550 (0.018)*	−0.636 (0.032)*	−1.545 (0.077)*	−2.148 (0.284)*
2	4	−0.579 (0.024)*	−0.680 (0.032)*	−1.640 (0.077)*	−2.174 (0.283)*
2	5	−0.587 (0.074)*	−0.717 (0.032)*	−1.715 (0.082)*	−2.202 (0.283)*
3	1	−0.310 (0.021)*	−0.517 (0.054)*	−1.257 (0.086)*	−2.130 (0.285)*
3	2	−0.529 (0.020)*	−0.664 (0.036)*	−1.547 (0.078)*	−2.208 (0.284)*
3	3	−0.628 (0.027)*	−0.663 (0.036)*	−1.602 (0.078)*	−2.205 (0.284)*
3	4	−0.681 (0.045)*	−0.664 (0.035)*	−1.616 (0.080)*	−2.220 (0.284)*
3	5	−0.506 (0.269)	−0.683 (0.036)*	−1.472 (0.234)*	−2.248 (0.284)*

Age categories: 1 = 1–5 years; 2 = 6–10 years; 3 = 11–15 years.
Size categories: 1 = 5–19 employees; 2 = 20–49 employees; 3 = 50–99 employees; 4 = 100–249 employees; 5 = ≥250 employees.
a. The reported intercept is the average value of the 60 estimated industry-time intercepts.
*Significant at the 0.05 level using Leamer's [1978] correction for sample size.

Mean and Variance of Growth Rates of Surviving Plants

- Mean growth rates decline both with size and age.
- Initial size has no effect on mean growth rate.
- Variance of growth rates decline both with size and age.

TABLE V

REGRESSION COEFFICIENTS FOR ALL PLANTS: 1967, 1972, 1977 ENTRANTS (STANDARD ERRORS IN PARENTHESES)

		Mean growth rate		Variance of growth rate	
		Single-plant	Multiplant	Single-plant	Multiplant
Intercept[a]		−0.126	−0.051	1.503	2.09
Age–initial-size (IS) versus current-size (CS) category					
2	IS > CS	−0.106 (0.022)*	−0.104 (0.017)*	−0.073 (0.024)	−0.001 (0.016)
2	IS < CS	0.043 (0.014)	0.012 (0.011)	0.094 (0.015)	0.033 (0.011)*
3	IS > CS	−0.054 (0.036)	−0.045 (0.034)	−0.016 (0.039)	−0.007 (0.026)
3	IS < CS	0.037 (0.023)	0.006 (0.019)	0.061 (0.020)	0.002 (0.016)
Age–current-size category					
1	2	−0.143 (0.010)*	−0.132 (0.020)*	−0.877 (0.048)*	−1.197 (0.155)*
1	3	−0.278 (0.013)*	−0.168 (0.019)*	−1.093 (0.047)*	−1.452 (0.151)*
1	4	−0.415 (0.017)*	−0.143 (0.019)*	−1.235 (0.047)*	−1.606 (0.150)*
1	5	−0.458 (0.056)*	−0.116 (0.021)*	−1.400 (0.054)*	−1.730 (0.150)*
2	1	−0.072 (0.011)*	−0.188 (0.025)*	−0.801 (0.053)*	−1.550 (0.153)*
2	2	−0.118 (0.013)*	−0.173 (0.020)*	−1.184 (0.047)*	−1.595 (0.150)*
2	3	−0.212 (0.017)*	−0.170 (0.020)*	−1.274 (0.047)*	−1.681 (0.150)*
2	4	−0.316 (0.025)*	−0.144 (0.020)*	−1.332 (0.049)*	−1.734 (0.150)*
2	5	−0.234 (0.100)*	−0.097 (0.021)	−1.401 (0.073)*	−1.812 (0.150)*
3	1	−0.090 (0.018)*	−0.152 (0.041)	−0.968 (0.058)*	−1.663 (0.157)*
3	2	−0.149 (0.020)*	−0.159 (0.027)*	−1.279 (0.049)*	−1.704 (0.151)*
3	3	−0.248 (0.028)*	−0.144 (0.027)*	−1.350 (0.049)*	−1.732 (0.151)*
3	4	−0.360 (0.045)*	−0.085 (0.026)	−1.353 (0.055)*	−1.774 (0.151)*
3	5	−0.156 (0.241)	−0.058 (0.027)	−1.205 (0.216)*	−1.828 (0.151)*

Age categories: 1 = 1–5 years; 2 = 6–10 years; 3 = 11–15 years.
Size categories: 1 = 5–19 employees; 2 = 20–49 employees; 3 = 50–99 employees; 4 = 100–249 employees; 5 = ≥250 employees.
[a] The reported intercept is the average value of the 60 estimated industry-time intercepts.
*Significant at the 0.05 level using Leamer's [1978] correction for sample size.

Mean and Variance of Growth Rates of All Plants

- Mean growth rates decline with size for single unit plants but for multi unit plants either U-shaped (small plants) or increases with size.
- Mean growth rates tend to decline with age for single unit plants but increase with age for multiple unit plants. Reduction in failure rates is overwhelming.
- Variance of growth rates decline both with size and age.

The relationship between plant failure, growth and size, age are roughly consistent with learning model of entry, exit and plant growth.