# Oligopoly

Static oligopoly model with $n$ firms producing homogenous product.

## Firm's Profit Maximization

Firm $i$'s profit maximization problem:

$$Max_{q_i} P(Q)q_i - C_i(q_i)$$

$P(Q)$: inverse demand curve: $p = P(Q)$
$Q = \sum_{k=1}^{n} q_k$ is total market output.
F.O.C. with respect to $q_i$:

$$q_i \frac{\partial P(Q)}{\partial q_i} + P(Q) - \frac{\partial C_i}{\partial q_i} = 0$$

# Strategic Interaction

Notice that

$$\frac{\partial P(Q)}{\partial q_i} = P'(Q)\frac{\partial Q}{\partial q_i} = P'(Q)\frac{\partial \sum_{k=1}^{n} q_k}{\partial q_i}$$

$$= P'(Q)\left[1 + \sum_{j \neq i} \frac{\partial q_j}{\partial q_i}\right]$$

$\frac{\partial q_j}{\partial q_i}$: the effect of firm $i$ quantity change on others firms' equilibrium quantities. Strategic interaction.

F.O.C. with respect to $q_i$:

$$q_i \frac{\partial P(Q)}{\partial q_i} + P(Q) - \frac{\partial C_i}{\partial q_i} = 0$$

Let $\theta_{ij} = \frac{\partial q_j}{\partial q_i}$. Then, after dividing by $p$ the F.O.C. becomes

$$\frac{q_i}{Q}\frac{Q}{p}\frac{dP(Q)}{dQ}(1 + \sum_{j \neq i} \theta_{ij}) + \frac{p - MC_i}{p} = 0$$

Let $\theta_i = 1 + \sum_{j \neq i} \theta_{ij} = 1 + v_i$. Then,

$$\frac{p - MC_i}{p} = \frac{s_i}{\eta}\theta_i$$

$\frac{p - MC_i}{p}$ is called the Lerner's index and is a measure of market power.

The parameter $\theta_i$ measures firm $i$'s conjecture about the reaction of other firms.

- $\theta_i = 0$ : Bertrand, perfect competition
- $\theta_i = 1$ : Cournot
- $\theta_i = 1, s_i = 1$ : Joint profit maximization (act as monopolist).

There are no theories when $\theta_i$ takes any values other than those.

# Estimation of the Oligopoly Model

Consider how to estimate the Oligopoly firms' optimal choice equation

$$\frac{p - MC_i}{p} = \frac{s_i}{\eta}\theta_i$$

- $\theta_i$ is unknown and needs to be estimated.
- $p$, $s_i$ (market share of firm $i$) are given in the data, and price elasticity of demand $\eta$ can be estimated from the demand curve, using cost shifters as IV's.
- However, there usually is no cost data on firms. We cannot estimate the marginal cost function from the data.

# Estimating $\theta_i$

How can we estimate $\theta$, the average conjectural parameter, if we only have market level data (price $p$ and total quantity $Q$)?

## Market Level Aggregation of Supply Equation

From the F.O.C. equation (1), we get

$$P + q_i \frac{\partial P(Q)}{\partial q_i} = MC_i$$

From the definition of $\theta_i$

$$P + \theta_i q_i P'(Q) = MC_i$$

Take an average over $N$ firms

$$P + P'(Q) \sum_{i=1}^{N} \frac{\theta_i q_i}{N} = \sum_{i=1}^{N} \frac{MC_i}{N}$$

# Specification of the Empirical Model

Rewrite as:

$$P + \theta \frac{\partial P}{\partial Q} Q = MC$$

where $\theta = \frac{1}{N} \sum_{i=1}^{N} \frac{\theta_i q_i}{Q}$ and MC is the average marginal cost.
Assumption: $\theta$ is invariant to the shares.
Suppose demand is $Q = \alpha_0 + \alpha_1 P + \epsilon$
and marginal cost is $MC_i = c_0 + c_1 q_i + c_2 w_i$

$$MC = c_0 + c_1 Q + c_2 W$$

Then, first estimate the demand equation to get $\alpha_0$ and $\alpha_1$. Then, estimate the supply equation.

$$P = c_0 + [c_1 - \frac{\theta}{\alpha_1}]Q + c_2 W + v$$

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It is impossible to separate out $c_1$ (economy of scale parameter) and $\theta$ (conduct parameter).
If prices are low, it is impossible to distinguish whether this is due to competition or marginal cost being low.

## Methods to overcome the identification problem

Several ways that people have dealt with the problem:

- Assume constant returns to scale ($c_1 = 0$)
- Impose some values to $\theta$ by assuming a specific market structure. (monopoly, perfect competition, Bertrand or Cournot competition).

- Find variables $Z$ that affects the slope of the demand curve. (example: average income, seasonality, etc.)
  Estimate the new demand curve

  $$Q = \alpha_0 + (\alpha_1 + \alpha_2 Z)P + \epsilon$$

  to get $\alpha$s. Then,

  $$P = c_0 + [c_1 - \frac{\theta}{\alpha_1 + \alpha_2 Z}]Q + c_2 W + v$$

  and given $\alpha$'s, we can separately estimate $c_1$ and $\theta$ by using $\frac{1}{\alpha_1+\alpha_2 Z}$ as a variable.
- Measure the marginal cost directly. (Genesove and Mullin (1998)

# Dynamic Oligopoly and Cartel (Porter (1983))

- $\theta$ may change over time, due to regime change from Cartel period and Price War period.
- High $\theta$ period: Cartel period. Low $\theta$ period: Price War period.

# Genesove and Mullin (1998)

- Looks at the sugar industry in late 1890 to 1920. This is because production of sugar is simple, where part of the marginal cost parameters are known and also has rich data on cost.
- With marginal cost parameters obtained, it is easy to estimate $\theta$.
- Also estimates the model in the standard way, without using the marginal cost data, and compares the parameters. Joint estimation of demand and cost parameters without cost information works well, i.e. parameter estimates are not that different from the ones estimated with known marginal cost parameters.

Data: weekly sugar production units, price and number of firms. The authors use quarterly data.

## Empirical Model

- Demand: $Q = \beta(\alpha - p)^{\gamma}$
- Marginal Cost: known to be Constant Returns to Scale
  $MC = c_0 + kp_{rw}$
  where $p_{rw}$ is the price of unprocessed sugar and $c_0$ is per unit labor and other costs.
- Supply: $P + \theta QP'(Q) = MC$

# Demand estimation

- Separately estimate demand for high season and other times.
- Instrument for price: Imports from Cuba

## Linear Demand

$$Q = [\alpha_{1H} + \alpha_{2H} P] I_H + [\alpha_{1L} + \alpha_{2L} P][1 - I_H] + \epsilon$$

$I_H$: 1 if high season. 0 otherwise.
Estimated demand parameters: $\alpha_H$, $\alpha_L$.

## Nonlinear Demand

$$Q = \beta(\alpha_H I_H + \alpha_L (1 - I_H) - p)^{\gamma}$$

$\gamma$ is set and $\alpha$s, $\beta$ are estimated.

# Supply estimation when MC information is not used

## Linear Demand

Using the estimated demand parameters $\alpha_{2H}$, $\alpha_{2L}$ estimate the remaining parameters $c_0$, $c_1$, $c_2$, $\theta$ (including MC parameters) using the supply equation.

$$P = c_0 + [c_1 - \theta(\frac{I_H}{\alpha_{2H}} + \frac{1 - I_H}{\alpha_{2L}})]Q + c_2 W + v$$

where $W$ is the cost shifter vector.

## Nonlinear Demand

Assume constant marginal cost $c$. Given $\gamma$, $\alpha$s, estimate $c$ and $\theta$ from

$$P = \frac{\theta[\alpha_H I_H + \alpha_L(1 - I_H)] + \gamma c}{\gamma + \theta} + v$$

# Supply Estimation when marginal cost is used.

No need for demand estimation. Set $\gamma$ and estimate

$$P = \frac{\theta[\alpha_H I_H + \alpha_L(1 - I_H)] + \gamma c_0}{\gamma + \theta} + \frac{\gamma c}{\gamma + \theta} k P_{RAW} + v$$

To get this, use $\frac{\partial Q}{\partial P} = \frac{\gamma Q}{\alpha_H I_H + \alpha_L(1 - I_L) - P} = \frac{1}{P'(Q)}$

# Results

## Direct estimate of conduct parameter using the MC information.

- Mean conduct parameter estimate: $\theta = 0.1$. Rather competitive conduct. Both monopoly and perfect competition are rejected.
- Elasticity-adjusted Lerner index $L_\eta = \eta \frac{P-MC}{P}$ is low, and declining over time. Could be due to decline in Cartell's market share and price wars. $L_\eta$ is lowest in quarter before high season and highest during the high season. Anticipation of future improvements in demand increases competition.

## Estimate of conduct parameter without using the MC data.

- $\hat{\theta} = 0.04$: underestimated
- $c_0$ overestimated (0.466 versus 0.26).
- $k$ is slightly underestimated (1.052 versus 1.075).
- Results are not sensitive to demand specifications.

## Estimate MC parameters under set conduct parameter

- Perfect competition ($\theta = 0$), Monopoly ($\theta = 1$) , Cournot ($\theta = \frac{1}{N}$), Cournot with capacity asymmetries.
- None works well.