# Perfect Competition

- An agent takes a market price as given (each firm faces a flat demand curve)
- The agent believes that her actions do not influence a market price
- The equilibrium concept is Walrasian
- Homogenous goods, perfect information, no externality, free entry /exit, no transaction costs

# An Empirical Model

Suppose that we have a cross sectional data on markets $m$, with equilibrium prices $p_m$ and quantities $Q_m$ and observable demand $x_m$ and supply $w_m$ shifters as data
Consider the following system:

- Demand for market $m$: $ln(Q_m) = f(p_m, x_m; \alpha) + \epsilon_m$
- Supply for market $m$ (marginal cost):

$$MC_m = g(Q_m, w_m; \beta) + v_m$$

- Equilibrium: $p_m = MC_m$

where
$x_m$: observable exogenous factors that affects demand
$\epsilon_m$: unobservables for demand (tastes, etc)
$w_m$: observable in the marginal cost function (technology,etc)
$v_m$: unobservables for marginal cost (technology, etc)
Assume that firms and consumers and market observe $\epsilon$ and $v$ but not the researchers. We need these unobservables. Otherwise, we will not be able to explain the data.

# OLS and its Bias

Estimate the following two systems of equations,

- Demand: $Q_m = \alpha_0 + \alpha_1 p_m + \alpha_2 x_m + \epsilon_m$
- Supply: $p_m = \beta_0 + \beta_1 Q_m + \beta_2 w_m + v_m$

Suppose we are interested in deriving the coefficients $\alpha_1$ and $\beta_1$ using the OLS.
An increase in $\epsilon_m$ increases demand. Then, given the shift of the demand curve, the equilibrium $p_m$ increases. Thus, $\epsilon_m$ and $p_m$ are positively correlated, which violates the assumption of the OLS.
Similarly $v_m$ could be correlated with $Q_m$, which violates the assumption of the OLS (in $Y = X\beta + \epsilon$, $X$ should be uncorrelated with the error term )

# IV estimation

- If $Y = X\beta + \epsilon$, $E[X'\epsilon] \neq 0$ then OLS is biased.
- If there are variables $Z$ which are correlated with $X$ but uncorrelated with $\epsilon$ then we can use them as instruments. That is, we can estimate $\beta$ using the equality $E[Z'\epsilon] = 0$. The sample analog is
$$Z'[Y - X\hat{\beta}] = 0$$
IV estimator: $\hat{\beta_{IV}} = [Z'X]^{-1}Z'Y$
- If the number of variables in $Z$ exceeds that of $X$ then $Z'X$ is not square, so not invertible. In this case, premultiply with $X'Z[Z'Z]^{-1}$ to get
$$X'Z[Z'Z]^{-1}Z'[Y - X\hat{\beta}] = 0$$

# 2SLS estimator

$$\hat{\beta_{2SLS}} = [X'Z(Z'Z)^{-1}Z'X]^{-1}X'Z(Z'Z)^{-1}Z'Y$$
$$= [\hat{X}'\hat{X}]^{-1}\hat{X}'Y$$

where

$$\hat{X} = Z(Z'Z)^{-1}Z'X = Z\hat{\gamma}$$

where $\hat{\gamma}$ is the OLS of $X$ on $Z$

- First stage: Regress $X$ on $Z$ and get the predictor of X, $Z\hat{\gamma}$
- 2nd stage:Use $\hat{X} = Z\hat{\gamma}$ as the independent variable for $Y$

# IV Estimation

- **demand function estimation**
  use $w_m$, the supply shifter as instruments for $p_m$.
- **supply function estimation**
  use $x_m$, the demand shifters as instruments for $Q_m$.

## Exclusion Restriction for Estimation

- At least one element of $w_m$ should be excluded from the demand function (one element of $w_m$ does not belong to $x_m$).
- At least one element of $x_m$ should be excluded from the supply function (one element of $x_m$ does not belong to $w_m$.

# 2SLS Estimation of Demand Equation.

Stage 1: Run OLS to derive the price predicted by $\omega_m$.

$$\hat{p}_m = \gamma_{OLS,0} + \gamma_{OLS,1}\omega_m$$

Stage 2: Run OLS using $\hat{p}_m$ instead of $p_m$.

$$\begin{aligned} Q_m &= \alpha_0 + \alpha_1 \hat{p}_m + \alpha_2 x_m + \epsilon_m \\ &= \alpha_0 + \alpha_1 \left(\gamma_{OLS,0} + \gamma_{OLS,1}\omega_m\right) + \alpha_2 x_m + \epsilon_m \end{aligned}$$

Suppose that either $\omega_m = x_m$ or $\omega_m$ is a subset of $x_m$. Then, $\gamma_{OLS}\omega_m$ and $x_m$ are perfectly collinear. Hence, the first term of the RHS and the second term are perfectly collinear. Hence, you cannot estimate the OLS.

# 2SLS Estimation of Supply Equation.

Stage 1: Run OLS to derive the quantity predicted by $x_m$.

$$\hat{Q}_m = \gamma_{OLS,0} + \gamma_{OLS,1} x_m$$

Stage 2: Run OLS using $\hat{Q}_m$ instead of $Q_m$.

$$\begin{aligned} p_m &= \beta_0 + \beta_1 \hat{Q}_m + \beta_2 x_m + \epsilon_m \\ &= \beta_0 + \beta_1 \left(\gamma_{OLS,0} + \gamma_{OLS,1} x_m\right) + \beta_2 w_m + v_m \end{aligned}$$

Suppose that either $x_m = \omega_m$ or $x_m$ is a subset of $\omega_m$. Then, $\gamma_{OLS} x_m$ and $\omega_m$ are perfectly collinear. Hence, the first term of the RHS and the second term are perfectly collinear. Hence, you cannot estimate the OLS.

# Structural Interpretation of the Results

Suppose we estimated the coefficients of the demand and supply equation. How can we interpret the results?
Modify the demand equation:

$$p_m = \gamma_0 + \gamma_1 Q_m + \gamma_2 x_m + \eta_m$$

where $\gamma_1 = \frac{1}{\alpha_1}$, $\gamma_2 = -\frac{\alpha_2}{\alpha_1}$.
Then, the equation looks very similar to the supply equation. The only way to distinguish between the demand equation and supply equation is what observable controls it includes. If the equation includes variables that we believe affect demand, then it is a demand equation.

# Structural Estimation

Recover parameters of the economic model that are policy invariant by imposing economic and statistical assumptions to the data. Economic assumption:

- demand and supply equations. Linear functional form,
- Exclusion Restriction: At least one element of $w_m$ is not included in demand equation. At least one element of $x_m$ is not included in supply equation.
- Exogeneity of demand and supply shifters: $x_m$ and $\epsilon_m, v_m$ are uncorrelated. $w_m$ and $v_m, \epsilon_m$ are uncorrelated.
- Equilibrium: $p_m^d = p_m^s$, $Q_m^d = Q_m^s$

# Policy Experiment

Suppose government imposes a price floor $\overline{p}$. The estimated parameters are invariant to the policy.
From the estimated demand equation, we can evaluate the effect of the policy. Predicted quantity is

$$\hat{Q}_m = \hat{\alpha}_0 + \hat{\alpha}_1 \overline{p} + \hat{\alpha}_2 x_m \quad (1)$$

# Reduced Form Estimation

Describe the statistical relationship between the variable that are exogenous to the model ($x$, $w$) and the endogenous variables ($p$, $Q$).

Linear reduced form model for price and quantity:

- $p_m = a_0 + a_1 x_m + a_2 w_m + \eta_{1,m}$
- $Q_m = b_0 + b_1 x_m + b_2 w_m + \eta_{2,m}$

We cannot evaluate the effect on the quantity by a binding price floor because the parameters are obviously not policy invariant. With price floor, from the equation 1 we know that $b_2$ should be zero.

# Structural Parameters and the Reduced Form

Can we recover the structural parameters from the reduced form coefficient estimates?
Solving the equilibrium of the model, we get.

$$p_m = \frac{\beta_1 \alpha_0}{1-\beta_1\alpha_1} + \frac{\beta_0}{1-\beta_1\alpha_1} + \frac{\beta_1\alpha_2}{1-\beta_1\alpha_1} x_m + \frac{\beta_2}{1-\beta_1\alpha_1} w_m + \frac{\beta_1}{1-\beta_1\alpha_1}\epsilon_m + \frac{1}{1-\beta_1\alpha_1} v_m$$

$$Q_m = \frac{\alpha_0}{1-\beta_1\alpha_1} + \frac{\alpha_1\beta_0}{1-\beta_1\alpha_1} + \frac{\alpha_2}{1-\beta_1\alpha_1} x_m + \frac{\alpha_1\beta_2}{1-\beta_1\alpha_1} w_m + \frac{1}{1-\beta_1\alpha_1}\epsilon_m + \frac{\alpha_1}{1-\beta_1\alpha_1} v_m$$

In this case, by looking at the constant terms and the coefficients on $x_m$ and $w_m$ we can recover the structural coefficients. But that is not always the case.

# The Role of Unobservables in Estimation

Also suppose we do not have unobservables. Then, the model says the price and quantity is:

$$p_m = \frac{\beta_1\alpha_0}{1-\beta_1\alpha_1} + \frac{\beta_0}{1-\beta_1\alpha_1} + \frac{\beta_1\alpha_2}{1-\beta_1\alpha_1}x_m + \frac{\beta_2}{1-\beta_1\alpha_1}w_m$$

$$Q_m = \frac{\alpha_0}{1-\beta_1\alpha_1} + \frac{\alpha_1\beta_0}{1-\beta_1\alpha_1} + \frac{\alpha_2}{1-\beta_1\alpha_1}x_m + \frac{\alpha_1\beta_2}{1-\beta_1\alpha_1}w_m$$

There is no data that has the exact linear relationship between the observable demand and supply variables and the price and quantity. Then, this model will be inconsistent with any market data, hence the model is not estimable.

If we include unobserved components $\epsilon_m, v_m$ then any discrepancies with the deterministic model and the data can be attributed to those error terms. Hence, the econometric model with the error term becomes estimable.