Indebted Sellers, Liquidity and Mortgage Standards[*]

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Abstract

The effects of households' indebtedness on their house-selling decisions are studied in a dynamic model with search in the housing market and defaultable long-term mortgages. In equilibrium, both sellers' asking prices and time-to-sell increase with the relative size of their outstanding debt. For sellers in financial distress, this behavior results in a positive relationship between debt and their individual risk of default. Prices and time-to-sell determine both the liquidity of the housing market and the aggregate risk of default. As such they determine the extent of profitable lending by competitive mortgagees. The theory demonstrates the implications of the distribution of indebtedness for the effects of aggregate shocks on house prices, the liquidity of the housing market, and the rate and severity of mortgage default. Calibrated to the U.S. economy, it also generates, as observed, positive correlations (i) across sellers of asking prices with loan-to-value ratios, and (ii) over time of both house prices and time-to-sell with mortgage loan-to-value ratios at origination.

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1   Introduction

In this paper we study the influence of a household's indebtedness on its house selling decisions. We measure indebtedness by the household's remaining mortgage relative to the market value of its house and refer to it as the household's *leverage*, or its *loan-to-value* ratio (LTV). To this end, we develop a dynamic model with a frictional housing market and defaultable long-term mortgage debt which gives rise in equilibrium to distributions across households of house prices, debt, and default probabilities. Mortgage standards (measured by the maximum LTV at origination) are determined by the expected default rate and the cost of default. These in turn depend on the extent of housing market liquidity; that is, the speed with which houses can be traded.

In our theory, indebted households are never forced to default outright. Yet, in equilibrium, prospective sellers with larger outstanding debt ask higher prices, sell more slowly and when in financial distress, default at higher rates. Such a relationship between debt levels and asking prices has been observed by Genesove and Mayer (1997, 2001) and Anenberg (2011). At the same time, the more liquid the housing market, the less likely sellers (of any debt level) are to default. House prices vary positively over time with liquidity, and so the theory also captures the observation that U.S. house prices move positively with mortgage LTV's at origination for first-time home buyers.[1]

We model a growing population of *ex ante* identical households that enter a single city when the value of doing so exceeds their outside option. Once there, they require housing and may live either as a renter or as a homeowner. Houses are identical and are produced and sold initially by a competitive development industry. Resident households remain in the city, either as renters or homeowners, until they leave as a result of exogenous shocks.

Within the city, houses are sold following the protocol of directed search (see Moen (1997)). Sellers offer houses in a range of sub-markets, each characterized uniquely by an announced asking price and distinct matching probabilities for buyers and sellers. Search is directed in the sense that sellers choose sub-markets optimally given the trade-off between the posted price and the matching probabilities implied by buyers' optimal search behavior.

House purchases are financed by mortgages issued by competitive lenders (*e.g.* banks). We assume that these lenders issue loans of fixed duration at an exogenous interest rate, but choose the *size* (and thus the LTV) of the mortgage at origination. Households that

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[1]Between 1987 and 2012, the average LTV at origination for first-time home buyers as measured by the American Housing Survey (2007, 2009, 2011) exhibited a correlation with the Case-Shiller Index of .802 in levels and .181 in growth rates.

do not own pay rent each period equal to a fixed and exogenous fraction of income.

Indebted homeowners are subject to random *financial distress* shocks which force them to either sell their homes through the search process or default and face foreclosure. A defaulting homeowner has her house seized by the lender, a foreclosure flag placed on her record, and is prohibited from participating in the housing market until the flag is lifted, which also occurs at random. Homeowners in distress are not committed to sell, and based on their specific situations decide whether and how to do so.

Homeowners thus choose optimally their likelihood of default. In principle, a homeowner in financial distress can avoid foreclosure by posting a sufficiently low price and driving her likelihood of selling to one. In equilbrium, however, they do not do this. Rather, selling homeowners with sufficiently high LTV's post prices that are steeply increasing in their outstanding mortgage debt and are thus more likely to default than are less indebted ones. Moreover, all selling homeowners choose prices associated with substantial probabilities of default.[2]

As housing market liquidity determines the probability of sale at any given price, it affects not only the expected default rate and homeowners' expected losses upon default, but also the expected returns to foreclosure for lenders. Foreclosed houses sell more quickly and at higher prices in a liquid market, lowering their expected carrying cost and thus the cost of default to lenders. Lenders thus relax lending standards in the sense that they offer larger loans (and higher LTV's at origination), when the housing market is expected to be more liquid in the future. This occurs when income and house prices are high.

We build on the model of Head, Lloyd-Ellis and Sun (2014) which considers the dynamics of house prices and construction with homogeneous households and complete financial markets.[3] Here, we introduce limited commitment and allow households to default at will, subject to the foreclosure procedure. This generates both a role for mortgage debt secured by homes and heterogeneity among households, *ex post*. Also, whereas Head, Lloyd-Ellis and Sun (2014) focus mainly on random search, directed search, through sellers' price setting decisions, is integral to our analysis.

We consider also the effects of temporary shocks to income on prices, population movements and construction as well as the aggregate dynamics of lending and foreclosure and

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[2]Some homeowners may choose to default outright, making no attempt to sell. This occurs in cases of sufficient negative home equity, a situation which may arise if house prices fall sufficiently in response to an aggregate shock.

[3]As such, it contributes to the growing literature on search frictions in the housing market (see, for example, Diaz and Jerez (2013), Branch, Petrosky-Nadeau and Rocheteau (2016), He, Wright and Zhu (2015), Head and Lloyd-Ellis (2012), and Wheaton (1990)).

the differential effects of shocks on households with different debt levels. Households with different mortgage LTV's respond very differently to shocks that result in differential capital gains and losses depending on debt.

This *home equity* effect increases the "momentum" in house prices both by reducing their initial response to the shock and by both prolonging and increasing their eventual response. Moreover, as highly levered households experience relatively large fluctuations in home equity in response to shocks, the city experiences a particularly large wave of defaults if a negative shock occurs when a relatively large number of homeowners already have high levels of outstanding debt.

Other papers that study the relationship between housing market liquidity and lending activities are Ungerer (2015), Hedlund (2016b), Hedlund (2016a) and Garriga and Hedlund (2017). The last three are more closely related to ours as they consider models featuring directed search, long-term mortgages, and limited commitment. These papers all focus, however, on macroeconomic issues and feature a different market arrangement in which heterogeneous buyers and sellers interact only indirectly via intermediaries.[4]

Here, we focus on the differential decisions of *sellers* exclusively, and find it useful to abstract from heterogeneity on the part of buyers. We do this not for tractability *per se*, but rather to focus specifically on sellers' strategic decisions and their implications for the behavior of lenders, thereby simplifying the analysis and interpretation of our findings. As our focus is on sellers in financial distress, we abstract from saving in forms other than home equity. Our thinking here is that by the time most sellers face the prospect of either selling or defaulting, they are unlikely to have significant savings otherwise.

Along these lines, in addition to having a different matching environment we focus also on housing markets at the city as opposed to national level and study finite mortgages at fixed interest rates rather than infinite-horizon mortgage contracts. While this adds complexity, we view our particular restrictions on contracts reasonable, as in the U.S. conventional mortgages typically have a 30-year term and about 70% of are at fixed interest rates. We also do not impose a constraint whereby households must sell for at least their outstanding mortgage balance as do both Garriga and Hedlund (2017) and Hedlund et al.

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[4]This setup renders these model block recursive and tractable, in spite of the presence of heterogeneity among both buyers and sellers. In general, directed search and sorting with two-sided heterogeneity raises challenging problems. A handful of papers characterize the steady states of such economies under specific conditions (see Shi (2001), Shimer and Smith (2000), Smith (2006), and Eeckhout and Kircher (2010)). Shi (2005) shows further that dynamics of sorting with two-sided heterogeneity can be tractable in under some assumptions, which can not be readily implemented in our theory.

(2017). As noted above, in our model homeowners are never *forced* to default. Rather, their pricing and default decisions are driven entirely by strategic considerations.

In a New Keynesian model along the lines of Iacoviello (2005) with credit-constrained consumers and housing market frictions, Ungerer (2015) shows that expansionary monetary policy leads to higher leverage among homeowners. A decrease in mortgage rates boosts demand for housing. With more buyers in the frictional market, lenders can sell foreclosed houses more quickly, effectively reducing the expected carrying cost of a foreclosed house and making lender more willing to finance larger fractions of house purchases.

Our model differs from that studied by Ungerer (2015) in several respects: First, there households face a collateral constraint following Kiyotaki and Moore (1997) and thus there is neither default nor liquidation of foreclosed houses in equilibrium. Second, as debt is one-period, aggregate shocks may lead to forced de-leveraging. Related to this, houses are divisible so that adjustments can be made easily to the quantity of housing owned. Finally, the housing stock is fixed and there is no construction.

Our paper is also related to those of Ngai and Tenreyro (2014) and Ngai and Sheedy (2017) in that we focus on *selling* behavior *per se.* These papers focus, respectively, on the effect of aggregate conditions and seasonal fluctuations in demand on the decisions of homeowners to put their houses on the market. In contrast, we focus specifically on the selling decisions of heterogeneous homeowners distinguished by their levels of mortgage debt, whether financially distressed or not. As such, while both the models and specific issues studied vary between our paper and theirs, we view our work as complementary.

The remainder of the paper is organized as follows. Section 2 describes the environment and Section 3 formalizes a directed search equilibrium. The calibration is described in Section 4 and a balanced growth path for the calibrated economy is characterized in Section 5. Section 6 considers dynamics in response to aggregate shocks, and makes comparisons to an economy without search that is described in an appendix. Section 7 concludes.

2 The Environment

Time is infinite and discrete, with time periods indexed by t. The economy consists of a single city, and the "rest of the world". The world is populated by a measure $Q(t)$ of *ex ante* identical households, growing exogenously at net rate μ. Each household lives forever and each period receives income, $y(t)$, in units of a single date t consumption good. Income follows a stationary stochastic process in logarithms.

Households in the city require housing, and may either rent or own *one* of a large

number of symmetric housing units. Households' preferences are represented by

$$\mathcal{U} = \mathrm{E}_t \left[\sum_{t=0}^{\infty} \beta^t \left[u(c(t)) + z(t) \right] \right], \tag{1}$$

where $c(t)$ denotes consumption and $z(t)$ *housing* in period t, respectively. We assume that $z(t) = z_H$ if the household owns the house in which they live and $z(t) = 0$ otherwise. The function $u(\cdot)$ is strictly increasing, strictly concave and twice continuously differentiable, with the boundary properties: $\lim_{c \to \infty} u'(c) = 0$ and $\lim_{c \to 0} u'(c)$ sufficiently large. All households have the common discount factor, $\beta \in (0, 1)$. Both consumption goods and housing services are non-storable. The only savings vehicles in the economy are houses, of which households may own only one at given time.[5]

At the beginning of each period, $\mu Q(t)$ new households arrive in the economy. Each of these households has a best alternative value to entering the city, denoted ε. These values are distributed across new households via a time-invariant distribution function, $G(\varepsilon)$, with support $[0, \bar{\varepsilon}]$. Households that enter the city are separated randomly and permanently into those that value home-ownership and those that do not. The former are referred to as *buyers* and the latter as *perpetual renters*. Each period there exists a critical alternative value, $\varepsilon_c(t)$, below which a new household strictly prefers to enter the city:

$$\varepsilon_c(t) = \psi V_b(t) + (1 - \psi) V_p(t). \tag{2}$$

In 2), $V_b(t)$ and $V_p(t)$ are the time t values of being a buyer and a perpetual renter, respectively, and $1 - \psi$ is the probability of the entrant becoming a perpetual renter.

New houses are built by an industry comprised of a large number of identical and competitive firms which we refer to as *developers*. Each new house requires one unit of land, which can be purchased in a competitive market at price $q(t) = \mathcal{Q}(N(t))$. A developer also incurs a construction cost $k(t) = \mathcal{K}(N(t))$, where $N(t)$ denotes the aggregate quantity of new houses built in period t. Houses require one period to build; *i.e.* those constructed in period t become available for sale at the beginning of period $t + 1$.

Home-ownership results immediately in both a utility benefit to the homeowner and a per-period maintenance cost. Houses depreciate over time, regardless of whether or not they are occupied, but depreciation is offset by the owner each period at maintenance cost d. Households in the city that do not own houses rent. We abstract from most aspects of the rental market, and assume that rent is equal to a fixed fraction of the city-level income;

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[5]As will be shown below, households that are homeowners are thus differentiated only by debt; and those that are searching for houses are all identical.

i.e. $R(t) = \varsigma y(t)$. Effectively, the supply of rental accommodation is totally elastic, and is not considered part of the city's housing stock.

At the end of each period, all households in the city, regardless of their ownership status, may experience a shock which induces them to leave the city permanently. For perpetual renters and buyers (regardless of whether or not they currently own a house) these shocks occur with probabilities $\pi_p \in (0,1)$ and $\pi_h \in (0,1)$, respectively. Households which exit the city receive continuation utility, $\overline{V}$. Exiting homeowners also have vacant houses that they may want to sell, depending in part on their outstanding mortgage debt, if any. These households also have the option of defaulting.

In the city, the housing market is characterized by directed search. We imagine it as being characterized by a large variety of potential *sub-markets* indexed by a posted price, p, and a pair of matching probabilities; one for buyers and the other for sellers. Within each sub-market, matching takes place via a constant returns to scale matching function, $\mathcal{M}(B, S)$, which is increasing in both arguments. Given this form, sub-markets may be indexed by (θ, p), where θ denotes *market tightness*, *i.e.* the ratio of the measures of buyers, B, and sellers, S, present in the sub-market.

Both buyers and sellers take (θ, p) for all sub-markets as given and decide which to enter. The matching probabilities for buyers $\gamma(\theta)$ and sellers $\rho(\theta)$ are given by

$$\gamma(\theta) = \frac{\mathcal{M}(B, S)}{B} = \mathcal{M}\left(1, \frac{1}{\theta}\right) \tag{3}$$

$$\rho(\theta) = \frac{\mathcal{M}(B, S)}{S} = \mathcal{M}(\theta, 1) = \theta\gamma(\theta). \tag{4}$$

Each buyer and seller may, without cost, enter only a single sub-market in a given period. Free-entry generates a trade-off between the house price and the matching probability across active sub-markets(*i.e* those with $\theta \in (0, \infty)$). Intuitively, higher-price sub-markets have lower levels of tightness as buyers (who are all identical) are willing to pay a higher price only if they are compensated with higher probability of matching with a seller.

The stock of searching buyers includes both newly entered households and those which have been searching unsuccessfully for some time. These households have no assets, and so are identical. Sellers, in contrast, are of a number of different types. First, developers sell newly built homes. Second, homeowners who receive exit shocks as described above may decide to sell. Note that these sellers are heterogeneous to the extent that they have different outstanding mortgages. Home-owners may also sell as a result of a foreclosure shock (described below), and again they are differentiated by their outstanding debt. Finally, lenders sell foreclosed houses (see below).

As houses are the only asset in the economy and households are both risk-averse and subject to idiosyncratic shocks, there is a role for debt to enable consumption smoothing. In addition, if house prices exceed per period income, then households *must* borrow to finance house purchases.

We assume that debt in the form of *mortgages* secured by houses is provided by a large number of perfectly competitive lenders, owned by risk-neutral investors who consume all profits and losses *ex post*.[6] To finance mortgages, these lenders trade one-period risk-free bonds at an exogenous interest rate, i, in an external bond market. They also incur a per period proportional service cost, ϕ, associated with the administration of mortgages.

A mortgage consists of a principle amount, m, a fixed interest rate, r, and a fixed, finite maturity, T. Specifically, contract (m^t, r^t, T) represents a mortgage of size, m^t; at interest rate, r^t; issued in period t. This contract specifies T equal per-period payments:

$$x(m^t, r^t) = \frac{r^t}{1 - (1 + r^t)^{-T}} \, m^t. \tag{5}$$

As the homeowner makes payments, the principle balance on a period t mortgage, m_n^t, evolves via

$$m_{n+1}^t = \left(1 + r^t\right) m_n^t - x\left(m^t, r^t\right) \tag{6}$$

where $n \in \{0, T-1\}$ and $m_0^t = m^t$. Since T and r^t are fixed, the payment, $x(\cdot)$, is unrelated to t after origination, and m_n^t, for $n \in \{0, T-1\}$, represents the mortgage balance at the beginning of the period in which the $n + 1$st payment will take place.[7]

A borrower can terminate her mortgage contract at any time by either paying off the remaining balance or defaulting. Mortgages, however, are issued only on new home purchases.[8] A mortgage termination is a default if the borrower does not repay all of the outstanding loan. Default leads to *foreclosure*, whereby the lender takes control of the house, remitting to the borrower any surplus value of the house in excess of the outstanding loan balance. Mortgages are *non-recourse*, meaning that in the event of a default lenders do not have access to either homeowners' current or future income, only to the house.

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[6]Alternatively, they could be owned by households who would receive their *ex post* profits and losses lump-sum. This would, however, complicate the analysis without changing our results substantively.

[7]That is, at the beginning of period t, m_n^t represents the remaining balance on a mortgage issued in period $t - n - 1$, for $n = 0, \ldots, T - 1$.

[8]That is, "re-financing" requires selling, repaying the debt, searching for a new house and taking out a new mortgage. Given the time required for these transactions, this is unlikely to be optimal except in the wake of large shocks. In any case, a limitation on refinancing is required to generate a distribution of outstanding mortgage debts on the balanced growth path. Otherwise, homeowners have incentive to refinance every period in order to smooth consumption, given their restricted options for saving.

In the event of default, a borrower's mortgage balance is set to zero and a foreclosure flag is placed on her credit record. The lender repossesses the borrower's house, puts it in real-estate-owned (REO) inventory, and decides whether and how to sell it starting the following period. As noted above, the defaulting homeowner receives the difference between the value of a house in REO inventory and the outstanding mortgage balance, if positive. Upon a successful sale, the lender loses a fraction $\chi \in (0, 1)$ of the revenue to cover an exogenous cost, taken to represent legal fees, costs of deferred maintenance, etc.. As a penalty for defaulting, buyers with foreclosure flags lose access to the mortgage market and are thus excluded from the housing market. Beginning with the following period, the foreclosure flag either (with probability π_f) remains on a buyer's record, or is removed.

Homeowners with outstanding mortgage debt receive, with probability π_d each period, a *financial distress* shock. We interpret these shocks as representing circumstances such as accidents or unexpected illnesses that render the household unable to continue mortgage payments. Recipients of such shocks are referred to as *distressed owners*. They must terminate their current mortgage contracts at the end of the current period and either pay their outstanding debt or default.

In equilibrium, the mortgage rate is given by $r^t = i + \phi + \varrho$. Here i and ϕ are exogenous as described above, and ϱ compensates for the risk of default (see below).
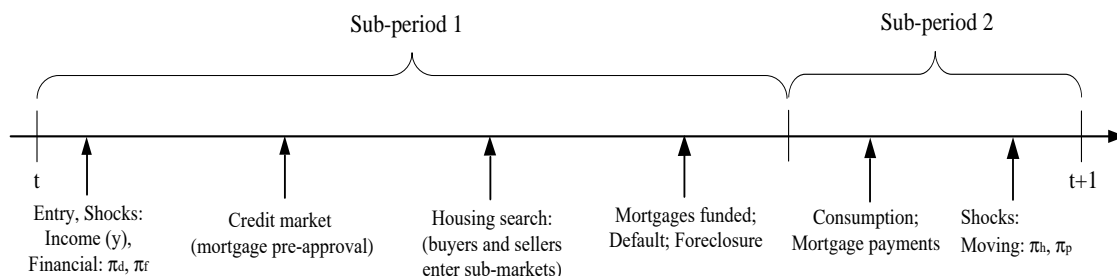

Figure 1: Time Line

Each period consists of two sub-periods. At the beginning of sub-period 1, new households with $\varepsilon \leq \varepsilon_c(t)$ enter the city. Period income, financial distress shocks and random removals of foreclosure flags are all revealed. Immediately thereafter, buyers visit lenders, and obtain pre-approval for mortgages in the event that they have an opportunity to buy a house later in the period. Thereafter, the housing market opens: Buyers and sellers decide

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on the sub-markets, (θ, p), in which to search and list their houses for sale, respectively. Pending the outcome of search, banks advance mortgage loan funds to sellers, and current mortgage holders (homeowners) holders decide whether or not to default.

In sub-period 2, households receive income, make payments (for maintenance, on new house purchases, mortgages and/or rents), and consume the remainder. At the end of the period, moving shocks are revealed for all households and those who receive them leave the city immediately. Figure 1 provides an illustration of the timing of decisions.

3 Equilibrium

We begin by describing in detail the behavior of agents and then define an equilibrium for the environment described above, which we refer to as our *baseline search* economy. Throughout $V(t)$'s will be used to denote agent and house values at the beginning of period t; while $W(t)$'s will be used to denote values at the beginning of the second sub-period. Sub-scripts will be used to distinguish agent and house states.

3.1 Households

Consider households' value functions sequentially for the two sub-periods of a typical time period t. From this point we will suppress the dependence of values on time where possible, with primes denoting future values (*i.e.* $V' \equiv V(t+1)$ at time t.)

3.1.1 The first sub-period

All household decisions are made during the first sub-period. Let V_b denote the value function for a *active buyer*. These households are either new entrants or those not currently owning a house and without a foreclosure flag, *i.e.* for whom $f = 0$:

$$V_b = \max_{(\theta, p)} \left[\gamma(\theta) W_o(m_0 - p) + (1 - \gamma(\theta)) W_b(0) \right]. \tag{7}$$

The buyer searches for a house to buy, choosing optimally to enter sub-market (θ, p). She is matched with a seller with probability $\gamma(\theta)$, in which case she proceeds to sub-period 2 as a new owner with value $W_o(m_0 - p)$. The price paid and initial loan balance m_0 (offered by the lender and specified below) determine her down-payment.[9] The argument of $W_b(a)$

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[9] We measure loan-to-value (LTV) ratios by the mortgage balance relative to the value of a house in REO inventory. Thus, while home buyers make different down-payments depending on the sub-market in which they purchase, at each point in time the LTV at origination is the same for all new mortgages.

indicates the buyer's *intra-period asset balance*, a. If $m_0 < p$, this is negative, indicating that the buyer must transfer the down-payment to the seller, out of her current income, y. In this case, $a = 0$.

With buyers free to choose among them, all active sub-markets must offer the same value, V_b. Using (7) yields

$$\theta = \gamma^{-1}\left(\frac{V_b - W_b(0)}{W_b(p, m_0) - W_b(0)}\right) \equiv \theta\left(p\right). \tag{8}$$

Thus, free-entry of buyers determines a relationship between the transaction price and market tightness across active sub-markets in equilibrium.

Let $V_o\left(m_n\right)$ denote the value for a resident homeowner with mortgage m^{t-n-1} at origination, who has made $n - 1$ payments and is *not* in financial distress:

$$V_o\left(m_n\right) = \max_{p_s} \left\{ \begin{array}{l} \rho(\theta(p_s))W_b(\max[0, p_s - m_n])+ \\ \left[1 - \rho(\theta(p_s))\right] \max_{D_n \in \{0,1\}} \left\{ \begin{array}{l} (1 - D_n)W_o(m_n)+ \\ D_n W_f(\max[0, \beta E\left[V'_{REO}\right] - m_n]) \end{array} \right\} \end{array} \right\}. \tag{9}$$

This homeowner's decision regarding whether and in which sub-market to sell can be represented by choice of asking price alone, given (8). If the she enters sub-market p_s, then with probability, $\rho(\theta(p_s))$, her house is successfully sold. In this case, she repays as much outstanding debt as possible, and keeps the remaining profit, if any, so that $a = \max[0, p_s - m_n]$. She then proceeds to the second sub-period as a buyer without the foreclosure flag and value $W_b(a)$.

COMENBACKHERE Note that the constraint $p_s \geq m_n$ is not imposed on on sellers. That is, the chosen selling price is not required to meet or exceed the outstanding debt. Effectively, mortgage lenders allow indebted homeowners to clear their debt (without the consequence of foreclosure) with an amount lower than the outstanding balance, as long as the owner makes the effort to list the house and successfully sells at the listed price.[10]

If the household chooses not to sell her house, or has failed to sell it, she then decides whether or not to default on her current mortgage. Here, for $n = 0, \ldots, T - 1$, $D_n = 1$ if a

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[10]In our quantitative exercises, we have found in *all* cases that the constraint $p \geq m_n$ is non-binding. It is not surprising that an indebted seller may find it optimal to set $p^s > m_n$, even when so-called "short-sales" are permitted. The gain to a seller from who sells at *any* $p_s < m_n$ is avoidance of the foreclosure flag, only. A seller choosing $p_s < m_n$ will maximize the probability of sale. Thus, their value going forward is bounded above by $W_b(0)$. In contrast, a seller who successfully sells for $p_s > m_n$ receives the residual profit from the sale. As long as there exists any price, $p_s > m_n$ such that $\rho(\theta(p_s))W_b(p_s - m_n) + (1 - \rho(\theta(p_s))) \geq W_b(0)$, a short-selling constraint will not be binding. This is true for all of our examples both on and off the balanced growth path given our calibration and the shocks we consider.

household who has made $n-1$ payments on a mortgage (issued in period $t-n-1$) chooses to default rather than making the nth payment; and $D_n = 0$ otherwise. The value of a homeowner who has not defaulted at the beginning of the second sub-period is $W_o(m_n)$, and that of a homeowner who *has* defaulted is $W_f(a)$, where $a = \max[0, \beta E[V'_{REO}] - m_n]$, at the beginning of the second sub-period. Such a homeowner effectively sells their house to the lender for the expected discounted value of a vacant house in the lender's inventory at the beginning of the next period, $\beta E[V'_{REO}]$. If this value is less than the household's outstanding mortgage debt, m_n, the household's assets are set to zero.[11] If the value of the vacant house exceeds the debt, the defaulting homeowner keeps the residual value. In either case, the home-owner acquires a foreclosure flag.

Next, consider a resident owner who receives a financial distress shock at the beginning of period t.[12] Such a homeowner must terminate her mortgage contract within the same period. If the house is sold, the homeowner receives the residual value net of debt and then becomes a buyer without a foreclosure flag, $W_b(\max[0, p_{sd} - m_n])$. If the house is not sold, the owner defaults, the foreclosure flag is placed on her credit record. In this case, the homeowner receives the residual value of the house net of the debt and enters the next sub-period with value $W_f(\max[0, \beta E[V'_{REO}] - m_n])$.[13] Thus the value of a distressed resident owner with debt m_n is given by:

$$V_f(m_n) = \max_{p_{sd}} \left\{ \begin{array}{l} \rho(\theta(p_{sd})) W_b(\max[0, p_{sd} - m_n]) \\ + [1 - \rho(\theta(p_{sd}))] W_f(\max[0, \beta E[V'_{REO}] - m_n]) \end{array} \right\}. \tag{10}$$

A resident homeowner who enters the period *with* a foreclosure flag and does not have it removed makes no decisions. She has access neither to credit nor housing market and has value

$$V_d = W_f(0). \tag{11}$$

That is, she enters the second sub-period in the same position as an owner who has just defaulted and has no intra-period asset balance.

A resident homeowner without a mortgage decides whether and how to sell. If her house sells successfully, she becomes a buyer with value $W_b(p_{nd})$. Otherwise, she remains

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[11]The expectation here is with respect to aggregate shocks which may affect the value of vacant houses.

[12]In the event of financial distress, it is always in an owner's best interest to attempt to sell if they have positive equity. If the housing market were perfectly liquid, distressed owners with positive equity would never default because they could sell immediately, repay their debt and keep their equity. According to the RealtyTrac report, however, less than 50% of homeowners who go into foreclosure have negative equity. In our model, time-consuming search and matching account for this feature of the housing market.

[13]Distressed resident owners may use only sales to pay outstanding mortgage debt. Allowing the use also of current labor income would complicate the analysis to no significant effect.

an owner without debt. Her value is

$$V_{nd} = \max_{p_{nd}}\{\rho(\theta(p_{nd}))W_b(p_{nd}) + [1 - \rho(\theta(p_{nd}))]\,W_{nd}(0)\}. \tag{12}$$

Next, consider homeowners who have left the city. Such households become irrelevant once they are no longer homeowners. As long as they are, however, they still make sales and default decisions. Such a homeowner has value:

$$V_L\left(m_n\right) = \max_{p_L}\left\{\begin{array}{l}\rho(\theta(p_L))\left\{u\left(\max_{p_L}\left[0, p_L - m_n\right] + y_L - R_L\right) + \beta\overline{V}\right\} + \\[1mm] [1 - \rho\left[\theta(p_L)\right]] \times \\[1mm] \max_{D_n^L}\left\{\begin{array}{l}(1 - D_{Ln})\left\{\begin{array}{l}u(y_L - R_L - x_n - d) \\ +\beta E\left[V_L'(m_{n+1})\right]\end{array}\right\} \\[2mm] +D_{Ln}\left\{u\left(\begin{array}{l}\max\left[0, \beta\left[EV_{REO}'\right] - m_n\right] \\ +y_L - R_L\end{array}\right) + \beta\overline{V}\right\}\end{array}\right\}\end{array}\right\}. \tag{13}$$

Here, y_L, R_L, and d are income, rent and maintenance costs paid by the exiting household while living outside the city.[14] Also, $x_n = x(m^t, r^t)$ denotes households' nth payment on their mortgage issued $n + 1$ periods prior. Once the homeowner has either sold her house or defaulted, she receives exogenous continuation value, $\overline{V}$.

The value of an owner who has left the city *without* debt prior to moving is given by:

$$V_{Lw} = \max_{p_{Lw}}\left\{\begin{array}{l}\rho(\theta(p_{Lw}))\left\{u(p_{Lw} + y_L - R_L) + \beta\overline{V}\right\} \\ + [1 - \rho\left[\theta(p_{Lw})\right]]\left\{u(y_L - R_L - d) + \beta E\left[V_{Lw}'\right]\right\}\end{array}\right\}. \tag{14}$$

Such a household's only decision is with regard to whether and at which price to sell.

Finally, perpetual renters make no decisions.[15] Such households have value:

$$V_p = W_p = u(y - R) + \pi_p\beta\overline{V} + (1 - \pi_p)\beta E\left[W_p'\right]. \tag{15}$$

With probability π_p, the perpetual renter receives a moving shock, leaves the city immediately and receives the continuation value $\beta\overline{V}$. Otherwise, she moves onto the next period as a renter, consuming her period income net of rent.

3.1.2 Households in sub-period 2

HouFsehold behavior in sub-period 2 is effectively trivial: All consume their income net of rent or mortgage payments and their intra-period asset balance, a. Above we summarized the payoff to these activities and to moving forward into the next period with the functions, $W_b(\cdot)$, $W_o(\cdot)$, $W_f(\cdot)$ and $W_p(\cdot)$. Expressions for these functions are given in Appendix A.

---

[14]These quantities are necesssary as long as the household remains a homeowner, because they impinge on its default and pricing decisions.

[15]Perpetual renters are included as they aid in matching calibration targets.

3.2 Developers

As noted above, the construction industry is comprised of a large number of competitive firms. Free entry into the industry ensures that in equilibrium the cost of building a house equals the expected value of a vacant house for sale in period $t + 1$:

$$\mathcal{Q}(N) + \mathcal{K}(N) = \beta E\left[V_c'\right]. \tag{16}$$

Here V_c is the value of a vacant house in developers' inventories at the beginning of the current period. It is given by

$$V_c = \max_{p_c} \left\{\rho\left(\theta(p_c)\right) p_c + \left[1 - \rho\left(\theta(p_c)\right)\right]\left[-d + \beta E\left[V_c'\right]\right]\right\}. \tag{17}$$

3.3 Lenders

At the beginning of the current period, the value of a house in lenders' REO inventories (as a result of a previous foreclosure) is:

$$V_{REO} = \max_{p_{REO}} \left\{\rho\left(\theta(p_{REO})\right)(1 - \chi) p_{REO} + \left[1 - \rho\left(\theta(p_{REO})\right)\right][-d + \beta E\left[V_{REO}'\right]]\right\}. \tag{18}$$

Comparing (17) and (18), we see that houses in REO inventory differ from other vacant houses in lenders lose fraction χ of their sales as a foreclosure cost.

Having access to funds at a fixed cost, lenders issue mortgages until they earn zero profit on each contract. In particular, the expected return net of expected foreclosure costs on mortgages will equal the opportunity cost of funds; the interest rate i on external bonds plus the servicing cost ϕ. Expected foreclosure costs are offset by the risk premium, ϱ.

Houses are identical and prospective buyers have no accumulated savings. As such, from the lender's perspective, all new borrowers are identical at the point of loan approval, which as noted above takes place before housing search. Lenders have no reason to distinguish between potential buyers and so we assume that they advance the same loan, m^t to all homeowners who purchase in period t, regardless of the price paid for the house.[16]

The present value of a mortgage issued $n + 1$ periods ago, $P(m_n)$, thus equals the current period payment x_n plus the discounted expected value of the mortgage in the next period. The latter is affected by the probabilities of the borrower receiving a moving and/or

---

[16]Rather than approving a fixed loan amount, lenders could approve a uniform schedule of loan sizes and interest rates to all prospective homeowners. In our environment this would lead to marginally more heterogeneity among *sellers* (it would not render prospective buyers heterogeneous), would greatly complicate both notation and computation, but would not change the results significantly.

financial distress shock, and the household's decisions regarding pricing and/or default in the event that they do. Note again that these decisions do *not* depend on the price at which the homeowner originally purchased.

Suppressing the time super-script, let $P(m_n)$, for $n \in \{0, \cdots, T-1\}$, be the value at the beginning of sub-period 2 of a mortgage issued in $n+1$ periods before and held by a resident homeowner. This is the value of a mortgage of original size m^{t-n-1} after $\max[0, n-1]$ payments have been made. Correspondingly, let $P_L(m_n)$, be the present value of such a mortgage held by an owner that has relocated.[17] Then, for $n \in \{0, 1, \cdots, T-1\}$,

$$
P(m_n) = x_n \mathbb{I}_{\{n \neq 0\}} + \frac{\mathbb{I}_{\{n \neq T-1\}}}{1+r} \times
$$

$$
E \left[\begin{array}{c} \pi_h \left[\begin{array}{c} \rho(\theta(p'_L)) \min\left[p'_L, m'_{n+1}\right] + \\ [1-\rho(\theta(p'_L))] \left\{ \begin{array}{c} D'_{Ln+1} \min\left[\beta V''_{REO}, m'_{n+1}\right] \\ + (1-D'_{Ln+1})P'_L(m'_{n+1})] \end{array} \right\} \end{array} \right] + \\ (1-\pi_h) \left[\begin{array}{c} \pi_d \left\{ \begin{array}{c} \rho(\theta(p'_{sd})) \min\left[p'_{sd}, m'_{n+1}\right] \\ + [1-\rho(\theta(p'_{sd}))] \min\left[\beta V''_{REO}, m'_{n+1}\right] \end{array} \right\} + \\ (1-\pi_d) \left\{ \begin{array}{c} \rho(\theta(p'_s)) \min\left[p'_s, m'_{n+1}\right] \\ + [1-\rho(\theta(p'_s))] \\ \times \left\{ \begin{array}{c} D'_{n+1} \min\left[\beta V''_{REO}, m'_{n+1}\right] \\ + (1-D'_{n+1})P'(m'_{n+1}) \end{array} \right\} \end{array} \right\} \end{array} \right] \end{array} \right] \tag{19}
$$

and for all $n \in \{1, \cdots, T-1\}$,

$$
P_L(m_n) = x_n + \frac{\mathbb{I}_{\{n \neq T-1\}}}{1+r} E \left[\begin{array}{c} \rho(\theta(p'_L)) \min\left[p'_L, m_{n+1}\right] + \\ [1-\rho(\theta(p'_L))] \left\{ \begin{array}{c} D'_{Ln+1} \min\left[\beta V''_{REO}, m_{n+1}\right] \\ + (1-D'_{Ln+1})P'_L(m_{n+1}) \end{array} \right\} \end{array} \right]. \tag{20}
$$

Here p'_s, p'_{sd} and p'_L are sales prices; and $D'_{n+1} \in \{0, 1\}$ and $D'_{Ln+1} \in \{0, 1\}$ are default decisions. All of these are household policies in period $t+1$ contingent on mortgage balance m_{n+1}. Also,

$$
\mathbb{I}_{\{n \neq 0\}} = \left\{ \begin{array}{ll} 0, & \text{if } n = 0 \\ 1, & \text{otherwise} \end{array} \right. \quad \text{and} \quad \mathbb{I}_{\{n \neq T-1\}} = \left\{ \begin{array}{ll} 0, & \text{if } n = T-1 \\ 1, & \text{otherwise.} \end{array} \right. \tag{21}
$$

are indicators identifying, respectively, mortgages on which the borrower is making regular repayments beginning with the period after origination, and mortgages that mature after the current repayment is made.

---

[17]For such owners, we have $n \geq 1$ as one repayment has already been made by the beginning of the first sub-period 2 following the household's relocation.

To compute the present value of a mortgage contract at origination, we proceed recursively. First, we compute $P(m_{T-1})$, and then use backward induction to obtain $P(m_n)$ for $n \in \{0, \ldots, T-2\}$. The value $P_L(m_n)$ is determined in a similar way except that relocated homeowners experience neither moving nor distress shocks.

If a borrower sells her house in period $t+1$, the amount that the lender will receive is the minimum of the sale proceeds and the outstanding debt m_n. The equilibrium mortgage loan size at orignination, m_0, is then determined using the mortgage lender's zero-profit condition:

$$P(m_0^t) - m_0^t = 0. \tag{22}$$

That is, the funds advanced in period t are equal the present value of a mortgage contract at the beginning of the next period $(t+1)$.

3.4 Laws of motion

We now describe the evolution of the distributions of households and houses across states. We express all in *per capita* terms (*i.e.* divided by the total economy population, $Q(t)$). At the beginning of period t, the households are divided into renters; perpetual renters, $F(t)$; buyers without a foreclosure flag, $B(t)$; buyers with a foreclosure flag, $B_f(t)$, and homeowners. This latter group are either residents of the city or have relocated and still own a house.

Within each type, households are further differentiated by their mortgage balance, if any. For $n = 0, \ldots, T-1$, let $H_n(t)$ denote the period t measure of resident homeowners who were issued a mortgage in period $t-n-1$ and thus have made n payments prior to period t. Similarly, $H_{Ln}(t)$ denotes the measure of homeowners who have relocated by period t holding such a mortgage. Let $H_\emptyset(t)$ and $H_{L\emptyset}(t)$, respectively denote resident and re-located homeowners who no longer have an outstanding mortgage. Note that there is no need to keep track of relocated households once they cease to be homeowners (because they have already either sold or defaulted).

Houses for sale are either held vacant by relocated homeowners, developers and mortgage lenders or offered by distressed homeowners. Denote the inventories of developers and mortgage lenders by $H_c(t)$ and $H_{REO}(t)$ respectively. Suppressing the time indicator, the total stock of houses for sale in the current period is then given by:

$$H_s = \underbrace{H_{L\emptyset} + \sum_{n=1}^{T-1} H_{Ln} + H_c + H_{REO}}_{\text{vacancies}} + \underbrace{\pi_d \sum_{n=0}^{T-1} H_n}_{\text{distressed sellers' homes}} . \tag{23}$$

16

Similarly, the total current period measure of buyers searching to trade in the housing market, B_{sum}, can be written:[18]

$$
\begin{aligned}
B_{sum} &= \sum_{n=0}^{T-1} [(1 - \pi_d)\theta(p_s) H_n + \pi_d \theta(p_{sd}) H_n + \theta(p_L) H_{Ln}] \\
&\quad + \theta(p_{nd}) H_{\emptyset} + \theta(p_{Lnd}) H_{L\emptyset} + \theta(p_c) H_c + \theta(p_{REO}) H_{REO}.
\end{aligned} \tag{24}
$$

Continuing to suppress the dependence of variables on time, we now write out the laws of motion for the stocks households and houses in the various states. To begin with, the *per capita* measure of permanent renters *next period* consists of those remaining from the current period and those who will have newly entered:

$$
(1 + \mu)F' = (1 - \pi_p)F + (1 - \psi)G(\varepsilon_c')\mu. \tag{25}
$$

Similarly, the measure of buyers with foreclosure flags next period includes those remaining from the current period who have neither moved nor had their flag removed randomly. To this is added the measure of resident homeowners who default this period. These homeowners may either have received a financial distress shock and failed to sell or have defaulted strategically. Thus, we have

$$
(1 + \mu)B_f' = (1 - \pi_h) \left\{ \begin{array}{l} \pi_f B_f + \\ \pi_d \sum_{n=0}^{T-1} (1 - \rho(\theta(p_{sd}))) H_n \} + \\ (1 - \pi_d) \sum_{n=0}^{T-1} (1 - \rho(\theta(p_s))) D_n H_n \end{array} \right\} \tag{26}
$$

where as above, p_{sd}, p_s, and D represent optimal pricing and default decisions. Note that in general these depend on homeowners' outstanding mortgages.

The measure of buyers *without* foreclosure flags at the beginning of next period consists of newly-entering buyers, previously flagged buyers whose flag has been removed, and non-relocating buyers from the current period who failed to buy a house. Thus, we have

$$
\begin{aligned}
(1 + \mu)B' &= \psi G(\varepsilon_c')\mu + (1 - \pi_f)B_f + \\
&\quad (1 - \pi_h) \left\{ \begin{array}{l} B - \rho(\theta(p_{Lw})) H_{L\emptyset} - \\ \rho(\theta(p_c)) H_c - \rho(\theta(p_{REO})) H_{REO} - \\ \sum_{n=1}^{T-1} \rho(\theta(p_L)) H_{Ln} \end{array} \right\}.
\end{aligned} \tag{27}
$$

---

[18]The measure of buyers in an active sub-market equals the measure of sellers in that sub-market multiplied by the the corresponding market tightness. For example, the measure of buyers searching for foreclosed houses sold by a lender equals the measure of REO houses, H_{REO}, multiplied by the tightness of the lender's optimally chosen submarket, $\theta(p_{REO})$.

The measure of indebted owners who have made n periodic payments by the beginning of period $t+1$ on a mortgage of size m^{t-n-1} at origination evolves (for $n > 0$) via:

$$(1+\mu)H'_n = (1-\pi_h)(1-\pi_d)(1-\rho\left(\theta(p_s)\right))(1-D_n)H_{n-1}. \tag{28}$$

That is, the indebted owners with an ongoing mortgage going into the next period are the indebted owners from the current period who do not move, experience financial distress, successfully sell their house, or default outright.

For $n = 0$, H'_0 is the measure of resident homeowners who successfully purchase a house, remain in the city and do not experience financial distress. This measure can be recovered from the number of sales in the current period:

$$(1+\mu)H'_0 = (1-\pi_h)\left\{ \begin{array}{l} (1-\pi_d)\sum_{n=0}^{T-1}\rho\left(\theta\left(p_s\right)\right)H_n \\ +\pi_d\sum_{n=0}^{T-1}\rho\left(\theta\left(p_{sd}\right)\right)H_n \\ +\sum_{n=1}^{T-1}\rho\left(\theta\left(p_L\right)\right)H_{Ln} \\ +\rho\left(\theta(p_{nd})\right)H_{\emptyset}+\rho\left(\theta(p_{Lw})\right)H_{L\emptyset} \\ +\rho\left(\theta(p_c)\right)H_c+\rho\left(\theta(p_{REO})\right)H_{TE"O} \end{array} \right\}. \tag{29}$$

Finally, the measure of resident owners without a mortgage evolves via

$$(1+\mu)H'_{\emptyset} = (1-\pi_h)\left\{ \begin{array}{l} (1-\pi_d)(1-\rho\left(\theta\left(p_d\right)\right))(1-D_{T-1})H_{T-1} \\ +(1-\rho\left(\theta(p_{nd})\right))H_{\emptyset} \end{array} \right\}. \tag{30}$$

This group is comprised of its previous members who have neither moved nor sold plus resident homeowners who make their last mortgage payment in the current period.

Proceeding similarly for relocated homeowners, H'_{Ln} is the measure who made their $n+1$st payment in the period t. Again, the loan volume at origination is m^{t-n-1} and $H_{L\emptyset}$ is the current period measure of relocated owners without debt:

$$\begin{aligned} (1+\mu)H'_{Ln} &= (1-\rho\left(\theta(p_L)\right))(1-D_{Ln-1})H_{Ln-1} \\ &\quad +\pi_h(1-\pi_d)(1-\rho\left(\theta(p_s)\right))(1-D_{n-1})H_{n-1}; \end{aligned} \tag{31}$$

$$(1+\mu)H'_{L0} = \pi_h\left\{ \begin{array}{l} (1-\pi_d)\sum_{n=0}^{T-1}\rho(\theta\left(p_s\right))H_n \\ +\pi_d\sum_{n=0}^{T-1}\rho(\theta\left(p_{sd}\right))H_n \\ +\sum_{n=0}^{T-1}\rho(\theta\left(p_L\right))H_{Ln} \\ +\rho(\theta(p_{nd})H_{\emptyset}+\rho(\theta(p_{Lw})H_{L\emptyset} \\ +\rho(\theta(p_c))H_c+\rho(\theta(p_{REO}))H_{REO} \end{array} \right\} \tag{32}$$

18

$$(1+\mu)H'_{L\emptyset} \;=\; \pi_h\left\{\begin{array}{l}(1-\rho(\theta(p_{nd})))H_\emptyset \\ +(1-\pi_d)(1-\rho(\theta(p_s)))(1-D_{T-1})H_{T-1}\end{array}\right\}$$
$$+(1-\rho(\theta(p_L)))(1-D_{LT-1})H_{LT-1}$$
$$+(1-\rho(\theta(p_{Lnd})))H_{L\emptyset}. \tag{33}$$

As depreciation is offset by maintenance, the *per capita* city housing stock evolves via

$$(1+\mu)H' = H + N, \tag{34}$$

where measure N houses are built in the current period and available for sale in the next.

The *per capita* stock of houses in developers' inventory at the beginning of the next period includes those that go unsold in the current period plus those that are newly built:

$$(1+\mu)H'_c = (1-\rho(\theta(p_c)))\,H_c + N. \tag{35}$$

Finally, the stock of houses in the REO inventory at the beginning of the next period, H'_{REO}, includes those that go unsold in the current period plus the new foreclosures:

$$(1+\mu)H'_{REO} \;=\; (1-\rho(\theta(p_{REO})))\,H_{REO}$$
$$+\pi_d\sum_{n=0}^{T-1}(1-\rho(\theta(p_{sd})))H_n$$
$$+(1-\pi_d)\sum_{n=0}^{T-1}(1-\rho(\theta(p_s)))D_n H_n$$
$$+\sum_{n=1}^{T-1}(1-\rho(\theta(p_L)))D_{Ln}H_{Ln}. \tag{36}$$

3.5 A Directed Search Equilibrium

Definition. Given a mortgage interest rate, r; rent level, R; terminal continuation value, $\overline{V}$; and a stochastic process for city-level income, y, a *directed search equilibrium* is, for all periods, a collection (suppressing the dependence on y) of

1. Household value functions:

$$V_b,\ W_b;\ V_o,\ W_o;\ V_f,\ W_f,\ V_{nd},\ W_{nd};\ V_L,\ V_{Lw},\ V_p,\ W_p \tag{37}$$

with associated policies (choices of sub-market to enter and whether to default):

$$p_s,\ p_{sd},\ p_{nd},\ p_L,\ p_{Lw},\ D_n,\ D_{Ln}, \qquad n=0,\ldots,T-1; \tag{38}$$

2. house values:

$$V_c, \; V_{REO} \tag{39}$$

with associated policies for developers and lenders:

$$p_c, \; p_{REO}; \tag{40}$$

3. an entry cut-off and mortgage contract:

$$\varepsilon_c, m_0; \tag{41}$$

4. and *per capita* measures of households and houses

$$\underbrace{F, \; B, \; B_f, \; H_n, \; H_{Ln}, \; H_\emptyset, \; H_{L\emptyset}}_{\text{households}}, \; n = 0, \dots T - 1; \qquad \underbrace{H, \; N, \; H_c, \; H_{REO}}_{\text{houses}}. \tag{42}$$

Such that:

1. New households enter the city optimally so that (2) holds;

2. All agents optimize such that the value and policy functions listed in (37) - (40) satisfy (7), (9) - (15) and (A.1) - (A.6);

3. Free entry of developers: N satisfies (16);

4. Free entry of lenders: m_0^t satisfies (22);

5. The stocks of households and inventories of houses evolve according to (25) - (34);

6. All buyers enter an active sub-market: $B = B_{sum}$.

Requirements 1-5 in the above definition are standard and have been described in detail above. Requirement 6 is an aggregate consistency condition. In equilibrium the measure of buyers without foreclosure flags must be consistent with the total measure of buyers actively participating in housing search.

As has been mentioned, sellers are heterogeneous, and each period their distribution is characterized by $(H_n, H_{Ln}, H_\emptyset, H_{L\emptyset}, H_c, H_{REO})$. The decision problems faced by households, developers and lenders are, however, not affected by this distribution. In fact, as can be seen from (7), and (9) - (22), all of the value and policy functions listed in (37) -

(40), together with the mortgage contract m_0, are independent of the stocks listed in (42). This is true despite the fact that the stocks themselves depend on individual decisions, and that the distribution of sellers does affect aggregate statistics.

Thus, the model is block recursive in the sense of Shi (2009). As discussed there, block recursiveness arises because heterogeneous sellers sort optimally into separate sub-markets through the directed search mechanism, taking the trade-off between the price and the matching probability as given. Given a particular target transaction price, all that matters for a seller's trading decision is the probability with which it will be matched with a buyer; the distribution of sellers over other price targets is irrelevant. *Vice versa*, for a given matching probability a seller cares only about the price at which it can sell.[19]

Below, we consider a *steady-state* (*i.e.* balanced growth path) conforming to the definition of equilibrium in Section 3.5, with the additional requirements that *per capita* income is constant and that all functions and values listed in (37) - (42) are time invariant. For this steady-state, we now derive some analytical results that will later be useful for understanding the optimal trading decisions of distressed sellers.

Using (10) the gain from selling to a distressed homeowner with debt, m_n, is:

$$\Psi\left(m_n\right) = W_b\left(p - m_n\right) - W_f\left(\max\left[0,\ \beta V_{REO} - m_n\right]\right). \tag{43}$$

We then have the following proposition, which is proved in Appendix B:

Proposition 1. *Let $p > m_n$. Then:*

i. *If $m_n \geq \beta V_{REO}$, then $\Psi'\left(m_n\right) < 0$;*

ii. *If $m_n < \beta V_{REO}$, $\Psi'\left(m_n\right) > 0$ for any given $p > \beta V_{REO}$ and $\Psi'\left(m_n\right) < 0$ for any given $p < \beta V_{REO}$.*

We also have the following:

Lemma 1. *Provided that $p > m_n$, a distressed seller's*

i. *trading probability, $\rho\left(\theta\left(p\right)\right)$, is strictly decreasing in p;*

ii. *trade surplus, $\Psi\left(m_n; p\right)$, is increasing in price p for any debt level m_n.*

---

[19]Block recursiveness greatly aids tractability by eliminating the role of the distribution of sellers in individual decisions.

A distressed seller chooses a sub-market to maximize her expected gain from trade. Given the matching function and free-entry of buyers, this decision (see 10) solves

$$\max_{p,\theta} \quad \rho(\theta) \Psi(m_n; p) \tag{44}$$

where

$$\theta(p) = \gamma^{-1} \left(\frac{V_b - W_b(0)}{V_o(p, m_0) - W_b(0)} \right) \tag{45}$$

follows directly from (8) evaluated at the steady-state. The proof of Lemma 1 is straightforward. Part (i) follows from matching function properties (3) and (4). Part (ii) from $u' > 0$. Thus, a higher selling price raises the gain from trade, but reduces selling probability. The optimal sub-market choice reflects this trade-off.

4 Calibration

We choose parameters to match several characteristics of U.S. city-level housing markets on a balanced growth path.[20] We begin by specifying the following functional forms:

$$
\begin{aligned}
u(c) &= \ln(c) \\
\mathcal{M}(B,S) &= \varpi B^\eta S^{1-\eta} \\
k &= \frac{1}{\kappa} N^{\frac{1}{\zeta}} \\
q &= \bar{q} N^{\frac{1}{\xi}}
\end{aligned}
\tag{46}
$$

where η is the elasticity of the measure of matches with respect to the measure of buyers and ξ represents the elasticity of new land supply with respect to land prices.

Table 1 lists parameter values for the baseline search economy. Parameters above the separating line are set to match the corresponding targets directly, while those below are determined jointly to match the targets in the right-most column. The time period is set to one year.[21] The discount factor β is set to reflect an annual real interest rate of 4%. To

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[20]We view this calibration as permitting the analysis of illustrative examples. To limit the complexity of our model, and for clarity, we abstract from certain aspects of the economy. For example, we abstract from house sales for reasons other than financial distress and intercity relocation; and we limit the time period to one year. These decisions and others prevent us from targeting certain aspects of U.S. housing markets at either the aggregate or city level.

[21]A time period of one year results in a longer average time to sell than observed. We concentrate, however, on distressed sales that end in foreclosure, *not*, as noted above, *all* sales. The foreclosure process typically takes substantially longer than a typical house sale.

Table 1: Calibration Parameter Values

Parameter	Value	Target	Data
		Parameters determined independently	
β	0.96	Annual interest rate	4.0%
π_p	0.120	Annual mobility of renters	12%
π_h	0.032	Annual mobility of owners	3.2%
ξ	1.75	Median price-elasticity of land supply	1.75
i	0.040	International bond annual yield	4.0%
T	30	Fixed-rate mortgage maturity (years)	30
μ	0.012	Annual population growth rate	1.2%
π_f	0.80	Average duration (years) of foreclosure flag	5
$\bar{q}$	0.96	Average land-price-to-income ratio	30%
m	0.08	Residential housing gross depreciation rate	2.5%
ζ	5	Median price elasticity of new construction	5
ς	0.16	Rent-price ratio	5%
		Parameters determined jointly	
χ	0.440	Loss severity rate	27%
ϕ	0.0246	Average down-payment ratio	20%
ϱ	0.0074	Average annual FRM-yield	7.20%
ψ	0.570	Fraction of households that rent	33.3%
π_d	0.060	Annual foreclosure rate	1.6%
z_H	0.3280	Average loan-to-income ratio at origination	2.72
ϖ	0.56	Average fraction of delinquent loans repossessed	33.5%
κ	0.137	Average housing price relative to annual income	3.2
η	0.1880	Relative volatility of sales growth	1.32
α_p	6.200	Relative volatility of population growth	0.17

determine the mortgage rate r, the annual yield on international bonds i is set at 4%. The values of ϕ and ϱ are determined jointly in calibration.

Income in the steady state is normalized to one. Thus, all present values and prices are measured relative to steady-state *per capita* income. The terminal continuation value, $\overline{V}$,

is equal to the steady-state value of being a perpetual renter, $\overline{V}_p$.

Several parameters and targets are chosen following Head, Lloyd-Ellis and Sun (2014): The rate μ is set equal to annual U.S. population growth during the 1990's. The value of π_p is set to match the annual fraction of renters that move between counties and π_h to match the annual fraction of homeowners that move between counties according to the Census Bureau.

The supply elasticity parameter is set to $\xi = 1.75$ following Saiz (2010), who estimates supply elasticities for 95 U.S. cities over the period 1970 to 2000. The estimates vary from 0.60 to 5.45 with a population-weighted average of 1.75 (2.5 unweighted). The steady-state unit price of land $\bar{q}$ is set such that the relative share of land in the price of housing is 30% (see Davis and Palumbo (2008) and Saiz (2010)). The elasticity of new construction with respect to the price of housing, ζ, is set equal to the median elasticity for the 45 cities studied by Green, Malpezzi and Mayo (2005), resulting in $\zeta = 5$.

The average house price is 3.2 times annual income, and the maintenance cost d is chosen to be 2.5% of that, following Harding, Rosenthal and Sirmansa (2007). Moreover, The value of ψ is chosen so that the overall ownership rate, $\overline{H}/(\overline{H} + \overline{B} + \overline{F}) = 66.7\%$, as reported by the Census Bureau, where

$$\overline{H} = \sum_{n=0}^{T-1} \overline{H}_n + \overline{H}_\emptyset$$

denotes the total measure of resident homeowners in the steady state.

The rent-to-income ratio is set to $\varsigma = 0.16$. The Lincoln Institute of Land Policy estimates the average rent-to-price ratio to have been roughly 5% prior to the housing boom leading up to the 2008 financial crisis. So, we set ς to the product of this rent-to-price ratio and the average house price.

The remainder of the parameters listed in Table 1 are determined to match jointly a number of targets based on the model. First, we set the average length of time following a foreclosure until a borrower is again allowed to access the mortgage market to five years, a time frame is consistent with the policies of Fannie Mae and Freddie Mac. Thus, we set the probability that a foreclosure flag remains on a borrower's credit record to $\pi_f = 0.8$.

According to the Federal Housing Finance Board, the average contract rate on conventional, fixed-rate mortgages between 1995 and 2004 was 7.2%. We target an average down-payment ratio of 20% and an annual default rate of 1.6%, which is close to the average annual foreclosure rate among all mortgages during the 1990s according to the National Delinquency Survey by the Mortgage Bankers Association.

Foreclosures cause losses to lenders due to transaction and time costs. Moreover, foreclosed houses typically sell at a discount relative to other houses with similar properties. The loss severity rate is defined as the present value of all losses on a given loan as a fraction of the balance on the default date. Economists have found the loss severity rates range from as small as 2% during calm period over 1995-1999 (Pennington-Cross, 2003) to as much as over 75% during the Great Recession (Andersson and Mayock, 2014). Here, we choose parameters so that in the event of a default, the value of

$$\frac{\min\{\beta \overline{V}_{REO}, m_n\}}{m_n} = 0.73 \tag{47}$$

on average. This implies an average loss severity rate of 27%.

Phillips and Vanderhoff (2004) find that 30% of defaulted conventional fixed-rate loans and 50% of defaulted conventional adjustable-rate loans transition to REO and Ambrose and Capone (1996, 1998) report that 32% to 38% of defaulted FHA loans transition to foreclosure. Thus, we choose parameters such that in the event of financial distress, the average probability of a successful sale is 66.5%. That is, 33.5% of the homeowners who experience financial distress ultimately end up in foreclosure in the steady-state.

Evidence from the American Housing Survey (AHS) suggests that prior to 2003 the average ratio of the original loan size to yearly income was 2.72. Accordingly, we target this value as the loan-to-income ratio at origination, $\overline{m}_0/\overline{y}$.

Finally, the *dynamics* of our model depend crucially on two elasticities: the elasticity of $G(\cdot)$, evaluated at ε_c, $\alpha_p = \varepsilon_c g(\varepsilon_c)/G(\varepsilon_c)$ (here g is the density of G) and the elasticity of the matching function with respect to the number of buyers, η. These two parameters are calibrated jointly by using estimates of the relative standard deviations of population growth and housing sales growth in response to income shocks as in Head, Lloyd-Ellis and Sun (2014).

5   The Balanced Growth Path

In the steady-state, all owners have strictly positive home equity, defined here as the difference between the average house price and the homeowner's outstanding debt. Resident owners who receive neither moving nor financial distress shocks do not attempt to sell their houses and relocated owners continue to make repayments until a successful sale occurs or their mortgage is completely repaid. Finally, there are no outright defaults. Thus, foreclosure occurs only as the result of financial distress.

Figure 2 depicts the steady-state distribution of house sellers across types. Nearly half of the sellers in the market are there as a result of financial distress. This is consistent with our calibration targets for mobility and default rates.[22]
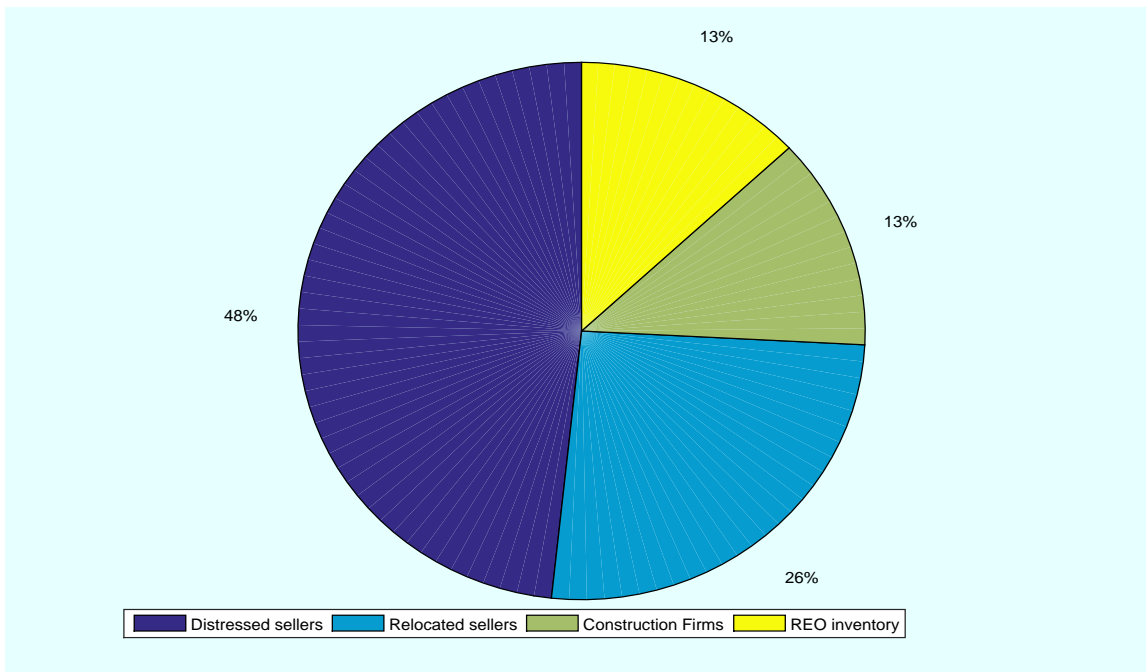

Figure 2: The composition of house sellers

Figure 3 presents the distributions of resident homeowners (upper panel) and sellers (lower panel) by mortgage status. In the steady-state, these distributions are determined by time and exogenous shocks alone. The measures of owners decrease with the number of payments for $n = 1, \cdots, 29$, owing to the effects of both moving and financial distress shocks which affect homeowners at constant rates over time. The large bin at $n = 30$ represents the stock of homeowners who have repaid their entire mortgage without experiencing either shock. While these homeowners no longer face a risk of financial distress, they remain subject to moving shocks and exit the city eventually. Similarly, the measure of distressed sellers decreases with the number of payments fulfilled, for $n = 1, \cdots, 29$, although there are no such sellers with $n = 30$ by construction.

The distribution of relocated sellers, in contrast, is driven by households' choice of selling probability. These households are not required to sell, and they are no longer hit by financial distress shocks. The fact that some enter sub-markets with high prices and low

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[22]Again, sales here result only from either financial distress or *intercity* relocation.
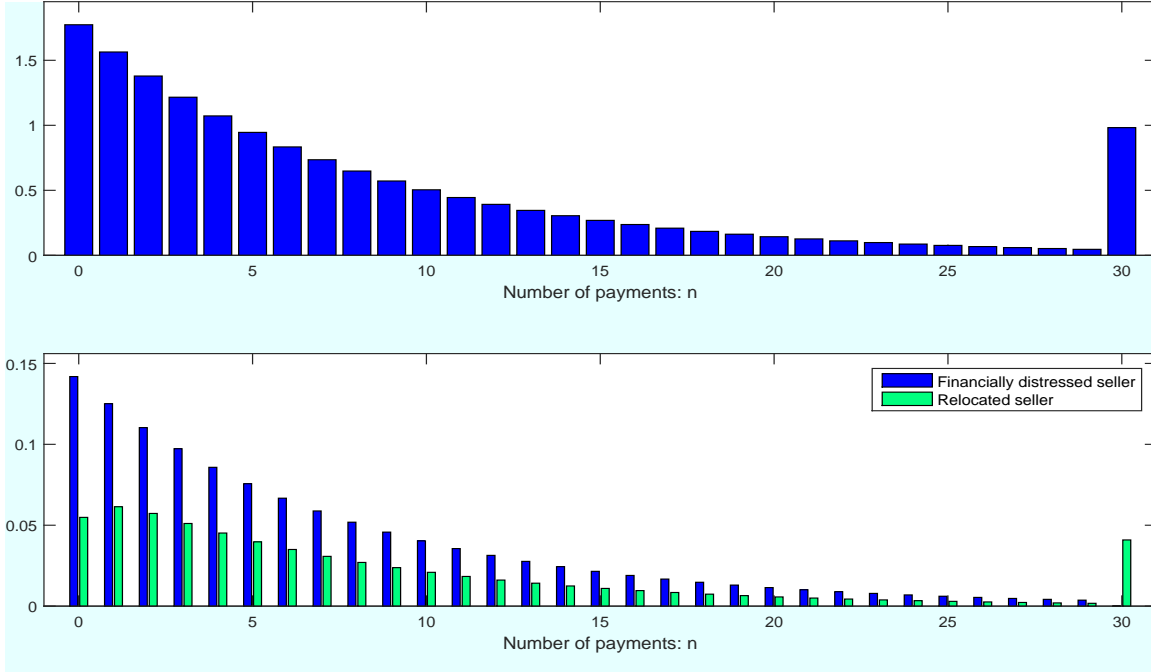
Figure 3: Steady-state distributions of mortgage status among resident homeowners (upper panel) and household sellers, both distressed and relocated (lower panel)

sales probabilities accounts for the hump-shape of the distribution. The spike at $n = 30$ arises from the fact that resident homeowners who have repaid their mortgages are still subject to moving shocks, at which point they become relocated sellers without debt.

Figure 4 illustrates the distribution of house prices in the steady state. Indebted sellers set the lowest prices, followed by relocated sellers and then by developers and lenders, who sell only new and foreclosed houses, respectively. Developers and lenders ask the highest prices because they are risk neutral and are concerned only with the expected return on a sale. Lenders ask slightly higher prices than developers, as they require a return sufficient to recoup the foreclosure cost (χ). For any level of outstanding mortgage, distressed sellers have greater incentive to sell than relocated ones, as they face the threat of foreclosure and exclusion from the housing market if they fail to sell. Both distressed and relocated sellers generally ask *higher* prices the larger is their outstanding debt.
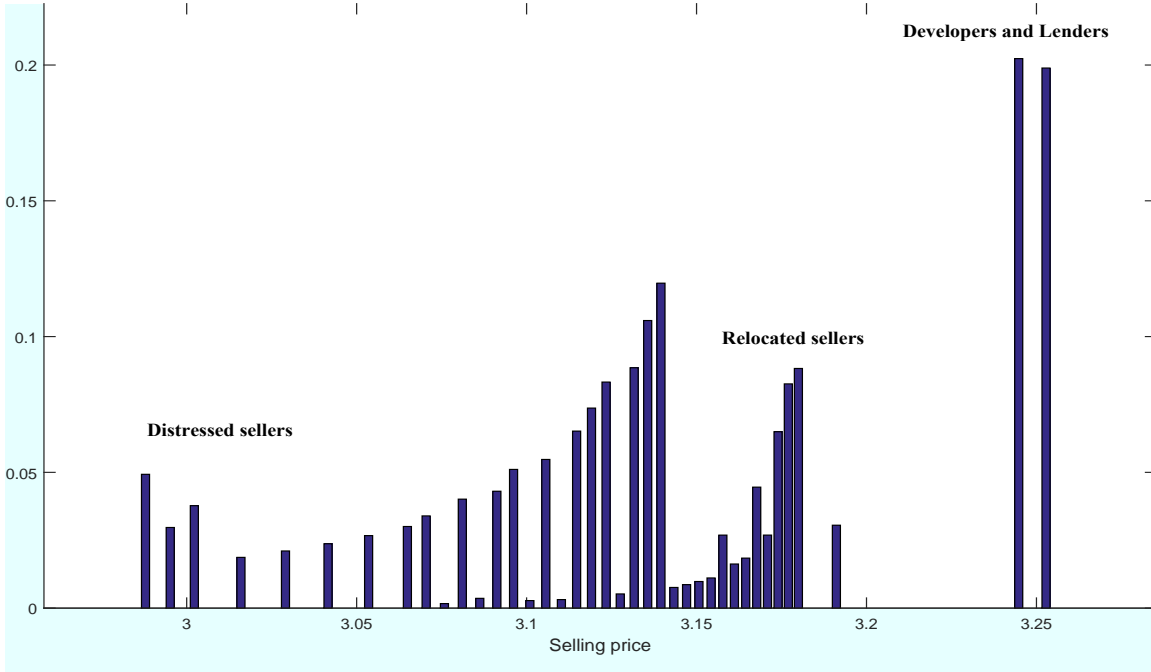
Figure 4: Steady-state distribution of housing prices

5.1 Leverage and seller behavior

Figure 5 illustrates the relationship between a seller's optimal choice of sub-market (which determines both her asking price and sales probability) and her debt position.[23] A distressed seller is more eager to sell than a relocated one and therefore, conditional on debt position (represented here by the LTV ratio as a percentage), posts a lower price and sells with a higher probability.

The cost of failing to sell is higher for distressed sellers for two reasons. First, a distressed seller has no choice but default if she fails to sell her house within the period, while a relocated one remains a homeowner and the default option for the future. Second, a relocated seller's continuation value contains $\overline{V}$, which is independent of her credit record. A distressed seller who defaults and remains in the city is excluded from the housing market until her foreclosure flag is lifted.

The average asking price is increasing in the LTV. For distressed sellers, however, the relationship between the two is initially negative, but becomes strongly increasing as

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[23]Here, to facilitated comparison to Genesove and Mayer (1997), we represent the asking (*i.e.* posted) price as a *mark-up* that is, as a ratio to V_{REO}. The asking price is proportional to the mark-up, as all vacant (non-foreclosure) houses have a common value.
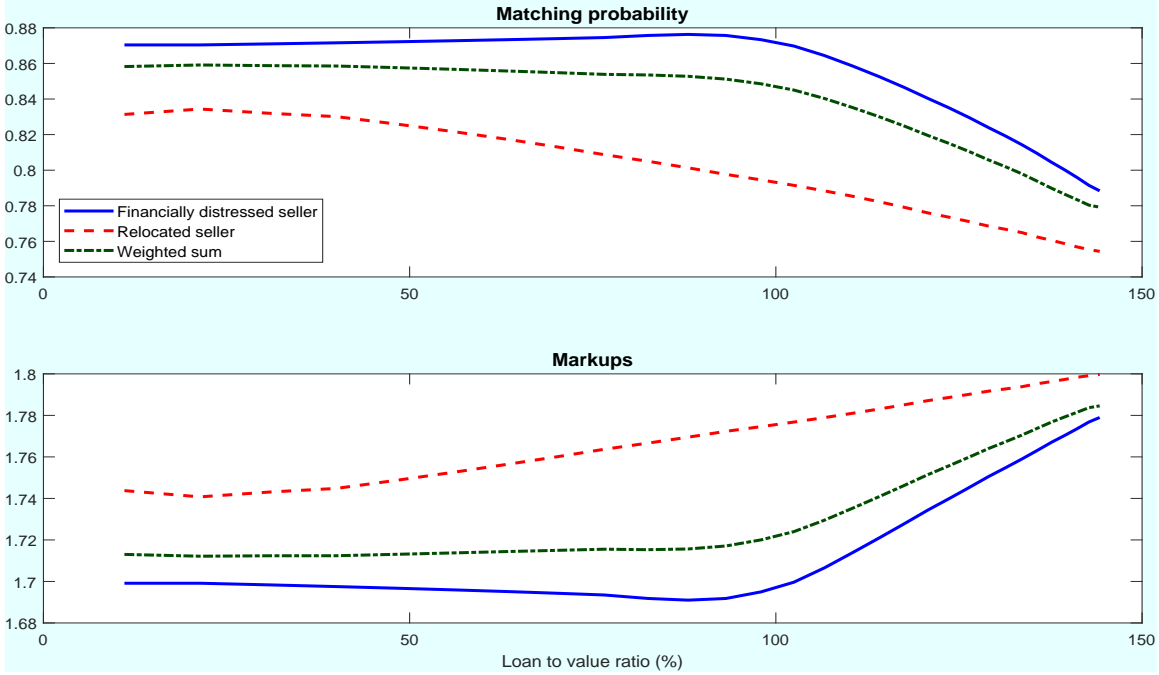
Figure 5: Leverage and seller behavior. The top panel shows the choices of selling probability by distressed and relocated sellers. Correspondingly, the bottom panel demonstrates the choices of selling price by the two types of sellers.

the LTV approaches 100. This relationship is consistent with the findings of Anenberg (2011), and for distressed sellers in particular, resembles closely those of Genesove and Mayer (1997) (see Figure 2, p. 267), in their empirical study of condominium sales in Boston during the 1990's. The closeness of the relationship between debt and the asking price here to their findings is striking, given that none of the quantities depicted for our economy are calibration targets.

The shape of the relationship depicted in Figure 5 can be understood from Proposition 1. When a seller is sufficiently indebted ($m_n \geq \beta V_{REO}$), the gain from trade $\Psi(m_n; p)$ is strictly decreasing in debt m_n, for any $p > m_n$. Heavily indebted sellers are, effectively, less concerned with the likelihood that they fail to sell than with the gain they receive if they succeed. The reason for this is that they receive residual profit only if they sell at a sufficiently high price. To them, the foreclosure cost is fixed; the marginal cost of defaulting on a larger debt is borne entirely by the lender.

A less indebted seller (with $m_n < \beta V_{REO}$) has greater incentive to sell, as failure to do so results in the loss of residual profit as well as the cost of the foreclosure tag. Moreover, for $p > \beta V_{REO}$, the gain from trade Ψ is strictly increasing in debt m_n. As such, a more

indebted seller (but with $m_n < \beta V_{REO}$) will chose a lower price/higher sales probability. Overall, for $m_n < \beta V_{REO}$, the effect of debt on the gain from trade (i.e., $\Psi'(m_n)$) is likely to be small in that m_n affects symmetrically the returns both to selling and failing to do so (see (B.4)). Thus the relationship between the asking price and the LTV is only weakly decreasing for $m_n < \beta V_{REO}$, but strongly increasing for higher LTVs.

The condition $p > \beta V_{REO}$ is not restrictive, at least for our baseline calibration. Here $\beta V_{REO} = 1.79$, while the minimum selling price chosen by a seller is 3.04. In general, βV_{REO} tends to be much lower than the choice of selling price by any seller due to the foreclosure and carrying costs associated with houses in REO inventory.

5.2   Matching and lending standards

We now consider the role of specific assumptions regarding matching. We consider first the effects of changing the matching coefficient ϖ and second the matching elasticity η, *ceteris paribus*. Our goal here is to examine how search frictions, which determine in part the liquidity of housing, affect mortgage lending standards directly. Overall, we see that mortgage lending standards are lower the more liquid is the housing market.

The top two panels of Figure 6 illustrate the effect of changes to ϖ on the average LTV (or alternatively, the down-payment ratio) at orgination and the probability of a mortgage ending in foreclosure, respectively.[24] Consider a mortgage issued in the current period. The probability, on the balanced growth path, of such a loan ending in foreclosure before being fully repaid is given by

$$\Pi_d = \sum_{n=1}^{T}(1-\pi_h)^n(1-\pi_d)^{n-1}\pi_d[1-\rho(\theta(p_{sd}(m_{n-1}))]. \tag{48}$$

where $\rho(\theta(p_{sd}(m_{n-1})))$ is the probability of successful sale for a distressed homeowner with outstanding mortgage m_{n-1} who prices optimally.

As is shown in Figure 6, LTVs at origination are increasing and default probabilities decreasing with the value of ϖ. The higher it is, the more likely *any* seller is to match, or equivalently, the more liquid the housing market. Thus, the expected default rate is lower because distressed homeowners are more likely to sell. At the same time, houses in REO inventory also sell more quickly. Overall, as both the likelihood and cost of default and foreclosure fall, lenders are advance larger loans, resulting in higher LTVs at origination.

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[24]*Households* may be thought of as differing in LTV at origination because they purchase houses at different prices but are advanced loans of the same size. The left-hand panels of Figure 6 depictthe average, maximum and minimum LTVs at origination.
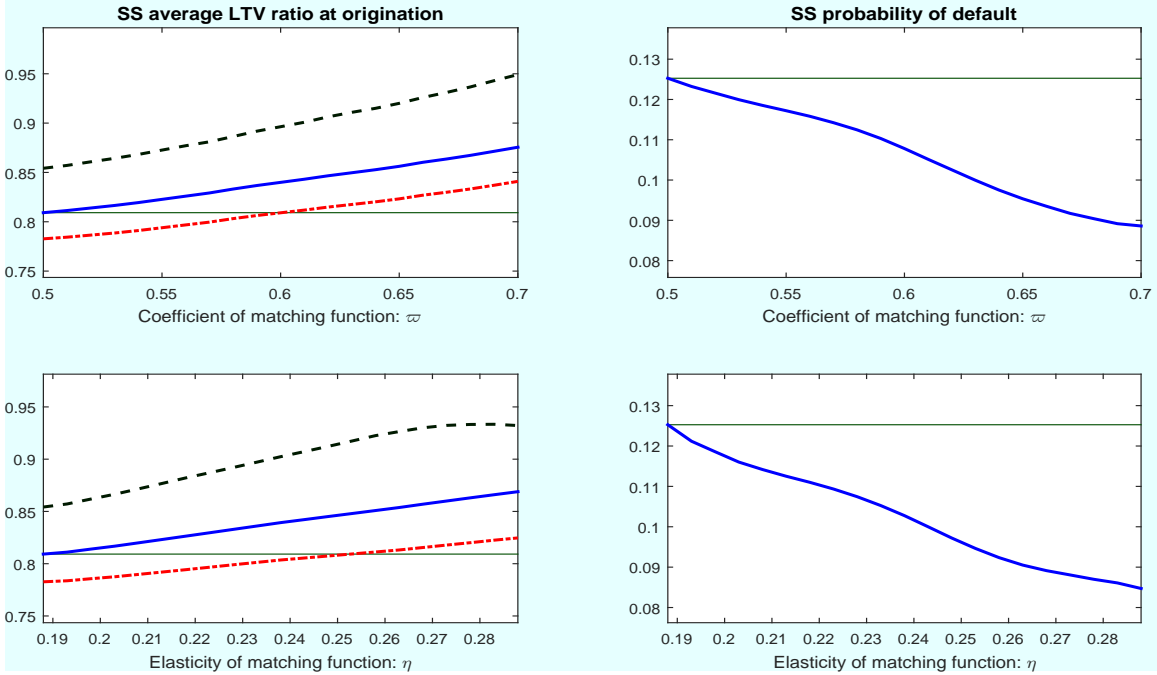
Figure 6: Effects of search friction parameters on average down-payments and default rates in the steady state. Left Column: Average down-payment (solid blue), the maximum LTV origination (dashed black) and the minimum LTV at origination (dot-dash red).

The bottom two panels of Figure 6 illustrate the results of varying the value of η, the elasticity of matches with respect to the measure of buyers. Buyers' share of the surplus from housing transactions increases with η (see Moen (1997) and Head, Lloyd-Ellis and Sun (2014)). This increases the value of living in the city, lowers the entry cutoff, ε_c, and increases entry by buyers. A higher η similarly lowers the return to construction by reducing developers' share. The housing market is thus tighter overall, and *all* houses sell with higher probability. Again, this lowers both the expected rate and cost of default, leading lenders to issue larger loans.

6 Equilibrium Dynamics

We now consider dynamics resulting from a shock to *per capita* income and compare them to those of an alternative *non-search* (NS) economy described in Appendix C. Specifically, we consider each economy on its balanced growth path experiencing an unanticipated shock

to $y(t)$. There are no additional shocks, and $y(t)$ returns to its long-run level, $\bar{y}$ via

$$\ln y(t) = \lambda \ln y(t-1), \qquad \text{where } \lambda = .96. \tag{49}$$

Here, λ and the initial shock, .02, are chosen so that (49) is an approximate first-order analog of the income process estimated in Head, Lloyd-Ellis and Sun (2014). Corresponding results for a negative income shock are presented in Appendix D.

6.1 Population growth, house prices and construction

Figure 7 illustrates the responses of city population growth, the average house price, and construction to the shock described above. All of these responses, for both the baseline and NS economies, are similar to those reported by Head, Lloyd-Ellis and Sun (2014).[25]

Briefly, a positive shock to local income induces immediate entry of households to the city and the population growth rate rises. The response of population growth is, however, much larger in the search economy. The responses of housing prices and construction rates differ across the two economies qualitatively as well as quantitatively. The search economy exhibits serial correlation in both price growth and construction. In contrast, in the non-search economy the average house price jumps immediately and then returns monotonically to its steady-state level. It is this initial jump in house prices, followed by a long and slow decline, effectively limits the entry of households to the city, accounting for the muted population response. Income in the city is higher, but houses, which increase in price initially, lose value quickly—limiting the city's attractiveness to those outside.

6.2 Market tightness and matching probabilities.

In the search economy, serial correlation in both house price growth and the construction rate is driven by the change in housing market liquidity. To illustrate this, Figure 8 depicts the responses of overall market tightness, and respective average matching probabilities of buyers and sellers. Following a positive shock to city income, the measure of searching buyers increases immediately due to household entry. Construction, however, takes time and so overall market tightness (the ratio of *total* buyers to sellers across all sub-markets) increases immediately. Tightness continues to rise for a prolonged period for two reasons: First, there is further entry of prospective buyers due to the persistence of the shock .

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[25]This is unsurprising as the baseline search economy has been constructed in part to preserve the basic dynamics of housing market variables in that paper. For this reason we discuss them only briefly here before moving on to a discussion of seller behaviour and the mortgage market, which are the focus here.

Figure 7: Impulse responses to a positive income shock: population, prices and construction

Second, buyers who do not match initially remain in the market. Although construction results in a persistent increase in the measure of sellers in the market, the former effect dominates and tightness both rises and remains above the steady-state for an extended period of time.

Higher market tightness implies higher (lower) matching probabilities for sellers (buyers) at any given trading price; that is, a more liquid housing market. The top-right and bottom-left panels in Figure 8 demonstrate: As housing markets become increasingly more liquid in the sense that it takes less and less time to sell, sales prices and house values continue to rise. This leads to serial correlation also in construction, as it is driven by the value of new houses.

As income returns to its steady-state level, entry of households to the city slows, fewer households enter, searching buyers match, and new houses come on the market. Thus, tightness falls and housing market liquidity falls. Eventually, house prices and construction return to their steady-state values.
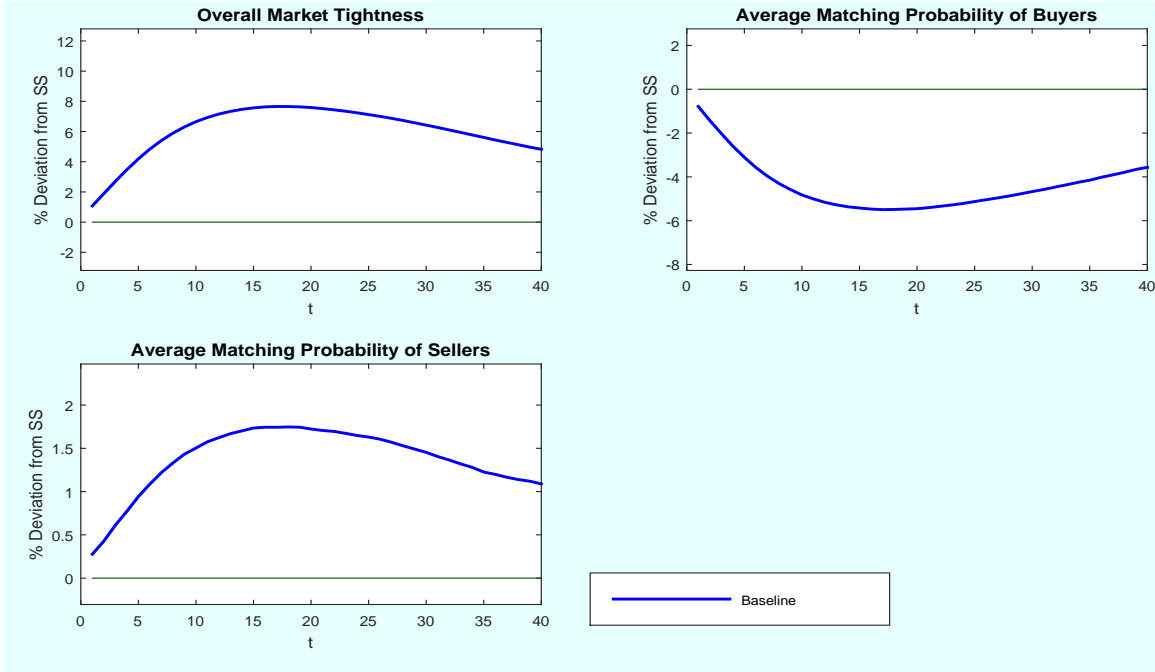
Figure 8: Impulse responses to a positive income shock: matching

6.3 The default rate, mortgage size, and LTV at origination

In the search economy, the persistent and positively auto-correlated increase in the selling probability (Figure 8, bottom-left) lowers the probability with which distressed sellers face foreclosure. Thus, the default rate moves opposite the selling rate, as shown in the top-left panel of Figure 9. The default rate for the NS economy (not shown), in contrast, is exogenous and thus unaffected by the shock.

Qualitatively, the responses of loan size at origination, m_0, resemble those of house prices for both economies (compare Figure 9, top-right; with Figure 7, bottom-left). *Quantitatively*, however, the response of m_0 reveals that lending standards (measured by LTV at origination) move in opposite directions in the two economies. Specifically, m_0 rises by more than the average house price in the baseline search economy, and by *less* in the NS economy. So, in response to a positive income shock, lending standards are relaxed in the former economy, and tightened in the latter.

Several forces contribute to the relaxation of lending standards in the search economy. First, the expected default rate on new mortgages declines persistently as houses become increasingly liquid. Since the mortgage market is competitive and the interest rate fixed, in equilibrium lower risk leads to loans being larger relative to the purchase price. We refer
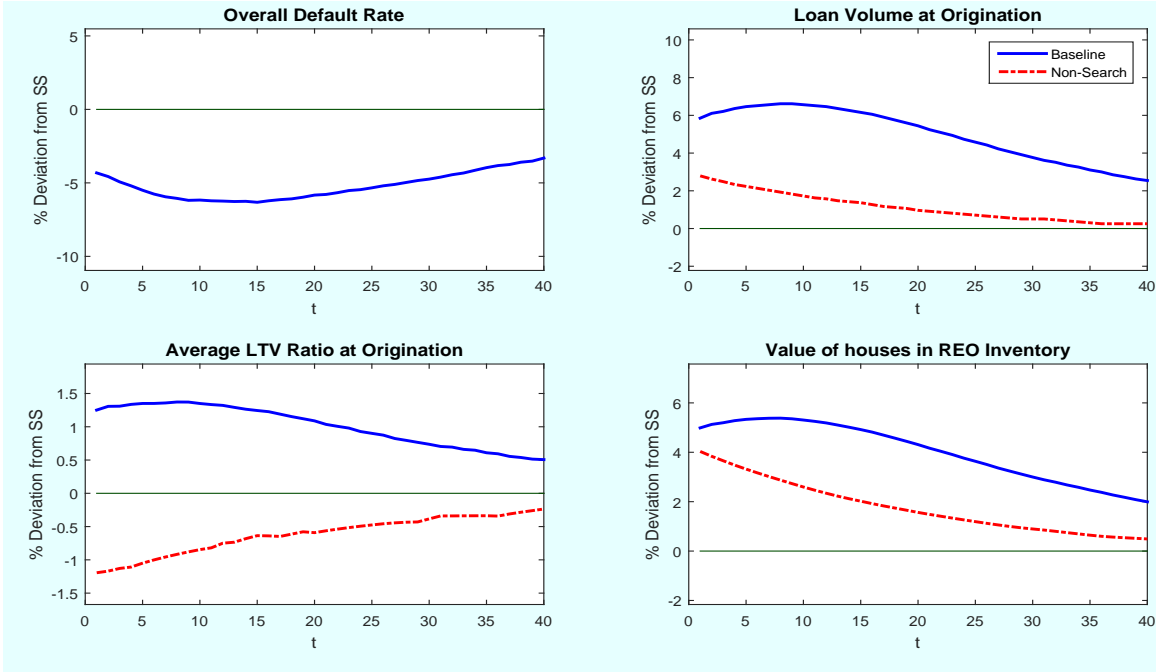
Figure 9: Impulse responses to a positive income shock: mortgage

to this as the *market tightness* effect.

Second, borrowers holding mortgages at the time of the shock experience *reductions* in their LTV's as a result of the increase in house values. This results in relatively large capital gains, as their housing investments are levered. As illustrated above, a decline in LTV is associated, *ceteris paribus*, with lower asking prices and higher sales probabilities, especially for sellers in financial distress. This mitigates the increase in house prices, attracts more buyers to the city, increases tightness throughout the housing market and thus further lowers the default rate.

This *home equity* effect contributes also to house price *momentum*, which has proved difficult to capture quantitatively (see Guren (2016) and Head, Lloyd-Ellis and Sun (2014)). Initially, this effect mitigates the jump in the average house price, as sellers whose LTV's are reduced raise prices by *less* than they otherwise would given the increase in market tightness. With a lag, it contributes to prices rising by *more*, as it leads to greater buyer entry thus making the overall response of tightness larger and more persistent.

Third, the proceeds of foreclosure sales rise and remain high for several periods reflecting the increases in both house values and the selling rate (see Figure 9, bottom-right) for the response of V_{REO}). This lowers the *cost* of default to lenders, increases the returns to lending and results in larger mortgages at origination.

Overall, reductions of both the expected default rate and the expected loss upon default motivate lenders in the search economy to increase the size of the loans they offer at origination. For the NS economy only the latter effect is present, and the LTV at origination *falls*. Without the endogenous response of liquidity, the expected default remains constant. House prices in the NS economy rise in response to the shock and are expected to fall monotonically. Thus, on origination, the lender's expected loss upon default rises. Given that the default rate does not fall to compensate, the lender must require a higher down-payment to cover the increase in default risk.

Finally, Figure 10 depicts co-movement of the average house price with the LTV at origination for both economies. The baseline search economy generates clearly a positive co-movement between the two variables while the NS economy predicts a negative one.
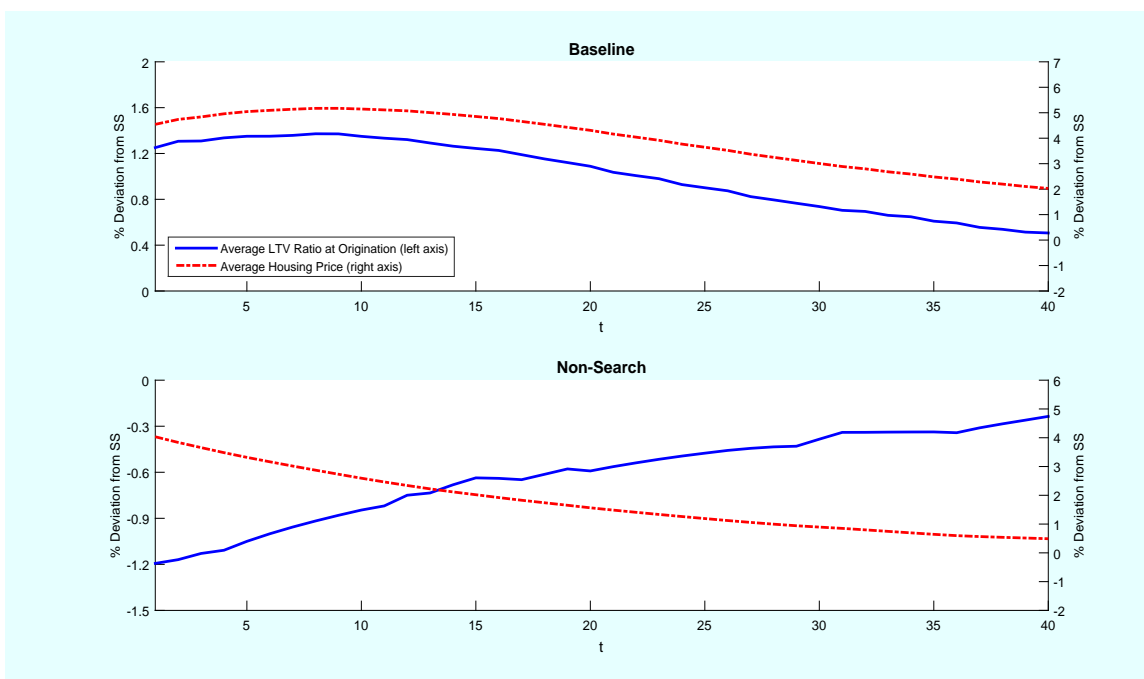

Figure 10: Co-movements between average down-payment ratio and average housing price in baseline and non-search economies.

6.4 Pricing decisions of indebted sellers

We now consider the effects of the shock on sellers' pricing decisions and their associated consequences. Figure 11 depicts the responses of the sellers' asking prices at four different

stages of mortgage-repayment.[26] The two top panels and the bottom-left depict pricing decisions of distressed sellers; the bottom-right those of newly relocated sellers.

All four panels reveal for the baseline search economy (the NS economy has no counterparts for these measures) a pattern consistent with the path of the average sales probability shown in the bottom panel of Figure 8.[27] That is, all panels display patterns consistent with the time-paths of tightness and the average sales rate.

Defying the overall pattern are distressed sellers who have just purchased in the period *before* the shock occurs ($n = 0$). As described above, the positive shock raises the value of houses and reduces these households' LTVs substantially, because they have they highest degree of leverage before the shock. When these households receive financial distress shocks, they face the prospect of losing this potential capital gain if they fails to sell, as their equity increases in value by much more than βV_{REO} (which increases with the *average* house price). The household thus has a strong incentive to sell, and so posts a low relatively low price and sells with a relatively high probability (see Figure 12).

Buyers who purchase *following* the shock experience no unanticipated capital gain, as current and future house prices, and future matching rates are taken into account when new mortgages are issued. This explains the large rise in the asking price (and drop in the selling-probability in Figure 12) for distressed sellers with $n = 0$ in subsequent periods. The choice of relocated sellers with $n = 1$ also displays a similar initial responses, albeit of smaller magnitude. These sellers neither face imminent foreclosure nor experience such a large capital gain because they are on average less levered than new homeowners.

Consider next the case of sellers one period away from having fully repaid their mortgage ($n = 29$) in the period before the shock. In the figure it is clear that these sellers *raise* their asking prices and thus experience a *lower* probability of a sale. This, of course, raises their default probability. Recall that in the event of a default, the lender keeps the outstanding mortgage balance and returns any remaining sale proceeds when the foreclosed house is. For sellers with $n = 29$, the outstanding balance is low precisely because the mortgage has been nearly repaid in full. Thus, the cost of default is low because these households recover a large portion of their equity after a default. Their return, even in default, is enhanced by the increase in the value of houses in REO inventory.

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[26]For example, the top-left panel depicts the pricing choice of a seller who has not yet made her first payment ($n = 0$), t periods following the shock. That is, it depicts the pricing decisions of a cross-section of sellers at the same stage of repayment but with loans originating at different times.

[27]Despite the connection, note that Figure 11 displays a panel where as Figure 8 depicts a time-series relationship. Each point in the last panel of Figure 8 represents a weighted average of the corresponding points in Figure 11 together with those for *all other* sellers, regardless of their mortgage status, if any.
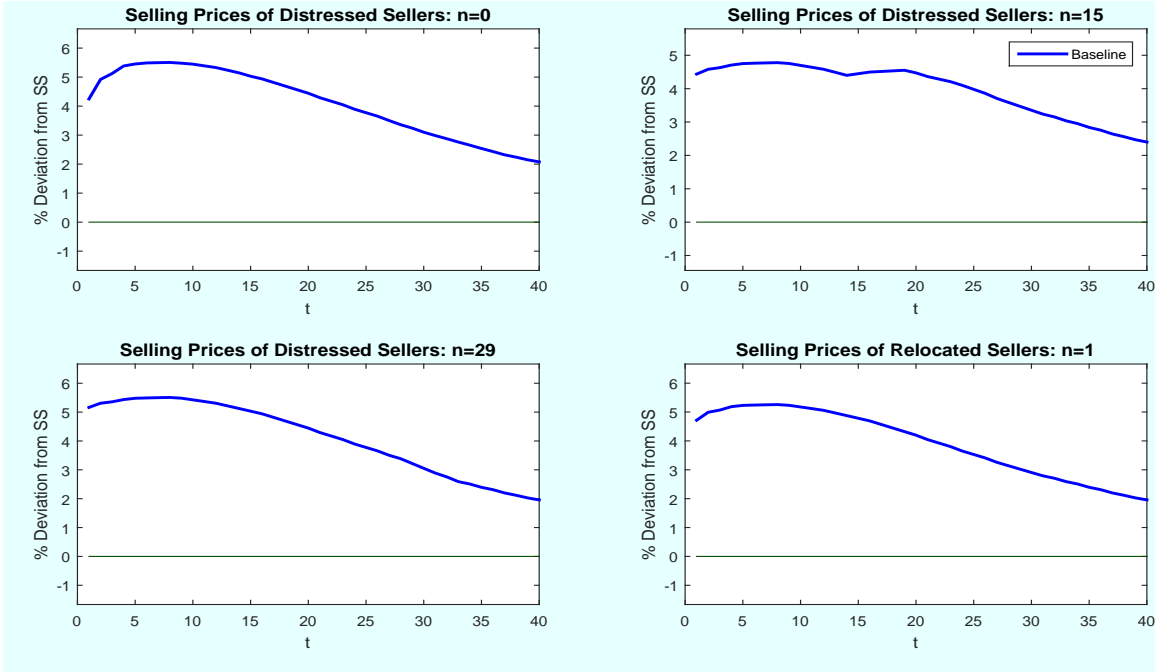
Figure 11: Impulse responses to a positive income shock: house-selling choices

The responses of sellers in the middle of their mortgage repayment term ($n = 15$ in the figure) lie between those of sellers at the beginning and end of their terms. The effects discussed above combine for these sellers and largely cancel, leaving the response to reflect largely the movements of the average sales probability.

Tables 2 and 3, respectively, contain the choices of sales probabilities associated with optimal pricing decisions and the implied default rates following a positive income shock. As is consistent with the steady-state results described above, sellers with relatively high leverage are much more likely to default than those with less.

6.5   Negative shocks

For the most part, negative income shocks have opposite effects of positive ones, qualitatively. The results are not entirely symmetric quantitatively, however, owing to the non-linear effects of default.[28] Specifically, following a negative income shock, non-distressed owners may experience such large *increases* in their LTV that they have negative home equity and thus choose to default outright. In this case, some non-distressed homeowners will attempt to sell before defaulting (see Figures 16 and 17). Tables 5 and 6 (see Ap-

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[28]Appendix D contains full results for a negative shock symmetric to the positive one considered above.
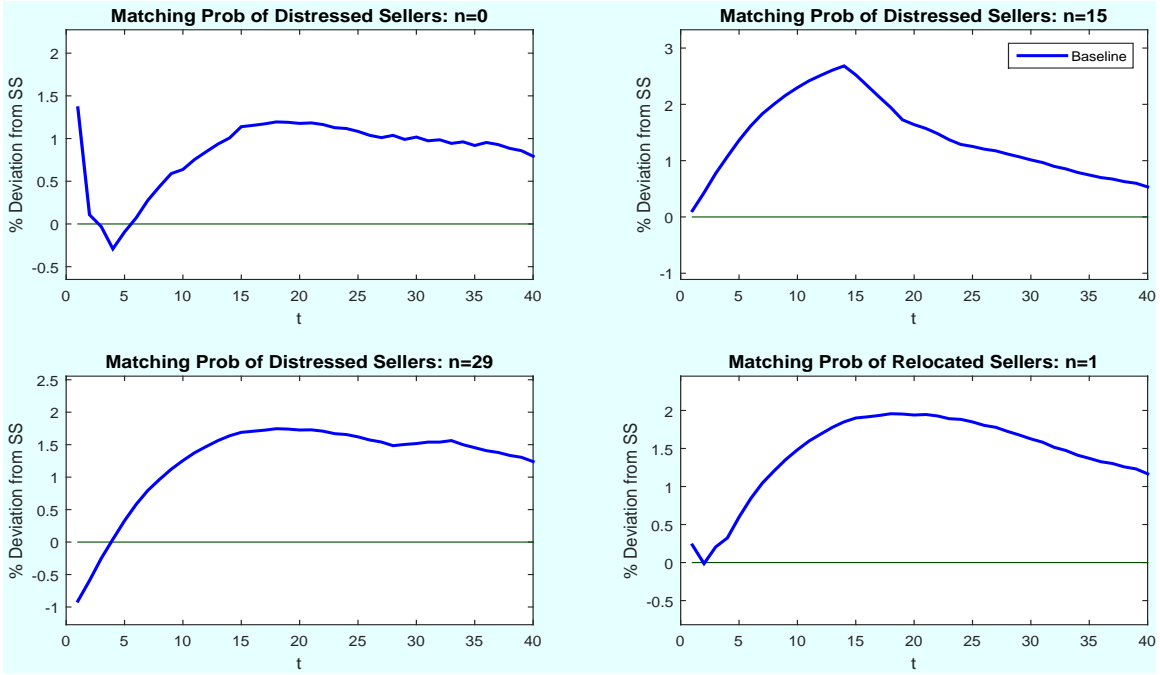
Figure 12: Impulse responses to a positive income shock: house-selling choices (probability)

Table 2: Sales probabilities (positive shock): distressed sellers having made n payments

	n=0	1	5	10	15	29
t=1	0.7995	0.8056	0.8178	0.8420	0.8599	0.8658
2	0.7834	0.8081	0.8234	0.8446	0.8626	0.8685
3	0.7859	0.7890	0.8260	0.8474	0.8684	0.8714
4	0.7857	0.7919	0.8291	0.8505	0.8716	0.8746
5	0.7881	0.7944	0.8317	0.8531	0.8744	0.8774
6	0.7892	0.7955	0.8329	0.8544	0.8757	0.8787
7	0.7912	0.7975	0.8164	0.8567	0.8780	0.8810
8	0.7925	0.7956	0.8176	0.8580	0.8794	0.8824
9	0.7938	0.7970	0.8190	0.8595	0.8810	0.8840
10	0.7945	0.7976	0.8197	0.8602	0.8817	0.8847

pendix D). Moreover, throughout the distribution, negative shocks lead to higher default probabilities than do positive ones.

Table 3: Default probabilities (positive shock): distressed sellers having made n payments

	n=0	1	5	10	15	29
t=1	0.2005	0.1944	0.1822	0.158	0.1401	0.1342
2	0.2166	0.1919	0.1766	0.1554	0.1374	0.1315
3	0.2141	0.2110	0.1740	0.1526	0.1316	0.1286
4	0.2143	0.2081	0.1709	0.1495	0.1284	0.1254
5	0.2119	0.2056	0.1683	0.1469	0.1256	0.1226
6	0.2108	0.2045	0.1671	0.1456	0.1243	0.1213
7	0.2088	0.2025	0.1836	0.1433	0.1220	0.1190
8	0.2075	0.2044	0.1824	0.1420	0.1206	0.1176
9	0.2062	0.2030	0.1810	0.1405	0.1190	0.1160
10	0.2055	0.2024	0.1803	0.1398	0.1183	0.1153

Out of the steady-state, the distribution of indebted sellers matters for the response of the economy to shocks. *Ceteris paribus*, a negative shock will cause more defaults the higher the proportion of relatively highly-leveraged homeowners. That is, during bad times a high-leverage economy will experience more severe defaults than a low-leverage one both from an *intensive margin* effect (costs per default are higher) and from an *extensive margin* effect (higher-leveraged households have greater incentive to default).

7 Conclusion

We develop a dynamic equilibrium model of housing transactions in which purchases are financed by long-term defaultable mortgages and use it to study (i) the effect of sellers' leverage on their pricing behavior and likelihood of default; and (ii) the effects of housing market liquidity on mortgage standards. House prices, default probabilities and mortgage standards both in and out of the steady-state are strongly influenced by changes in housing market liquidity.

Sellers' asking prices are decreasing in and relatively insensitive to increase in leverage when LTVs are low, but become steeply increasing in leverage at higher debt ratios. This result accords well with the empirical findings of Anenberg (2011), Genesove and Mayer (1997) and others. Moreover, seller behavior also differs with leverage along the

dynamic path in response to shocks. Housing market liquidity also influences mortgage standards significantly. In particular, the theory generates positive co-movements between house prices and LTVs at origination, a finding qualitatively consistent with observations regarding lending standards both during the period leading up to the recent house price collapse in the U.S., and during the current and on-going period of house price growth in Canada. These co-movements would be negative in the absence of endogenous housing market liquidity (*e.g.* in a non-search environment).

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A Household Values in the Second Sub-period

A buyer who enders the second sub-period having defaulted in the past and retaining a foreclosure flag lives as a renter and remains as such until either she moves out of the city or has her foreclosure flag lifted. Her value, $W_f(a)$ is

$$W_f(a) = u(y + a - R) + \pi_h \beta \overline{V} + (1 - \pi_h)\beta \left\{ \pi_f \, E\left[V_d'\right] + (1 - \pi_f)E\left[V_b'\right] \right\} \tag{A.1}$$

For such a buyer who has just defaulted in the previous sub-period, $a = \max[0, \beta E[V_{REO}] - m]$. If, however, the buyer entered the period with a foreclosure flag, $a = 0$. In either case, conditional on staying in the city, with probability π_f the foreclosure flag remains and the household moves onto the following period with expected value V_d', (11). With probability $1 - \pi_f$, the foreclosure flag is lifted and this household will enter the next period as a buyer with value V_b'.

A buyer without a foreclosure flag at the beginning of sub-period 2 is either a resident owner who just successfully sold or a buyer who failed to purchase in sub-period 1. Such a buyer may have a positive intra-period asset balance, a, coming from sale proceeds net of the outstanding mortgage debt in the previous sub-period. She will move on with value V_b' and participate in the housing market in the next period if not hit by the moving shock at the end of the current period. Here second sub-period value is

$$W_b(a) = u(y + a - R) + \pi_h \beta \overline{V} + (1 - \pi_h)\beta E\left[V_b'\right]. \tag{A.2}$$

A resident homeowner with a mortgage has the principle balance m_n. The owner's periodic income is used to cover repayment, maintenance cost and consumption. Let $W_o(m_n)$ denote the value of such an owner. It follows that for $n \in [0, T-2]$,

$$\begin{aligned} W_o(m_n) &= u(y - x_n - d) + z_H + \pi_h \beta E\left[V_L'(m_{n+1})\right] + \\ &\quad (1 - \pi_h)\left\{ \pi_d \beta E\left[V_f'(m_{n+1})\right] + (1 - \pi_d)\beta E\left[V_o'(m_{n+1})\right] \right\}. \end{aligned} \tag{A.3}$$

If the owner receives a moving shock, she exits the city immediately and continues with value $V_L'(m_{n+1})$. Her mortgage debt does not vanish on relocation. Conditional on not relocating, in the next period the owner receives a financial distress shock with probability π_d. In this case, she continues as a distressed resident owner with debt $V_f'(m_{n+1})$. Otherwise, she enters the next period as a non-distressed owner with value $V_o'(m_{n+1})$.

For $n = T - 1$, a resident homeowner with a mortgage has second sub-period value

$$W_o(m_{T-1}) = u(y - x_{T-1} - d) + z_H + \pi_h \beta V_{Lw}' + (1 - \pi_h)\beta E\left[V_{nd}'\right]. \tag{A.4}$$

In this case, the current mortgage payment is the homeowner's last. Thus, she will continue on with value V'_{Lw} if hit by the moving shock (in which case she leaves the city owning a house but having no debt) and with value V'_{nd} if she remains in the city.

Let $W_o(p, m_0)$ denote the second sub-period value of a new homeowner, who has purchased a house in the preceding sub-period. This owner makes a *down-payment* equal to the purchase price minus the new mortgage, $p - m_0$. Periodic mortgage payments begin the following period. Thus,

$$
\begin{aligned}
W_o(p, m_0) &= u(y - (p - m_0) - d) + z_H + \pi_h \beta V'_L(m_0) + \\
&\quad (1 - \pi_h)\left\{ \pi_d \beta E\left[V'_f(m_0)\right] + (1 - \pi_d)\beta E\left[V'_o(m_0)\right] \right\}.
\end{aligned} \tag{A.5}
$$

Finally, homeowners without mortgages do not suffer financial distree. They remain in the city until they experience a moving shock. Their second sub-period value is given by

$$
W_{nd} = u(y - d) + z_H + \pi_h \beta V'_{Lw} + (1 - \pi_h)\beta E\left[V'_{nd}\right]. \tag{A.6}
$$

B   Proof of Proposition 1

Proof. Using (10), in the steady state, for any given $p > d$ (see footnote 10), the gain from trade for a such a seller as a function of her outstanding debt, m_n, is given by:

$$
\Psi(m_n) = W_b(p - m_n) - W_f(\max[0, \beta V_{REO} - m_n]) \tag{B.1}
$$

$$
= \begin{cases} W_b(p - m_n) - W_f(\beta V_{REO} - m_n), & \text{if } m_n < \beta V_{REO} \\ W_b(p - m_n) - W_f(0), & \text{if } m_n \geq \beta V_{REO} \end{cases} \tag{B.2}
$$

$$
= \pi_f(1 - \pi_h)\beta\left[W_f(0) - V_b\right] + \\
\begin{cases} u(y - R + p - m_n) - u(y - R + \beta V_{REO} - m_n), & \text{if } m_n < \beta V_{REO} \\ u(y - R + p - m_n) - u(y - R), & \text{if } m_n \geq \beta V_{REO} \end{cases} \tag{B.3}
$$

Differentiating (B.3) with respect to the level of debt, m_n, we have

$$
\Psi'(m_n) = \begin{cases} u'(y - R + \beta V_{REO} - m_n) - u'(y - R + p - m_n), & \text{if } m_n < \beta V_{REO} \\ -u'(y - R + p - m_n), & \text{if } m_n \geq \beta V_{REO} \end{cases} \tag{B.4}
$$

where here $\Psi'(\cdot)$, $u'(\cdot)$ denote differentiation. Then the results of Proposition 1 follow from $u' > 0$ and $u'' < 0$. □

While this proof relies on the assumption that consumption goods are non-storable, the result is in fact more general. In particular, the derived properties of $\Psi'(d)$ require only that $W_b'(p - m_n) > 0$ and $W_f'(\beta V_{REO} - m_n) - W_b'(p - m_n) > 0$ for $m_n < \beta V_{REO}$. The former condition requires that the value of a buyer who has just sold is strictly increasing in her asset holdings. The latter requires the slope of the value of a buyer who has failed to sell at asset position $\beta V_{REO} - m_n$ to exceed that of the value of an unflagged buyer at $p - m_n$ for lower levels of debt. Observing that $p > \beta V_{REO}$ in general, this requirement is not restrictive for value functions such as $W_f(\cdot)$ and $W_b(\cdot)$, which are strictly concave.

C   An Economy without Search

To illustrate the role of search frictions in the economy's dynamics, we consider also an environment in which the housing market is perfectly competitive. Here, houses are perfectly liquid in that buyers (without a foreclosure flag) and sellers are able to trade immediately and neither developers nor lenders hold houses in inventory.

In this setting, financial distress is extreme — at the beginning of period t, with probability π_d an indebted resident owner may experience a default shock which forces her to default immediately. A borrower not hit by such a shock may choose to default only in the case in which their housing equity becomes negative.[29]

C.1   Value functions

Household decisions in sub-period 2 are identical to those in the search economy. Sub-period 1 household values here are distinguished by the superscript w. A buyer without the foreclosure flag purchases a house at competitive price p_t and immediately becomes an owner with value $V_o^w(m_0)$. An indebted resident owner who does not receive a default shock decides whether and how to sell and whether or not to default. As before, let $D^w \in \{0, 1\}$ be the default indicator. If the owner sells, she repays as much of her outstanding debt as possible, keeps any remaining profit, and becomes a buyer without the foreclosure flag.

---

[29]Note, however, that as default is costly, not all owners with negative equity will default.

If she decides not to sell, then she decides whether to default:

$$V_o^w(m_n) = \max \left\{ \underbrace{W_b^w(\max[0, p - m_n]),}_{\text{sell}} \max_{D^w \in \{0,1\}} \left\{ \underbrace{\begin{array}{c} (1 - D^w)W_b(m_n) + \\ D^w W_f^w(\max[0, \beta E[V_{REO}'^w] - m_n]) \end{array}}_{\text{don't sell}} \right\} \right\}, \qquad \text{(C.1)}$$

where $V_{REO}'^w = (1 - \chi)V_c'^w$ is the value of a vacant house in the next period, net of the foreclosure cost, χ.

An indebted owner who experiences a distress shock immediately defaults. Such an owner has the value:

$$V_f^w(m_n) = W_f^w(\max[0, \beta E[V_{REO}'^w] - m_n]). \qquad \text{(C.2)}$$

A resident owner without debt decides whether or not to sell and has value:

$$V_{nd}^w = \max \left\{ W_b^w(\max[0, p - m_n]), W_o^w(m_n) \right\}. \qquad \text{(C.3)}$$

Relocated owners with and without mortgage debt make similar selling and default decisions and have values $V_L^w(m_n)$ and V_{Lnd}^w, respectively:

$$V_L^w(m_n) =$$
$$\max \left\{ \underbrace{u(\max[0, p - m_n] + y^L - R^L) + \beta \overline{V} +}_{\text{sell}} \max_{D_t^w \in \{0,1\}} \left\{ \underbrace{\begin{array}{c} ((1 - D_L^w)(u(y^L - R_t^L - x_n - d) + \beta E[V_L'^w(m_{n+1})]) + \\ + D_L^w(u(\max[0, \beta E[V_{REO}'^w] - m_n] + y^L - R^L) + \beta \overline{V} \end{array}}_{\text{don't sell}} \right\} \right\} \qquad \text{(C.4)}$$

$$V_{Lnd}^w = \max \left\{ \underbrace{u(p + y^L - R^L) + \beta \overline{V}}_{\text{sell}}, \underbrace{u(y^L - R^L - d) + \beta E[V_{Lnd}'^w]}_{\text{don't sell}} \right\}. \qquad \text{(C.5)}$$

The values of vacant houses to both developers and lenders are given respectively by:

$$V_c^w = p \qquad \text{(C.6)}$$
$$V_{REO}^w = (1 - \chi)p. \qquad \text{(C.7)}$$

For mortgage contract $\iota = (m^t; r^t)$, the present mortgage values at the beginning of sub-period 2 after $n-1$ payments are given, for relocated and resident homeowners, respectively, are given by:

$$P_L(m_n) = x_n + \frac{\mathbb{I}_{\{n \neq T-1\}}}{1+i+\phi} \times$$

$$E\left\{\max\left\{\begin{array}{l} \min[p', m_{n+1}], \\ \max_{D'^w_{Ln+1}}\left\{\begin{array}{l} [D'^w_{n+1}\min[\beta V'''^w_{REO}, m_{n+1}] \\ +(1-D'^w_{n+1})P'_L(m_{n+1}) \end{array}\right\} \end{array}\right\}\right\} \tag{C.8}$$

for $n \in \{1, T-1\}$, and

$$P^\iota_t(m_n) =$$

$$x_n\mathbb{I}_{\{n \neq 0\}} + \frac{\mathbb{I}_{\{n \neq T-1\}}}{1+i+\phi} \tag{C.9}$$

$$\times E\left\{\begin{array}{l} \pi_h \max\left\{\begin{array}{l} \min[p, m_{n+1}], \\ \max_{D^w_{Ln+1}}\left\{\begin{array}{l} D'^w_{Ln}\min[\beta V'''^w_{REO}, m_{n+1}] \\ +(1-D'^w_{Ln})P'_L(m_{n+1}) \end{array}\right\} \end{array}\right\} \\ +(1-\pi_h)\left\{\begin{array}{l} \pi_d\min[\beta V'''^w_{REO}, m_{n+1}] + \\ (1-\pi_d)\left\{\max\left\{\begin{array}{l} \min[p', m_{n+1}], \\ \max_{D^w_{n+1}}\left\{\begin{array}{l} D'^w_{n+1}\min[\beta V''_{REO}, m_{n+1}, \\ (1-D'^w_{n+1})P^\iota_{t+1}(m_{n+1}) \end{array}\right\} \end{array}\right\}\right\} \end{array}\right\} \end{array}\right\}$$

for all $n \in \{0, \cdots, T-1\}$, where, D'^w_{n+1} and D'^w_{Ln+1} are households default choices in the next period, conditional on the aggregate shocks and having mortgage balance, m_{n+1}.

C.2 Laws of motion

For the non-search economy, we have the following laws of motion:

$$(1+\mu)F' = (1-\pi_p)F + (1-\psi)G(\varepsilon'_c)\mu. \tag{C.10}$$

$$(1+\mu)B'_f = (1-\pi_h)\left\{\begin{array}{l} \pi_f B_f + \pi_d \sum_{n=0}^{T-1} H_n \\ +(1-\pi_d)\sum_{n=0}^{T-1}(1-I_n)D_n H_n \end{array}\right\} \tag{C.11}$$

where $I_n = 1$ if the owner chooses to sell, and 0 otherwise.

$$(1+\mu)B' = \psi G(\varepsilon'_c)\mu + (1-\pi_f)B_f + (1-\pi_h)\sum_{n=0}^{T-1} I_n H_n. \tag{C.12}$$

$$(1+\mu)H'_n = (1-\pi_h)(1-\pi_d)(1-I_{n-1})(1-D_{n-1})H_{n-1}; \tag{C.13}$$

$$(1+\mu)H'_0 = (1-\pi_h)(1-\pi_d)B; \tag{C.14}$$

$$(1+\mu)H'_\emptyset = (1-\pi_h)\left\{ \begin{array}{c} (1-\pi_d)(1-I_{T-1})(1-D_{T-1})H_{T-1} \\ +(1-I_T)H_\emptyset \end{array} \right\}. \tag{C.15}$$

$$(1+\mu)H'_{Ln} = (1-I_{Ln-1})(1-D_{Ln-1})H_{Ln-1}$$
$$+ \pi_h(1-\pi_d)(1-I_{n-1})(1-D_{n-1})H_{n-1}; \tag{C.16}$$

$$(1+\mu)H'_{L0} = \pi_h(1-\pi_d)B; \tag{C.17}$$

$$(1+\mu)H'_{L\emptyset} = \pi_h\left\{ \begin{array}{c} (1-\pi_d)(1-I_{T-1})(1-D_{T-1})H_{T-1} \\ +(1-I_T)H_\emptyset \end{array} \right\}$$
$$+ (1-I_{LT-1})(1-D_{LT-1})H_{LT-1} + (1-I_{LT})H_{L\emptyset}. \tag{C.18}$$

$$(1+\mu)H'_c = N. \tag{C.19}$$

$$(1+\mu)H'_{LREO} = \pi_d\sum_{n=0}^{T-1}H_n + \sum_{n=1}^{T-1}(1-I_{Ln})D_{Ln}H_{Ln}$$
$$+ (1-\pi_d)\sum_{n=0}^{T-1}(1-I_n)D_nH_n. \tag{C.20}$$

C.3   Equilibrium

The definition of equilibrium is similar to that for the search economy except that the housing market now clears each period in the Walrasian sense. All households (other than permanent renters) who begin the period without a house are buyers. If the measure of buyers exceeds the sum of the measures of new and foreclosed houses, then the price of housing adjusts until the appropriate measure of current homeowners chooses to sell. A shortage of buyers (and thus $p = 0$) is avoided by the continual entry of buyers without homes driven by population growth.

C.4   Calibration

Where possible, we use the same functional forms and calibrate the non-search economy to the same targets as the baseline search economy. All parameters remain at their values in Table 1 except for π_d, z_H and ψ, which are adjusted ot match the relevant targets. Similarly, the construction cost parameter is adjusted so that $P^* = 3.2$ given the rate of population growth.

Table 4: Calibration Parameter Values: Non-Search Economy

Parameter	Value	Target	Data
		Parameters determined independently	
β	0.96	Annual interest rate	4.0%
π_p	0.120	Annual mobility of renters	12%
π_h	0.032	Annual mobility of owners	3.2%
ξ	1.75	Median price-elasticity of land supply	1.75
i	0.040	International bond annual yield	4.0%
T	30	Fixed-rate mortgage maturity (years)	30
μ	0.012	Annual population growth rate	1.2%
π_f	0.80	Average duration (years) of foreclosure flag	5
$\bar{q}$	0.96	Average land price-income ratio	30%
m	0.08	Residential housing gross depreciation rate	2.5%
ζ	5	Median price elasticity of new construction	5
ς	0.16	Rent-price ratio	5%
		Parameters determined jointly	
χ	0.460	Loss severity rate	27%
ϕ	0.0246	Average down-payment ratio	20%
ϱ	0.0074	Average annual FRM-yield	7.20%
ψ	0.570	Fraction of households that rent	33.3%
π_d	0.016	Annual foreclosure rate	1.6%
z_H	0.3280	Average loan-to-income ratio at origination	2.72
κ	0.137	Average price of a house	3.2
α_p	6.200	Relative volatility of population growth	0.17

D Effects of a Negative Income Shock

Table 5: Sales probabilities: Distressed borrowers following a negative shock

	n=0	1	5	10	15	29
t=1	0.7721	0.7783	0.797	0.8279	0.8614	0.8794
2	0.7953	0.7767	0.7953	0.8262	0.8596	0.8775
3	0.7924	0.7955	0.7924	0.8231	0.8563	0.8741
4	0.7899	0.7930	0.7899	0.8204	0.8535	0.8712
5	0.7902	0.7902	0.7871	0.8175	0.8504	0.8681
6	0.7876	0.7907	0.7846	0.8148	0.8477	0.8653
7	0.7861	0.7891	0.8043	0.8133	0.8460	0.8636
8	0.7847	0.7877	0.8028	0.8118	0.8445	0.862
9	0.7834	0.7864	0.8014	0.8104	0.843	0.8605
10	0.7825	0.7855	0.8006	0.8095	0.8421	0.8595

Table 6: Default probabilities: Distressed borrowers following a negative shock

	n=0	1	5	10	15	29
t=1	0.2279	0.2217	0.2030	0.1721	0.1386	0.1206
2	0.2047	0.2233	0.2047	0.1738	0.1404	0.1225
3	0.2076	0.2045	0.2076	0.1769	0.1437	0.1259
4	0.2101	0.2070	0.2101	0.1796	0.1465	0.1288
5	0.2098	0.2098	0.2129	0.1825	0.1496	0.1319
6	0.2124	0.2093	0.2154	0.1852	0.1523	0.1347
7	0.2139	0.2109	0.1957	0.1867	0.1540	0.1364
8	0.2153	0.2123	0.1972	0.1882	0.1555	0.1380
9	0.2166	0.2136	0.1986	0.1896	0.1570	0.1395
10	0.2175	0.2145	0.1994	0.1905	0.1579	0.1405

Figures 13—15 depict the responses of aggregates. Figures 16 and 17 depict indebted sellers' pricing choices and the resulting selling probabilities. Figure 18 compares house prices and lending standards in the search and no-search economies.
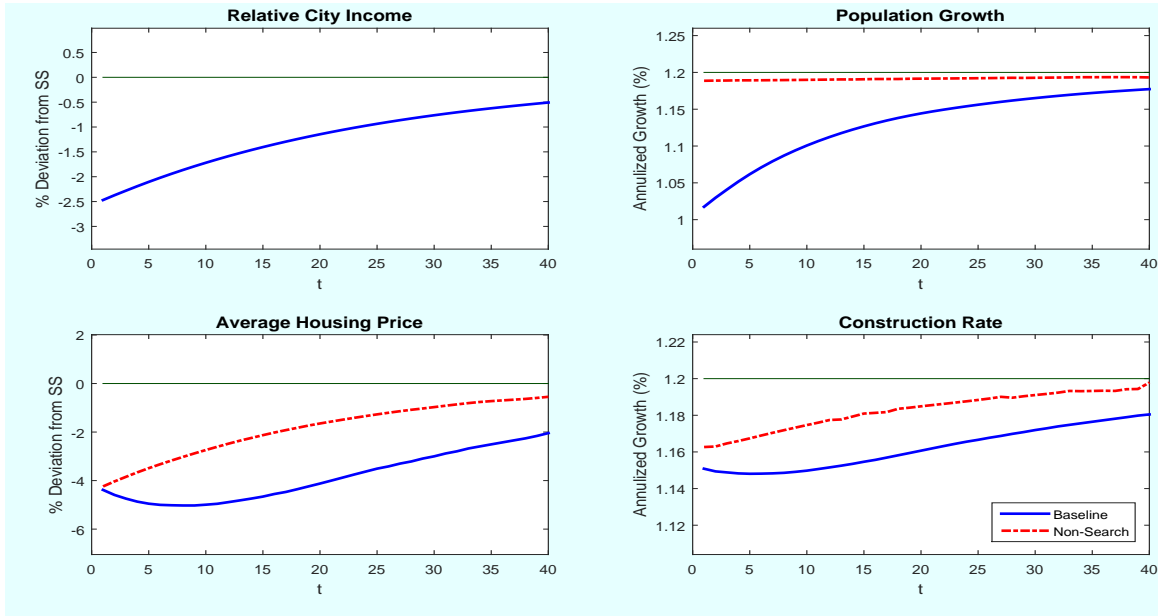

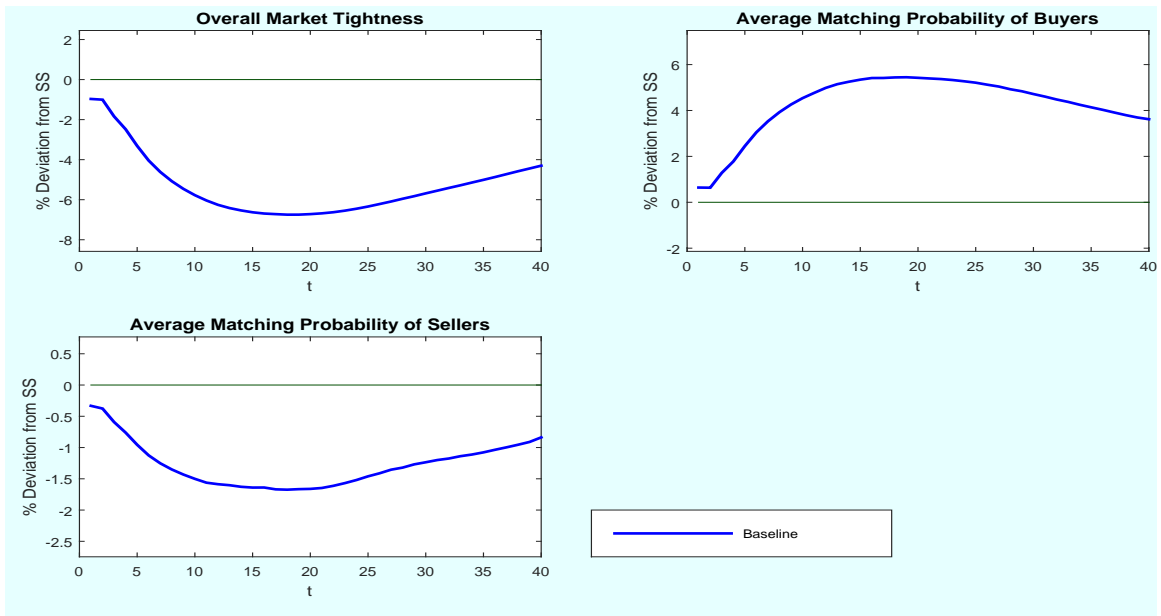Figure 13: Negative income shock: Responses of population, prices and construction
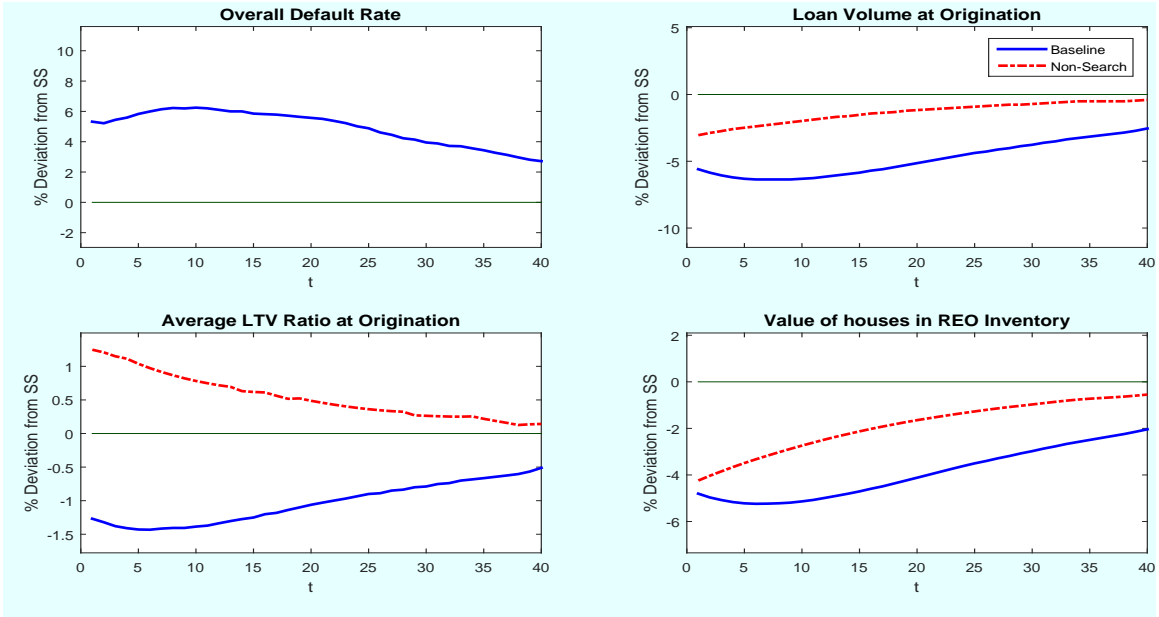

Figure 14: Negative income shock: Matching

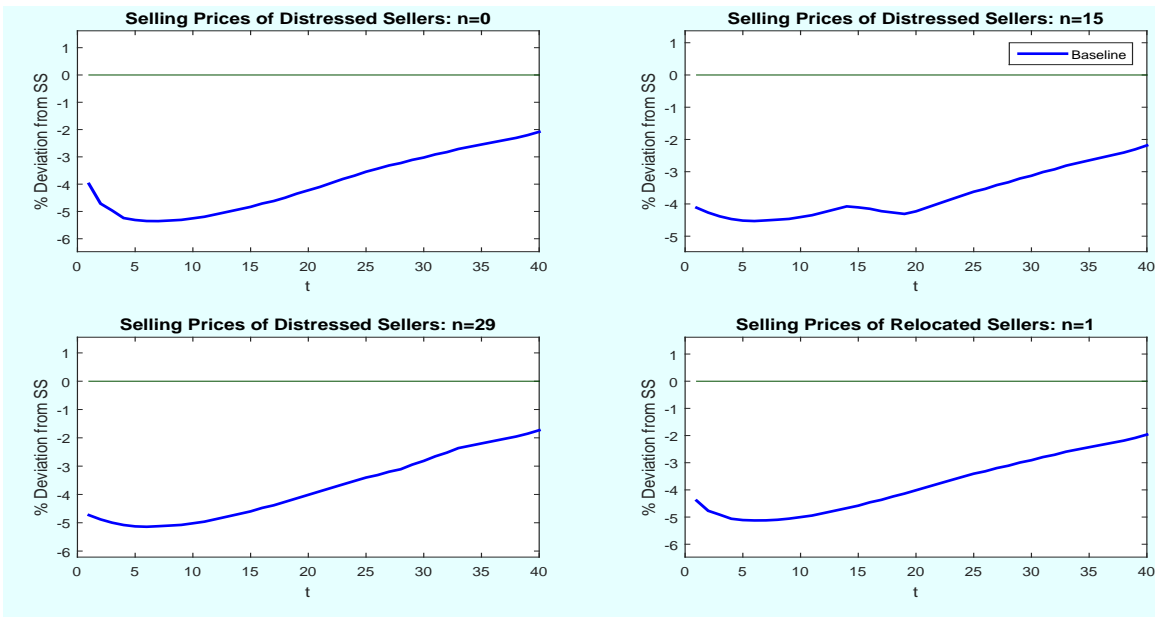Figure 15: Negative income shock: Mortgages

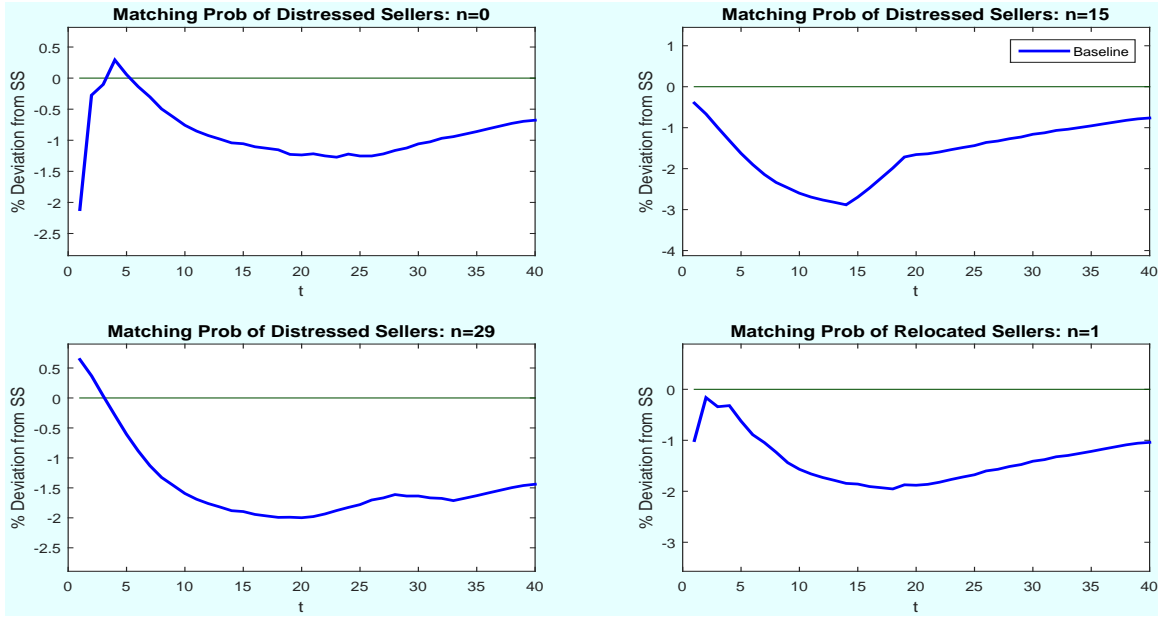Figure 16: Negative income shock: Sellers' choices

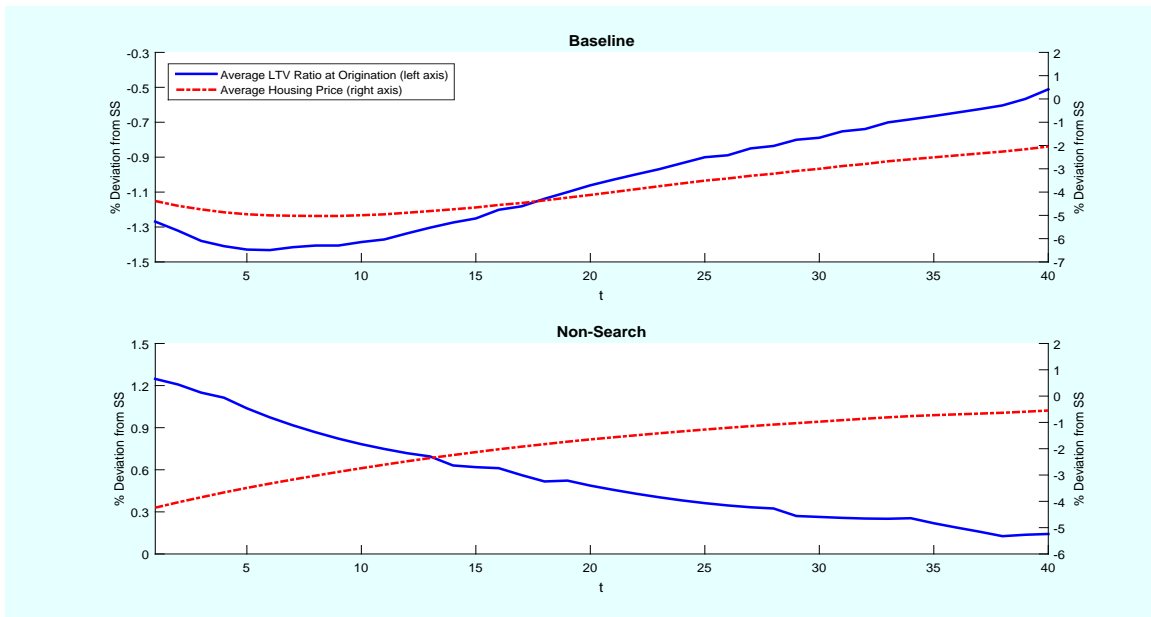Figure 17: Negative income shock: Sales probabilities

Figure 18: The down-payment ratio and the average house price; baseline vs. no-search.