Default, Mortgage Standards, and Housing Liquidity

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Abstract

The influence of households' indebtedness on their house-selling decisions is studied in a tractable dynamic general equilibrium model with housing market search and defaultable long-term mortgages. In equilibrium, sellers' behavior varies significantly with their indebtedness. Specifically, both asking prices and time-to-sell increase with the relative size of sellers' outstanding mortgages. In turn, the *liquidity* of the housing market associated with equilibrium time-to-sell determines the mortgage standards offered by competitive banks. When calibrated to the U.S. economy the model generates, as observed, negative correlations over time between both house prices and time-to-sell with downpayment ratios.

JEL Classification: E30, G21, R10, R31

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1 Introduction

In this paper we study the influence of households' mortgage debt on their house-selling decisions and the effects of the resulting housing market *liquidity*, by which we mean the ease with which houses can be sold, on mortgage lending standards. To this end, we develop a dynamic model with a frictional housing market and long-term mortgage debt. The model gives rise in equilibrium to distributions of both house prices and debt, as well as to endogenous default probabilities that differ across households. Aggregate shocks drive mortgage standards through their effect on liquidity, and this together with the mortgage terms offered to buyers affect the selling decisions of households.

Our theory demonstrates that (i) house-selling decisions depend critically on sellers' levels of home equity, and (ii) liquidity in the housing market is an important factor in determining the ease with which households can borrow. A version of the economy calibrated to U.S. data captures, qualitatively, both the observed relationship between households' mortgage loan-to-value (LTV) ratios and asking prices and the negative correlation between house prices and mortgage lending standards over time.

Our paper is motivated first by the observation that mortgage lending standards are in some sense "counter-cyclical". Specifically, Figure 1 illustrates that house prices are positively correlated with LTV's at origination (or negatively with down-payment ratios) for first-time home buyers[1]. This phenomenon has drawn particular attention for the period leading up to the U.S. sub-prime mortgage crisis and subsequent house price collapse. Over this time period, U.S. housing markets were "booming" in the sense that prices were rising, sales volume increasing and time-to-sell declining (Ngai and Sheedy, 2015). At the same time, mortgage lending standards were relaxed, specifically and significantly in the sense of lowered down-payment requirements.[2] It has been argued that these changes occured beyond those associated with changes in regulatory constraints (see, *e.g.* Belsky

[1] Actual down-payments don't necessarily reflect mortgage standards. First-time buyers, however, are the most likely to be affected by down-payment constraints.

[2] Lending standards may include mortgage approval rates, down-payment ratios, document requirements, interest rates, *etc.*. We focus on down-payment ratios, or LTV at origination. **?** show that many subprime loans in this period were characterized by such high LTV's. **?** construct a series for loan-to-value ratios (LTVs) faced by first-time home buyers and show that the overall LTV ratio increased from 2000 to 2005. **?** use data compiled for over 200 U.S. cities between 2000 and 2008 to find that interest-only (IO) mortgages were used sparingly in cities in which an elastic housing supply kept housing prices in check, but were common in cities with an inelastic supply in which housing prices rose sharply and then crashed. **?** document and show that lending standards (denying rates) declined more in areas that experienced larger credit booms (more applicants) and greater price appreciation. **?** find that regions with high latent demand from 2001 to 2005 experienced large relative decreases in denial rates, increases in mortgages originated, and increases in housing price appreciation, despite the fact that the same regions experienced significantly negative relative income and employment growth over this time period.

and Richardson, 2010 and Dow, 2015). Here, we explore the incentive of profit maximizing lenders to relax lending standards in response to a "hot" housing market, by which we mean specifically one in which prices are high and time-to-sell is low by historical standards.

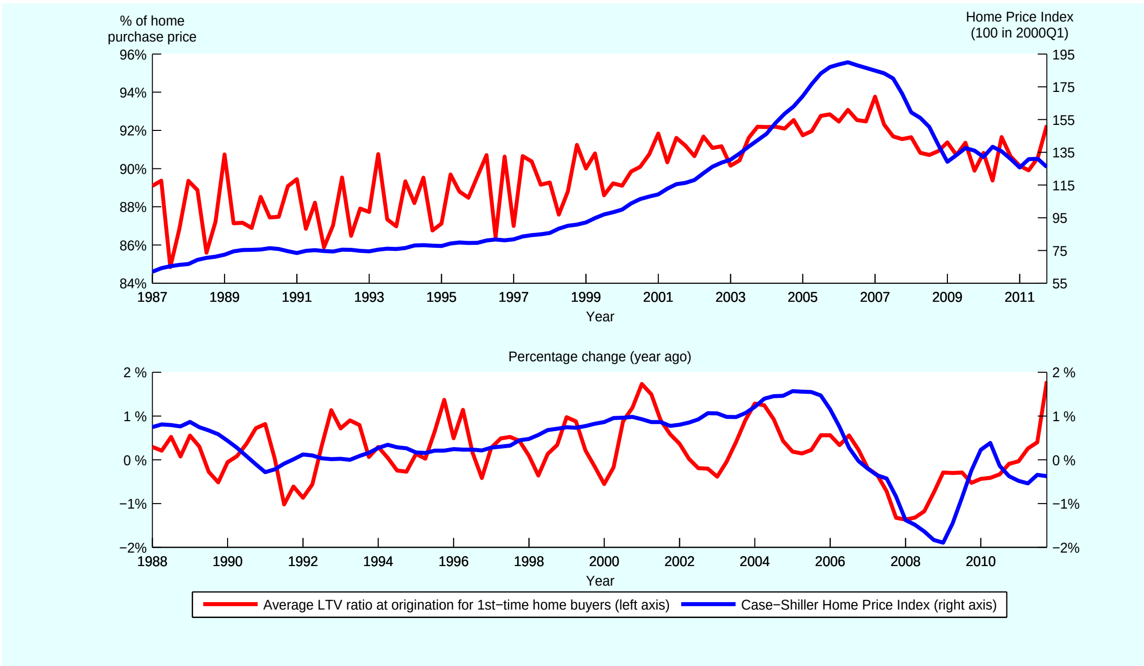

Figure 1: Values and percentage changes (from one year earlier) in average first-time home buyers' down-payment ratios and S&P/Case-Shiller U.S. National Home Price Index. Source: American Housing Survey (AHS) 2007, 2009, 2011 national data.

It has also been observed by **??** and Anenberg (2011), that house sellers' leverage affects both their asking prices and time-to-sell. Specifically, sellers with high LTV's post higher asking prices, wait longer to sell, and sell ultimately at higher prices. This observation suggests that mortgage debt affects not only prices, but also households' incentive to sell, as more heavily indebted sellers are evidently willing to wait for a buyer who will pay a higher price.

Our theory extends that of **?** (HLS) which focuses on the dynamics of house prices, sales, construction, and population growth in an environment with homogeneous buyers and sellers and complete financial markets. Here, we introduce a form of limited commitment which allows households to default under certain circumstances. This generates a role for mortgage debt secured by homes and generates heterogeneity among households, *ex post*. Also, while HLS focuses mainly on random search, competitive search is integral to our analysis and important for our results.[3]

We consider *ex ante* identical households which live either in a single city (on which we focus) or in a largely unmodeled rest-of-the-world All residents of the city require housing, and may either rent or own one of a large number of identical houses which are produced and sold initially by a competitive construction industry. Households enter the city when the value of doing so, in part driven by fluctuations in income at the city level, exceeds their outside option. Once there, households remain in the city either as renters or homeowners until they leave as a result of exogenous shocks.

Following the usual protocol of competitive search, sellers offer houses for sale in a variety of sub-markets, inside each of which prospective buyers and sellers are randomly matched. Each sub-market is characterized by a unique combination of a posted price and pair of matching probabilities (for buyers and sellers). These probabilities determine the expected time-to-buy and time-to-sell for buyers and sellers, respectively. Search is directed in the sense that buyers and sellers choose sub-markets optimally given the trade-off between the posted price and the matching probabilities.

House purchases are financed by mortgages offered by competitive banks which control the terms offered. Specifically, banks decide the *size* of the mortgage to offer, and this determines the LTV at origination. Mortgages are of a fixed length and at an exogenous interest rate, which we model as determined by aggregate conditions rather than those within the city. Households who do not own pay rent each period equal to a fixed and exogenous fraction of income.

Mortgage holders are subject randomly to *financial distress* shocks which force them to either sell their homes through the search process or default and face foreclosure. Households are not committed to sell, and based on their specific situations decide whether and how to do so. Thus, households determine optimally their likelihood of default on mortgage debt. If a household defaults, its house is seized by the mortgage company, a foreclosure flag is placed on the its record, and it is prohibited from participating in the housing market until the flag is lifted, which also occurs randomly.

Thus, households face foreclosure in equilibrium when they fail to sell their houses in time to avoid default. In principle, a homeowner could drive the probability of default arbitrarily low by posting a sufficiently low price. In our calibration, however, financially distressed households do not do this. Rather, they choose prices associated with substantial probabilities of default, and which generally rise with the size of their outstanding mortgage debt. Moreover, some households choose to default outright, making no attempt

[3] HLS does consider a case with competitive search, but only as a robustness check.

to sell. This occurs for households with negative home equity, which can occur if changing conditions drive home prices sufficiently low.

Because house *sellers* are heterogeneous there arises in inequilibrium a distribution of house prices which evolves over time owing to aggregate shocks.[4] Home *buyers* remain identical, however, as we assume that goods are non-storable and rule out household saving. Free-entry of these homogeneous buyers into the housing market gives rise to the aforementioned trade-off between house prices and matching probabilities. Heterogeneous sellers separate themselves optimally into various submarkets based on their individual states. As a result, the individual decision problem is independent of the distribution of sellers and the model is *block recursive* as in **?** and **?**.

Regardless of the aggregate state, above a certain LTV, the prices posted in equilibrium by selling homeowners are steeply increasing in their mortgage size, a result consistent with the empirical findings of (**?**, GenesoveMayer97). More highly levered sellers thus are more likely to default than are less levered ones. Negative shocks (to city-wide income, for example) thus cause particularly severe waves of default and foreclosure if they occur when the economy has a high proportion of highly levered homeowners.

Housing market liquidity affects mortgage lending standards through both the expected default rate and lenders' expected losses upon default. The more liquid the housing market, the higher the probability which which indebted households sell and thus the lower the rate of default and foreclosure. Similarly, mortgage companies also sell foreclosed houses more quickly, lowering the expected carrying cost of a foreclosed house and thus the cost of default. Also, houses typically sell at higher prices in a hotter market, regardless of who sells, further lowering the cost of default. Thus, mortgage companies are willing to offer larger mortgages when the housing market is more liquid. All together, these results imply a negative correlation over time between house prices and LTV's at origination.

The paper contributes to the growing literature on search frictions in the housing market (see, for example, **?**, **?**,, **?**, **?**, and **?**). None of these papers, however, models mortgage contracts and examines the long-term housing-lending relationship on which we focus. Two papers that do study theoretically the relationship between housing market liquidity and lending standards are **?** and **?**, with the latter being the most closely related.

? also studies a model that features directed search, long-term mortgages, and limited commitment. Hedlund's model, however, features a different market arrangment in which buyers and sellers to not interact directly but via the interaction of "real estate agents"

[4]Houses are sold by construction firms, mortgage companies, and home-owners differentiated by both their reasons for selling and levels of mortgage debt.

who buy houses from heterogenous sellers and then sell them, along with newly constructed houses, to heterogeneous buyers. Like ours, this setup renders the model block recursive and tractable.[5] Comparison of the two models, however, sheds light on the role of the elasticity of housing supply in generating our main results. In Section 5, we study a calibrated version of our model featuring a directed search framework based on Hedlund's, which we refer to as *directed search with middlemen*. In summary, we find that our framework is better able to account for a negative correlation between house prices and lending standards owing to its different implications for variation in the elasticity of housing supply than those of this alternative environment.

In addition to the matching environment, our model also differs from that of **?** in that we study finite mortgages at fixed interest rates rather than flexible-rate infinite-horizon mortgage contracts. Our motivation here is principally realism, as in the U.S. conventional mortgages typically have a 30-year term and about 70% of these mortgages are at fixed interest rates.[6] While finite-horizon contracts add complexity, they enable us to study how optimal house-trading decisions vary across households at different stages of mortgage repayment. Another important difference between our paper and Hedlund's is that we focus on housing and mortgage markets at the city level, whereas he considers the business cycle dynamics of housing and mortgage markets at the national level.

in a New Keynesian model along the lines of **?** with credit-constrained consumers and housing market frictions, **?** shows that expansionary monetary policy leads to higher leverage among homeowners. In his model, a decrease in mortgage interest rates boosts demand for housing. With more buyers in the frictional market, lenders can liquidate foreclosed houses more quickly, effectively reducing the expected carrying cost of a foreclosed house and making lender more willing to finance larger fractions of house purchases.

Our paper differs from **?** in several respects: First, in that paper debt is one-period rather than long-term. Thus, we are able to trace out the default decision of indebted households at every stage of their mortgage repayment process. Second, in **?** there is no default in equilibrium and thus no liquidation of houses. Financial frictions take the form of Kiyotaki-Moore collateral constraint with lenders offering debt to the extent that default is prevented in equilibrium. In contrast, in our theory lending standards reflect

[5] Directed search and sorting with two-sided heterogeneity is a challenging problem to tackle. There are a handful of papers that characterize the steady state of such an economy under certain conditions (see **?**, **?**, **?**, and **?**). **?** further shows that dynamics of sorting with two-sided heterogeneity can be tractable in some settings. Nevertheless, our model would be intractable if allowing for endogenous saving decisions along with endogenous mortgage choices.

[6] This percentage, however, has declined in recent years.

default probabilities and the foreclosure inventory is a component of housing supply. As a result, market liquidity affects both the expected carrying cost of a foreclosed house *and* the expected default rate. Both factors contribute to the negative correlation between house prices and LTV's at origination. Finally, in **?** houses are divisible, the housing stock is fixed and there is no construction sector.

By focusing on the *selling* decisions of households, our paper is related to those of Ngai and Sheedy (2015) and Ngai and Tenreyo (2014). Those papers focus, respectively, on the effect of aggregate conditions and seasonal fluctuations in demand on the decisions of homeowners to put their houses on the market. In contrast, we focus specifically on the selling decisions of heterogeneous homeowners distinguished by their levels of mortgage debt, whether finanically distressed or not. As such, while both the models and specific issues studied vary between our paper and theirs, we view our work as complementary.

The remainder of this paper is organized as follows. Section 2 introduces the environment of our baseline search economy. Section 3 formalizes the equilibrium for the baseline. Section 4 presents an alternative model with frictionless housing markets rather than competitive search. Section 5 introduces a second alternative model, using the directed search with middlemen setup of **?**. Section 6 provides justifications for our calibration choices. Section 7 presents the steady-state. Section 8 discusses the dynamic implications of our model in response to aggregate shocks. Section 9 concludes.

2 The Environment

Time is infinite and discrete, with time periods indexed by t. The economy consists of a single housing market, which we refer to as *the city*, and the "rest of the world". The aggregate economy is populated by a measure Q_t of *ex ante* identical households, which grows exogenously at net rate μ. Each household lives indefinitely and supplies one unit of labor inelastically every period. In period t, the unit of labor supplied earns income y_t, in units of a single date t consumption good. Income follows a stationary stochastic process in log-levels.

Households in the city require housing, and may either rent or own *one* of a large number of symmetric housing units. Households' preferences are represented by

$$\mathcal{U} = \mathrm{E}_t \left[\sum_{t=0}^{\infty} \beta^t \left[u(c_t) + z_t \right] \right], \tag{1}$$

where c_t denotes consumption and z_t *housing* in period t, respectively. We assume that $z_t = z^H$ if the household owns the house in which they live and $z_t = 0$ otherwise. The function $u(\cdot)$ is strictly increasing, strictly concave and twice continuously differentiable, with the boundary properties: $u(0) = 0$, $\lim_{c\to\infty} u'(c) = 0$ and $\lim_{c\to 0} u'(c)$ sufficiently large. All households have the common discount factor, $\beta \in (0, 1)$. Both consumption goods and housing services are non-storable and there is no technology for households to save across periods.[7]

At the beginning of each period, measure μQ_t of new households arrive in the economy. Each of these households has a best alternative value to entering the city, denoted ε. These values are independently and identically distributed across new households according to the distribution function $G(\varepsilon)$, with support $[0, \bar{\varepsilon}]$. Households that enter the city are, immediately upon doing so, separted randomly and permanently into two groups; those that value home-ownership and those that do not. The former we refer to as *buyers* and the latter as *perpetual renters.* Each period there exists a critical alternative value, ε_t^c, below which a new household strictly prefers to enter the city:

$$\varepsilon_t^c = \psi U_t + (1 - \psi) W_t^p, \tag{2}$$

where U_t and W_t^p are of lifetime values of being a buyer and a perpetual renter, respectively, and $1 - \psi$ is the probability of the entrant becoming a perpetual renter.

New houses are built by a construction industry comprised of a large number of identical and competitive firms. Each new house requires one unit of land, which can be purchased in a competitive market at price $q_t = \mathcal{Q}(N_t)$. The builder also incurs construction cost $k_t = \mathcal{K}(N_t)$, where N_t denotes the quantity of new houses built in period t. Houses require one period to build; those constructed in period t become available for sale at the beginning of period $t + 1$.

Home-ownership both provides a utility benefit to the owner and requires costly maintenance immediately. Houses depreciate over time, regardless of whether or not they are occupied. Depreciation is, however, offset by the owner at maintenance cost m each period. Households in the city that do not own houses rent. We abstract from most aspects of the rental market, and assume that rent is equal to a fixed fraction of the city-level income, $R_t = \varsigma y_t$. The supply of rental accommodation is totally elastic is not considered part of the city's housing stock.

[7] This assumption renders buyers in the housing market homogenous both their labor incomes are identical. This simplifies the analysis significantly.

At the end of each period, all households in the city, regardless of their ownership status, experience shocks which induce them to leave the city. For perpetual renters and buyers (regardless of whether or not they currently own a house) these shocks occur with probabilities $\pi_p \in (0,1)$ and $\pi_h \in (0,1)$, respectively. All households which exit the city receive continuation utility, L. Exiting homeowners also have vacant houses that they may want to sell, depending in part on their outstanding mortgage debt, if any. These households also have the option of defaulting.

The housing market is characterized by competitive search. We imagine it as being characterized by a large variety of potential *submarkets* indexed by a price, p, and a pair of matching probabilities; one for buyers and one for sellers. Within each submarket, matching takes place via a *matching function,* : $\mathcal{M}(B,S)$, which is increasing in both arguments and has constant returns to scale. Given this form we can index submarkets by (θ, p), where θ denotes the *market tightness* (*i.e.,* the ratio of the measure of buyers, B, to that of sellers, S) and p the posted transaction price.

Both buyers and sellers take (θ, p) of all submarkets as given and decide which to enter in search of a trade. The matching probabilities for buyers $\gamma(\theta)$ and sellers $\rho(\theta)$ are given by

$$\gamma(\theta) = \frac{\mathcal{M}(B,S)}{B} = \mathcal{M}\left(1, \frac{1}{\theta}\right) \tag{3}$$

$$\rho(\theta) = \frac{\mathcal{M}(B,S)}{S} = \mathcal{M}(\theta, 1) = \theta\gamma(\theta). \tag{4}$$

Each buyer and seller can enter only a single submarket in a given period, and there is no cost of entry. Free-entry generates endogenously a trade-off between the house price and the matching probability across active submarkets. Intuitively, higher-price submarkets have lower levels of tightness as buyers (who are all identical) are willing to pay a higher price only if they are compensated with higher probability of matching with a seller.

The stock of searching buyers includes both newly entered households and those which have been searching unsuccessfully for some time. As noted above, these households are identical. Sellers, however, are of a number of different types. First, construction firms sell newly built homes. Second, homeowners who receive exit shocks as described above may decide to sell. Note that these buyers are heterogeneous to the extent that they have different outstanding mortgages. Home-owners may also sell as a result of a foreclosure shock (described below), and again they are differentiated by their outstanding mortgage. Finally, mortgage companies sell foreclosed houses (see below).

In our calibration, prices typically exceed per period income and as there is no saving, households must borrow to finance house purchases.[8] *Mortgages* are provided by a large number of perfectly competitive firms owned by risk-neutral investors who consume all profits and losses *ex post*.[9] To finance its loans, these *mortgage companies* trade one-period risk-free bonds at an exogenous interest rate, i, in an international bond market. They also incur a proportional service cost, ϕ, per period associated with the administration of household mortgages.

The debt contract is a standard fixed-rate mortgage with finite maturity T. Let m_0 and r_m represent the size of a loan at origination and the mortgage rate, respectively. Contract $(m_0; r_m, T)$, specifies a constant payment per period:

$$x(m_0) = \frac{r_m}{1 - (1 + r_m)^{-T}}\, m_0. \tag{5}$$

As the homeowner makes payments, the principle balance, d, evolves via

$$d(m_0, n+1) = (1 + r_m)\, d(m_0, r_m, n) - x\,(m_0, r_m) \tag{6}$$

where $n \in \{0, T-1\}$ and $d(m_0, 0) = m_0$. Since T and r_m are fixed exogenously, both $x(\cdot)$ and $d(\cdot,\cdot)$ are unrelated to t after origination, and (m_0, n) is sufficient to represent the state of an ongoing mortgage.

A borrower can terminate his/her mortgage contract at any time by paying off the remaining balance, $d(m_0, n)$. A termination is a default, if the borrower does not repay the outstanding mortgage balance. Default leads to *foreclosure*, whereby the mortgage company takes control of the house, remitting to the borrower any surplus value of the house in excess of the outstanding loan balance. The lender does not have direct access to the homeowner's current and/or future income,

Homeowners with outstanding mortgage debt receive, with probability π_d each period a *financial distress* shock. We interpret these shocks as representing circumstances such as accidents or unexpected illness that render the household unable to continue mortgage payments. Recipients of such shocks are referred to as *distressed owners*. They must terminate their current mortgage contract within the same period and either pay their outstanding debt or default.

[8] In the absence of saving, households would prefer to borrow to smooth consumption even if the house price were less than period income.

[9] Alternatively, these firms could be owned by households to whom they would transfer their *ex post* profits and losses lump-sum. This formulation would, however, complicate the computation without changing our results significantly.

In the event of default, a borrower's mortgage balance is set to zero and a foreclosure flag is placed on his/her credit record. The mortgage company repossesses the borrower's house, puts it in real-estate-owned (REO) inventory, and decides whether and how to sell it beginning with the following period. As noted above, the defaulting homeowner receives the difference between the value of a house in REO inventory and the outstanding mortgage balance, if positive. Upon a successful sale, the mortgage company loses a fraction $\chi \in (0, 1)$ of the revenue as costs, *e.g.,* legal fees. As a penalty for defaulting, buyers with foreclosure flags lose access to the mortgage market and are thus excluded from the housing market. Beginning with the following period, the foreclosure flag either remains on a buyer's record (with probability $\pi_f \in (0, 1)$) or is removed.[10]

In equilibrium, the mortgage rate is given by $r_m = i + \phi + \varrho$, where i and ϕ are exogenously given as described above. The component ϱ, however represents an endogenous risk premium, which compensates for the risk of default, which occurs with positive probability.

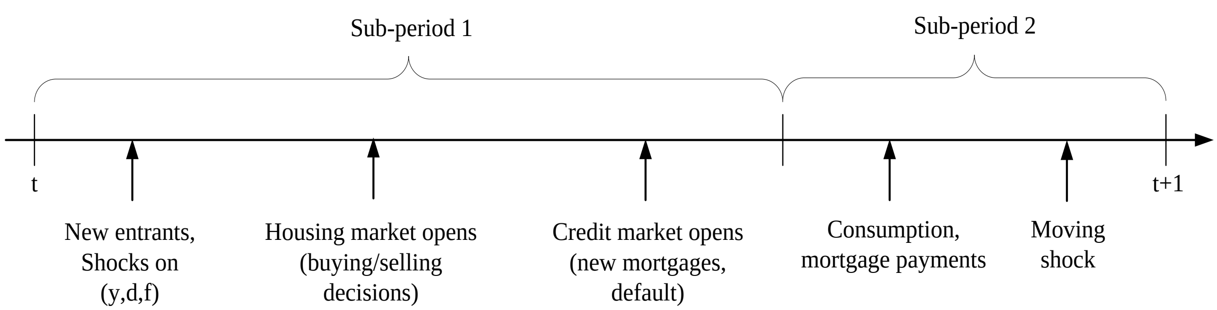

Figure 2: Time Line

Each period consists of two sub-periods. At the beginning of sub-period 1, new households with $\varepsilon \leq \varepsilon_t^c$ enter the city. Income shocks, financial distress shocks and shocks on the foreclosure flag are all revealed. Immediately thereafter, the housing market opens: Buyers and sellers decide the submarkets, (p, θ), in which to search and list their houses for sale, respectively. After the housing market closes, the mortgage sector becomes active: New owners take out mortgages to finance their purchases and current mortgage holders decide whether or not to default.

In sub-period 2, households receive income, make payments (maintenance, down payments, mortgage payments or rents), and consume the remainder. At the end of the

[10] According to the policies of Fannie Mae and Freddie Mac, foreclosure filings stay on a borrower's credit record for a finite number of years.

period, moving shocks are revealed for all households and those who receive them leave the city immediately. Figure 2 provides a more detailed illustration of the timing of decisions.

3 Equilibrium

We begin by describing in detail the behavior of agents and defining an equilibrium for the environment described above, which we will refer to as our *baseline search* economy.

3.1 Households

Consider households' value functions sequentially for the two sub-periods of a typical time period t:

3.1.1 The first sub-period

House trading and mortgage default decisions are both made in the first sub-period. Let U_t denote the value function for a *buyer.* These households are either new entrants or those not owning a house and without a foreclosure flag, *i.e.* for whom $f_t = 0$:

$$U_t = \max_{(p,\theta)} \left[\gamma(\theta)V_t^o(p, m_0) + (1 - \gamma(\theta))W_t(0)\right]. \tag{7}$$

In sub-period 1, a buyer will search for a house to buy, choosing optimally to enter sub-market (p, θ). The buyer is matched with a seller with probability $\gamma(\theta)$, in which case she proceeds to sub-period 2 as a new owner with value $V_t^o(p, m_0)$. The price paid and loan volume m_0(determined by the mortgage company and specified below) determine the homeowner's LTV. With probability $1-\gamma(\theta)$, the buyer fails to a match, remains a buyer, and proceeds to sub-period 2 with value $W_t(f_t = 0)$.

With buyers free to enter any sub-market, all those which are active must offer the same level of U_t to all buyers. Rewriting (7) yields

$$\theta = \gamma^{-1}\left(\frac{U_t - W_t(0)}{V_t^o(p, m_0) - W_t(0)}\right) \equiv \theta\,(p)\,. \tag{8}$$

Thus, the free-entry of buyers determines the relationship between the transaction price and the market tightness across sub-markets.

Let $S_t\,(m_0, n)$ denote the value for a resident owner with debt m_0 at origination, who has made n payments and is *not* in financial distress:

$$\begin{aligned} S_t(m_0, n) &= \max_{(p^s,\theta^s)} \{\rho(\theta^s) W_t(\max[0, p^s - d(m_0, n)]) \\ &\quad + (1-\rho(\theta^s)) \Big[\max_{D_t \in \{0,1\}} \{(1-D_t) V_t(m_0, n) \\ &\quad + D_t W_t^f(\max[0, \beta E_t V_{t+1}^{REO} - d(m_0, n)])\Big]\Big\} \qquad (9) \\ \textit{subject to} \qquad \theta^s &= \Omega(p^s). \qquad (10) \end{aligned}$$

This household decides whether and in which sub-market, (p^s, θ^s), to sell her house. With probability, $\rho(\theta^s)$, the house is successfully sold. In this case, this individual repays as much outstanding debt as possible, and keeps the remaining profit, $a = \max[0, p^s - d(m_0, n)]$, if any. The household then proceeds to the second sub-period as a buyer without the foreclosure flag who has value $W_t(a)$.

If the household chooses not to sell her house, or has failed to sell it, she then decides whether or not to default on her current mortgage contract. Here $D_t = 1$ if the household chooses to default, and $D_t = 0$ otherwise. The value of a homeowner who has not defaulted at the begining of the second sub-period is $V_t(m_0, n)$. A homeowner who has defaulted has value $W_t^f(\max[0, \beta E_t V_{t+1}^{REO} - d(m_0, n)])$ at the beginning of the second sub-period. Such a homeowner effectively "sells" their house to the mortgage company for the expected discounted value of a vacant house in the mortgage company's inventory at the beginning of the next period, $\beta E_t V_{t+1}^{REO}$. If this value is less than the household's outstanding mortgage debt, $d(m_0, n)$, the household's assets are set to zero. The expectation here is taken with respect to aggregate shocks which affect the value of vacant houses. If the value of the vacant house exceeds the debt, the defaulting homeowner keeps the residual value. In either case, the home-owner acquires a foreclosure flag.

Next, consider a resident owner who receives a financial distress shock at the beginning of period t.[11] Such a homeowner must terminate her mortgage contract within the same period. If the house is sold, the homeowner receives the residual value net of debt and then becomes a buyer without a foreclosure flag, $W_t(\max[0, p^{sd} - d(m_0, n)])$. If the house is not

[11] In the event of financial distress, it is always in an owner's best interest to attempt to sell if they have positive equity. In an economy with perfectly liquid housing markets, distressed owners with positive equity would never default because they can immediately sell their houses and repay their mortgage. According to the RealtyTrac report, however, less than 50% of homeowners who go into foreclosure have negative equity. Time-consuming search and matching is responsible for this observation. The appraisal value of a house is not equal to its true liquidation value in decentralized housing markets. Appraisal value is typically estimated based on the most recent sale prices of houses with similar characteristics, whereas liquidation value takes not only the price of a house but also the probability of sale into consideration.

sold, the owner defaults, the foreclosure flag is placed on her credit record. In this case, the homeowner receives the residual value of the house net of the debt and enters the next sub-period with value $W_t^f(\max[0, \beta E_t V_{t+1}^{REO} - d(m_0, n)])$.[12] Thus the value of a distressed resident owner with debt $d(m_0, n)$ is given by:

$$
\begin{aligned}
S_t^f(m_0, n) &= \max_{(p^{sd}, \theta^{sd})} \left\{ \rho(\theta^{sd}) W_t(\max[0, p^{sd} - d(m_0, n)]) \right. \\
&\quad \left. + \left(1 - \rho(\theta^{sd})\right) W_t^f(\max[0, \beta E_t V_{t+1}^{REO} - d(m_0, n)]) \right\} \qquad (11) \\
\textit{subject to} \qquad \theta^{sd} &= \Omega\left(p^{sd}\right). \qquad (12)
\end{aligned}
$$

Note that in (11) a distressed seller's choice of asking price is not constrained to exceed the outstanding mortgage debt.

A resident homeowner without a mortgage decides whether and how to sell her house. If the house is successfully sold, the owner moves on as a buyer with value $W_t(p^{sw})$. Otherwise, she moves onto the next sub-period as an owner without debt $\bar{V}_t$. Such a homeowner has value

$$
\begin{aligned}
\bar{S}_t &= \max_{(p^{sw}, \theta^{sw})} \{\rho(\theta^{sw}) W_t(p^{sw}) + (1 - \rho(\theta^{sw}))\bar{V}_t\} \qquad (13) \\
\textit{subject to} \qquad \theta^{sw} &= \Omega\left(p^{sw}\right). \qquad (14)
\end{aligned}
$$

Next, consider homeowners who have left the city. Such households become essentially irrelevant once they are no longer homeowners. A homeowner that has left the city with an outstanding mortgage debt has value:

$$
\begin{aligned}
V_t^L(m_0, n) &= \max_{(p^L, \theta^L)} \left\{ \rho(\theta^L) \left\{ u(\max\left[0, p^L - d(m_0, n)\right] + y_t^L - R_t^L) + \beta L \right\} \right. \\
&\quad + (1 - \rho(\theta^L)) \max_{D_t^L} (1 - D_t^L) \left\{ u(y_t^L - R_t^L - x(m_0) - m) \right. \\
&\quad + \beta E_t V_{t+1}^L(m_0, n+1) \qquad (15) \\
&\quad \left. \left. + D_t^L \left\{ u(\max\left[0, \beta E_t V_{t+1}^{REO} - d(m_0, n)\right] + y_t^L - R_t^L) + \beta L \right\} \right\} \right\} \\
\textit{subject to} \qquad \theta^L &= \Omega\left(p^L\right). \qquad (16)
\end{aligned}
$$

Such a household makes decisions regarding whether to default and in what sub-market, if any, to offer her house for sale. The possible scenarios are similar to those described

[12] Note that distressed resident owners can use proceeds from sales, but not labor income, to pay off outstanding mortgage debt. Relaxing this constraint would complicate the model and does not change the results significantly.

above for households resident in the city. Here, y_t^L, R_t^L, and m are income received and rent and maintenance costs paid by the exiting household while it is in the alternative location.[13] Once the homeowner has either sold her house or defaulted, she receives exogenous continuation value, L.

The value of an owner who has left the city *without* debt prior to moving is given by:

$$\begin{aligned} \bar{V}_t^L &= \max_{(p^{Lw},\theta^{Lw})} \left\{\rho(\theta^{Lw})\left\{u(p^{LW}+y_t^L-R_t^L)+\beta L\right\}\right. \\ &\quad \left. +(1-\rho(\theta^{Lw}))\left\{u(y_t^L-R_t^L-m)+\beta E_t\bar{V}_{t+1}^L\right\}\right\} \quad (17) \\ \textit{subject to} \quad \theta^{Lw} &= \Omega\left(p^{Lw}\right). \quad (18) \end{aligned}$$

Such a household's only decision is with regard to whether and how to sell her house.

Finally, the value of a vacant house in a construction firm's inventory V_t^c, and the value of a foreclosed house V_t^{REO} are given, respectively by

$$\begin{aligned} V_t^c &= \max_{(p^c,\theta^c)} \left\{\rho(\theta^c)p^c+(1-\rho(\theta^c))[-m+\beta E_t V_{t+1}^c]\right\} \quad (19) \\ \textit{subject to} \quad \theta^c &= \Omega\left(p^c\right). \quad (20) \end{aligned}$$

$$\begin{aligned} V_t^{REO} &= \max_{(p^{REO},\theta^{REO})} \left\{\rho(\theta^{REO})(1-\chi)p^{REO}+(1-\rho(\theta^{REO}))[-m+\beta E_t V_{t+1}^{REO}]\right\} \quad (21) \\ \textit{subject to} \quad \theta^{REO} &= \Omega\left(p^{REO}\right). \quad (22) \end{aligned}$$

Note that in (21), it can be seen that the mortgage company loses fraction χ of the proceeds of its sales as a cost of foreclosure.

3.1.2 Households in sub-period 2

Since there is no saving, households' behaviour in sub-period 2 is essentially trivial: They simply consume their income net of rent and mortgage payments. Here, we establish the value functions for the various household states at the beginning of this sub-period, which were used in the expressions above.

A perpetual renter remains a renter (never seeking to purchase a house) the entire time

[13] These quantities are necesssary as long as the household remains a homeowner, because they impinge on its default and pricing decisions.

she stays in the city. Such a household's value is given by:

$$\begin{aligned} W_t^p &= u(c_t) + \pi_p \beta L + (1-\pi_p)\beta E_t W_{t+1}^p && (23) \\ \textit{subject to} \qquad c_t &= y_t - R_t && (24) \end{aligned}$$

With probability π_p, the perpetual renter is hit by the moving shock, leaves the city immediately and receives the continuation value L. Otherwise, she moves onto the next period as a renter. Her consumption is simply income net of rent.

A buyer with the foreclosure flag on her credit record has access neither to credit nor the housing market. She will remain renting until she moves out of the city or the foreclosure flag is lifted from her record. Let $W_t^f(a)$ be the value of such a buyer with asset a at the beginning of sub-period 2. The balance a represents this buyer's intra-period asset holding. This balance is strictly positive if (i) the resident owner has defaulted on her mortgage in the preceding sub-period 1; and (ii) the value of a foreclosed house in the REO inventory is above the outstanding mortgage debt. Otherwise, $a = 0$. Thus,

$$\begin{aligned} W_t^f(a) &= u(c_t) + \pi_h \beta L + (1-\pi_h)\beta \left[\pi_f\ E_t W_{t+1}^f(0) + (1-\pi_f) E_t U_{t+1}\right] && (25) \\ \textit{subject to} \qquad c_t &= y_t + a - R_t. && (26) \end{aligned}$$

Conditional on staying in the city, with probability π_f the foreclosure flag remains and the household moves onto the following period with expected value $W_{t+1}^f(0)$ (such households are inactive in the first-subperiod of period $t+1$). With probability $1-\pi_f$, the foreclosure flag is lifted and this household will continue on with value U_{t+1} as a buyer searching for a house in the sub-period 1 of period $t+1$.

A buyer without the foreclosure flag at the beginning of sub-period 2 is either a resident owner who just successfully sold her house or a buyer who has failed to purchase a house in sub-period 1. Such a buyer may have a positive intra-period asset balance, a, coming from sale proceeds net of the outstanding mortgage debt in the previous sub-period. She will move on with value U_{t+1} and participate in the housing market in the next period if not hit by the moving shock at the end of the current period. The value of such a buyer is given by

$$\begin{aligned} W_t(a) &= u(c_t) + \pi_h \beta L + (1-\pi_h)\beta E_t U_{t+1} && (27) \\ \textit{subject to} \qquad c_t &= y_t + a - R_t && (28) \end{aligned}$$

A resident homeowner with a mortgage has the principle balance $d(m_0, n)$, where $n \in [0, T-1]$. The owner's periodic income is used to cover repayment, maintenance cost and consumption. Let $V_t(m_0, n)$ denote the value of such an owner. It follows that for $n \in [0, T-2]$,

$$\begin{aligned} V_t(m_0,n) &= u(c_t) + z^H + \pi_h \beta E_t V^L_{t+1}(m_0, n+1) \\ &\quad +(1-\pi_h)\left[\begin{array}{c} \pi_d \beta E_t S^f_{t+1}(m_0, n+1) \\ +(1-\pi_d)\beta E_t S_{t+1}(m_0, n+1) \end{array}\right] \end{aligned} \tag{29}$$

$$\textit{subject to} \quad c_t = y_t - x(m_0) - m \tag{30}$$

If the owner receives a moving shock, she exits the city immediately and continues with value $V^L_{t+1}(m_0, n+1)$. Note that her mortgage debt does not vanish because she has relocated. Conditional on not relocating, in the next period the owner receives a financial distress shock with probability π_d. In this case, she continues as a distressed resident owner with debt $S^f_{t+1}(m_0, n+1)$. Otherwise, she enters the next period as a non-distressed owner with value $S_{t+1}(m_0, n+1)$.

For $n = T-1$, a resident homeowner with a mortgage has value

$$V_t(m_0, T-1) = u(c_t) + z^H + \pi_h \beta \bar{V}^L_{t+1} + (1-\pi_h)\beta E_t \bar{S}_{t+1} \tag{31}$$

$$\textit{subject to} \quad c_t = y_t - x(m_0) - m \tag{32}$$

In this case, the current mortgage payment is the homeowner's last. Thus, she will continue on with value $\bar{V}^L_{t+1}$ if hit by the moving shock and with value $\bar{S}_{t+1}$ otherwise.

A new owner who has purchased a house in the preceding sub-period pays the difference between the purchase price and total debt m_0, that is, the *down-payment*. The periodic mortgage payment begins from the following period. The debt carried by this household into the next period is $d(m_0, 0)$. Let $V^0_t(p, m_0)$ denote the value of a new homeowner:

$$\begin{aligned} V^o_t(p_t,m_0) &= u(c_t) + z^H + \pi_h \beta V^L_{t+1}(m_0, 0) \\ &\quad +(1-\pi_h)\left[\begin{array}{c} \pi_d \beta E_t S^f_{t+1}(m_0, 0) \\ +(1-\pi_d)\beta E_t S_{t+1}(m_0, 0) \end{array}\right] \end{aligned} \tag{33}$$

$$\textit{subject to} \quad c_t = y_t - (p_t - m_0) - m \tag{34}$$

Finally, owners without mortgage debt do not suffer financial distress shocks. They remain in the city until experiencing a moving shock. The value of such an owner is given by

$$\bar{V}_t = u(c_t) + z^H + \pi_h \beta \bar{V}^L_{t+1} + (1-\pi_h)\beta E_t \bar{S}_{t+1} \tag{35}$$

$$\textit{subject to} \qquad c_t = y_t - m \tag{36}$$

3.2 Construction firms

The construction industry is comprised of a large number of competitive firms. Building a new house requires one unit of land at the price $q_t = \mathcal{Q}(N_t)$, where N_t is the measure of new houses built in period t and available at $t+1$. In addition, the process of actually building the house results in construction costs $k_t = \mathcal{K}(N_t)$. Free entry into the industry ensures that in equilibrium the cost of building a house equasl the expected value of a vacant house for sale in period $t+1$:

$$\mathcal{Q}(N_t) + \mathcal{K}(N_t) = \beta E_t V^c_{t+1}. \tag{37}$$

3.3 Mortgage companies

Because the mortgage firm has access to funds at a fixed cost, it will issue mortgages until it earns zero profit on each contract. In particular, the expected return net of expected foreclosure costs on mortgages will equal the opportunity cost of funds, that is, the interest rate i of the international bonds plus the servicing premium ϕ. Houses are identical, households cannot save over time, and regular repayments of all new mortgages start from the period following that in which the house is purchased and the mortgage initiated. As such, all new borrowers are identical to the mortgage company at the point of loan origination. Therefore, the mortgage company loans of the same size, $m_{0,t}$, to all new borrowers in period t, regardless of the price they pay for their house.

Let $P^\iota_t(m_0, n)$ be the present value of mortgage $\iota = (m_0; r_m)$ held by a resident home-owner at the beginning of sub-period 2 of period t after n payments have been made, for $n \in \{0, \cdots, T-1\}$. Correspondingly, let $P^{L\iota}_t(m_0, n)$, for $n \in \{1, \cdots, T-1\}$ be the present value of such a mortgage held by an owner that has relocated ($n \geq 1$ for such owners because one repayment has been made by the beginning of the first sub-period 2 after the household relocated). Then, for $n \in \{0, 1, \cdots, T-1\}$,

$$
\begin{aligned}
& P_t^{\iota}(m_0, n) \\
= \; & x(m_0)\mathbb{I}_{\{n \neq 0\}} + \frac{\mathbb{I}_{\{n \neq T-1\}}}{1+i+\phi} \times \qquad (38) \\
& E_t \left[\begin{array}{l}
\pi_h \left[\begin{array}{l}
\rho(\theta_{t+1}^{L*}) \min \left[p_{t+1}^{L*}, d(m_0, n+1) \right] \\
+(1-\rho(\theta_{t+1}^{L*})) \left\{ \begin{array}{l}
D_{t+1}^{L*}(m_0, n) \min \left[\beta V_{t+2}^{REO}, d(m_0, n+1) \right] \\
+ \; (1 - D_{t+1}^{L*}(m_0, n)) P_{t+1}^{L\iota}(m_0, n+1)]
\end{array} \right\}
\end{array} \right] \\
+(1-\pi_h) \times \\
\left[\begin{array}{l}
\pi_d \left\{ \begin{array}{l}
\rho(\theta_{t+1}^{sd*}) \min \left[p_{t+1}^{sd*}, d(m_0, n+1) \right] \\
+ \; (1-\rho(\theta_{t+1}^{sd*})) \min \left[\beta V_{t+2}^{REO}, d(m_0, n+1) \right]
\end{array} \right\} \\
+(1-\pi_d) \times \left\{ \begin{array}{l}
\rho(\theta_{t+1}^{s*}) \min \left[p_{t+1}^{s*}, d(m_0, n+1) \right] \\
+(1-\rho(\theta_{t+1}^{s*})) \\
\times \left\{ \begin{array}{l}
D_{t+1}^{*}(m_0, n) \min \left[\beta V_{t+2}^{REO}, d(m_0, n+1) \right] \\
+ \; (1 - D_{t+1}^{*}(m_0, n)) P_{t+1}^{\iota}(m_0, n+1)
\end{array} \right\}
\end{array} \right\}
\end{array} \right]
\end{array} \right]
\end{aligned}
$$

and for all $n \in \{1, \cdots, T-1\}$,

$$
\begin{aligned}
P_t^{L\iota}(m_0, n) \; = \; & x(m_0) + \frac{\mathbb{I}_{\{n \neq T-1\}}}{1+i+\phi} \\
& \times E_t \left[\begin{array}{l}
\rho(\theta_{t+1}^{L*}) \min \left[p_{t+1}^{L*}, d(m_0, n+1) \right] \\
+(1-\rho(\theta_{t+1}^{L*})) \left\{ \begin{array}{l}
D_{t+1}^{L*} \min \left[\beta E_t V_{t+2}^{REO}, d(m_0, n+1) \right] \\
+(1 - D_{t+1}^{L*}) P_{t+1}^{L\iota}(m_0, n+1)
\end{array} \right\}
\end{array} \right] \qquad (39)
\end{aligned}
$$

where p_{t+1}^{sd*}, θ_{t+1}^{sd*}, p_{t+1}^{s*}, θ_{t+1}^{s*}, p_{t+1}^{L*}, θ_{t+1}^{L*}, D_{t+1}^{*}, D_{t+1}^{L*} are household policies in period $t+1$ contingent on having mortgage balance $(m_0, n+1)$. Moreover, $I_{\{n \neq 0\}}$ and $I_{\{n \neq T-1\}}$ are index functions such that

$$
\mathbb{I}_{\{n \neq 0\}} = \left\{ \begin{array}{ll} 0, & \text{if } n = 0 \\ 1, & \text{otherwise} \end{array} \right. \qquad (40)
$$

$$
\mathbb{I}_{\{n \neq T-1\}} = \left\{ \begin{array}{ll} 0, & \text{if } n = T-1 \\ 1, & \text{otherwise} \end{array} \right. . \qquad (41)
$$

The first index function $I_{\{n \neq 0\}}$ reflects that a borrower starts making regular repayments from the period after origination. The second, $I_{\{n \neq T-1\}}$, indicates that the mortgage matures after the current repayment is made. The present value $P_t^{\iota}(m_0, n)$ is equal to the period-t repayment $x(m_0)$ plus the discounted expected value of the mortgage in period

$t+1$. The latter is affected by the probability of the borrower receiving either a moving financial distress shock, and the decisions the household will make regarding pricing and/or default in event that such shocks are realized. Note that these decisions do not depend on the price which the homeowner originally paid for the house, only their outstanding loan and expectations of the future relative returns to sale and default.

To compute the present value of a mortgage contract ι at origination, we proceed recursively. First, we compute the value for $n = T-1$, and then use backward induction to obtain $P_t^{\iota}(\cdot,\cdot)$ for $n \in [0, T-2]$. The value $P_t^{L\iota}(m_0, n)$ is determined in a similar way except that a relocated borrower does not experience any moving or distress shock.

If a borrower sells her house in period $t+1$, the amount that the mortgage company will receive is the minimum of the sale proceeds and the outstanding debt $d(m_0, n+1)$. The mortgage company's zero-profit condition is given by

$$P_t^{\iota}(m_0, 0) - m_0 = 0. \tag{42}$$

Thus, (42) implicitly determines the equilibrium value of $m_{0,t}$ in period t.

3.4 Laws of motion

We now describe the evolution of the distribution of households across states. All measures are normalized by the total population, Q_t. At the beginning of period t, we have the *per capita* measures of perpetual renters F_t, buyers without a foreclosure flag, B_t, buyers with a foreclosure flag, B_t^f, indebted owners, Φ_t, indebted and relocated owners Φ_t^L, the construction firm's inventory Φ_t^c, and the stock foreclosed houses Φ_t^{REO}.

The stock of perpetual renters in period t consists of those remaining from the previous period and those who have newly entered:

$$(1+\mu)F_t = (1-\pi_f)F_{t-1} + (1-\psi)G(\varepsilon_t^c)\mu. \tag{43}$$

The stock of buyers with the foreclosure flag at the beginning of period t includes those remaining from previous period. These have neither moved nor had their flag removed randomly. To this are added resident homeowners who defaulted in period $t-1$. These homeowners may either have received a financial distress shock and failed to sell or have defaulted strategically. Thus, we have

$$(1+\mu)B_t^f = (1-\pi_h)\left\{\begin{array}{l} \pi_f B_{t-1}^f + (1-\pi_d) \\ \times\sum_{n=0}^{T-1}\left\{\begin{array}{l} (1-\rho(\theta_{t-1}^s(m_{0,t-1-n},n)))D_{t-1}(m_{0,t-1-n},n) \\ \times\Phi_{t-1}(m_{0,t-1-n},n) \end{array}\right\} \\ +\pi_d\sum_{n=0}^{T-1}(1-\rho(\theta_{t-1}^{sd}(m_{0,t-1-n},n)))\Phi_{t-1}(m_{0,t-1-n},n)\} \end{array}\right\} \quad (44)$$

where $\Phi_{t-1}(m_{0,t-1-n},n)$ is the measure of mortgage holders who had made n payments by the beginning of $t-1$ on a mortgage of size $m_{0,t-1-n}$ at origination.

The measure of buyers *without* foreclosure flags at the beginning of period t consists of newly-entering buyers, previously flagged buyers whose flag has been removed, and non-relocating buyers from the previous period who failed to buy a house. Note that the measure of buyers who successfully matched in the previous period equals the sum of the measures of the prospective sellers of various types multiplied by their corresponding matching probabilities. Thus, we have

$$\begin{aligned}(1+\mu)B_t &= \psi G(\varepsilon_t^c)\mu + (1-\pi_f)B_t^R + (1-\pi_h) \\ &\quad\times\left\{\begin{array}{l} B_{t-1} - \rho(\theta_{t-1}^{Lw})\Phi_{t-1}^L(\emptyset,\emptyset) \\ -\rho(\theta_{t-1}^c)\Phi^c - \rho(\theta_{t-1}^{REO})\Phi^{REO} \\ -\sum_{n=0}^{T-1}\rho\left(\theta_{t-1}^L(m_{0,t-1-n},n)\right)\Phi_{t-1}^L(m_{0,t-1-n},n) \end{array}\right\}\end{aligned} \quad (45)$$

Let $\Phi_t(m_0,n)$ be the measure of indebted owners who have made n periodic payments by the beginning of period t on a mortgage of size $m_{0,t-n}$ at origination. For $n>0$, this measure evolves via:

$$\begin{aligned}(1+\mu)\Phi_t(m_0,n) &= (1-\pi_h)(1-\pi_d)(1-\rho(\theta_{t-1}^s(m_{0,t-n},n-1))) \\ &\quad\times(1-D_{t-1}(m_{0,t-n},n-1))\Phi_{t-1}(m_{0,t-n},n-1).\end{aligned} \quad (46)$$

That is, the current period owners with an ongoing mortgage are the indebted owners from the previous period who neither moved nor experienced financial distress, *and* neither successfully sold their house (whether by choice or bad luck) nor defaulted strategically.

For $n=0$, $\Phi_t(m_0,0)$ is the measure of resident homeowners who purchased a house in the last period, have stayed in the city and have not experienced financial distress. This measure can be recovered from the number of sales in the previous period. Thus we have

$$(1+\mu)\Phi_t(m_0,0) = (1-\pi_h)\left\{\begin{array}{l} (1-\pi_d) \\ \times\sum_{n=0}^{T-1}\rho(\theta_{t-1}^{s}(m_{0,t-1-n},n))\Phi_{t-1}(m_{0,t-1-n},n) \\ +\pi_d\sum_{n=0}^{T-1}\rho(\theta_{t-1}^{sd}(m_{0,t-1-n},n))\Phi_{t-1}(m_{0,t-1-n},n) \\ +\sum_{n=0}^{T-1}\rho(\theta_{t-1}^{L}(m_{0,t-1-n},n))\Phi_{t-1}^{L}(m_{0,t-1-n},n) \\ +\rho(\theta_{t-1}^{sw})\Phi_{t-1}^{sw}+\rho(\theta_{t-1}^{Lw})\Phi_{t-1}^{Lw} \\ +\rho(\theta_{t-1}^{c})\Phi^{c}+\rho(\theta_{t-1}^{REO})\Phi^{REO} \end{array}\right\}. \tag{47}$$

Finally, the measure of resident owners without a mortgage $\Phi_t(\emptyset,\emptyset)$ is given by

$$(1+\mu)\Phi_t(\emptyset,\emptyset) = (1-\pi_h)\left\{\begin{array}{l} (1-\pi_d)(1-\rho(\theta_{t-1}^{s}(m_{0,t-T},T-1))) \\ \times(1-D_{t-1}(m_{0,t-T},T-1))\Phi_{t-1}(m_{0,t-T},T-1) \\ +(1-\rho(\theta_{t-1}^{sw}))\Phi_{t-1}(\emptyset,\emptyset) \end{array}\right\}. \tag{48}$$

This group is comprised of its previous members who have not moved plus resident homeowners who made their last mortgage payment in period $t-1$.

Proceeding similarly for relocated homeowners, $\Phi_t^L(m_0,n)$ is the measure who have made n periodic payments at the beginning of period t. Again, the loan volume at origination is $m_{0,t-n}$ and $\Phi_t^L(\emptyset,\emptyset)$ the measure of relocated owners without debt:

$$\begin{aligned}(1+\mu)\Phi_t^L(m_0,n) &= (1-\rho(\theta_{t-1}^{sL}(m_{0,t-n},n-1)))(1-D_{t-1}^{L}(m_{0,t-n},n-1)) \\ &\quad\times\Phi_{t-1}^{L}(m_{0,t-n},n-1) \\ &\quad+\pi_h(1-\pi_d)(1-\rho(\theta_{t-1}^{s}(m_{0,t-n},n))) \\ &\quad\times(1-D_{t-1}(m_{0,t-n},n))\Phi_{t-1}(m_{0,t-n},n)\end{aligned} \tag{49}$$

$$(1+\mu)\Phi_t^L(m_0,0) = \pi_h\left\{\begin{array}{l} (1-\pi_d) \\ \times\sum_{n=0}^{T-1}\rho(\theta_{t-1}^{s}(m_{0,t-1-n},n))\Phi_{t-1}(m_{0,t-1-n},n) \\ +\pi_d\sum_{n=0}^{T-1}\rho(\theta_{t-1}^{sd}(m_{0,t-1-n},n))\Phi_{t-1}(m_{0,t-1-n},n) \\ +\sum_{n=0}^{T-1}\rho(\theta_{t-1}^{L}(m_{0,t-1-n},n))\Phi_{t-1}^{L}(m_{0,t-1-n},n) \\ +\rho(\theta_{t-1}^{sw})\Phi_{t-1}^{sw}+\rho(\theta_{t-1}^{Lw})\Phi_{t-1}^{Lw} \\ +\rho(\theta_{t-1}^{c})\Phi^{c}+\rho(\theta_{t-1}^{REO})\Phi^{REO} \end{array}\right\} \tag{50}$$

$$
\begin{aligned}
(1+\mu)\Phi_t^L(\emptyset,\emptyset) &= \pi_h \left\{ \begin{array}{l} (1-\rho(\theta_{t-1}^{sw}))\Phi_{t-1}(\emptyset,\emptyset) \\ +(1-\pi_d)(1-\rho(\theta p_{t-1}^{s}\left(m_{0,t-T},T-1\right))) \\ \times(1-D_{t-1}\left(m_{0,t-T},T-1\right))\Phi_{t-1}\left(m_{0,t-T},T-1\right) \end{array} \right\} \\
&\quad +(1-\rho(\theta_{t-1}^{sL}\left(m_{0,t-T},T-1\right)))(1-D_{t-1}^{L}(\cdot))\Phi_{t-1}^{L}\left(m_{0,t-T},T-1\right) \\
&\quad +(1-\rho(\theta_{t-1}^{sLw}))\Phi_{t-1}^{L}(\emptyset,\emptyset). \qquad (51)
\end{aligned}
$$

As depreciation is offset by maintenance, the total city housing stock evolves via

$$
H_{t+1} = H_t + N_t, \qquad (52)
$$

where N_t is newly built houses available for sale in period t.

Let Φ_t^c be the stock of houses in construction firms' inventory at the beginning of period t. This includes unsold houses from the previous period and newly built ones:

$$
(1+\mu)\Phi_t^c = \left(1-\rho(\theta_{t-1}^c)\right)\Phi_{t-1}^c + N_t. \qquad (53)
$$

The stock of houses in the REO inventory at the beginning of period t, Φ_t^{REO}, includes unsold houses from the previous period and new foreclosures:

$$
\begin{aligned}
& (1+\mu)\Phi_t^{REO} \\
= \; & \left(1-\rho(\theta_{t-1}^{REO})\right)\Phi_{t-1}^{REO} \\
& +\pi_d \sum_{n=0}^{T-1}(1-\rho(\theta_{t-1}^{sd}\left(m_{0,t-1-n},n\right)))\Phi_{t-1}\left(m_{0,t-1-n},n\right) \\
& +(1-\pi_d)\sum_{n=0}^{T-1}(1-\rho(\theta_{t-1}^{s}\left(m_{0,t-1-n},n\right)))D_{t-1}\left(m_{0,t-1-n},n\right)\Phi_{t-1}\left(m_{0,t-1-n},n\right) \\
& +\sum_{n=1}^{T-1}(1-\rho(\theta_{t-1}^{sL}\left(m_{0,t-1-n},n\right))D_{t-1}^{L}\left(m_{0,t-1-n},n\right)\Phi_{t-1}^{L}(m_{0,t-1-n},n). \qquad (54)
\end{aligned}
$$

Finally, for accounting purposes, the total measure of buyers searching to trade in the housing market, B_t^{sum}, can be derived as:

$$
\begin{aligned}
B_t^{sum} \;=\; & (1-\pi_d)\sum_{n=0}^{T-1}\theta_t^s\left(m_{0,t-n},n\right)\Phi_t\left(m_{0,t-n},n\right) \\
& +\pi_d\sum_{n=0}^{T-1}\theta_t^{sd}\left(m_{0,t-n},n\right)\Phi_t(m_{0,t-n},n) \\
& +\sum_{n=0}^{T-1}\theta_t^{sL}\left(m_{0,t-n},n\right)\Phi_t^L(m_{0,t-n},n) \\
& +\theta_t^{sw}\Phi_t(\emptyset,\emptyset)+\theta_t^{sLw}\Phi_t^L(\emptyset,\emptyset)+\theta_t^c\Phi_t^c+\theta_t^{REO}\Phi_t^{REO}. \qquad (55)
\end{aligned}
$$

In particular, the measure of buyers in an active submarket equals the measure of sellers in that submarket multiplied by the the corresponding market tightness. For example, the measure of buyers searching for foreclosed house sold by the mortgage company equals the measure of REO houses Φ_t^{REO} multiplied by the tightness of the mortgage company's optimally chosen submarket, θ_t^{REO}

3.5 Definition of equilibrium

Definition. An *equilibrium* is a collection of value functions,

$$
\begin{aligned}
&\{U_t,\ S_t\left(\cdot,\cdot\right),S_t^f\left(\cdot,\cdot\right),\ \bar{S}_t,\ V_t^L\left(\cdot,\cdot\right),\ \bar{V}_t,\ V_t^c, \\
&\ V_t^{REO},W_t^p,\ W_t(\cdot),\ W_t^f(\cdot),\ V_t\left(\cdot,\cdot\right),\ V_t^o\left(\cdot,\cdot\right),\bar{V}_t^L\} \qquad (56)
\end{aligned}
$$

associated policy functions,

$$
\begin{aligned}
&\{p_t^s\left(\cdot,\cdot\right),\ \theta_t^s\left(\cdot,\cdot\right),\ D_t\left(\cdot,\cdot\right),\ p^{sd}\left(\cdot,\cdot\right),\theta^{sd}\left(\cdot,\cdot\right),\ p_t^{sw*},\ \theta_t^{sw*}, \\
&\ p_t^L\left(\cdot,\cdot\right),\ \theta_t^L\left(\cdot,\cdot\right),\ D_t^L\left(\cdot,\cdot\right),\ p_t^{Lw*},\ \theta_t^{Lw*},\ p^{c*},\theta^{c*},p^{REO*},\theta^{REO*}\}, \qquad (57)
\end{aligned}
$$

an entry cut-off value, mortgage contract, rent,

$$\left\{\varepsilon_t^c,m_{0,t},R_t\right\}, \qquad (58)$$

and *per capita* measures of households and houses

$$\{F_t,B_t,B_t^R,\Phi_t\left(\cdot,\cdot\right),\Phi_t^L\left(\cdot,\cdot\right),\Phi_t^c,\Phi_t^{REO},N_t,H_t,B_t^{sum}\}. \qquad (59)$$

Given the mortgage interest rate r_m and the stochastic process for the general income y_t, the above functions and values satisfy:

1. New households enter the city optimally so that (2) holds;
2. All agents optimize such that the value and policy functions listed in (56) - (57) satisfy (7), (9) - (35);
3. Free entry of construction firms: N_t satisfies (37);
4. Free entry of mortgage companies: $m_{0,t}$ satisfies (42);
5. The stocks of households and inventories $\left\{F_t, B_t, B_t^R, \Phi_t(\cdot,\cdot), \Phi_t^L(\cdot,\cdot), \Phi_t^c, \Phi_t^{REO}, H_t\right\}$ evolve according to (43) - (52);
6. Consistency: $B_t = B_t^{sum}$.

Requirements 1-5 in the above definition are standard and have been described in detail above. Requirement 6 states that in equilibrium the measure of buyers without foreclosure flags must be consistent with the total measure of buyers actively participating in housing search.

As has been mentioned, sellers are heterogeneous, and their distribution is characterized by $\left\{\Phi_t(\cdot,\cdot), \Phi_t^L(\cdot,\cdot), \Phi_t^c, \Phi_t^{REO}\right\}$. None, however, of the decision problems faced by households, construction firms and mortgage companies are affected by this distribution. In fact, as can be seen from (7), and (9) - (42), all of the value and policy functions listed in (56) - (57), together with the mortgage contract $m_{0,t}$, are independent of the stock variables listed in (59). This is true despite the fact that the stocks themselves depend on individual decisions, and that the distribution of sellers does affect aggregate statistics.

Thus, the model is block recursive in the sense of **?**. As discussed there, block recursivity arises in our economy because through the competitive search mechanism heterogeneous sellers select themselves optimally into separate submarkets, taking the trade-off between the price and the matching probability as given. Given a particular target transaction price, the only factor that matters for a seller's trading decision is the probability with which it will be matched with a buyer; the distribution of sellers over other price targets is irrelevant. Vice versa, for a given matching probability that the seller cares only about the price at which it will sell.

Block recursivity greatly aids tractability by eliminating the role of the distribution of sellers in individual decisions. It is especially useful here as it enables us to examine the dynamics of the model in response to aggregate shocks.

4 An Economy without Search

To accentuate the role of search frictions in our model, we also consider an economy in which the housing market is perfectly competitive. In this economy, houses are perfectly liquid in that buyers (without a foreclosure flag) and sellers are able to trade immediately. In this case, neither construction firms nor mortgage companies hold houses inventory.

In this economy, financial distress is extreme—at the beginning of period t, with probability π_d^f an indebted resident owner may experience a default shock which forces her to default immediately. A borrowers not hit by such a shock may choose to default only in the case in which their housing equity becomes negative.[14]

4.1 Value functions.

Household decisions in sub-period 2 are identical to those in the search economy. The subperiod-1 household values in this non-search economy are distinguished by the superscript n. A buyer without the foreclosure flag purchases a house at competitive price p_t and has the value U_t^n:

$$U_t^n = V_t^o(p_t, m_0). \tag{60}$$

An indebted resident owner who does not receive a default shock decides whether and how to sell and whether or not to default. Let $H_t^s = 1$ and $D_t^n = 1$ be indicators of the selling and default decisions, respectively. If the homeowner sells, she repays as much of her outstanding debt as possible, keeps any remaining profit, and becomes a buyer without the foreclosure flag. If she decides not to sell, then $H_t^s = 0$ and she decides whether to default:

$$S_t^n(m_0, n) = \max_{H_t^s, D_t^n \in \{0,1\}} \left\{ \begin{array}{l} (1 - H_t^s) W_t(\max[0, p_t - d(m_0, n)]) \\ +H_t^s \left\{ \begin{array}{l} (1 - D_t^n) V_t(m_0, n) \\ +D_t^n W_t^f(\max\left[0, \beta E_t V_{t+1}^{nREO} - d(m_0, n)\right]) \end{array} \right\} \end{array} \right\}. \tag{61}$$

An indebted owner who experiences a distress shock immediately defaults. Such an owner has the value $S_t^{nf}(m_0, n)$:

$$S_t^{nf}(m_0, n) = W_t^f(\max[0, \beta E_t V_{t+1}^{nREO} - d(m_0, n)]). \tag{62}$$

[14] Note, however, that as default is costly, not all owners with negative equity will default.

A resident owner without debt decides whether or not to sell and has value:

$$\bar{S}_t^n = \max_{H_t^{sw} \in \{0,1\}} \left\{ (1 - H_t^{sw}) W_t(p_t) + H_t^{sw} \bar{V}_t \right\}. \tag{63}$$

Relocated owners with and without mortgage debt make similar selling and default decisions and values $V_t^{nL}(m_0, n)$ and $\bar{V}_t^{Ln}$, respectively:

$$\begin{aligned} &V_t^{nL}(m_0, n) \\ &= \max_{H_t^{sL}, D_t^{nL} \in \{0,1\}} \left\{ \begin{array}{l} (1 - H_t^{sL})(u(\max[0, p_t - d(m_0, n)] + y_t^L - R_t^L) + \beta L) \\ + H_t^{sL}((1 - D_t^{nL})(u(y_t^L - R_t^L - x(m_0) - m) + \beta E_t V_{t+1}^{nL}(m_0, n+1)) \\ + D_t^{nL}(u(\max\left[0, \beta E_t V_{t+1}^{nREO} - d(m_0, n)\right] + y_t^L - R_t^L) + \beta L) \end{array} \right\} \end{aligned} \tag{64}$$

$$\bar{V}_t^{nL} = \max_{H^{sLw} \in \{0,1\}} \left\{ \begin{array}{l} (1 - H_t^{sLw})(u(p_t + y_t^L - R_t^L) + \beta L) \\ + H_t^{sLw}(u(y_t^L - R_t^L - m) + \beta E_t \bar{V}_{t+1}^{nL}) \end{array} \right\}. \tag{65}$$

The values of vacant houses in a construction firms' and mortgage companies' inventories are given, respectively by:

$$V_t^{nc} = p_t \tag{66}$$

$$V_t^{nREO} = (1 - \chi) p_t. \tag{67}$$

For mortgage contract $\iota = (m_0; r_m)$, the present mortgage values at the beginning of sub-period 2 after n payments are given, for relocated and resident homeowners, respectively, are given by:

$$\begin{aligned} P_t^{L\iota}(m_0, n) = x(m_0) + &\frac{\mathbb{I}_{\{n \neq T-1\}}}{1 + i + \phi} \\ &\times E_t \left\{ \begin{array}{l} (1 - H_{t+1}^{sL*}) \min[p_{t+1}, d(m_0, n+1)] \\ + H_{t+1}^{sL*} \left\{ \begin{array}{l} [D_{t+1}^{nL*} \min\left[\beta V_{t+2}^{nREO}, d(m_0, n+1)\right] \\ + (1 - D_{t+1}^{nL*}) P_{t+1}^{L\iota}(m_0, n+1) \end{array} \right\} \end{array} \right\} \end{aligned} \tag{68}$$

for $n \in \{1, T-1\}$ and

$$
\begin{aligned}
& P_t^{\iota}(m_0, n) \\
&= x(m_0)\mathbb{I}_{\{n \neq 0\}} + \frac{\mathbb{I}_{\{n \neq T-1\}}}{1+i+\phi} \\
&\times E_t \left\{ \begin{array}{l}
\pi_h \left\{ \begin{array}{l} (1 - H_{t+1}^{sL*}) \min\left[p_{t+1}, d(m_0, n+1)\right] \\
+H_{t+1}^{sL*} \left\{ \begin{array}{l} D_{t+1}^{nL*}(m_0, n) \min\left[\beta V_{t+2}^{nREO}, d(m_0, n+1)\right] \\ +(1 - D_{t+1}^{nL*}(m_0, n)) P_{t+1}^{L\iota}(m_0, n+1) \end{array} \right\} \end{array} \right\} \\
+(1-\pi_h) \left\{ \begin{array}{l} \pi_d^n \min\left[\beta V_{t+2}^{nREO}, d(m_0, n+1)\right] \\
+(1-\pi_d^n) \\
\times \left\{ \begin{array}{l} (1 - H_{t+1}^{s*}) \min\left[p_{t+1}, d(m_0, n+1)\right] \\
+H_{t+1}^{s*} \left\{ \begin{array}{l} D_{t+1}^{n*}(m_0, n) \\ \times \min\left[\beta V_{t+2}^{nREO}, d(m_0, n+1)\right] \\ +(1 - D_{t+1}^{n*}(m_0, n)) \\ \times P_{t+1}^{\iota}(m_0, n+1) \end{array} \right\} \end{array} \right\} \end{array} \right\}
\end{array} \right\}
\end{aligned} \tag{69}
$$

for all $n \in \{0, \cdots, T-1\}$, where, H_{t+1}^{sL*}, H_{t+1}^{s*}, D_{t+1}^{n*}, D_{t+1}^{nL*} are policies that households follow in period $t+1$ conditional on the aggregate shocks and having mortgage balance $(m_0; n+1)$.

4.2 Equilibrium.

The definition of equilibrium is similar to that for the search economy except that the following housing market clearing condition replaces the consistency requirement:

$$
B_t = S_t^{sum},
$$

where B_t is the demand of houses and S_t^{sum} is the total supply of houses:

$$
\begin{aligned}
S_t^{sum} &= \sum_{n=0}^{T} (1 - H_t^s(m_{0,t-n}, n))\Phi_t(m_{0,t-n}, n) \\
&\quad + \sum_{n=0}^{T} (1 - H_t^{sL}(m_{0,t-n}, n))\Phi_t^L(m_{0,t-n}, n) + \Phi_t^c + \Phi_t^{REO}.
\end{aligned}
$$

Finally, the corresponding *per capita* laws of motion are listed in Appendix A.

5 Directed Search with Middlemen

In this section, we embed the mechanism of *directed search with middlement* introduced by ?.[15] In Section 8 we contrast the dynamics that arise under this mechanism from those displayed by our baseline economy. In particular, this exercise is helpful for illustrating the importance of the elasticity of housing supply for our results. Here, we briefly describe the modifications to our environment required to embed this search process in our economy.

5.1 The environment.

The principal difference between this alternative economy from our baseline pertains to the search process. Let there be a continuum of competitive *real estate firms*. The housing market is divided into two two separate parts, which we refer to respectively as *buying* and *selling* markets. Real estate firms send agents (RE's henceforth) to both markets to trade bi-laterally with homeowners and prospective buyers. In the *buying* market, RE's sell homes to prospective homeowners; In the *selling* market, RE's buy houses from homeowners who desire to sell.

A real estate firm incurs a cost κ^b (κ^s) for sending an agent to the buying (selling) market. Each RE can sell/buy at most one house at a time. The two markets open sequentially, with the selling market opening first. Each of these markets are characterized by competitive search: There are a continuum of potential sub-markets distinguished by price and tightness. Both real estate firms and households take as given the combination of price and tightness across all submarkets.

In addition to those which they acquire in the selling market, real estate firms also purchase new houses from construction firms at a competitive price P_t^c. We refer to this as the *shadow price of housing,* as does ?. Thus, there exists a competitive market for new housing to which only real estate construction firms have access. Real estate firms do not hold inventories of houses. That is, by controlling the measures of RE's that they send to each market, and by purchasing firms for re-sale immediately from contruction firms, real estate firms can maintain a *zero net flow* of housing. Effectively, a law of large numbers assumption renders real estate firms purchase and sale of houses deterministic, although individual RE's match randomly within sub-markets.

In all other aspects, the environment for this alternative economy remains the same as that for our baseline economy. In particular, buyers remain identical and sellers hetero-

[15] Hedlund uses this environment to study tractably an environment with two-sided heterogeneity.

geneous?

5.2 Value functions

All individual value functions and mortgage company decisions remain the same as in our baseline because nothing has changed from the households' and the mortgage company's perspectives. Individual choices of submarkets are, however, now denoted $\left(p^b, \theta^b\right)$ and (p^s, θ^s), to distinguish participation in the buying and selling markets, respectively.

For $i = b, s$, let $\Omega^i\left(p^i, \theta^i\right)$ denote the measure of REs that a real estate firm sends to the buying and selling markets. Let N_t^d denote the measure of houses that a real estate firm acquires from construction firms in the competitive housing market. A real estate firm solves the following static problem:

$$\max_{\left\{\Omega_t^b(p_t^b,\theta_t^b),\Omega_t^s(p_t^s,\theta_t^s),N_t^d\right\}} \int \left[\gamma(p_t^b,\theta_t^b)p_t^b - \kappa^b\right]\Omega_t^b(p_t^b,\theta_t^b) \tag{70}$$

$$-\int \left[\rho(p_t^s,\theta_t^s)p_t^s - \kappa^s\right]\Omega_t^s(p_t^s,\theta_t^s) - P_t^c N_t^d$$

$$\textit{subject to} \qquad N_t^d + \int \rho(p_t^s,\theta_t^s)\Omega_t^s(p_t^s,\theta_t^s) \geq \int \gamma(p_t^b,\theta_t^b)\Omega_t^b(p_t^b,\theta_t^b) \tag{71}$$

Profit maximization implies that

$$\gamma(p_t^b,\theta_t^b)\left(p_t^b - P_t^c\right) \;\leq\; \kappa^b \;\text{ and }\; \Omega_t^b(p_t^b,\theta_t^b) \geq 0 \;\text{ with comp. slackness} \tag{72}$$

$$\rho(p_t^s,\theta_t^s)\left(P_t^c - p_t^s\right) \;\leq\; \kappa^s \;\text{ and }\; \Omega_t^s(p_t^s,\theta_t^s) \geq 0 \;\text{ with comp. slackness.} \tag{73}$$

As we can see, P_t^c effectively serves as the shadow price of houses. That is, it is the multiplier on (**??**) in a real estate firm's optimization problem.

As in the baseline, construction firms sell newly constructed houses such that supply of new houses N_t satisfies

$$k_t(N_t) + q_t(N_t) = P_t. \tag{74}$$

This equation effectively replaces (37).

5.3 Equilibrium

The definition of equilibrium corresponds to that described in Section 3.5, with the following amendments:

– The competitive market for new housing clears: $N_t^d = N_t$;

– The condition $B_t = B_t^{sum}$ is replaced by (**??**) holding with equality.

Note that because all buyers are homogenous, there will only be one active buying submarket (p^b, θ^b) in equilibrium.

6 Calibration

We now choose parameters for both the baseline search and non-search economies to match selected facts of the U.S. economy in a steady-state equilibrium with balanced growth. In this steady-state, the housing stock grows at the rate of population growth and all other components of the equilibrium remain constant.

For the baseline search economy, we choose the follwoing functional forms:

$$\begin{aligned} u(c_t) &= \log(c_t) \\ \mathcal{M}(B,S) &= \varpi B^{\eta} S^{1-\eta} \\ k_t &= \frac{1}{\kappa} N_t^{\frac{1}{\zeta}} \\ q_t &= \bar{q} N_t^{\frac{1}{\xi}} \end{aligned} \tag{75}$$

where η is the elasticity of the measure of matches with respect to the measure of buyers and ξ represents the elasticity of new land supply with respect to land prices.

Table 1 lists parameter values for the baseline search economy. Parameters above the separating line are set to match the corresponding targets directly. Parameters below the line are determined jointly to match the set of parameters mentioned in the table. A time period is defined as one year.[16] The discount factor β is set to reflect an annual interest rate of 4%. Income in the steady state is normalized to one. Thus, all present values and prices are measured relative to steady-state *per capita* income. The continuation value upon leaving the city, L, is equal to the steady-state value of being a perpetual renter, V_{ss}^{p}. To determine the mortgage rate r_m, the annual yield on international bonds i is set at 4%. The values of ϕ and ϱ are determined jointly in calibration.

[16]Setting a time period as one year is due to specifics of our model. In this setting, households cannot save (so that all buyers are homogeneous) and thus house buyers can only finance the down payment with their periodic labor income. Empirically, the average housing price is about 12.8 times of quarterly income. If one period takes a quarter, then a buyer's periodic income is too small to afford a typical down payment of 15% - 20%. In fact, borrowers could only afford a down payment less than 7.9% of the average housing price if the time period was set as a quarter.

Table 1: Calibration Parameter Values

Parameter	Value	Target	Data
		Parameters determined independently	
β	0.96	Annual interest rate	4.0%
π_p	0.120	Annual mobility of renters	12%
π_h	0.032	Annual mobility of owners	3.2%
ξ	1.75	Median price-elasticity of land supply	1.75
i	0.040	International bond annual yield	4.0%
T	30	Fixed-rate mortgage maturity (years)	30
μ	0.012	Annual population growth rate	1.2%
π_f	0.80	Average duration (years) of foreclosure flag	5
$\bar{q}$	0.96	Average land-price-to-income ratio	30%
m	0.08	Residential housing gross depreciation rate	2.5%
ζ	5	Median price elasticity of new construction	5
ς	0.05	Average rent-to-price ratio	5%
		Parameters determined jointly	
χ	0.440	Loss severity rate	46%
ϕ	0.0246	Average down-payment ratio	20%
ϱ	0.0074	Average annual FRM-yield	7.20%
ψ	0.570	Fraction of households that rent	33.3%
π_d	0.060	Annual foreclosure rate	1.6%
z^H	0.3280	Average loan-to-income ratio at origination	2.72
ϖ	0.56	Average fraction of delinquent loans repossessed	33.5%
κ	0.137	Average housing price relative to annual income	3.2
η	0.1880	Relative volatility of sales growth	1.32
α_p	6.200	Relative volatility of population growth	0.17

The following parameters and targets are chosen following **?**: The rate μ is chosen to match the annual population growth during the 1990s. The value of π_p is set to match the annual fraction of renters that move between counties and π_h to match the annual fraction of home owners who move between counties according to the Census Bureau.

The supply elasticity parameter is set to $\xi = 1.75$ following **?**. There, the supply elasticity for 95 U.S. cities is estimated for the period between 1970 and 2000. The estimates vary from 0.60 to 5.45 with a population-weighted average of 1.75 (2.5 unweighted). The steady-state unit price of land $\bar{q}$ is set such that the relative share of land in the price of housing is 30% (see **?** and **?**). The elasticity of new construction with respect to the price of housing, ζ, is set equal to the median elasticity for the 45 cities studied by **?**, $\zeta = 5$.

The maintenance cost m is chosen to be 2.5% of the steady-state housing price according to **?**. Moreover, the average house price is 3.2 times annual income. The value of ψ is calibrated so that the ownership rate in the city $\sum \Phi_{ss}/(\sum \Phi_{ss} + B_{ss} + B^R_{ss} + F_{ss}) = 66.7\%$, where $\sum \Phi_{ss}$ denotes the total measure of homeowners in the steady state. The Census Bureau reports the ownership rate among households whose head is between age 35 and 44 is roughly 66.7%.

We set the average rent-to-price ratio ς, based on empirical findings of the Lincoln Institute of Land Policy. They estimate annual rents for owner-occupied units based on data from the Census Bureau. These estimated rents are then divided by the average self-reported value of owner-occupied units to obtain the rent-to-price ratio. This number was fairly stable and hovered around 5% prior to the most recent housing boom leading up to the 2008 financial crisis.

The remainder of the parameters listed in Table 1 are determined to match jointly a number of targets based on the model. First, we set the average length of time following a foreclosure until a borrower is again allowed to access the mortgage market to five years. This time frame is consistent with the policies of Fannie Mae and Freddie Mac, which guarantee most U.S. mortgages. Thus we set the probability that a foreclosure flag remains on a borrower's credit record to $\pi_f = 0.8$.

According to the Federal Housing Finance Board, the average contract rate on conventional, fixed-rate mortgages between 1995 and 2004 was 7.2%. We target an average down-payment ratio of 20% and an annual default rate of 1.6%, which is close to the average annual foreclosure rate among all mortgages during the 1990s according to the National Delinquency Survey by the Mortgage Bankers Association.

The loss severity rate is defined as the present value of all losses on a given loan as a fraction of the balance on the default date. According to **?**, these losses are caused by both transaction and time costs associated with the foreclosure process, and by the fact that foreclosed properties tend to sell at a discount relative to other houses with similar properties. Using a data set of $90,000$ first-lien liquidated loans, the authors estimate that loss severity rates range from around 35% among more recent mortgages to as much as 60% among older loans. Based on these numbers, we choose parameters so that in the event of a default, the value of

$$\frac{\min\{\beta V^{REO}, d\}}{d} = 0.54 \tag{76}$$

on average. This implies an average loss severity rate of 46%.

? examine sub-prime mortgage data and find that 50% of delinquent loans with loan-to-value ratios between 80% and 90% end up being repossessed, compared to 55% of delinquent loans with loan-to-value ratios between 90% and 100%, and 59% of delinquent loans with loan-to-value ratios above 100%. In addition, **?** find that 30% of defaulted conventional fixed-rate loans and 50% of defaulted conventional adjustable rate loans transition to REO and **??** report that 32% to 38% of defaulted FHA loans transition to foreclosure. Based on these numbers, we choose parameters such that in the event of financial distress, the average probability of a successful sale is 66.5%. That is, 33.5% of the homeowners who experience financial distress ultimately end up in foreclosure in the steady-state.

Evidence available from the American Housing Survey (AHS) suggests that prior to 2003 the ratio of the original loan size to yearly income averages 2.72. Accordingly, we choose parameters such that the steady-state loan-to-income ratio at origination is given by $m_{0,ss}/y_{ss} = 2.72$.

Finally, dynamics of our model depend crucially on two elasticities: the elasticity of $G(\cdot)$, evaluated at ε^c, $\alpha_p = \varepsilon^c G'(\varepsilon^c)/G(\varepsilon^c)$ and the elasticity of the matching function with respect to the number of buyers, η. These two parameters are calibrated jointly by using estimates of the relative standard deviations of population growth and housing sales growth in response to income shocks as in **?**.

For the non-search economy, all parameters remain at their values in Table 1 except for π_d^f, z^H and ψ, which are adjusted so that the steady-state statistics match the relevant targets again. In the steady-state of the non-search economy, the construction cost parameter is adjusted so that $P^* = 3.2$ given the rate of population growth. Appendix B provides lists of parameter values re-calibrated to the non-search and the directed-search-with-middlemen economies respectively.

7 The Steady-State

We now characterize the steady-state of the baseline search economy. In the steady-state, *per capita* income (which is exogenous) remains constant over time. The steady-state is defined based on the definition of equilibrium established in Section 3.5, plus the requirement that all functions and values listed in (56) - (59) are time invariant.

In the steady-state, all owners have strictly positive home equity. Resident owners who receive neither moving nor financial distress shocks do not attempt to sell their houses, regardless of their outstanding mortgage balances. All relocated owners continue to make repayments until a successful sale occurs or the mortgage is completely paid off. Finally, all distressed owners attempt to sell their houses. As such, there no strategic defaults in the steady-state, in the sense that foreclosure occurs only as the result of the a financial distresse shock followed by an unsuccessful sale.

Figure 3 depicts the steady-state distribution of house sellers across types. Nearly half of the sellers in the market are there as a result of financial distress. Note that this is consistent with the mobility and default rate targets from the calibration.

Figure 4 presents the distributions resident homeowners (upper panel) and sellers (lower panel) by mortgage status. The distribution of homeowners is purely driven by exogenous shocks.The measures of owners decrease with the number of payments fulfilled, for $n = 1, \cdots, 29$, owing to the effects of both moving and financial distress shocks which affect homeowners at constant rates over time. The large bin at $n = 30$ represents all homeowners who have repayed their entire mortgage before experiencing either shock. While these homeowners no longer face a risk of financial distress, they remain subject to moving shocks and exit the city eventually with probability one. Similarly, the measure of distressed sellers decreases with the number of payments fulfilled, for $n = 1, \cdots, 29$, although there are no such sellers with $n = 30$ by construction.

The distribution of relocated sellers, in constrast, is driven by households' choice of selling probability. These households are not required to sell, and they are no longer hit by relocation shocks. The fact that some enter sub-markets with high prices and low sales probabilities accounts for the hump-shape of the distribution. The spike at $n = 30$ arises from the fact that resident homeowners who have paid off their mortgages are still subject to moving shocks, at which point they become relocated sellers without a mortgage.

Figure 5 illustrates the distribution of housing prices in the steady state.

7.1 Leverage and seller behavior

Figure 6 illustrates the relationship between a seller's optimal choice of sub-market, which determines both her asking price and sales probability on selling price and probability, and her debt position. Overall, a distressed seller is more eager to sell than a relocated seller and therefore, conditional on debt position (represented here by the LTV ratio), posts a lower price and sells with a higher probability. The cost of failing to sell are higher for distressed sellers for two reasons. First, a distressed seller has no choice but default if she fails to sell her house within the period, while a relocated seller retains the choice of whether to default in the next period. Second, a relocated seller receives continuation value L, which is independent of her credit record, while a distressed seller who has defaulted on her mortgage and remains in the city is excluded from the housing market for the next five

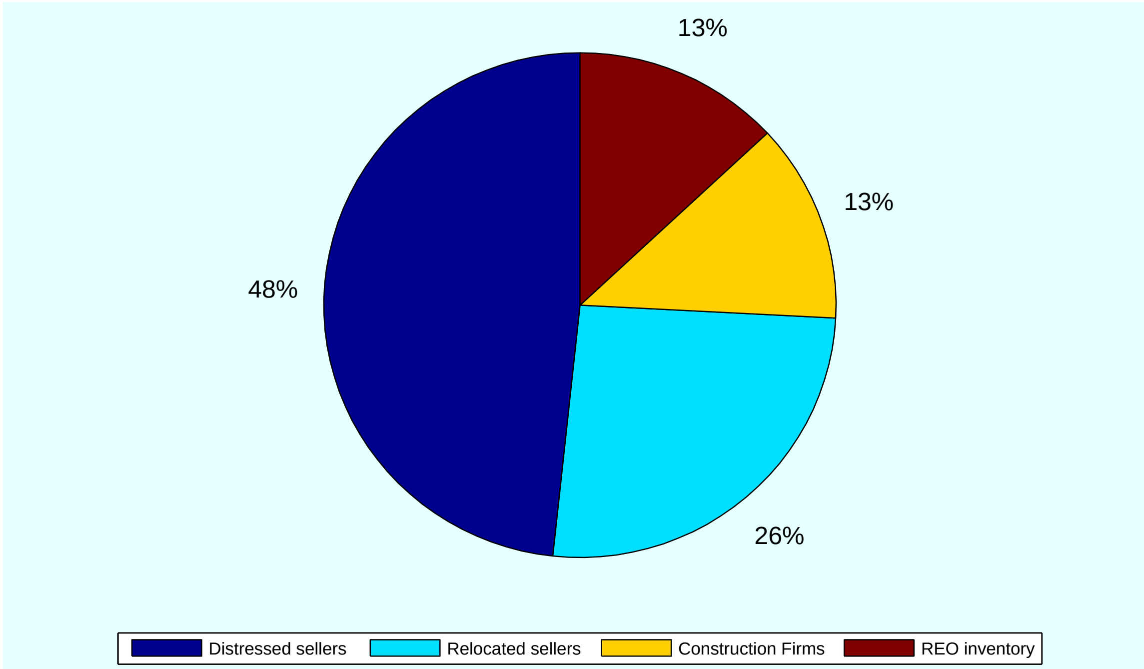

Figure 3: Composition of house sellers

periods on average.

Note also the relationship between the posted asking price and LTV. For both types of seller, the posted prices is initially (very) weakly decreasing in LTV. At some point, and this is most dramatic for distressed sellers, the relationship becomes strongly increasing. For distressed sellers in particular, the relationship resembles closely that reported by **?** (see Figure 2, p. 267). in their empirical study of condominium sales in Boston during the 1990's and is also consistent with the findings of Anenberg (2011) and others. Figure 7 combines their results with ours in common units.[17] The closeness of the relationship is striking, given that none of the quantities depicted for our economy are calibration targets.[18]

To understand leverage-price relationship illustrated in Figures 6 and 7, consider the case of a distressed seller. In the steady state, the gain from trade for a such a seller as a

[17] In our economy, the posted (asking) price is proportional to the mark-up, as all vacant (non-foreclosure) houses have a common value.

[18] It is reasonable to believe that the curvature of the price choice of relocated sellers would resemble more of the Genesove-Mayer result if the continuation value, L, did depend on one's credit record in the city.

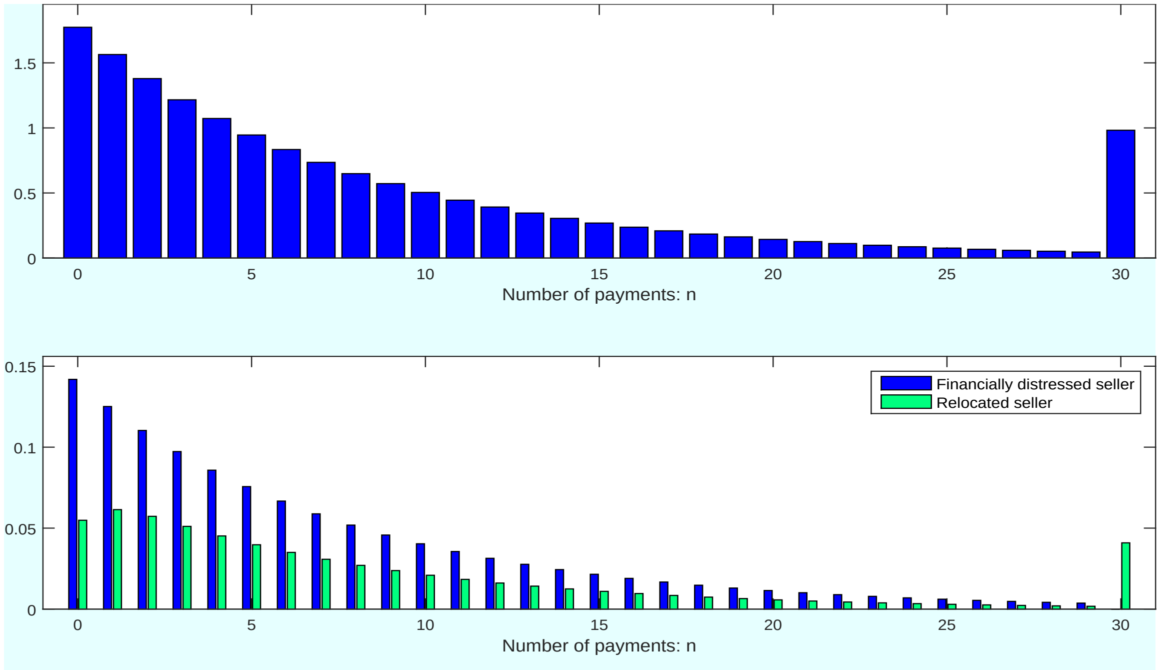

Figure 4: Steady-state distributions of mortgage status respectively among resident owners (upper panel) and household sellers (lower panel)

function of her outstanding debt, d, is given be:

$$\Psi(d) = W(p-d) - W^f\left(\max\left[0,\ V^{REO}-d\right]\right) \tag{77}$$

$$= \begin{cases} W(p-d) - W^f\left(V^{REO}-d\right), & \text{if} \quad d < V^{REO} \\ W(p-d) - W^f(0), & \text{if} \quad d \geq V^{REO} \end{cases} \tag{78}$$

$$= \pi_f(1-\pi_h)\beta\left[W^f(0)-U\right] + \begin{cases} u(y-R+p-d) - u\left(y-R+V^{REO}-d\right), & \text{if} \quad d < V^{REO} \\ u(y-R+p-d) - u(y-R), & \text{if} \quad d \geq V^{REO}, \end{cases} . \tag{79}$$

This result follows from (11) with the dependence of the outstanding debt (d) on the specifics of loan (m_0, n) suppressed. Differentiating (79) with respect to d, we have

$$\Psi'(d) = \begin{cases} u'\left(y-R+V^{REO}-d\right) - u'(y-R+p-d), & \text{if} \quad d < V^{REO} \\ -u'(y-R+p-d), & \text{if} \quad d \geq V^{REO}, \end{cases} \tag{80}$$

Given that $u' > 0$ and $u'' < 0$, we have the following:

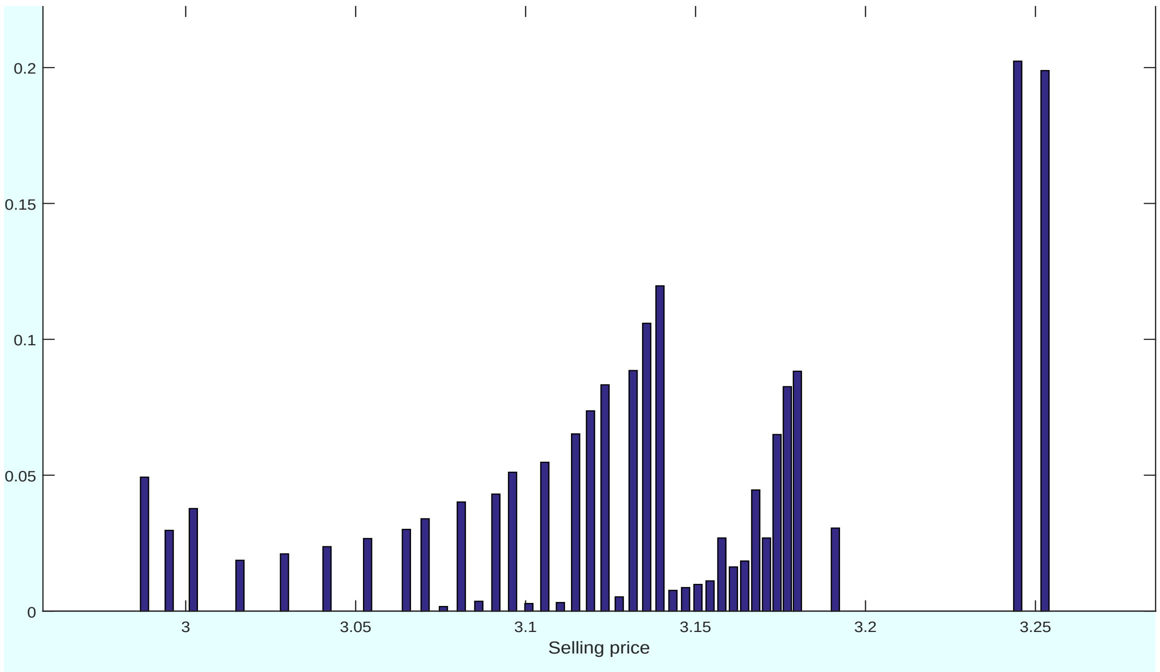

Figure 5: Steady-state distribution of housing prices

Proposition 1 *(i) If $d \geq V^{REO}$, then $\Psi'(d) < 0$; (ii) If $d < V^{REO}$, $\Psi'(d) > 0$ for any given $p > V^{REO}$ and $\Psi'(d) < 0$ for any given $p < V^{REO}$.*

In the steady-state a distressed seller chooses a sub-market to maximize her expected gain from trade. Given the matching function and free-entry of buyers, the optimal sub-market decision in (11) is equivalent to

$$\max_{p,\theta} \ \rho(\theta)\,\Psi(d;p) \tag{81}$$

where

$$\theta = \Omega(p) = \gamma^{-1}\frac{U - W(0)}{V^o(p, m_0) - W(0)} \tag{82}$$

follows directly from (8) evaluated at the steady-state. It is straightforward to show (i) that $\rho(\Omega(p))$ is strictly decreasing in p given the properties of the matching function listed in (3) and (4); and (ii) that the gain from trade $\Psi(d;p)$ increases with price p for any debt level, d, since $u' > 0$. Thus, a higher selling price raises the gain from trade, but reduces selling probability. The optimal sub-market choice reflects this trade-off.

The shape of the relationship depicted in Figures 6 and 7 can be understood using Proposition 1. When a seller is sufficiently indebted ($d \geq V^{REO}$), the gain from trade

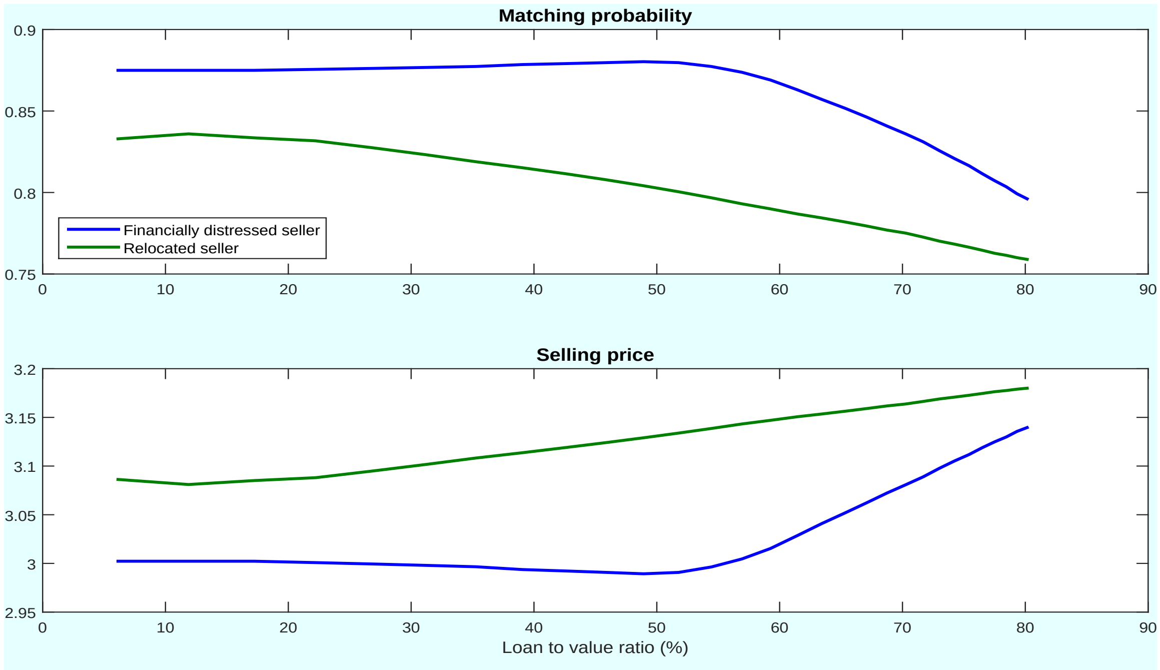

Figure 6: Leverage and seller behavior. The top panel shows the choices of selling probability by distressed and relocated sellers. Correspondingly, the bottom panel demonstrates the choices of selling price by the two types of sellers.

$\Psi(d;p)$ is strictly decreasing in debt, d, for any given price, p. Essentially, heavily indebted sellers with more debt worry less about making sure that trade happens than about raising the gain from trade if it does. The reason for this is that they will receive no residual profit from a trade unless it is at a high price. The foreclosure cost is *fixed*, and the cost of defaulting on a larger debt is borne by the lender, rather than the seller herself.

For a less indebted seller (*i.e.* one with $d < V^{REO}$), has greater incentive to sell, as failure to do so results in the loss of residual profit as well as the cost of the foreclosure tag. Moreover, for $p > V^{REO}$, the gain from trade Ψ is strictly increasing in debt d. As such, a more indebted seller (but with $d < V^{REO}$ still) will chose a lower price/higher sales probability. Overall, for $d < V^{REO}$, the effect of debt on the gain from trade (*i.e.*, $\Psi'(d)$) is likely to be small in that d affects the returns both to selling and failing to do so in a symmetric way (see (80)). Thus the relationship is essentially flat for lower LTVs but rapidly increasing for higher LTVs.

As a final note on the leverage-price relationship, it is worthwhile clarifying that the condition $p > V^{REO}$ is not particularly restrictive. For example, in our baseline calibration, the steady-state value of $V^{REO} = 1.79$, while the minimum selling price chosen by a seller

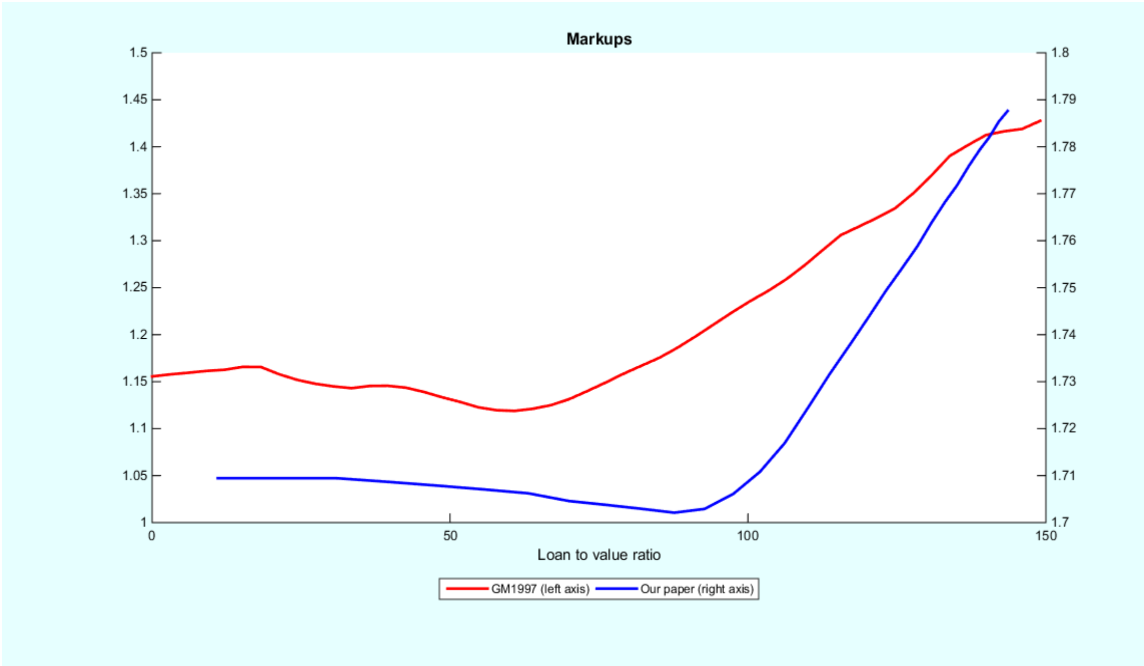

Figure 7: The red curve (left axis) depicts the ratio of asking price to assessed value as measured by Genesove and Mayer (1997) plotted against sellers' LTV. The blue curve (right) depicts the same relationship for the ratio of the posted price to the value of a house in REO inventory for our baseline search economy.

is 3.04. In general, the V^{REO} tends to be much lower than the choice of selling price by any seller due to the foreclosure and carrying costs associated with houses in REO inventory.

Note that while our proof of Proposition 1 relies on the assumption that consumption goods are not storable, the result is in fact more general. In particular, the same properties for $\Psi'(d)$ arise as long as $W'(p-d) > 0$ and $W^{f\prime}\left(V^{REO}-d\right) - W'(p-d) > 0$ for $d < V^{REO}$. The former condition requires that the value of a buyer without the foreclosure flag is a strictly increasing function of her asset holdings. The latter requires that the slope of the value of a buyer with the flag at asset position $V^{REO}-d$ be greater than that of the value of an unflagged buyer at $p-d$ for lower levels of debt. Considering that $p > V^{REO}$ in general, this requirement does not seem overly restrictive for value functions such as $W^f(\cdot)$ and $W(\cdot)$, which are strictly concave. Indeed, we also gain support for this argument given the match between the predictions of our calibrated economy and the evidence provided by **?**.

7.2 Matching and lending standards

We conduct two comparative statics exercises in order to determine the role of specific assumptions regarding matching. Holding all the other parameters constant, we consider the effects of changing the matching coefficient ϖ and the elasticity η. In our calibration, these parameters determine the fundamental trading conditions of the housing market. Our goal here is to examine how search frictions, which determine in part the liquidity of housing, affect mortgage lending standards directly.

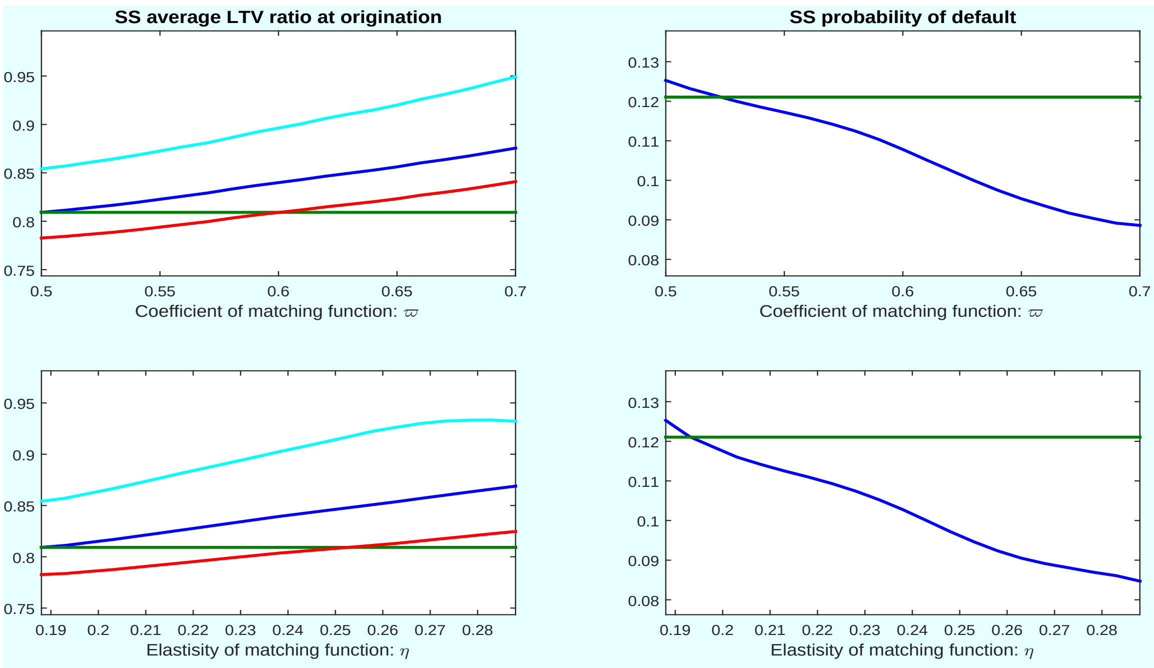

Figure 8: Effects of search frictions on average down-payment ratios and default rates in the steady state. In the left column, the three colored curves represent the following: the average down-payment ratio in blue, the maximum down-payment ratio in red and the minimum down-payment ratio in green.

The top two panels of Figure 8 illustrate the effect of changes to ϖ on the average LTV at orgination and the probability of a mortgage ending in foreclosure, respectively.[19] Consider a mortgage issued in period t. The probability of such a mortgage ending in

[19] In equilibrium, households differ in LTV at origination because they purchase houses at different prices but are advanced loans of the same size. Both of the left-hand panels of Figure 8 depict the effects of matching parameters on average, maximum and minimum LTV's at origination, respectively.

foreclosure, $\Pi_{d,t}$, is given by

$$\begin{aligned}\Pi_{d,t} &= \sum_{i=1}^{T}(1-\pi_h)^i(1-\pi_d)^{i-1}\pi_d[1-\rho(\theta^{sd}_{t+i}(m_{0,t},i-1))] \\ &\quad+\sum_{i=1}^{T}(1-\pi_h)^{i-1}\pi_h d(m_0,i-1)[1-\rho(\theta^{L}_{t+i}(m_{0,t},i-1))]\end{aligned} \tag{83}$$

where $\rho\left(\theta^{sd}_{t+i}(\cdot,\cdot)\right)$ and $\rho\left(\theta^{L}_{t+i}(\cdot,\cdot)\right)$ are the trading probabilities in the optimal sub-markets chosen at period $t+i$ for resident and relocated borrowers who have made $i-1$ payments, respectively. The first term is the summation of probabilities of default over the entire duration of mortgage conditional on staying in the city. Similarly, the second term is the summation of probabilities of default conditional on having relocated elsewhere.

As is shown in Figure 8, LTV's at origination are increasing and default probabilites decreasing with the value of ϖ. *Ceteris paribus*, the higher the value of ϖ, the more likely a seller is to match, or equivalently, the more liquid the housing market. Thus, the expected default rate is lower because fewer homeowners experiencing distress fail to sell their houses. At the same time, houses in REO inventory also sell more quickly. Overall, with both the likelihood and cost of default and foreclosure reduced, mortgage firms are willing to advance larger loans, resulting in higher LTV's at origination.

The bottom two panels in Figure 8 demonstrate similar results for varying the value of η. Intuitively, the surplus resulting from housing transactions that accrues to buyers increases with η, the elasticity of the measure of matches with respect to the measure of buyers.[20] A higher η thus implies a higher value of being a buyer, which in turn increases the value of living in the city, lowering the entry cutoff, ε^c. Similarly the return to construction is lower as firms receive less of the surplus associated with new houses. Overall, the housing market is tighter in the steady-state, and *all* houses sell with relatively higher probability. Again, this lowers the expected default rate, leading mortgage firms to issue larger loans.

Overall, these exercises demonstrate that mortgage lending standards are lower the more liquid is the housing market. We now turn to the effects of aggregate shocks which induce liquidity to vary endogenously over time.

[20] See **?** and **?**.

8 Equilibrium Dynamics

We now consider the dynamics resulting from aggregate shocks in equilibrium. We compare our baseline search economy to our two alternatives, the *non-search* (NS) and direct search with middlemen (DSM) economies.

To begin with, we posit an $AR(1)$ process for the log of income, $\ln y_t$:

$$\ln y_t = a_1 \ln y_{t-1} + \epsilon_t, \qquad \epsilon_t \sim N(0, \sigma_\epsilon). \tag{84}$$

We set $a_1 = 0.96$ and $\sigma_\epsilon = 0.02$.

8.1 Population growth, house prices and construction

Figure 9 illustrates the responses of city population growth, the average house price, and construction to a shock to local income which evolves via (84). For each of the three endogenous variables, the responses to the shock in both the baseline and NS economy are similar to those reported by **?**. This is not surprising as our baseline economy has been constructed in part to preserve the dynamics of basic housing market variables generated in that paper. For this reason we discuss them only briefly here before moving on to a discussion of seller behaviour and the mortgage market, which are the focus of this paper.[21]

Briefly, a positive shock to local income induces immediate entry of households to the city and the population growth rate rises. The response of population growth is, however, much larger in the search economy.[22] The responses of housing prices and construction rates differ both qualitatively and quantitatively across the two economies. The search model generates serial correlation in both price growth and construction. In contrast, the non-search economy does not generate such dynamics, rather, the house price jumps immediately, and by a large amount, and then returns monotonically to its steady-state level. This initial jump in house prices, followed by a long decline, effectively limits the entry of households to the city.

The equlibrium response of population growth in the DSM model is qualitatively similar to that of the baseline search model. This economy generates,however, significantly less serial correlation (roughly half) in the house price, and effectively none in construction. Overall, the responses of prices and construction rates in DSM resemble more closely of

[21] For a detailed discussion, see Section V.A. of **?**.

[22] In their experiment **?** adjusted the elasticity of alternative values so that the variance of population growth in the search and non-search economies was equal. Here we do not do this, in order to highlight the difference in the responses of house prices in the two models.

those of the NS economy than those of the baseline.

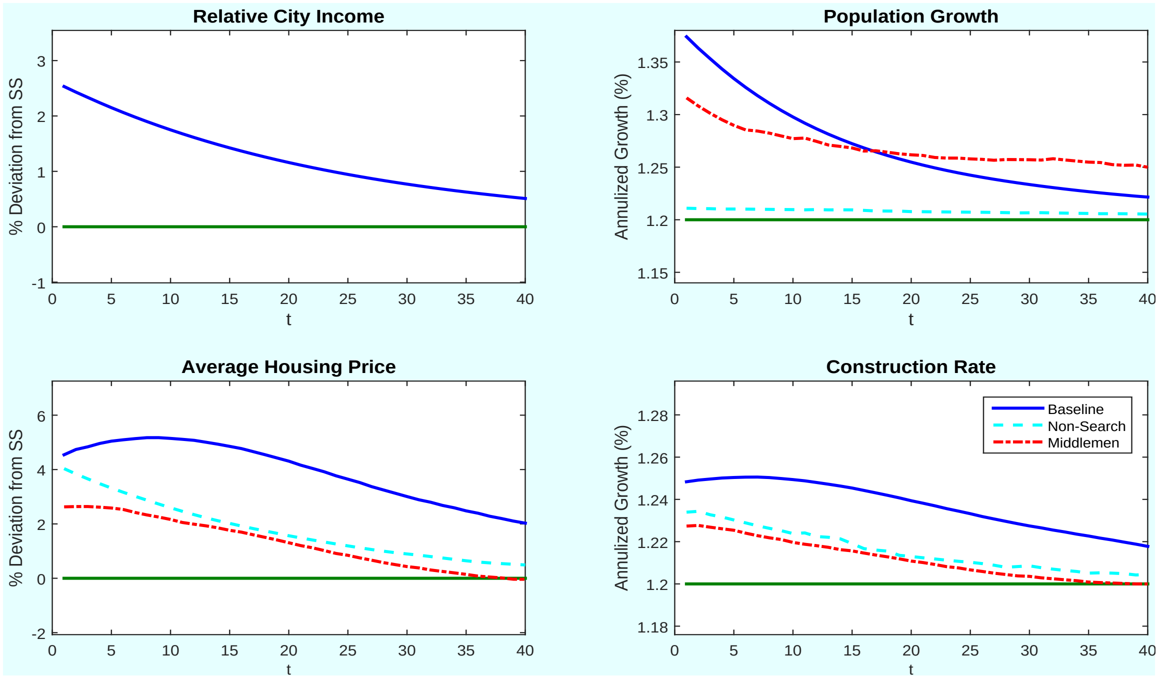

Figure 9: Impulse responses to a positive income shock: population, prices and construction

8.2 Market tightness and matching probabilities.

In both of the search economies considered, serial correlation in both house price growth and the construction rate is driven by the change in housing market liquidity due to search and matching. To illustrate this, Figure 10 depicts responses of overall market tightness, and respective average matching probabilities of buyers and sellers for both the baseline and DSM.[23] There are some key differences in results between the two economies in this figure.

Beginning with the baseline, the results illustrate that changes in housing market liquidity leads to serial correlation in both tightness and the matching probabilities. Following a positive shock to city income, the increase in household entry leads to an increase in housing search by prospective buyers. Construction, however, takes time and so overall market tightness (*i.e.*, the ratio of the *total* numbers of buyers to sellers across all submarkets) increases immediately. Tightness continues to rise for a prolonged period for two

[23] These phenomena do not occur in the NS economy. This accounts for the lack of momentum in the impulse responses.

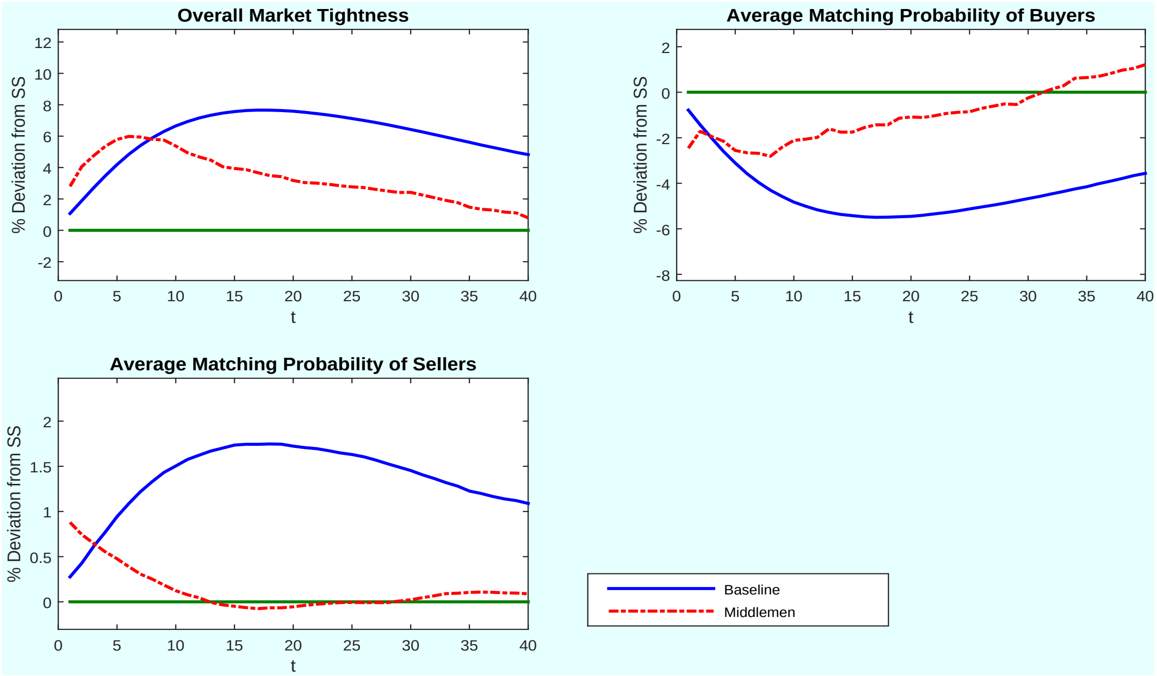

Figure 10: Impulse responses to a positive income shock: matching

reasons: First, there is further entry of prospective buyers due to the persistence of the income shock. Second, buyers who do not match initially remain in the market. Although construction results in a persistent increase in the measure of sellers in the market, the former effect dominates and tightness both rises and remains above the steady-state for an extended period of time.

Given the matching function, higher market tightness implies higher (lower) matching probabilities for sellers (buyers) at any given trading price. The top-right and bottom-left panels in Figure 10 demonstrate such relationships very clearly. As houses become increasingly more "liquid" in the sense that it takes increasingly less time to sell them, their values, and thus their sales prices continue to rise. This leads to serial correlation both in house price growth and construction, as the latter is driven by the value of new houses. As income returns to its steady-state level, entry of households to the city slows. As fewer households enter, searching buyers match, and new houses come on the market, tightness falls. Eventually, house prices and construction return to their steady-state values.

Next, consider the effect of an income shock in the DSM economy. In this case, overall market tightness is defined as the ratio of the total number of buyers in the buying market to the number of sellers in the selling market. The latter consists of household sellers and REOs but *not* construction firms, as they trade directly with real estate firms in a competitive market for new housing. The separation of buying and selling markets effectively breaks the link between the matching rates for buyers and sellers, as the numbers of real estate agents participating in the two markets market adjust instantaneously and independently. Qualitatively, the response of the overall market tightness and average matching probability for buyers in the DSM economy are similar to those in the baseline. Quantitatively, however, the two economies are significantly different. In particular, the serial correlations of both tightness and the matching probability are much reduced in DSM relative to the baseline. Moreover, the overall response of the matching probability for buyers is much smaller in DSM.

The most significant contrast between the two economies, however, lies in that of the average matching rate for sellers. In the baseline, this probability not only rises on impact following the shock, but also continues to rise for an extended period. The DSM economy, in contrast, exhibits no momentum in the matching rate for sellers; it rises on impact and then returns monotonically to its steady-state.

In the DSM economy, the ability of RE firms to can acquire new houses from construction firms in a competitive market and adjust the number of agents they send to the buying market accordingly effectively renders housing supply much more elastic than in the baseline. In response to a positive income shock, the RE firm sends more agents into the buying market to take advantage of the rise in housing demand, supplying them with houses acquired in the competitive market for new homes. Of course, the RE firm could (and to some extent does) acquire additional houses by sending more agents to the frictional selling market. This, however, is costly both because of the direct cost, κ^s, and because the presence of these agents lowers tightness and reduces the rate at which they successfully match.

With much of the rise in housing demand met by houses acquired from the competitive market for new housing, the matching probability for house sellers falls after the initial increase. Serial correlation is reduced due to the effective elasticity of housing supply, which ultimately behaves more similarly to that in the NS economy than in the baseline.

8.3 The default rate, mortgage size, and LTV at origination.

In baseline search economy, the average selling probability for sellers increases on impact and continues to rise for several periods before gradually reverting to its long-run level. An increase in the selling rate benefits distressed sellers substantially by lowering the probability with which they face foreclosure. As such, the default rate moves opposite the selling rate, as shown in the first panel of Figure 11. Similarly, the default rate in the DSM economy mirrors its (very different) response of the selling rate. The overall default rate for the NS economy is exogenously given and thus does not deviate from its steady-state level.

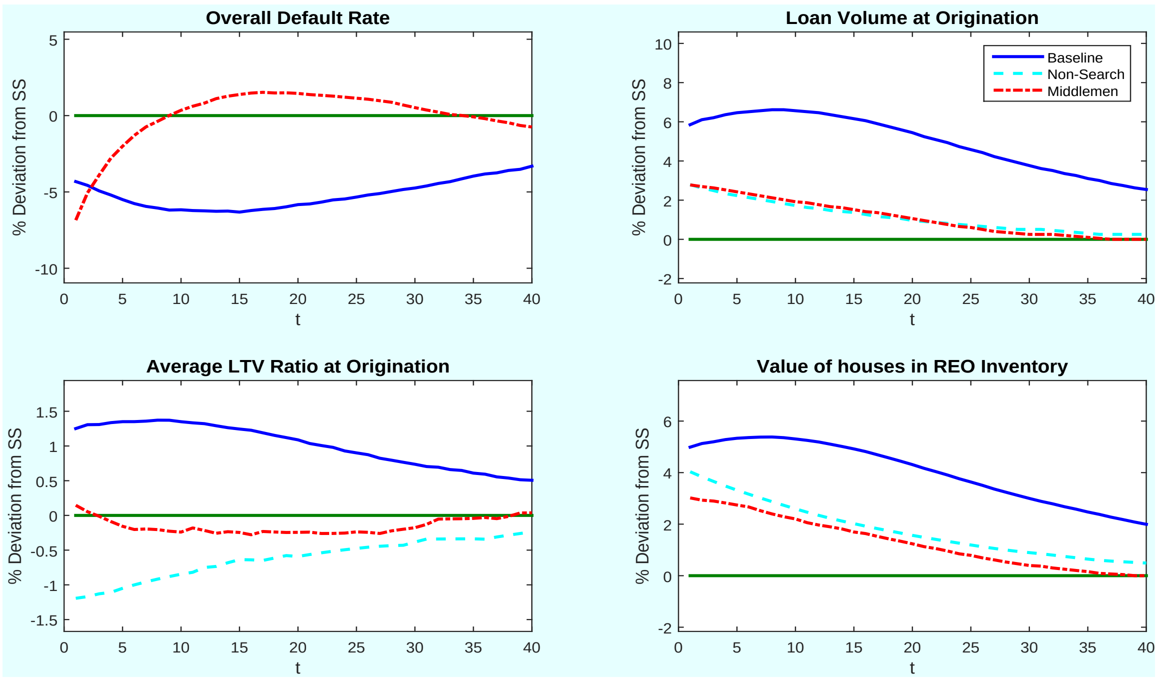

Figure 11: Impulse responses to a positive income shock: mortgage

The responses of loan volume at origination (*i.e.*, m_0) for the baseline differ qualitatively from those of both the NS and DSWM economies, both of which experience nearly identical responses (see Figure 11). Reconsidering the bottom-left panel of Figure 9, it is clear that the response of loan size largely follows that of the housing price. For example, in the baseline, loan size increases on impact and exhibits momentum following the house price.

Recall that loan size, m_0, is determined by (42) and in general depends on the expected default rate and carrying costs in addition to house prices. The close tracking of equilibrium loan size to the house price illustrates that ultimately the value of houses *must* be reflected in mortgage size, in all three economies. Thus, as discussed above, it is movements in housing demand relative to construction (supply) that drive home values, including the component associated with default risk.

The responses of LTV at origination differ significantly across the three economies. In the baseline, the initial LTV rises immediately follwoing the shock. Several forces contribute to this result: First, the expected default rate on new mortgages declines and remains low for an extended time as houses become increasingly liquid. Similarly, lenders' exposure to risk associated with mortgages issued in earlier periods declines as well. Since the mortgage market is competitive and the interest rate fixed, in equilibrium lower risk translates into loans being larger relative to the purchase price. We refer to this as the *market tightness* effect.

Second, borrowers holding mortgages at the time of the shock experience a relatively large increase in home equity (and a corresponding *reduction* in LTV) as a result of the increase in house values. As illustrated earlier, a decline in LTV is associated with lower asking prices and higher sales probabilities, especially for sellers in financial distress. This *home equity* effect also lowers the default rate and hence the riskiness of lenders' portfolios of outstanding mortgages. Again, competition results in this being passed through to buyers in the form of larger mortgages.

Third, the proceeds of foreclosure sales rise and remain high for several periods reflecting the increases in both house values and the selling rate. This increases the value of houses in REO inventory (V_t^{REO}). This lowers the *cost* of default to lenders and again results in greater returns to lending and larger mortgages in equilibrium. See the bottom-right panel of Figure 11 for the response of V_t^{REO}.[24]

Overall, reductions of both the expected default rate and the expected loss upon default motivate the mortgage company to relax lending requirements in the sense that increase the size of the loan they offer at origination. These effects are enhanced because mortgages are long term, and thus the LTV's on all existing mortgages are instantaneously and persistently *reduced*. Eventually, as tightness and the selling rate return to their steady-states, LTV's at origination do as well, following the path of the average house price.

The dynamics of the LTV at origination in the DSM economy are driven by a similar mechanism, although they exhibit very different dynamics. That is, just as in the baseline economy they are driven by movements in the default rate and the value of REO inventory. These in turn are driven by the dynamics of tightness and the sales probability. As described above, however, these variables move differently in the DSM economy. As such, the DSM economy does not exhibit a significant or persistent relaxation of lending standards (*i.e.* an increase in loan size) in response to a positive income shock.

In the NS economy the LTV at origination *falls* significantly in response to the shock,

[24] As the response of V_t^{REO} is almost identical to that of housing prices, it is driven mostly by changes in house prices, as opposed to the higher selling rate. This is not surprising given that the carrying cost (*e.g.*, maintenance costs) is less than 1.5% of the average house price in the calibration.

and gradually returns to the steady-state monotonically thereafter. In this model the expected default rate is exogenously given and the mechanism discussed above for both the baseline and DSM economies is not operative. As house prices rise in response to the shock and are expected to fall monotonically back to their steady-state levels in the future, the mortgage company's expected loss upon default of a mortgage is higher at origination. Given that the default rate does fall to compensate, the mortgage company must require a higher down-payment to cover the increase in default risk.

8.4 The co-movement of house prices LTV's

Figure 12 depicts co-movements between the average housing price and the LTV at origination across all three economies. The baseline model generates a clearly positive co-movement between the two variables while the non-search economy predicts a strong negative relationship. The DSM economy exhibits a small a positive co-movement immediately following the shock. This, however, yields to an extended period of negative co-movement in later periods. Also, while not reported here, the DSM economy exhibits a uniformly negative co-movement between the house price and the LTV at origination in response to a *negative* income shock. In contrast, the baseline search economy predicts consistently positive co-movements. The NS economy, of course, generates negative co-movements in all cases. The responses of the economies to negative income shocks can be found in Appendix C.

8.5 The pricing decisions of indebted sellers.

Figure 13 depicts the responses of the sellers' asking prices at four different stages of the mortgage-repayment process.[25] The first three panels depict choices of distressed sellers and the last for newly relocated sellers. The figure depicts responses for both the baseline and DSM economies; the NS economy has no counterparts for these measures. For the baseline economy, all four panels demonstrate a pattern consistent with an average sales probability as shown in the bottom panel of Figure 10.[26] That is, all panels display patterns consistent with the time-paths of tightness and the average sales rate.

[25] For example, the first panel of the figure depicts the pricing choice of a seller who has not yet made her first payment ($n = 0$), t periods following the shock. That is, it depicts the pricing decisions of a cross-section of sellers at the same stage of repayment but with loans originating at times.

[26] Despite the connection, note that Figure 13 displays a panel where as Figure 10 depicts a time-series relationship. Each point in the last panel of Figure 10 represents a weighted average of the corresponding points in Figure 13 together with those for all sellers with n's not shown in the figure, construction firms, and mortgage companies holding REO inventories.

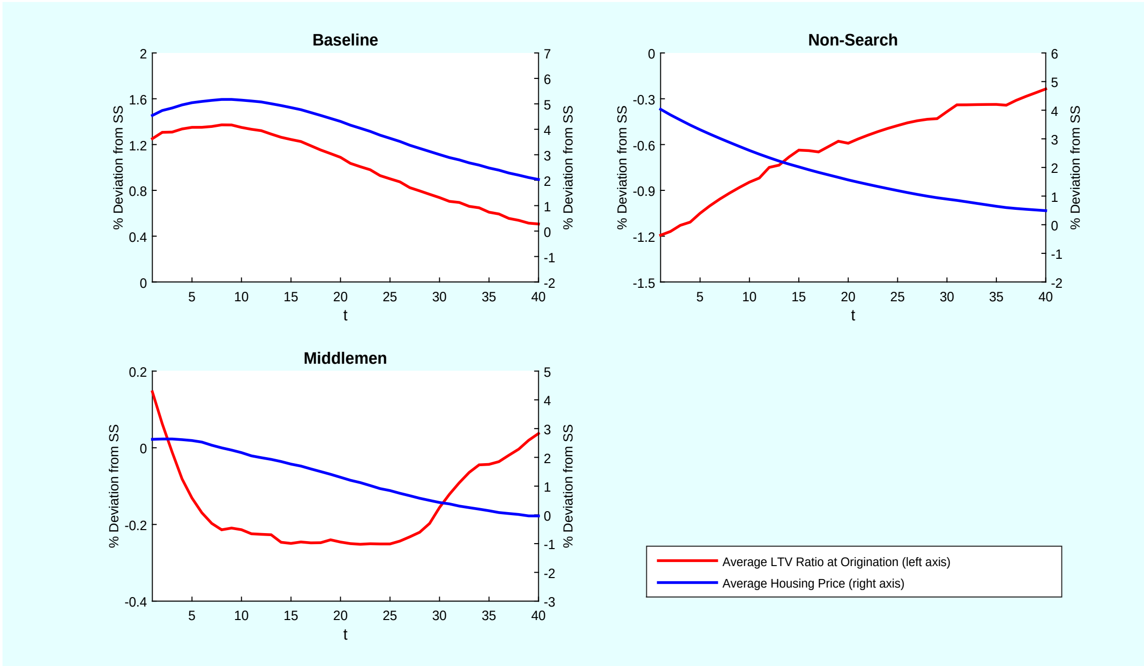

Figure 12: Co-movements between average down-payment ratio and average housing price in baseline, non-search and middlemen economies.

The main departure from this pattern involves distressed sellers who have just purchased and taken out a mortgage in the period *before* the shock occurs ($n = 0$). As described above, the raises the value of houses and reduces these households' LTV's substantially. When such a household receives a financial distress shock, it faces the prospect of losing this potential capital gain if it fails to sell and ends up in foreclosure. They have strong incentive to sell, and thus post a low price and, equivalently, have a high sales probability.[27] Buyers who purchase following the shock experience no random capital gain, as current and future house price as well as future matching rates are taken into account when the mortgage is issued. This explains the large rise in asking price (and drop in the selling-probability) for distressed sellers with $n = 0$ in subsequent periods. The choice of relocated sellers with $n = 0$ also displays a similar initial responses, albeit of smaller magnitude. These sellers neither face imminent foreclosure nor experience such a large capital gain because they are on average less levered than new homeowners.

Consider next the case of sellers one period away from paying off their mortgage ($n = 29$) in the period before the shock. In the figure it is clear that these sellers *raise* their asking prices and thus experience a *lower* probability of a sale. This, of course, raises

[27] The increase in the sales probability is particularly significant given the entry of buyers.

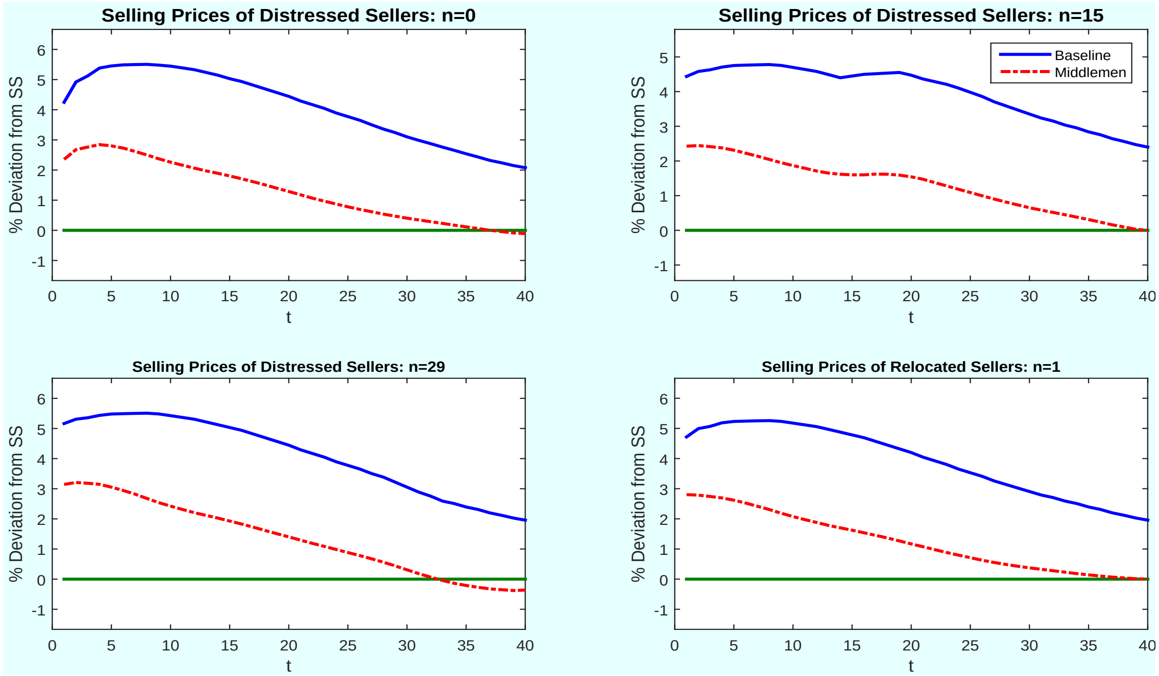

Figure 13: Impulse responses to a positive income shock: house-selling choices

their default probability. Recall that in the event of a default, the mortgage company keeps the outstanding mortgage balance and returns any remaining sale proceeds when the foreclosed house is. For sellers with $n = 29$, the outstanding balance is low precisely because the mortgage has been nearly repaid in full. Thus, the cost of default is low because these households can still recover a large portion of their equity after a default.

The responses of sellers with $n = 15$ lies in between those of sellers with $n = 0$ and $n = 29$. The effects discussed above combine for these sellers and largely cancel, leaving the response to reflect largely movements of the average sales probability. Similar mechanisms drive the responses of different types of sellers in the DSM economy. The results differ from those of the baseline economy again because of the differences in the responses of the average sales rate discussed above.

Responses of these variables to a negative income shock are contained in Appendix C. For the baseline, the dynamics are nearly symmetric to the responses to a positive income shock. One significant exception is that immediately a negative income shock, non-distressed owners may experience such a large increase in LTV that they have negative home equity and thus choose to default on their loans strategically. Therefore, the set of house sellers also includes such non-distressed home owners.

Tables 2 and 5, respectively, contain the choices of sales probabilities associated with optimal pricing decisions and the implied default rates following a positive income shock. Overall, sellers with relatively high leverage are much more likely to default on mortages than those with less leverage. Out of the steady-state. the distribution of indebted sellers matters for the response of the economy to shocks. All else equal, a negative shock occuring when the economy has a high proportion of high-leverage home owners will cause much more severe defaults at the aggregate level than will one occuring when leverage is lower overall.

Table 2: Selling probabilities of borrowers made n payments upon a postivie shock.

	n=0	1	5	10	15	29
t=1	0.7995	0.8056	0.8178	0.8420	0.8658	0.8658
2	0.7834	0.8081	0.8234	0.8446	0.8685	0.8685
3	0.7859	0.7890	0.8260	0.8474	0.8714	0.8714
4	0.7857	0.7919	0.8291	0.8505	0.8746	0.8746
5	0.7881	0.7944	0.8317	0.8531	0.8774	0.8774
6	0.7892	0.7955	0.8329	0.8544	0.8787	0.8787
7	0.7912	0.7975	0.8164	0.8567	0.8810	0.8810
8	0.7925	0.7956	0.8176	0.8580	0.8824	0.8824
9	0.7938	0.7970	0.8190	0.8595	0.8840	0.8840
10	0.7945	0.7976	0.8197	0.8602	0.8847	0.8847

Table 3: Default probability of borrowers made n payments upon a positive shock.

	n=0	1	5	10	15	29
t=1	0.2005	0.1944	0.1822	0.158	0.1342	0.1342
2	0.2166	0.1919	0.1766	0.1554	0.1315	0.1315
3	0.2141	0.2110	0.1740	0.1526	0.1286	0.1286
4	0.2143	0.2081	0.1709	0.1495	0.1254	0.1254
5	0.2119	0.2056	0.1683	0.1469	0.1226	0.1226
6	0.2108	0.2045	0.1671	0.1456	0.1213	0.1213
7	0.2088	0.2025	0.1836	0.1433	0.1190	0.1190
8	0.2075	0.2044	0.1824	0.1420	0.1176	0.1176
9	0.2062	0.2030	0.1810	0.1405	0.1160	0.1160
10	0.2055	0.2024	0.1803	0.1398	0.1153	0.1153

Table 4: Selling probability of borrowers made n payments upon a negtive shock.

	n=0	1	5	10	15	29
t=1	0.7721	0.7783	0.797	0.8279	0.8614	0.8794
2	0.7953	0.7767	0.7953	0.8262	0.8596	0.8775
3	0.7924	0.7955	0.7924	0.8231	0.8563	0.8741
4	0.7899	0.7930	0.7899	0.8204	0.8535	0.8712
5	0.7902	0.7902	0.7871	0.8175	0.8504	0.8681
6	0.7876	0.7907	0.7846	0.8148	0.8477	0.8653
7	0.7861	0.7891	0.8043	0.8133	0.8460	0.8636
8	0.7847	0.7877	0.8028	0.8118	0.8445	0.862
9	0.7834	0.7864	0.8014	0.8104	0.843	0.8605
10	0.7825	0.7855	0.8006	0.8095	0.8421	0.8595

Table 5: Default probability of borrowers made n payments upon a negative shock.

	n=0	1	5	10	15	29
t=1	0.2279	0.2217	0.2030	0.1721	0.1386	0.1206
2	0.2047	0.2233	0.2047	0.1738	0.1404	0.1225
3	0.2076	0.2045	0.2076	0.1769	0.1437	0.1259
4	0.2101	0.2070	0.2101	0.1796	0.1465	0.1288
5	0.2098	0.2098	0.2129	0.1825	0.1496	0.1319
6	0.2124	0.2093	0.2154	0.1852	0.1523	0.1347
7	0.2139	0.2109	0.1957	0.1867	0.1540	0.1364
8	0.2153	0.2123	0.1972	0.1882	0.1555	0.1380
9	0.2166	0.2136	0.1986	0.1896	0.1570	0.1395
10	0.2175	0.2145	0.1994	0.1905	0.1579	0.1405

9 Conclusion

We develop a tractable dynamic general-equilibrium model of housing purchases financed by long-term mortgages and use it to study (i) the effect of sellers' degree of leverage on their pricing behavior and liklihood of default; and (ii) the effects of housing market liquidity on mortgage standards. The model generates endogenous responses of house prices, market liquidity, mortgage standards, and default probabilities in response to income shocks.

We find that sellers' asking prices are decreasing in and relatively insensitive to increase in leverage when LTV's are low, but become steeply increasing in leverage at higher debt ratios. This result matches well the curvature of the leverage-price relationship estimated by Genesove and Mayer (1997), Anenberg (2011) and others. Moreover, seller behavior also differs in leverage along the dynamic path. Second, housing market liquidity influences mortgage standards significantly. In particular, the theory generates a consistent positive co-movement between house prices and LTV's at origination. This observation is qualitatively consistent with observations regarding lending standards both during the period leading up to the recent house price collapse in the U.S., and during the current and on-going period of house price growth in Canada.

An alternative model without search (and a frictionless housing market) fails to capture these phenomena. An alternative model based on the directed search with middlement approach of Hedlund (2014) contains a similar theoretical mechanism. This theory fails, however, to capture quantitatively the dynamics of house prices and sales probabilities, and misses the relationship between prices and lending standards entirely.

A Laws of motion for the non-search economy

For the non-search economy, we have the following laws of motion:

$$(1+\mu)F_t = (1-\pi_f)F_{t-1} + (1-\psi)G(\varepsilon_t^c)\mu \tag{n.43}$$

$$\begin{aligned}&(1+\mu)B_t^R \\ =&(1-\pi_h)\left\{\begin{array}{l}\pi_f B_{t-1}^R + \pi_d^f \sum_{n=0}^{T-1} \Phi_{t-1}(m_{0,t-1-n}, n) \\ +(1-\pi_d^f)\sum_{n=0}^{T-1} H_t^s(m_{0,t-1-n}, n) D_{t-1}(m_{0,t-1-n}, n)\Phi_{t-1}(m_{0,t-1-n}, n)\end{array}\right\}\end{aligned} \tag{n.??}$$

$$\begin{aligned}(1+\mu)B_t &= \psi G(\varepsilon_t^c)\mu + (1-\pi_f)B_t^R \\ &+ (1-\pi_h)\sum_{n=0}^{T}(1-H_t^s(m_{0,t-1-n}, n))\Phi_{t-1}(m_{0,t-1-n}, n)\end{aligned} \tag{n.45}$$

$$\begin{aligned}(1+\mu)\Phi_t(m_0, n) &= (1-\pi_h)(1-\pi_d^f)H_t^s(m_{0,t-n}, n-1) \\ &\times (1-D_{t-1}(m_{0,t-n}, n-1))\Phi_{t-1}(m_{0,t-n}, n-1)\end{aligned} \tag{n.46}$$

$$(1+\mu)\Phi_t(m_0, 0) = (1-\pi_h)(1-\pi_d^f)B_{t-1} \tag{n.47}$$

$$(1+\mu)\Phi_t(\emptyset,\emptyset) = (1-\pi_h)\left\{\begin{array}{l}(1-\pi_d^f)H_t^s(m_{0,t-n}, n-1) \\ \times(1-D_{t-1}(m_{0,t-n}, n-1))\Phi_{t-1}(m_{0,t-T}, T-1) \\ +H_t^s(m_{0,t-n}, n-1)\Phi_{t-1}(\emptyset,\emptyset)\end{array}\right\} \tag{n.48}$$

$$
\begin{aligned}
(1+\mu)\Phi_t^L(m_0,n) \;&=\; H_t^{sL}(m_{0,t-n},n-1)(1-D_{t-1}^{sL}(m_{0,t-n},n-1))\Phi_{t-1}^L(m_{0,t-n},n-1) \\
&\quad + \pi_h(1-\pi_d^f)H_t^s(m_{0,t-n},n)(1-D_{t-1}(m_{0,t-n},n))\Phi_{t-1}(m_{0,t-n},n)
\end{aligned} \tag{n.??}
$$

$$
(1+\mu)\Phi_t^L(m_0,0) \;=\; \pi_h(1-\pi_d^f)B_{t-1} \tag{n.??}
$$

$$
\begin{aligned}
(1+\mu)\Phi_t^L(\emptyset,\emptyset) \;&=\; \pi_h \left\{ \begin{array}{c} (1-\pi_d^f)H_t^s(m_{0,t-T},T-1) \\ \times(1-D_{t-1}(m_{0,t-T},T-1))\Phi_{t-1}(m_{0,t-T},T-1) \\ +H_t^s(\emptyset,\emptyset)\Phi_{t-1}(\emptyset,\emptyset) \end{array} \right\} \\
&\quad + H_t^{sL}(m_{0,t-T},T-1)(1-D_{t-1}^L(m_{0,t-T},T-1)) \\
&\quad \times \Phi_{t-1}^L(m_{0,t-T},T-1) + H_t^{sLw}(\emptyset,\emptyset)\Phi_{t-1}^L(\emptyset,\emptyset)
\end{aligned} \tag{n.51}
$$

$$
(1+\mu)\Phi_t^c = N_t \tag{n.53}
$$

$$
\begin{aligned}
(1+\mu)\Phi_t^{REO} \;&=\; \pi_d \sum_{n=0}^{T-1} \Phi_{t-1}(m_{0,t-1-n},n) \\
&\quad + \sum_{n=1}^{T-1} H_t^{sL}(m_{0,t-1-n},n)D_{t-1}^L(m_{0,t-1-n},n)\Phi_{t-1}^L(m_{0,t-1-n},n) \\
&\quad + (1-\pi_d^f)\sum_{n=0}^{T-1} H_t^s(m_{0,t-1-n},n)D_{t-1}(m_{0,t-1-n},n)\Phi_{t-1}(m_{0,t-1-n},n).
\end{aligned} \tag{n.54}
$$

B Calibration Parameters for Alternative Settings

Table 6: Calibration Parameter Values: Non-Search Economy

Parameter	Value	Target	Data
Parameters determined independently			
β	0.96	Annual interest rate	4.0%
π_f	0.120	Annual mobility of renters	12%
π_h	0.032	Annual mobility of owners	3.2%
ξ	1.75	Median price-elasticity of land supply	1.75
i	0.040	International bond annual yield	4.0%
T	30	Fixed-rate mortgage maturity (years)	30
μ	0.012	Annual population growth rate	1.2%
π_f	0.80	Average duration (years) of foreclosure flag	5
$\bar{q}$	0.96	Average land price-income ratio	30%
m	0.08	Residential housing gross depreciation rate	2.5%
ζ	5	Median price elasticity of new construction	5
R/P	0.05	Average rent to price ratio	5%
Parameters determined jointly			
χ	0.460	Loss severity rate	46%
ϕ	0.0246	Average down-payment ratio	20%
ϱ	0.0074	Average annual FRM-yield	7.20%
ψ	0.570	Fraction of households that rent	33.3%
π_d	0.016	Annual foreclosure rate	1.6%
z^H	0.3280	Average loan-to-income ratio at origination	2.72
κ	0.137	Average price of a house	3.2
α_p	6.200	Relative volatility of population growth	0.17

Table 7: Calibration Parameter Values: DS with Middlemen

Parameter	Value	Target	Data
		Parameters determined independently	
β	0.96	Annual interest rate	4.0%
π_f	0.120	Annual mobility of renters	12%
π_h	0.032	Annual mobility of owners	3.2%
ξ	1.75	Median price-elasticity of land supply	1.75
i	0.040	International bond annual yield	4.0%
T	30	Fixed-rate mortgage maturity (years)	30
μ	0.012	Annual population growth rate	1.2%
π_f	0.80	Average duration (years) of foreclosure flag	5
$\bar{q}$	0.96	Average land price-income ratio	30%
m	0.08	Residential housing gross depreciation rate	2.5%
ζ	5	Median price elasticity of new construction	5
R/P	0.05	Average rent to price ratio	5%
		Parameters determined jointly	
χ	0.440	Loss severity rate	46%
ϕ	0.0246	Average down-payment ratio	20%
ϱ	0.0074	Average annual FRM-yield	7.20%
ψ	0.570	Fraction of households that rent	33.3%
π_d	0.060	Annual foreclosure rate	1.6%
z^H	0.4280	Average loan-to-income ratio at origination	2.72
η	0.166	Average fraction of delinquent loans repossessed	33.5%
κ^b	0.08	Maximum buying premium of average price	2.5%
κ^s	0.64	Maximum selling discount of average price	20%
κ	0.137	Average price of a house	3.2
α_p	6.200	Relative volatility of population growth	0.17

C IRFs upon a negative local income shock

Figure 14: Impulse responses to a negative income shock: population, prices and construction

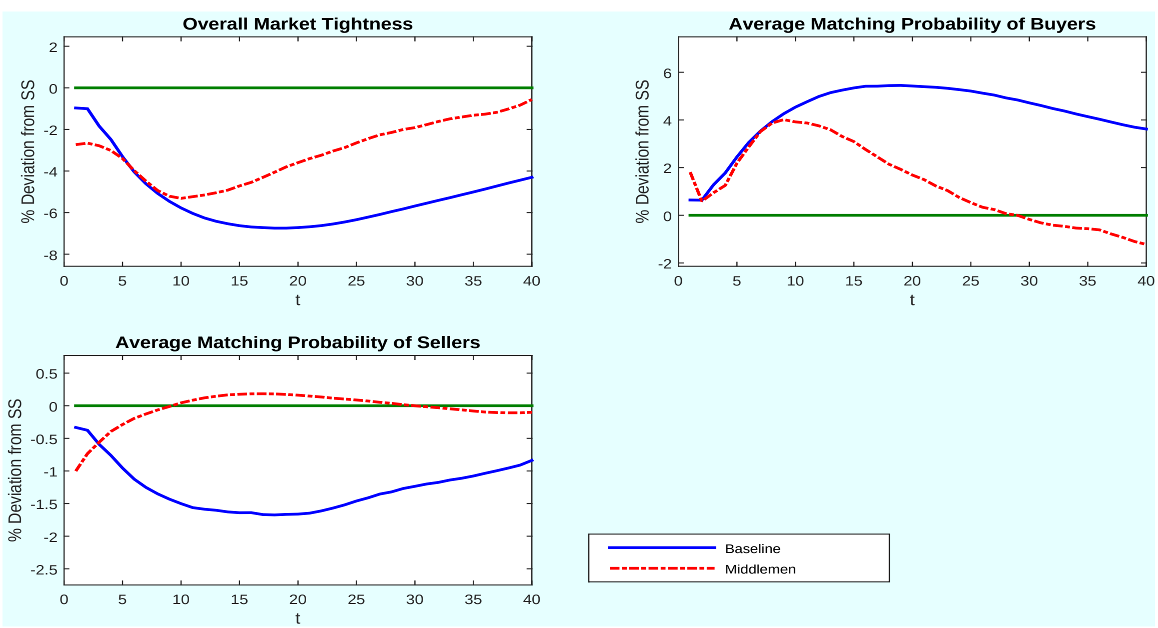

Figure 15: Impulse responses to a negative income shock: matching

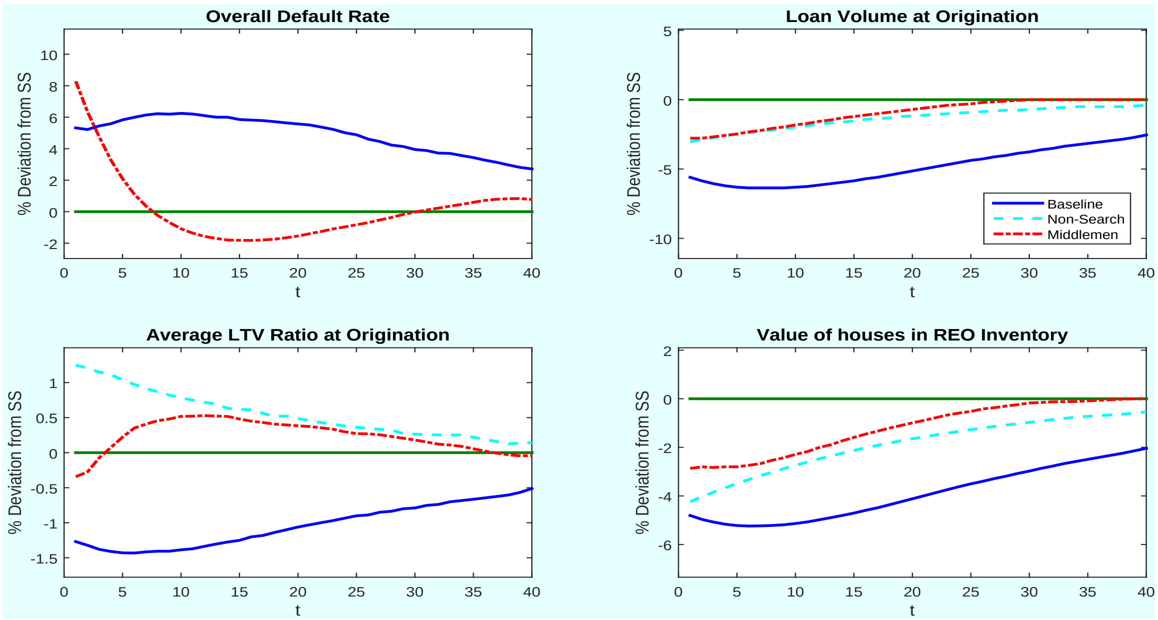

Figure 16: Impulse responses to a negative income shock: mortgage

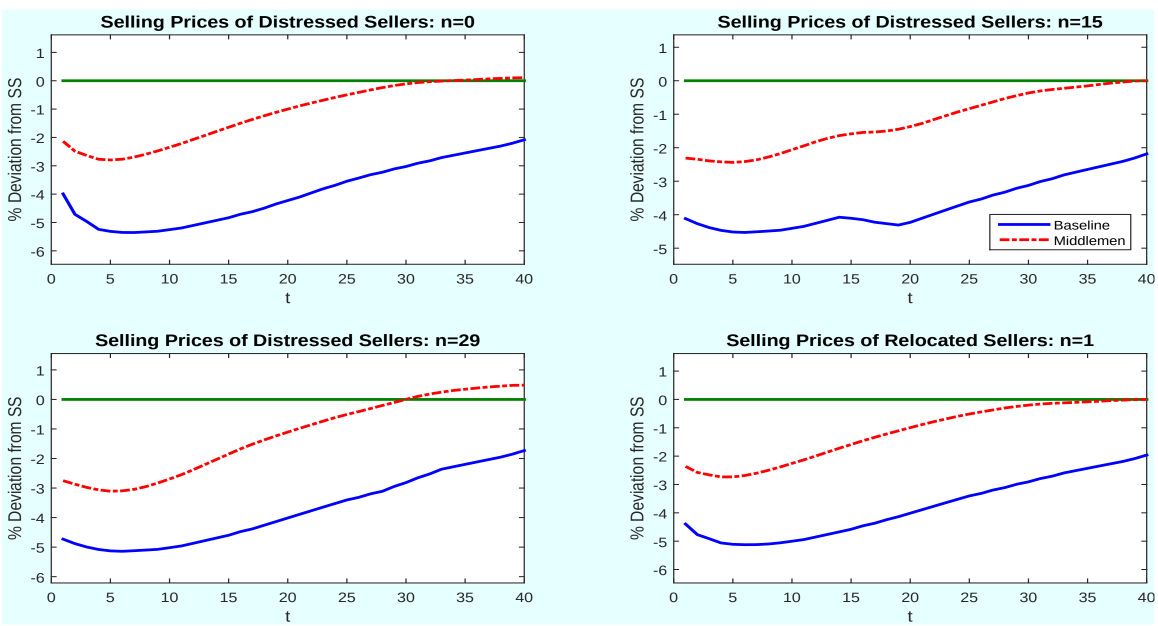

Figure 17: Impulse responses to a positive income shock: house-selling choices

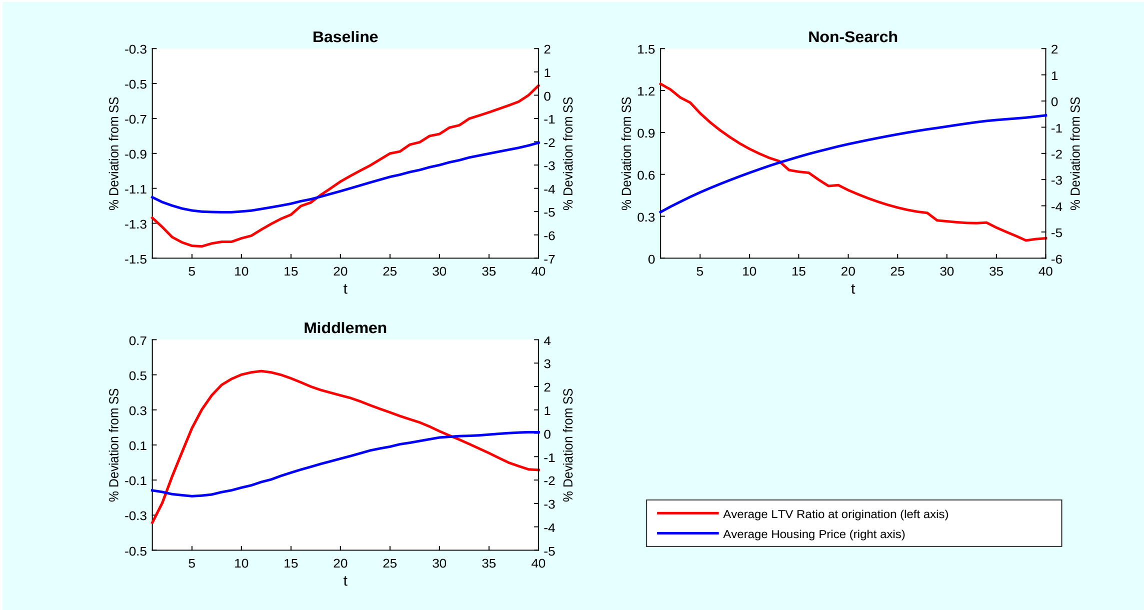

Figure 18: Co-movements between average down-payment ratio and average housing price in baseline, non-search and middlemen economies.