Inequality, Frictional Assignment and Home-ownership *

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Abstract

A theory of the distribution of housing tenure in a city is developed. Heterogeneous houses are built by a competitive development industry and either rented competitively or sold to households which differ in their income and sort over housing types through a directed search process. In the absence of either financial or supply restrictions, higher income households are more likely to own and lower quality housing is more likely to be rented. The composition of the housing stock and the rate of home-ownership depend on the distribution of income, the age of the population and construction costs. When calibrated to match average features of housing markets *within* U.S. cities, observed differences in these variables account well for the variation observed *across* cities in home-ownership and the price-rent ratio. A policy designed to improve housing affordability significantly raises home-ownership among lower income households while lowering the quality supplied to high income households.

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1   Introduction

In this paper, we develop a dynamic equilibrium theory of the distribution of housing tenure in a growing city. When calibrated to match several robust features of housing markets *within* U.S. cities, the theory's predictions for the effects of median income, inequality, household age and land costs for average home-ownership and the average cost of owning relative to renting are qualitatively consistent with observed relationships *across* cities. Policies designed to increase the affordability of housing (but not targeting ownership specifically) nevertheless increase significantly both ownership and housing quality for low income households; while lowering quality (but not ownership) for high income ones.

In our theory, households differ by income and wealth and choose both the quality of the houses in which they live and whether to own or rent them. Owners of existing vacant houses that differ in quality choose to rent them competitively or sell them through a process of directed search. Developers choose the quality of newly constructed housing to meet the needs of the growing population. While the demand for both housing quality and ownership *per se* are increasing in income, the distribution of housing tenure is driven primarily by rent vs. sell decisions on the part of sellers.

According to U.S. Census data, home-ownership is strongly increasing in income, but is significant for all income quintiles. Also, the composition of rental housing is systematically different from that which is owner-occupied. For example, owned units often consist of single-family detached dwellings, while rental units are more commonly part of multi-unit buildings. The average owner-occupied unit is roughly double the size of the typical rental unit. Rental and owner-occupied units are typically located in different parts of a city, with rental units closer to the urban core (see Glaeser and Gyourko, 2007). Moreover, recent estimates suggest that price-rent ratios for houses with similar characteristics rise with housing quality (see Bracke, 2015; Garner and Verbrugge, 2009; Hill and Syed, 2016; and Halket and Pignatti Morano di Custoza, 2015).

Both rates of home-ownership and average price-rent ratios vary dramatically also *across* cities (Glaeser, Resseger, and Tobin, 2009), as do both median income and income inequality. For a sample of 366 U.S. MSA's in the 2010 census, median incomes range from \$31,264 to \$86,286; income Gini's from .388 to .537; home-ownership rates from 51.2% to 81.1%; and average price-rent ratios from 8.7 to 54.2.[1]

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[1]There are several ways to compute price-rent ratios (see Appendix C). For our particular calculation, we used the ratio of mean price-asked to mean annual rent. Note that this refers to a

To study these phenomena, we construct a model of a city comprised of housing units differentiated by quality (taken to represent size, proximity to amenities, etc.) and populated by households with stochastic lifetimes and differentiated permanently by income/wealth. New houses may be of any type and are built by a development industry comprised of a large number of firms with free entry. Construction/land costs increase with quality and, once built, a house's quality is permanently fixed. All households require housing and prefer owning to renting, with the preference for ownership increasing in house quality.

Ownership patterns in equilibrium solve a frictional assignment problem in the sense of Shi (2001, 2005). Vacant houses of any quality may be either rented in Walrasian markets or offered for sale to households through a process of directed search.[2] Unmatched households rent until they find an affordable home to own of their preferred quality. How likely they are to find such a house (and whether they will find one at all) depends critically on sellers' choices regarding whether to offer their vacant houses for sale or rent.

The economy has a balanced growth path in which the housing stock grows at the same rate as the city's population, the rate of home-ownership is constant and the distributions of both houses by quality and households by housing tenure are time invariant. Along this path, there is one-to-one assignment of house quality to household income/wealth for both owners and renters. For higher income households, marginal utility is low for non-housing consumption and high for housing. They therefore live in better (and more expensive) houses. Due to the complementarity between house quality and ownership, these households are willing to pay proportionately more to own than to rent. As a result, high quality houses are offered for sale (rather than for rent) at a relatively high rate, so those who can afford them are more likely to own. Conversely, a larger share of lower quality housing is supplied to the rental markets, and so lower income households (who search for low quality houses) are more likely to rent.

The model is calibrated to match several median features of U.S. MSA housing markets, including average time to sell, average ownership duration, the average price-rent ratio and the distribution of ownership across income levels.[3] Under this baseline calibration, within the city rental housing is generally of lower quality than

comparison of city-wide averages of the prices of owner-occupied houses to rents, not to the relative price and rent of a specific unit.

[2]Search is motivated by the idea that while houses of a given quality are in some sense alike, they have idiosyncratic differences which appeal only to certain households.

[3]The assumed income distribution is log-normal, with inequality measured by the Gini coefficient.

owner-occupied housing and price-rent ratios rise with house quality as higher income households pay relatively more to own than to rent. We then use this calibrated version of the model to characterize the steady-state effects of variation in median income, inequality, average age and land/construction costs per unit of housing quality.

Empirical relationships between both ownership rates and average price-rent ratios and median income, inequality, age and construction/land costs are characterized using cross-city regressions, controlling for other factors affecting the desirability of living in a given city. We find that these relationships are qualitatively consistent with those predicted by the theory. To study the quantitative contribution of these different factors, we use the model to generate predicted cross-city variation in outcomes resulting from the observed variation in these fundamentals. We find that differences in construction/land costs and average amenities across cities are crucial in accounting for observed cross-city variation in both ownership and price-rent ratios.

Given the observed variation in fundamentals, the model generates substantial variation in the *affordability* of housing across cities. Having less affordable housing reduces housing quality throughout the distribution, but affects the tenure decision mainly for relatively low income households. We consider the implications of policies aimed at improving the affordability of housing by subsidizing the provision of relatively low quality/size units to both the rental and owner-occupied markets. Such policies improve the well-being of lower income households relative to that of higher income ones, mainly by increasing housing quality. These policies nevertheless do increase home-ownership in spite of not targeting it directly. High income households (who effectively bear the cost of the policy) continue to own at roughly the same rate, but live in lower quality houses. They compensate for this to some extent by increasing their non-housing consumption.

Several other studies have emphasized the role of search frictions in housing markets (Wheaton, 1990; Krainer, 2001; Albrecht, Anderson, Smith and Vroman, 2007; Diaz and Jerez, 2013; Head, Lloyd-Ellis and Sun, 2014; Ngai and Sheedy, 2017; Hedlund, 2015; Garriga and Hedlund, 2017; and many others). The chief difference between our papers and these is that we emphasize the interaction between the rent-versus-sell decisions of developers and moving homeowners and the rent-versus-buy decisions of heterogeneous buyers in determining the allocation of housing units of differing qualities (see also Halket and Pignatti Morano di Custoza, 2015).

Our theory abstracts from both financial and house size constraints, both of which might be seen as natural candidates for determinants of home-ownership. We do this because they tend to cause only the poorest households to rent and then only because

they are *unable* to own. We find this unsatisfactory first, because home-ownership is observed to be significant except for the very poorest households and second because renting is significant throughout the income and wealth distributions. In our theory, ownership patterns are driven not by exogenous constraints but rather by the optimal decisions of buyers and sellers faced with a choice between competitive rental markets and frictional markets for owner-occupied houses.

The remainder of the paper is organized as follows. Section 2 describes a simple version of the model to illustrate the principal mechanism for determining housing tenure in equilibrium and to provide intuition. Section 3 describes the expanded environment which is used for comparisons across cities, both qualitative and quantitative. Section 4 defines a stationary balanced growth path for this economy. In Section 5 the calibration is described and the implied characteristics of housing markets within the city described. Section 6 describes variation across cities with regard to housing tenure and the average price-rent ratio in both the data and the model. The affordability of housing and the effects of policies designed to improve it are considered in Section 7. Section 8 concludes and outlines future work.

2   A Simplified Model

Here we provide a simple model that illustrates the basic mechanisms at work in our theory and is useful for providing intuition. In the next section we describe the full model for which we characterize numerically equilibria of calibrated versions.

2.1   The Environment

Consider a one-period economy that consists of a single *city*, populated by a unit measure of households that differ with regard to their income, y, which is distributed according to F with support $[\underline{y}, \overline{y}]$. Households consume both goods and housing services. In particular, they must live in a single *house*. Houses differ with regard to their characteristics, and we represent these by a single index of *quality*, $q \in \mathbb{R}_+$.

Households may either rent or own their home, and have identical preferences:

$$u(c, q, z) = (1 - \alpha) \ln c + \alpha (1 + z\psi) \ln q, \quad z \in \{0, 1\}, \quad \psi > 0; \qquad (1)$$

where c is goods consumption, q the quality of the household's residence, and z is an indicator of housing tenure. That is, $z = 1$ if the household owns its home; $z = 0$ if it

rents. Thus, $\psi \ln q$ represents an *ownership premium* which, importantly, is increasing in the quality of the house owned. A household renting a house of quality q receives the amenity value, $\alpha \ln q$, but not the ownership premium.

Houses of different qualities are built using a construction technology through which the cost of land and construction required is increasing in the quality of the house. Let $T(q)$ denote the cost of building a house of quality q, where

$$T(q) = \tau q. \tag{2}$$

Construction is undertaken by an industry comprised of a large number of identical, risk neutral *developers* who maximize expected profits, which they retain. Developers decide, in sequence, which houses to build and whether to sell or rent them.

Consider the single time period as being divided into two *stages*. In the first, there is free entry of developers who decide which houses to build, taking the technology (2) as given. In the second stage, these houses can *either* be sold through a process of directed search following Moen (1997), or rented in competitive markets to households. At the same time, households choose a quality level and search for a house to buy in that segment. After the outcome of matching in the frictional housing market, unmatched households rent houses from developers who have held houses out of the sales market. Houses that were offered for sale but not purchased remain vacant.[4]

At the end of the second stage there are competitive rental markets associated with each house quality, q, and in each of these, $x(q)$ denotes the rent. Developers are willing to effectively commit some houses to the rental market because the matching process (described below) allows them to predict perfectly the measures of unmatched buyers who will be looking for houses of each type to rent. As such, committing a house to the rental market provides developers with a sure return.

In contrast, putting a house up for sale is a risky venture. A developer may build and offer a house for sale in *one* of many potential *sub-markets*, each characterized by a price, p, and a quality, q. A pair of probabilities, γ and λ, representing respectively the probabilities of a seller finding a buyer (γ) and a buyer finding a seller (λ), arise from random matching within each sub-market.

In each sub-market, the number of successful matches is given by

$$\mathcal{M}(B, S) = \kappa B^{\eta} S^{1-\eta}, \tag{3}$$

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[4]Although the risk faced by sellers may seem extreme in this static model, in the full dynamic version it will just be the risk of not selling for one period.

where B and S indicate the measures of searching buyers and sellers present in the sub-market. We refer to $\theta \equiv B/S$ as the *tightness* of the particular sub-market.[5] The matching function $\mathcal{M}$ exhibits constant returns to scale and is increasing and strictly concave in both its arguments. The probabilities with which households and vacant houses, respectively, are successfully matched are functions of θ:[6]

$$\lambda(\theta) = \frac{\mathcal{M}(B, S)}{B} = \kappa\theta^{\eta-1} \qquad \text{and} \qquad \gamma = \frac{\mathcal{M}(B, S)}{S} = \theta\lambda(\theta) = \kappa\theta^{\eta}. \qquad (4)$$

The elasticity of matches for sellers with respect to tightness here is a constant, η.

Upon matching, transactions occur at the posted price. Households direct their search to a particular segment/sub-market, (q, p), knowing both the price they will pay in a successful match and the probability of attaining such a match, given the matching function and their beliefs regarding the number of prospective buyers relative to the number of houses offered for sale in submarket (q, p), denoted $\theta(q, p)$.

2.2 Equilibrium

The value of a newly built house of quality q at the start of the second stage is

$$V(q) = \max\left\{\underbrace{x(q)}_{\text{rent}}, \ \underbrace{\max_{p}\gamma(\theta(q, p))p}_{\text{sell}}\right\}. \qquad (5)$$

The first term in brackets is the value of renting the house and the second is that of holding it vacant for sale. The maximization operator in the second term reflects the optimal choice of sub-market by the seller. The seller, taking as given the search behavior of buyers and other sellers, anticipates how market tightness, and consequently the matching probability, responds to the price.

Free entry into construction implies $V(q) = \tau q$, and competition among developers in rental markets implies $x(q) = \tau q$. Sellers may freely decide whether to rent or hold a house vacant-for-sale. Moreover, since all sellers value identically vacant houses of a particular quality, the must be indifferent across *active* sub-markets (meaning those into which a positive measure of sellers has entered). This gives rise to an

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[5]The matching technology captures the idea that a household must be *well-matched* with a house to receive the ownership premium.

[6]Below we show that the support of F can be restricted so that in equilibrium, these probabilities remain strictly less than one for both buyers and sellers.

equilibrium relationship between price and tightness across active sub-markets for houses in a given market segment:

$$\gamma(\theta)p = x(q) = \tau q. \tag{6}$$

The two principal decisions in the economy are reflected in (6): developers build houses until the expected profit from doing so is equal to zero; and offer them for sale until the expected return from doing so is equal to that from renting, taking as given the search decisions of households.

At the beginning of the second stage, the expected utility of a household with income y can be expressed as

$$W(y) = \max_{q_R, q_S, p, \theta} \left[\lambda(\theta)u(y - p, q_S, 1) + (1 - \lambda(\theta))u(y - \tau q_R, q_R, 0) \right], \tag{7}$$

subject to (6). If a household with income y rents at the end of the period, it chooses its segment optimally, denoted $q_R(y)$. Similarly, the household searches optimally *across* and *within* segments, represented by decision rules $q_S(y)$ and $p(y)$. Given $q_S(y)$, the choice of sub-market, $p(y)$, determines tightness $\theta(q_S(y), p(y))$.

With the support of F suitably restricted so that matching rates are bounded by one, it is possible to characterize analytically households' optimal decisions:[7]

$$q_R(y) = \frac{\alpha y}{\tau} \tag{8}$$

$$q_S(y) = \left[\frac{\alpha(1 + \psi)\Delta}{(1 + \alpha\psi)\tau} \right] y^{\frac{1}{1+\psi}} \tag{9}$$

$$p(y) = \left[\frac{\alpha(1 + \psi)}{1 + \alpha\psi} \right] y, \tag{10}$$

where

$$\Delta = \frac{e^{\frac{\eta}{1-\eta}}}{1 + \psi} \left[\frac{\tau}{\alpha} \right]^{\frac{\psi}{1+\psi}} [1 + \alpha\psi]^{\frac{1+\alpha\psi}{\alpha(1+\psi)}}. \tag{11}$$

From (8) is it clear that renting households spend a fraction, α, of their income on rent. Condition (9) characterizes optimal search *across* segments, whereas (10) and characterizes optimal search *within* a market segment.

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[7]The calculations are straightforward and available on request. To ensure that both $\gamma(\theta)$ and $\lambda(\theta)$ lie in $[0, 1]$ it is sufficient to assume $\Delta^{\frac{1+\psi}{\psi}} \leq y \leq \left[\Delta/\kappa^{\frac{1}{1-\eta}} \right]^{\frac{1+\psi}{\psi}}$.

A formal definition of equilibrium is provided in Appendix A.1. Developers supply housing to both the rental and sales markets, taking as given the optimal search strategies of renters and buyers reflected in (8)–(10). In equilibrium rents adjust so that the supply of houses of quality $q_R(y)$ produced for the rental market will equal the demand $[1 - \lambda(\theta(y))]f(y)$, for all y in the support of $F(y)$. Since $\lambda(\theta(y))f(y)$ houses of quality $q_S(y)$ are sold in the owner-occupied market but the probability of selling any given house is $\gamma(\theta(y))$, free entry of developers results in $\lambda(\theta(y))f(y)/\gamma(\theta(y)) = f(y)/\theta(y)$ houses being produced for that market.

In general, households' search strategies and hence the distribution of housing tenure depend on the properties of the ownership utility premium and the matching function. For this example, the following proposition (proved in Appendix A.2) summarizes several equilibrium results which we view as essential predictions of any model of housing tenure *within* a city.

Proposition 1.

   (i) *Quality increases with income for both rental and owner-occupied housing.*

  (ii) *The probability of ownership increases with household income.*

 (iii) *The price-rent ratio increases with housing quality.*

That the price-rental ratio rises with housing quality is consistent with the empirical findings of several studies, including Halket and Pignatti Morano di Custoza (2015), who describe the combination of these two observations as renting being relatively cheap where rentals are "scarce".

Turning now to aggregate statistics and the cross-city implications of the model, in this simplified environment, it is straightforward to prove (see Appendix A.3):

Proposition 2. *If $(1 - \eta)\psi < \eta(1 + \psi)$ then*

   (i) *aggregate (i.e. city-level) home-ownership is increasing in mean income and decreasing in inequality and construction costs, τ; and*

  (ii) *the ratio of the average price to the average rent is increasing in inequality and construction costs, τ.*

The basic nature of these results carry over to more general environments, provided that the ownership premium increases with quality and the market for owned housing is relatively illiquid. We now turn to the construction of a more complete, dynamic model to capture quantitatively the observed properties of housing tenure in U.S. cities.

3 The Full Model

Consider a dynamic economy in discrete time, again consisting of a single city. The city is populated by a growing number of households with stochastic life-times. Each period new households enter the city's housing market either through migration from elsewhere or by its members attaining an age at which they form independent households. The rate of entry/household formation is constant, and denoted by ν. Households die with probability δ each period.[8] The population of the city, L_t, thus evolves:

$$L_{t+1} = (1 + \nu - \delta)L_t. \tag{12}$$

As above, households differ *ex ante* with regard only to their lifetime income which is distributed according to CDF F with positive support. For simplicity we think of this being represented by a constant income stream, y. Similarly, households again must live in a single house with quality $q \in \mathbb{R}_+$; developers construct houses using technology (2) and once built, a house's quality does not change. Let $M_t(q)$ denote the measure of houses with characteristic q that have been constructed up until the beginning of period t.

Households may again either rent or own their homes, and maximize expected utility over their infinite lifetime,

$$U = \sum_{t=0}^{\infty} \beta^t \left[(1 - \alpha)u(c_t) + \alpha h(z_t, q_t) \right], \quad 0 < \alpha < 1, \tag{13}$$

where β is the household's discount factor adjusted for the probability of survival. That is, $\beta/(1 - \delta)$ reflects the pure rate of time preference. The discount factor satisfies $\beta = (1 - \delta)/(1 + \rho)$ with ρ the exogenous world interest rate.[9]

We now use the following form for preferences:

$$u(c) = \ln c \quad \text{and} \quad h(z, q) = \ln q + zg(q), \tag{14}$$

where $g(q)$ is the *ownership premium* with $g'(q) > 0$. Again a homeowner is *well-matched* in period t if $z_t = 1$. Each period, with probability π, a well-matched owner

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[8]Rather than dying, households could randomly leave the city for elsewhere. This would make little difference for our results, although it would require a re-interpretation of certain parameters.

[9]This assumption is necessary for there to be a stationary balanced growth path. One justification is that ρ is set in a stationary "rest of the world" where households have identical rates of time preference and death.

receives an idiosyncratic shock which results in them no longer enjoying ownership of their home (*i.e.* $z_t = 0$). In this case, the household's ownership premium *from that particular house* falls permanently to zero. The household can, however, obtain an ownership premium in the future by finding, buying, and occupying a different suitable house. This *mobility shock* is intended to capture a household's evolving taste for the idiosyncratic features of a house, which generates turnover in the owner-occupied market.[10]

Construction of new houses is again undertaken by an industry of identical development firms with free entry with costs proportional to quality.[11] Construction of a house takes one period, and the developer must specify a quality q that the house retains forever. Development firms are owned by households and remit their profits (if any) lump-sum. Because of free entry, firms build houses of each type as long as the discounted future value of a house exceeds the current cost of construction.

Rental markets are again perfectly competitive, and $x_t(q)$ represents the rent for a house of quality q. This is paid to the owner of the rented house, which may be either a (mismatched) household or a developer.

As in the simple model, house sales in each market segment take place through a directed search process. The only difference here is that sellers may be either developers or households themselves. Houses are offered for sale in sub-markets characterized by a posted price and a pair of matching probabilities. We assume that the matching function is CRS and that the matching rates (λ and γ) are functions of tightness, $\theta = B/S$, satisfying[12]

Assumption 1. *The matching probabilities have the following properties:*

(i) $\lambda(\theta) \in [0, 1]$ *and* $\gamma(\theta) \in [0, 1]$ *for all* $\theta \in [0, \infty]$;

(ii) $\lambda'(\theta) < 0$ *and* $\gamma'(\theta) > 0$ *for all* $\theta \in (0, \infty)$; *and*

(iii) $\lim_{\theta \to 0} \gamma(\theta) = 0$ *and* $\lim_{\theta \to \infty} \gamma(\theta) = \bar{\gamma} \leq 1$.

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[10]In general, mobility risk may be specified to depend on house quality. For example, a case in which $\pi'(q) < 0$ is consistent with the finding of Piazzesi, Schneider and Stroebel (2013) that in the San Francisco Bay area, less expensive market segments tend to be less "stable" (*i.e.*, moving shocks occur more frequently). For simplicity, however, here we hold π constant across house types.

[11]Since q is itself an index, the assumption of linearity (2) is not very restrictive. For example, any homogeneous function of q will generate identical results with an appropriate adjustment of the preference specification.

[12]The likelihood of a match could, in principle, depend on the quality of the house. This could reflect, for instance, higher quality houses being more diverse and thus specifically appealing to a smaller fraction of buyers who visit them. Here we assume that the matching probabilities depend only on market tightness.

A household can be well-matched with (and receive an ownership premium from) only one house at a time. There is no restriction, however, on how many houses a household can own.

Occupancy of houses results in depreciation which we assume to be completely offset by maintenance, the cost of which is proportional to quality. That is, ζq represents the per period cost of maintaining a house of quality q. Houses depreciate when rented or owned, but not while vacant.[13]

At each point in time, the total stock of housing in the city is given by

$$M_t = \int_0^\infty M_t(q)dq \tag{15}$$

Each of these houses may be either owned by or rented to its occupant, or held vacant for sale. Let N_t denote the measure of owner-occupied houses (or, equivalently, the measure of homeowners), R_t denote the measure of houses for rent (or of renting households), and S_t the measure of houses vacant for sale. We then have

$$M_t = N_t + S_t + R_t \tag{16}$$
$$L_t = N_t + R_t. \tag{17}$$

Thus, in each period, the measure of houses in the city exceeds that of resident households by vacancies, S_t. Note that for each fixed house quality, q, equations analogous to (16) and (17) hold also.

Finally, there exist competitive markets in a complete set of one-period-ahead state-contingent claims paying off in units of the non-storable consumption good. These enable households to fully insure their idiosyncratic risks in the housing market (associated with λ and γ) and losing their ownership premium (associated with π). Household are also required to purchase/issue contingent claims to settle their estate in the event of death (associated with δ). Households face no financial constraint on purchasing a house beyond that implied by their life-time budgets.[14]

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[13] The inclusion of maintenance costs is important only for our calibration in Section 5.

[14] Household could also face idiosyncratic shocks to their income, y_t. These would make no difference given our assumption of full insurance, and so they are omitted.

4 Equilibrium

4.1 The supply-side decision problem

As in the simplified model, the most important choice in the economy is the rent vs. sell decision, in this case made by households as well as developers. Let $V_t(q)$ denote the value of a vacant house of quality q at the beginning of period t. Such a house may be either rented or held vacant for sale in the current period. Its value is

$$V_t(q) = \max \left\{ \underbrace{x_t(q) - \zeta q + \frac{V_{t+1}(q)}{1+\rho}}_{\text{rent}}, \quad \underbrace{\frac{1}{1+\rho} \max_p \left[\gamma(\theta_t(q,p))p + (1 - \gamma(\theta_t(q,p)))V_{t+1}(q) \right]}_{\text{sell}} \right\}. \quad (18)$$

Of course, (18) is the analog of (5) for the dynamic economy.

All sellers value identically vacant houses of a particular quality. Then, as before, indifference among sellers across *active* sub-markets gives rise to an equilibrium relationship between price and tightness for houses in a given market segment:

$$\gamma(\theta_t(q,p)) = \frac{(1+\rho)V_t(q) - V_{t+1}(q)}{p - V_{t+1}(q)}. \quad (19)$$

Moreover, as sellers may freely decide whether to rent or hold a house vacant-for-sale, we have

$$x_t(q) = \zeta q + \frac{(1+\rho)V_t(q) - V_{t+1}(q)}{1+\rho}. \quad (20)$$

Conditions (19) and (20) equate house values across segments and active sub-markets (analogous to (6) for the simplified model). Free entry then implies that house values and rents are determined by construction costs. House values and rental costs can therefore be represented by time-invariant functions of house quality:

$$V(q) = (1+\rho)\tau q \quad (21)$$

$$x(q) = \zeta q + \frac{\rho}{1+\rho}V(q) = (\zeta + \rho\tau)q. \quad (22)$$

Moreover, tightness for active sub-markets is a time-invariant function of p and q:

$$\gamma(\theta(q,p)) = \frac{\rho V(q)}{p - V(q)}. \quad (23)$$

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4.2 The household decision problem

As households are risk-averse, have separable utility and markets are complete, households carry out financial transactions to smooth consumption completely. Specifically, at the beginning of each period, through the purchase and sale of contingent claims, households insure themselves against risks associated with preference shocks (which determine whether a homeowner continues to like his/her house), matching outcomes and death, all of which are random.

Consider first a well-matched homeowner. This household may purchase/issue w_S units of a security that each pay one unit of the consumption good in the next period contingent on receiving a preference shock and becoming a renter, and w_N units of a security that pay contingent on remaining a well-matched homeowner. A homeowner may also sell up to $V(q)$ units of contingent claims which pay-off in the event that the household dies at the end of the period. The payment of these claims is financed by the sale of the household's then-vacant house.

Similarly, a household renting while searching to buy a house may purchase w_B units of insurance that pay contingent on buying a house, and w_R units of a security that pay contingent on having failed to buy and continuing to rent. Finally, we impose that a household searching to buy in sub-market p of segment q must purchase $p - V(q)$ units of a claim that pays off contingent on the household committing to buy a house but then dying at the of the period.

The prices of the contingent securities, in the order defined, are denoted ϕ_S, ϕ_N, ϕ_D, $\phi_B(q,p)$, $\phi_R(q,p)$, and $\phi_D(q,p)$. The last three prices depend on the house quality and the price as they insure against outcomes in a particular sub-market of a particular housing market segment.

At the beginning of each period, renters, depending on their *total wealth*,[15] choose: (i) a type/quality of house to rent, q_R; (ii) whether or not to search for a house to buy; and, if searching; (iii) a particular sub-market, p, (associated with a particular house type, q_S) in which to search; and (iv) a consumption level, c, and savings in the form of a portfolio of claims, $\{w_B, w_R\}$. For simplicity, the decision not to search for a house to purchase will be represented by the choice of $q_S = 0$ and $p = 0$. Accordingly,

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[15]Total wealth, w, is defined to include all available financial resources. In particular, the present discounted value of future labor income, $\frac{y}{1-\beta}$, is contained in w. See Appendix B.1 for details.

$\theta(0,0) = \infty$ and $\lambda(\theta(0,0)) = 0$. A renter with wealth w then has value:

$$W^R(w) = \max_{\substack{c,w_R,w_B, \\ q_R,q_S,p}} \left\{ \begin{array}{l} (1-\alpha)\ln c + \alpha \ln q_R \\ +\beta \left[\lambda(\theta(q_S,p))W^N(q_S,w_B-p) + (1-\lambda(\theta(q_S,p)))\,W^R(w_R) \right] \end{array} \right\} \quad (24)$$

subject to:

$$c + \phi_B(q_S,p)w_B + \phi_R(q_S,p)w_R + \phi_D(q_S,p)(p-V(q_S)) + x(q_R) = w, \quad (25)$$

where $W^N(q_S, w_B - p)$ is the value of entering next period as an owner of a house of quality q_S with wealth $w_B - p$. This value is given by:

$$W^N(q,w) = \max_{c,w_N,w_S,w_D} \left\{ \begin{array}{l} (1-\alpha)\ln c + \alpha \left[\ln q + g(q) \right] \\ +\beta \left[\pi W^R(w_S + V(q)) + (1-\pi)W^N(q,w_N) \right] \end{array} \right\} \quad (26)$$

subject to:

$$c + \phi_S w_S + \phi_N w_N + \phi_D w_D + \zeta q \;=\; w \quad (27)$$

$$w_D + V(q) \;\geq\; 0. \quad (28)$$

4.2.1 Consumption and portfolio choice

Appendix B.1 contains the solution to households' portfolio allocation problem and the derivation of the optimal consumption by income. Let q_R, q_S, and p denote the optimal rental and search choices for a household with permanent income y. This household's (constant) per period consumption is given by

$$\begin{aligned} c = y &- \frac{[1 - \beta(1-\pi)]\,x(q_R) + \beta\lambda(\theta(q_S,p))x(q_S)}{1 - \beta\,(1 - \pi - \lambda(\theta(q_S,p)))} \\ &- \frac{\beta\lambda(\theta(q_S,p))\,[1 - \beta(1-\pi)]\,(p - V(q_S))}{(1-\delta)\,[1 - \beta\,(1 - \pi - \lambda(\theta(q_S,p)))]}. \end{aligned} \quad (29)$$

This is the highest attainable constant consumption sequence satisfying the present value budget constraint given the cost of insuring against both preference shocks and the matching risks associated with housing-related transactions.

4.2.2　Renting

The rent decision is determined by the *intratemporal* Euler equation associated with the choice of q_R in (24):

$$\frac{1-\alpha}{c}\frac{\partial x(q_R)}{\partial q} = \frac{\alpha}{q_R} \qquad \Rightarrow \qquad q_R = \frac{\alpha c}{(1-\alpha)(\zeta + \rho\tau)}. \tag{30}$$

For a non-searching household with income y, (29) and (30) imply that a constant share, α, of income is allocated to rent: $x(q_R) = \alpha y$. This is consistent with the evidence of Davis and Ortalo-Magné (2011) and implies that households at each income level choose rental housing of different qualities with $q_R(y)$ increasing in y.

4.2.3　Home-ownership

Maximizing with respect to q_S and p in (24) reflects optimal search decisions with regard to house type and sub-market, where tightness $\theta(q, p)$ is determined by (23). A pair of intertemporal Euler equations (one for segment choice, q_S, the other for choice of sub-market, p) characterize the solution to the household's search problem, derived in Appendix B.2:

$$\alpha\frac{\partial h(1, q_S)}{\partial q} = \frac{(1-\alpha)}{c}\left\{\frac{[1-\beta(1-\pi)]p - (\beta\pi + \delta)V(q_S)}{(1-\delta)V(q_S)}\frac{\partial V(q_S)}{\partial q} + \zeta\right\} \tag{31}$$

$$p - V(q_s) = \mathcal{L}(\theta(q_S, p))\left\{x(q_R) - x(q_S) + \frac{\alpha\left[h(1, q_S) - h(0, q_R)\right]}{(1-\alpha)/c}\right\} \tag{32}$$

where $\mathcal{L}(\theta(q, p))$ reflects housing liquidity in sub-market p of segment q. More precisely,

$$\mathcal{L}(\theta) = \frac{(1-\delta)(1-\eta(\theta))}{1 - \beta(1 - \pi - \eta(\theta)\lambda(\theta))}, \tag{33}$$

where $\eta(\theta) = \theta\gamma'(\theta)/\gamma(\theta)$. The right-hand side of (32) is the price premium the searching household is willing to pay to acquire owner-occupied housing.

Household optimization is then represented by the decision rules, $c(y)$, $q_R(y)$, $q_S(y)$ and $p(y)$, These satisfy (29), (30), (31) and (32), with market tightness, $\theta(y) \equiv \theta(q_S(y), p(y))$, determined by (23).

4.3 A Balanced Growth Path

The measure of renters with income level y evolves according to:

$$R_{t+1}(y) = (1 - \lambda(\theta(y)))(1 - \delta)R_t(y) + \pi(1 - \delta)N_t(y) + \nu f(y)L_t. \tag{34}$$

Here the first term is the measure of unsuccessful, surviving searchers who remain as renters, the second term that of mismatched surviving owners who enter the renter pool and the last is that of new entrants into the housing market.

Similarly, the measure of owners with income level y evolves according to:

$$N_{t+1}(y) = (1 - \pi)(1 - \delta)N_t(y) + \lambda(\theta(y))(1 - \delta)R_t(y). \tag{35}$$

This consists of surviving owners who remain well-matched and surviving renters who successfully match and buy a home. Dividing all quantities by the population, L_t, and using lower case letters to represent *per capita* values, the relative measures along a balanced growth path can be expressed as

$$r(y) = \frac{\nu + \pi(1 - \delta)}{\nu + (1 - \delta)[\pi + \lambda(\theta(y))]}f(y) \tag{36}$$

$$n(y) = \frac{(1 - \delta)\lambda(\theta(y))}{\nu + (1 - \delta)[\pi + \lambda(\theta(y))]}f(y). \tag{37}$$

Given (36) and (37), the (normalized) stocks of owner-occupied and rental housing of type q are $n(q_S^{-1}(q))$ and $r(q_R^{-1}(q))$, where $q_S^{-1}(q)$ and $q_R^{-1}(q)$ denote the income levels for households that buy and rent in segment q, respectively. Similarly, the (normalized) stock of vacant houses for sale in segment q is $r(q_S^{-1}(q))/\theta(q, p(q_S^{-1}(q)))$. The per capita total stock of housing of each type therefore satisfies:

$$m(q) = n\left(q_S^{-1}(q)\right) + r\left(q_R^{-1}(q)\right) + \frac{r\left(q_S^{-1}(q)\right)}{\theta(q, p(q_S^{-1}(q)))}. \tag{38}$$

We then have the following:

Definition 1. *A Directed Search Equilibrium Balanced Growth Path is a list of time-invariant functions of income $y \in \mathbb{R}_+$ and house quality, $q \in \mathbb{R}_+$:*

i. *household values, $W^R(w)$ and $W^N(q, w)$, and decision rules: $w_R(y)$, $w_B(y)$, $w_N(y)$, $w_S(y)$, $w_D(y)$, $c(y)$, $q_R(y)$, $q_S(y)$, and $p(y)$;*

ii. *house values, $V(q)$, and rents, $x(q)$;*

 iii. *a function for market tightness, $\theta(q, p)$;*

 iv. *shares of households renting and living in owner-occupied housing $r(y)$ and $n(y)$;*

  v. *per capita stocks of housing, $m(q)$;*

such that

  1. *W^R and W^S satisfy the household Bellman equations, (24) and (26), with the associated policies $q_R(y)$, $q_S(y)$, $p(y)$, $c(y)$, $w_R(y)$, $w_N(y)$, $w_B(y)$, $w_S(y)$ and $w_D(y)$ satisfying (29)—(32) and (B.16)—(B.20);*

  2. *free entry into new housing construction and rental markets: that is, $V(q)$ and $x(q)$ satisfy (21) and (22);*

  3. *optimal price posting strategies by sellers of houses in the owner-occupied market: that is, $\theta(q, p)$ satisfies (23);*

  4. *the per capita measures of households, $r(y)$ and $n(y)$, satisfy (36) and (37);*

  5. *the stock of each type of housing grows at the rate of population growth: that is, $m(q)$ satisfies (38).*

5 A Calibrated Economy

We now parameterize the model to match several characteristics of the median MSA in a sample of U.S. cities. We then assess the extent to which the predictions of the model are consistent with observations of the within-city distributions of houses and households across rental and owner-occupied markets. In Section 6, we compare the cross-city predictions of the model with the data.

5.1 Baseline Calibration

We use the following functional form for the ownership premium:

$$g(q) = \psi_0 + \frac{\psi_1 q}{1 + e^{\psi_2 - \psi_3 q}} \qquad q > 0, \tag{39}$$

and define $g(0) \equiv 0$. That is, we assume that the ownership premium is a constant plus a logistic function of housing quality with the modification that the so-called

carrying capacity is proportional to quality. The shape of this function is important for fitting the within-city ownership profile by income.[16]

Our matching function is the so-called "telephone line" function derived from a congestion and coordination problem (see Cox and Miller, 1965; and Stevens, 2007):

$$\mathcal{M}(B,S) = \frac{\kappa BS}{B+S}.$$ (40)

Here $\kappa \in (0,1)$ is the probability that the household can obtain the ownership premium from the house with which the prospective owner is matched. Note that (40) implies matching probabilities for both buyers and sellers satisfying Assumption 1.

We model income as a quarterly flow distributed log-normally:

$$y \sim \log\mathcal{N}(\mu, \sigma^2).$$ (41)

This implies that incomes are bounded below by zero and that their distribution can be summarized by its first two moments. It also implies a one-to-one relationship between the standard deviation and the implied Gini coefficient.[17]

Table 1 contains calibrated parameter values, together with the economy statistics with which each is most closely associated. The mean of log quarterly income, μ, is chosen so that median annual income is normalized to unity and the standard deviation, σ, is set so that the Gini coefficient corresponds to the average across MSA's. Given a value of δ chosen to deliver an annual death rate of 5%, the entry rate, ν, is chosen so that steady-state population growth is 1%. Given these parameters, the moving probability, π, is chosen to match an average ownership duration of 10 years and the matching efficiency parameter, κ, is set to generate an average time-to-sell of 1.25 quarters. The maintenance cost, ζ, and preference parameter, α, are set so that the model generates an average city-level price-rent ratio and mean rent to median income ratio equal to the corresponding averages across MSA's.

The parameters of the ownership premium, $\{\psi_0, \psi_1, \psi_2, \psi_3\}$, were chosen to minimize the sum of the squared differences between the equilibrium ownership rates by

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[16]We settled on this particular functional form by first calibrating the ownership premium using a small number of grid points on $\mathbb{R}_+$ and interpolating function values at other points with cubic hermite polynomials. A suitable fit required only four *knots* (grid points and corresponding function values), and the resulting ownership premium was an increasing function with a slight sigmoid curve. Given that preferences are logarithmic, $q = 0$ is never desired by either a renter or prospective homeowner in equilibrium, and as such is never attained.

[17]Below we use observed Gini coefficients computed for MSA's by the U.S. Census Bureau.

income quintile and those reported in Table 1 which are computed using census summary tables for all U.S. households in 2010.[18] Table 2 displays the extent to which the calibrated economy successfully matches these targets. Figure 1 plots the calibrated home-ownership premium as a function of house quality. Since there is a strong correspondence between household income and housing quality, the slight sigmoid shape should not be too surprising given the ownership rates by income quintile.

Table 1: Calibrated Parameters

statistic	targeted value	calibrated parameter	calibrated value
median annual income (normalization)	1	μ	-1.3863
Gini coefficient for income	0.445	σ	0.8348
population growth rate (%)	1.0	ν	0.0152
probability of death/exit (%)	5.0	δ	0.0127
average ownership duration (years)	10	π	0.0250
average time to sell (quarters)	1.25	κ	0.9523
rental segment choice of median household	1	τ	2.3103
average price-rent ratio	24	ζ	0.0282
annual interest rate (%)	4.0	β	0.9776
ratio of mean rent to median income	0.186	α	0.2039
average ownership rate, Q1 (%)	44	ψ_0	0.0232
average ownership rate, Q2 (%)	56	ψ_1	0.0528
average ownership rate, Q3 (%)	67	ψ_2	3.0954
average ownership rate, Q4 (%)	77	ψ_3	1.2923
average ownership rate, Q5 (%)	87		

5.2 The City in Equilibrium

Figure 2 plots the implied ownership premium against income. The premium rises quite steeply with income for households above the median; reflecting, of course, the complementarity between ownership and housing quality which rises with income.

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[18]See https://www.census.gov/data/tables/time-series/demo/income-poverty/cps-hinc/hinc-05.2010.html). For the US economy as a whole, ownership rates for the bottom quintiles are likely higher than for households residing in MSA's. A disproportionate fraction of lower-income households live outside MSA's and own at a higher rate than those in MSA's.

Table 2: Calibration Results

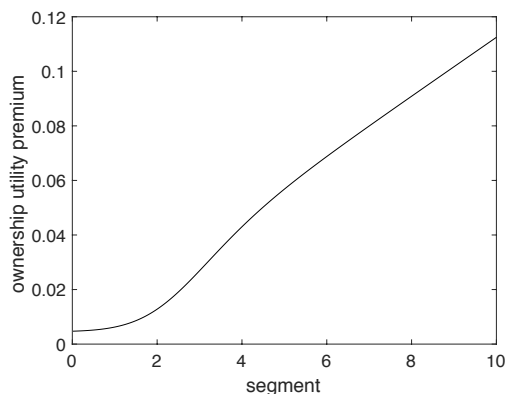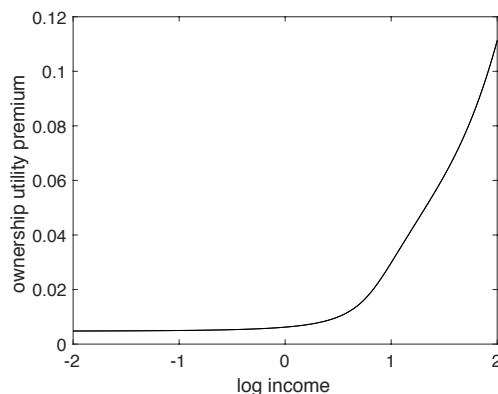| statistic	target value	model value
average ownership rate, Q1	0.44	0.4396
average ownership rate, Q2	0.56	0.5617
average ownership rate, Q3	0.67	0.6675
average ownership rate, Q4	0.77	0.7716
average ownership rate, Q5	0.87	0.8696

Figure 1: Ownership premium by house quality.

Figure 2: Ownership premium by household income.

Figure 3 illustrates the housing decisions of households by income. Clearly, quality is strictly increasing in income for both rental (q_R) and owner-occupied (q_S) housing. At low incomes, households search to own houses of slightly lower quality than those they rent. At income levels above the median, however, the strong complementarity between ownership and quality causes households of a given income to own houses of higher quality than they rent.

Figure 4 plots home-ownership by income. The supply of owner-occupied housing to the very poorest households is low: these households optimally choose to search in sub-markets with low prices and hence low matching rates. Beyond a point, however, home-ownership rises rapidly and then flattens out at high incomes. Intuitively, an increasing ownership rate manifests because a high income household is willing to incur a price premium for a shorter time-to-buy.
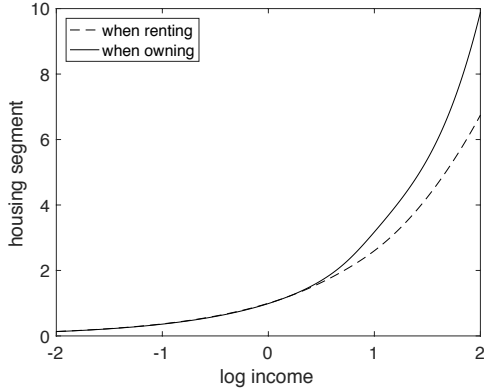
21

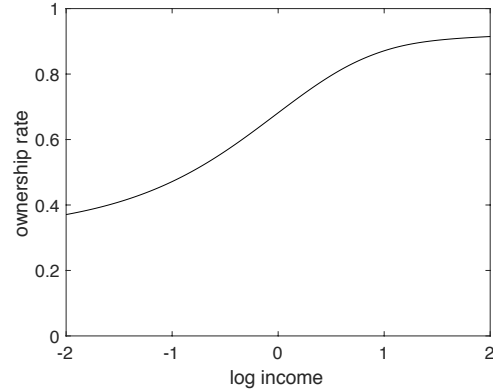Figure 3: Market segment vs. income by housing tenure.

Figure 4: Ownership rate vs. household income.

Figure 5 plots the price-rent ratio by household income. The figure compares, for a household with a given permanent income, the price of the house for which they search to buy to their annual rental cost while searching. The relationship reflects the sum of two endogenous outcomes. First, it reflects differences in households' choice of house quality when renting versus owning (see Figure 3). Given the complementarity between ownership and quality, a higher income household searches to buy a house of higher quality than it rents while searching. Second, higher income households search in sub-markets with lower buyer-seller ratios (those with high prices and short times-to-buy). To isolate this second effect, Figure 6 plots the price-rent ratio by market segment. The price-rent ratio rises with quality reflecting the price premia higher income households pay in order to buy more quickly.

The price-rent ratio for a house of a given quality is low, yet the calibrated model delivers an aggregate price-rent ratio of 24. This reflects households' heterogeneous search strategies in the owner-occupied market. Figure 7 displays the histogram of housing market transactions by income for both sales and rentals. High income households are responsible for a large share of transactions in the owner-occupied market, whereas the opposite is true for the rental market. Both of these contribute to a high overall price-rent ratio. The distributions of transactions by income play a crucial role in the calculation of aggregate statistics and are important determinants of the cross-city implications derived below.

Overall, the balanced growth path of our calibrated economy has the following robust endogenous features:

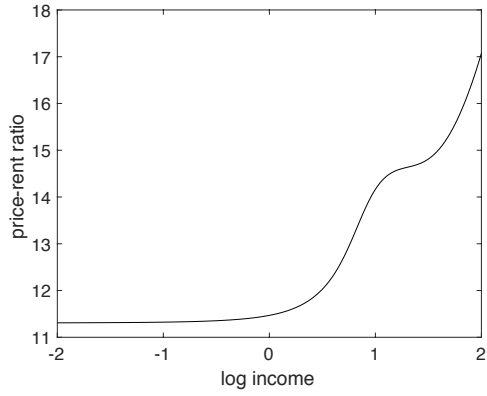- Within the city, home-ownership is increasing in income.

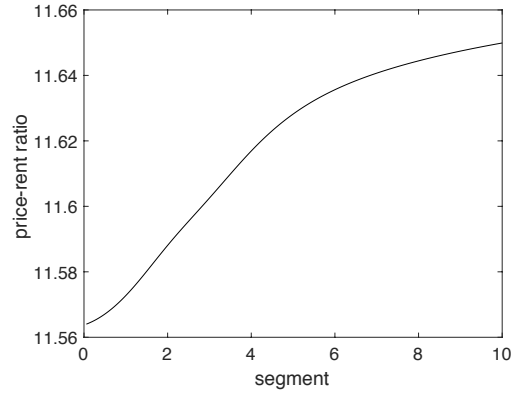Figure 5: Price-rent ratio by household income.

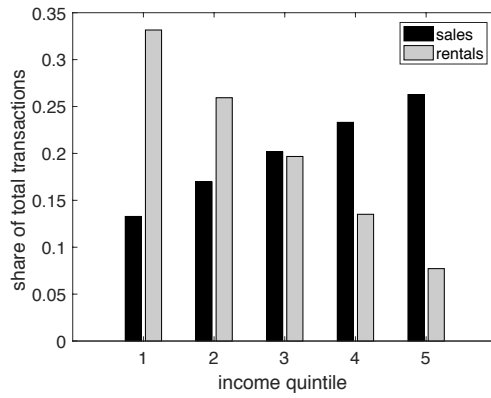Figure 6: Price-rent ratio by housing segment.

Figure 7: Histograms of house purchases and rentals by income.

- Rental housing is generally of low quality relative to that which is owner-occupied, and renters are of relatively low income.
- The price-rent ratio is higher for high quality houses.

These results are consistent with both the implications of Proposition 1 for the simplified model and, as noted above, with observations for U.S. MSA's. Again, these findings are driven in large part by the increasing ownership premium ($g'(q) > 0$). Here, this property is implied by our calibration to ownership rates by income quintile.

5.3 Comparisons across equilibrium balanced-growth paths

We now consider the effects of changes in median income, income inequality (*i.e.* the Gini coefficient), median age (or population growth) and land/construction costs on both home-ownership and the relative cost of owner-occupied vs. rental housing (measured by the average price-rent ratio) along the balanced growth path. As above, these relationships depend on the properties of the ownership premium, $g(q)$; if $g'(q) = 0$, then neither home-ownership nor the price-rent ratio vary across cities, regardless of other parameters.

5.3.1 Median income

Figures 8 and 9 plot home-ownership and the average price-rent ratio, respectively, against median income. Both relationships are increasing in the region depicted. As city incomes rise holding inequality constant, households that own are willing to pay more in expected terms to do so given the higher ownership premia associated with their desired house types. This induces developers and movers to sell rather than rent their vacant houses at a higher rate.

Recall from Figure 5 that the price-rent ratio rises with income. As a result, if the ownership profile were flat, a first-order stochastic increase in the distribution of incomes would raise the overall price-rent ratio. As noted above, however, the share of rentals is proportionally larger for lower income households than for higher income ones. Such shift in the income distribution therefore tends to have a large and positive impact on *both* the average price and the average rent. Under our calibration, the average price-rent ratio varies non-monotonically with median income. Around the (normalized) median city income level, however, the price-rent ratio rises with income as the price effect dominates.
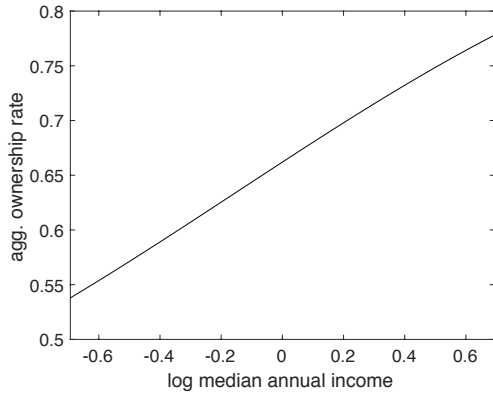
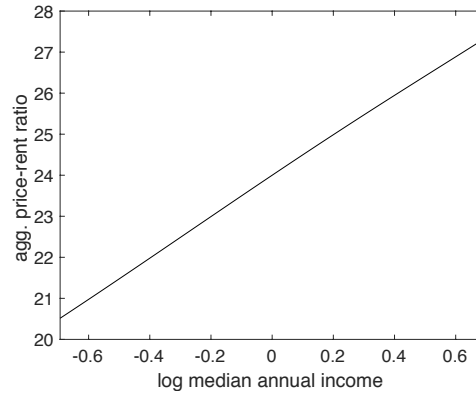Figure 8: Cross-city relationship between home-ownership and median income.

Figure 9: Cross-city relationship between the price-rent ratio and median income.

5.3.2 Inequality

Figures 10 and 11, respectively, plot home-ownership and the average price-rent ratio against the income Gini coefficient, holding median income constant. Due to the relationship between ownership and household income, increasing income inequality, reduces ownership at the bottom by more than it raises it at the top. Thus, overall home-ownership falls as inequality increases.

At the same time, increased income inequality *raises* the average price-rent ratio through a composition effect. Having more low-income households results in more "low-q" houses being built. While this lowers *both* prices and rents, the effect on the former is mitigated by the fact that low income households transact infrequently in the owner-occupied market. Similarly, having more high-income households results in more "high-q" houses being built and drives *up* both prices and rents. In this case, however, the effect on the *latter* is minor as high-income households rarely rent. Overall, the increase in the prices of high quality homes (which are purchased) and the reduction in rents (as low income households rent low quality houses) together result in an increase in the average price-rent ratio.
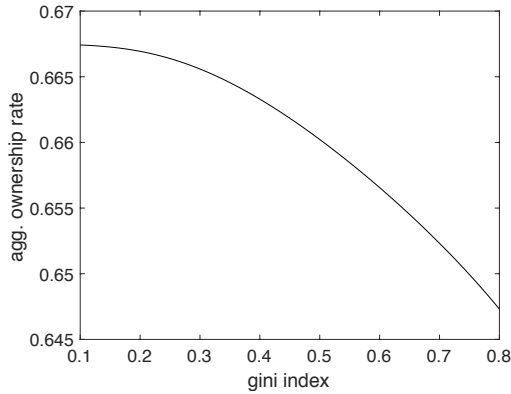
25

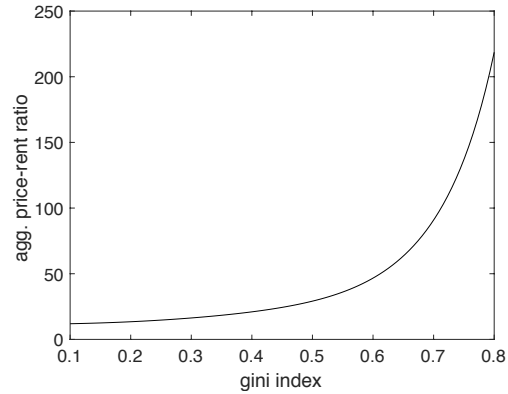Figure 10: Cross-city relationship between home-ownership and inequality.

Figure 11: Cross-city relationship between the price-rent ratio and inequality.

5.3.3 Median Age

To the extent that death rates do not vary much across cities, variation in median age largely reflects variation in entry rates and steady-state population growth. Figures 12 and 13, depict the relationships between median age and home-ownership and the average price-rent ratio, respectively, resulting from variation in the entry rate, ν. Ownership increases monotonically with median age. An older city has a lower rate of entry, and as such a smaller fraction of the population renting while searching for an initial house. This accounts directly for the effect of age on ownership. The impact of median age on the price-rent ratio is small and, while for this calibration it is negative, for others it can be positive.

5.3.4 Construction Costs and City-wide Amenities

The parameter τ represents the cost of building per unit of housing quality. As such, it reflects exogenous city-wide amenities (*e.g.* climate) and costs (*e.g.* regulatory hurdles) as well as endogenous choices of developers (*e.g.* land, size, construction materials, etc.).

Increases in τ lower home-ownership through two channels, and quantitatively the effect can be quite dramatic. First, for a given distribution of income, an increase in τ induces directly all households to choose lower quality housing both as renters and owners. Since the ownership premium is increasing in q, these houses command
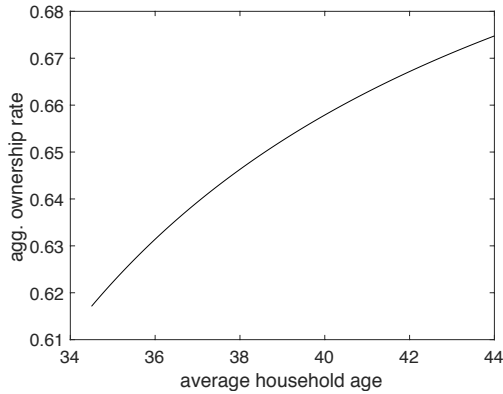
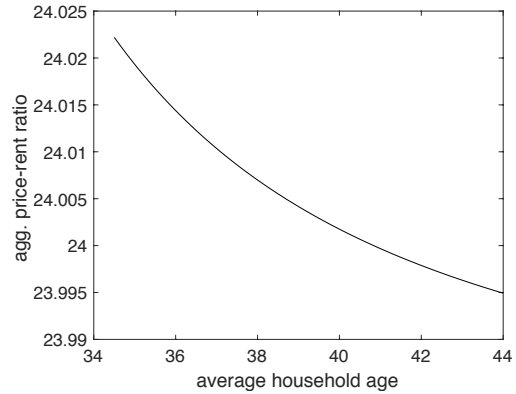Figure 12: Cross-city relationship between home-ownership and median age.

Figure 13: Cross-city relationship between the price-rent ratio and median age.

lower prices relative to rents; thus accounting for a negative relationship between τ and the price-rent ratio. At the same time owners of vacant houses who sell must be compensated for relatively low sales prices with reduced time-to-sell. Given the constant returns in matching, reduced time-to-sell means *increased* time-to-buy, and so throughout the income distribution households rent at a higher rate.

This direct effect is reinforced by households at the low end of the income distribution becoming *permanent renters* in the sense that they do not enter any available sub-market as a buyer. In this case, a minimum size/quality for *owner-occupied* houses will arise endogenously in equilibrium. Owners of vacant houses with q smaller than this will always choose to rent rather than sell, as the sales rate will be insufficient to warrant forgoing immediate rent.

6 Cross-City Variation in the Data and the Theory

We now document observed variation across U.S. cities with regard to home-ownership and the relative costs of owning and renting (price-rent ratios) and consider the extent to which it is associated with variation in median incomes, inequality, age and land values, controlling for several other factors. We then compare the corresponding variation generated by the model to these characteristics of the data.

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6.1   Variation Among a Sample of U.S. Cities

Our base sample consists of the 366 primary MSA's from the 2010 American Community Survey (ACS).[19] For all of these MSA's ownership rates, average price-rent ratios, median incomes, Gini coefficients, median age and population density can be computed from the ACS. Since it affects average lot size, population density should be inversely related average city-wide housing quality.

We take the view that land values are the main source of variation in overall construction costs across cities. Land values are taken from Albouy *et al.* (2017) who compute them at the MSA level using land transactions data, adjusted to account for geographic selection in location and limited sample sizes. Their calculations are based on the 1999 OMB definitions of MSA's. There is not an exact match between MSA's in the two samples for several reasons. For example, there were several new primary MSA's in the 2010 census resulting from population growth, some MSA's were subdivided and others experienced name changes. After matching the MSA's as closely as possible and, where they were subdivided, applying the same land values to each, we were left with 332 MSA's.[20]

To isolate the role of costs per unit of housing quality we must control for average quality (density and amenities). The amenity controls that we use are "natural" amenities taken from Albouy (2016). This data is also based on the 1999 OMB definitions of MSA's. Moreover, in this case some of the observations are for consolidated MSA's, which combine congruent primary MSA's. In order to maximize our sample size, we use the amenity controls computed for the consolidated MSA's to approximate those for each constituent primary MSA.

In the first column of Table 3 the dependent variable is the ownership rate by MSA, computed as the ratio of owner-occupied units to total (owner and renter) occupied units. As predicted by our model, the ownership rate is positively associated with median income and age and negatively associated with inequality and density. These four variables alone account for over 50% of the variation in the ownership rate across cities. In the second column we add land values ($ per acre) and the amenity controls.[21] As predicted by the model, ownership is negatively and significantly associated with land values, controlling for amenities and density.

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[19]An MSA is an urban agglomeration containing at least 50,000 households. These 366 MSA's contain over 83% of US household.

[20]See Appendix C for details.

[21]Full estimation results are provided in Appendix D.

Table 3: U.S. Metropolitan Statistical Areas (2010)

(Median income not adjusted for local non-housing cost of living)

	Ownership Rate		Price-Rent ratio			
			(price-asked)		(estimated-value)	
Log Median Income	0.022	0.076	23.560	14.497	22.141	13.088
Robust s.e.	(0.021)	(0.021***)	(2.458***)	(2.283***)	(1.972***)	(1.653***)
Clustered s.e.	(0.024)	(0.030**)	(2.410***)	(2.833***)	(2.057***)	(2.114***)
Gini index	-0.511	-0.449	89.809	90.286	53.287	65.427
Robust s.e.	(0.116***)	(0.130***)	(13.445***)	(13.677***)	(9.682***)	(8.240***)
Clustered s.e.	(0.155***)	(0.138***)	(18.925***)	(13.395***)	(16.261***)	(8.523***)
Age	0.007	0.007	0.205	0.032	0.239	0.122
Robust s.e.	(0.001***)	(0.001***)	(0.084**)	(0.077)	(.057***)	(0.039***)
Clustered s.e.	(0.001***)	(0.001***)	(0.119*)	(0.083)	(.096**)	(0.044***)
Log Density	-0.028	-0.024	-1.631	-3.585	0.226	-1.447
Robust s.e.	(0.005***)	(0.005***)	(0.679**)	(0.680***)	(0.492)	(0.407***)
Clustered s.e.	(0.009***)	(0.006***)	(0.991)	(0.787***)	(0.947)	(0.336***)
Log Land Value		-0.010		2.680		2.321
Robust s.e.		(0.004**)		(0.606***)		(0.392***)
Clustered s.e.		(0.005**)		(0.736***)		(0.651***)
Amenity Controls	No	Yes	No	Yes	No	Yes
$\overline{R}^2$	0.52	0.63	0.26	0.56	0.42	0.79
# obs	366	332	366	332	366	332

Notes: (1) Sources and definitions of variables may be found in Appendix C.

(2) Standard errors are provided in parenthesis:

heteroskedasticity-robust and clustered at the state level.

(3) *, ** and *** indicate statistical significance at the 10, 5, and 1% levels, respectively.

For the remaining columns of Table 3 the dependent variable is the average price-rent ratio by MSA. In all cases, *rent* refers to the mean gross rent of renter-occupied housing units. In the second two columns *price* refers to the mean price asked for vacant units for sale whereas in the last two columns, it refers to the mean estimated value of all owner-occupied housing units. Again, as predicted by the model, the estimates in the second two columns imply that average price-rent ratios are positively and significantly associated with median income, inequality and land values.[22] The effect of median age is small and less robust.

Table 4 presents the same regressions but where median incomes are adjusted using a local cost of living index (COLI) for non-housing expenditures. This index is not available for every MSA, so this reduces the base sample size to 219 MSA's.

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[22]We also considered controlling for city-size as measured by the total number of households in each MSA. The results are robust to the inclusion of this variable and, while statistically significant in some cases, it does not add any explanatory power to the regressions.

When combined with the land value and amenity data, the sample size is further reduced to 199 MSA's. Nonetheless, the correlations with median income, the Gini index, age and land values remain much the same as in Table 3 and the impacts of controlling for amenities are very similar.

Overall, these results conform very well qualitatively with the cross-city predictions of our model. In Appendix D, we show that this conformity extends to smaller urban areas (micropolitan areas and urban clusters). While these estimates cannot be interpreted as causal, they do suggest that our stylized model replicates qualitatively the patterns observed in the data.

Table 4: U.S. Metropolitan Statistical Areas (2010)

(Median income adjusted for local non-housing cost of living)

	Ownership Rate		Price-Rent ratio			
			(price-asked)		(estimated-value)	
Log Median Income	0.089	0.093	4.613	5.874	7.163	5.945
Robust s.e.	(0.030***)	(0.025***)	(3.582)	(3.020*)	(3.208**)	(2.105***)
Clustered s.e.	(0.031***)	(0.030***)	(2.924)	(3.120*)	(2.527***)	(2.720**)
Gini index	-0.380	-0.448	27.740	56.147	3.119	46.824
Robust s.e.	(0.134***)	(0.160***)	(19.005)	(18.544***)	(14.495)	(11.085***)
Clustered s.e.	(0.157**)	(0.148***)	(25.661)	(17.899***)	(25.131)	(14.960***)
Age	0.006	0.008	0.215	-0.066	0.324	0.097
Robust s.e.	(0.001***)	(0.001***)	(0.118*)	(0.091)	(0.086***)	(.061)
Clustered s.e.	(0.001***)	(0.001***)	(0.133)	(0.101)	(0.118***)	(.062)
Log Density	-0.038	-0.025	1.605	-2.766	2.681	-1.175
Robust s.e.	(0.005***)	(0.005***)	(0.875*)	(0.735***)	(0.754***)	(0.500**)
Clustered s.e.	(0.008***)	(0.006***)	(1.200)	(0.808***)	(1.193**)	(0.468**)
Log Land Value		-0.012		3.851		3.979
Robust s.e.		(0.005***)		(0.685***)		(0.494***)
Clustered s.e.		(0.005***)		(0.769***)		(0.874***)
Amenity controls	No	Yes	No	Yes	No	Yes
$\overline{R}^2$	0.52	0.65	0.07	0.49	0.24	0.77
# obs	219	199	219	199	219	199

Notes: See Table 3

6.2   Cross-city Variation in the Theory

We now use the model to generate predicted cross-city variation in ownership rates and price-rent ratios resulting from the observed variation in the observable MSA-level
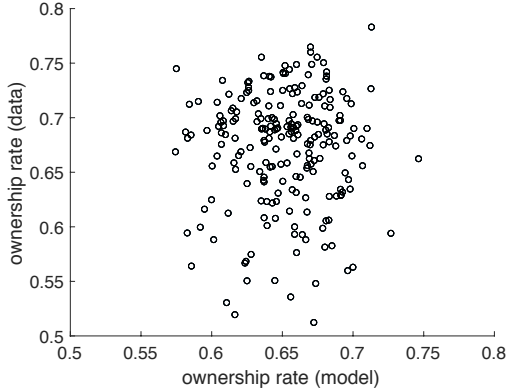
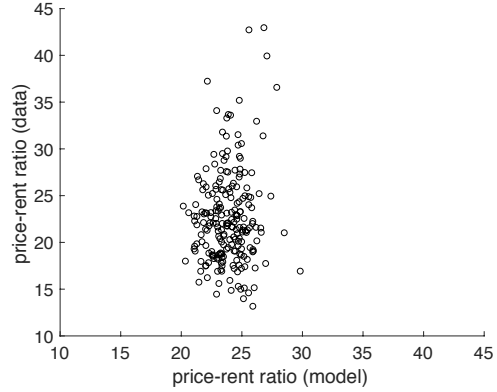Figure 14: Ownership in the data and the model; no variation in costs.

Figure 15: Price-rent ratio in the data and the model; no variation in costs.

characteristics and compare these predictions with their actual values. In our first experiment, we only allow median income, the income Gini coefficient and median age to vary across cities, holding all other parameters fixed at their baseline calibration levels. In a subsequent experiment, we fit values of the cost parameter, τ, city-by-city in order to match jointly the observed variation in home-ownership and average price-rent ratios across cities and evaluate the relationship between these values and actual land costs and amenity measures.

Figures 14 and 15 compare the actual and predicted home-ownership rates and average price-rent ratios for our sample of MSA's. In these figures, the median income, Gini coefficient and average age in the model are set to match those observed for the corresponding city in the 2010 census.[23] Although their variation is similar, the positive correlation between actual and predicted ownership rates is not very strong and overall the model tends to under-predict actual ownership rates. The correspondence between actual and predicted price-rent ratios is even weaker with the variation predicted by the model being much less than that in the data.

We conjecture that the failure of the model to capture the variation observed across cities is driven principally by the assumption that construction/land costs and amenities reflected in τ are common. It is unclear, however, how to incorporate variation in these factors into the model using a single parameter. Instead, we compute city-specific τ's to minimize the weighted distance between actual and predicted

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[23]The sample here is of the 199 U.S. MSA's for with COLI's are available. Average age is controlled by population growth, and is subject to the requirement that growth be neither less than 0% nor greater than 5% annually.
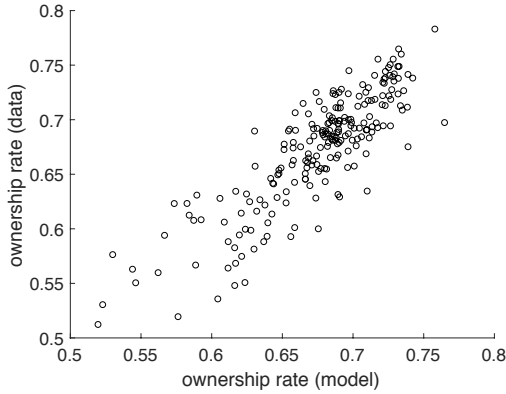
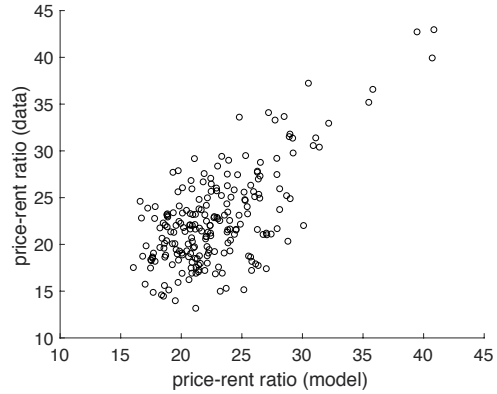Figure 16: Ownership in the data and the model with variation in costs.

Figure 17: Price-rent ratio in the data and the model with variation in costs.

values of home-ownership and the average price-rent ratio, where the weights are inversely proportional to their variance.[24] Figures 16 and 17 depict the results.

Not surprisingly, this procedure generates a much closer correspondence between the actual and predicted outcomes both in terms of variation and correlation. However, this is relevent only to the extent that the inferred values of τ are related to actual construction cost and amenity measures across MSA's. Table 5 documents the results of estimating a least-squares regression across MSA's of our inferred τ's on average land values (from Albouy *et al.* 2017), population density and various natural amenity measures (from Albouy 2016).

The first column of Table 5 includes only measures of natural amenities in the explanatory variables. As may be seen, these are all statistically significant. Adding populations density in the second column significantly increases the explanatory power and so does adding land values as in third column. Both these variables are statistically significant and account for most of the variation in τ. The last column replaces the direct amenity measures with that constructed by Albouy (2016). As may be seen, this more parsimonious representation accounts for just as much of the overall variation as before.

Overall, these results demonstrate that variation in income, inequality and population growth play an important role in determining differences in the ownership rate across cities. Variation in land costs and average amenities, however, are crucial for understanding these relationships, especially with respect to the price-rent ratio.

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[24]For details on the procedure used, see Appendix C.

Table 5: Relationship between Inferred τ's and Land Costs and Amenities

	Log of Inferred τ							
Cdd65	0.058	(0.017***)	0.060	(0.014***)	0.050	(0.014***)	–	–
Cdd65	0.058	(0.017***)	0.060	(0.014***)	0.050	(0.014***)	–	–
Hdd65	0.039	(0.007***)	0.032	(0.006***)	0.019	(0.007***)	–	–
Sun_a	0.451	(0.098***)	0.207	(0.094**)	0.136	(0.088)	–	–
Slope_pct	0.010	(0.003***)	0.011	(0.003***)	0.008	(0.003***)	–	–
Inv_water	0.725	(0.157***)	0.231	(0.164)	0.017	(0.178)	–	–
Latitude_w	0.013	(0.004***)	0.006	(0.003)	0.002	(0.003)	–	–
Log Density	–	–	0.071	(0.010***)	0.048	(0.013***)	0.025	(0.012**)
Log Land value	–	–	–	–	0.058	(0.012***)	0.032	(0.015**)
Log Amenity Index	–	–	–	–	–	–	0.514	(0.112***)
Constant	0.232	(0.169*)	0.077	(0.125)	0.070	(0.137)	0.436	(0.115)
R^2	0.30		0.42		0.47		0.47	
# obs	199		199		199		199	

Notes: Heteroskedasticity-robust standard errors in parentheses.

7   The Affordability of Housing

In the model, everyone can "afford" housing of *some* quality, as any $q > 0$ can feasibly be supplied. Moreover, all households can afford to *buy* housing, although as noted above, some households may not so choose.[25] Nevertheless, in the model the distribution of income and the cost of supplying housing interact to determine the type of housing that households at various positions in the income distribution can afford either to rent or to buy. Exogenous wealth inequality generates inequality of well-being, in part attributable to differential access to housing. As such, redistribution can be beneficial depending on one's definition of social welfare. In order to study these issues, we consider the implications of policies designed to increase the *affordability* of housing, using a standard definition taken from the National Association of Realtors (NAR).

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[25]In the absence of supply constraints, these "permanent renters" will be only those with incomes below a particular cutoff level which depends on parameters. In the presence of supply constraints, however, examples may be constructed in which households in any region of the income choose to rent permanently.

7.1 The Housing Affordability Index (HAI)

The Housing Affordability index (HAI) is defined as:

$$\text{HAI} = \frac{y_m}{y_q} \times 100; \tag{42}$$

where y_m denotes median (annual) household income,[26] and y_q denotes *qualifying* income.[27] The latter is taken to represent the annual income flow required to afford, "reasonably", the mortgage payment on the median-priced home.[28]

Our notion of "reasonableness", with regard to affording a mortgage payment, is based on the following assumptions:

(i) The household makes a 20% down-payment. Of course, in our model, households need not do this, and they will not given the concavity of utility and the existence of complete financial markets.

(ii) The mortgage interest rate is 4% annually; the monthly rate is thus $i_m = \frac{.04}{12}$.

(iii) The amortization period is 30 years or 360 months.

(iv) The household spends no more than 25% of its income on its mortgage payment.

Let p_m denote the price of a house median quality (equivalently in the model, the median price of a house), then we have:

$$y_q = 4 \times \underbrace{\frac{(.8p_m)i_m}{1 - \left(\frac{1}{1+i_m}\right)^{360}}}_{\text{monthly payment}} \times 12. \tag{43}$$

A lower HAI indicates that housing is *less* affordable.

In our baseline calibration the HAI = 189. That is, a household with median income has a quarterly income flow 1.89 times that of the qualifying income. Using the values of τ fitted to match the variation in construction/land costs across U.S. MSA's (see Figures 16 and 17 in Section 6), the HAI ranges from $\underline{\text{HAI}} = 131$ to $\overline{\text{HAI}} = 250$. Figure 18 depicts a kernel density estimate for the distribution of HAI's

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[26] As we assume a log-normal distribution of income, y, with mean μ, and interpret it as a quarterly flow, $y_m = 4e^{\mu}$.

[27] Note that this HAI measures, specifically, the affordability of *home-ownership*.

[28] The method used by the NAR in constructing HAI's for various housing markets is described online at: https://www.nar.realtor/topics/housing-affordability-index/methodology.
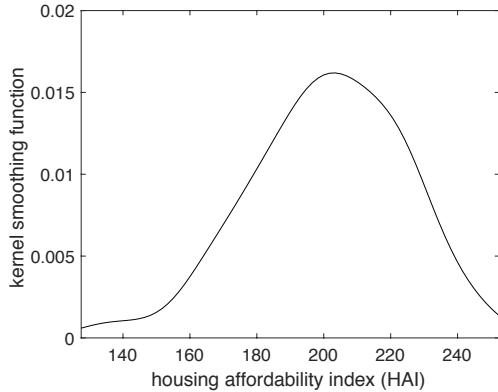
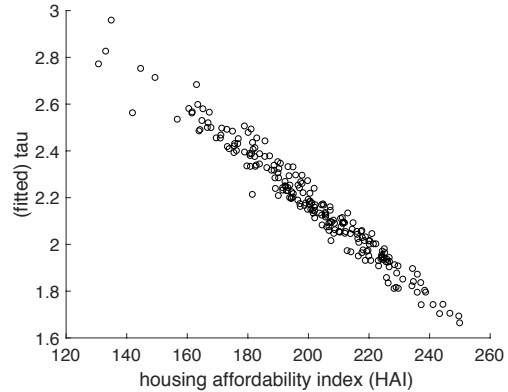Figure 18: Housing affordability across cities (kernel density estimate).

Figure 19: Fitted measure of construction costs and housing affordability.

across the 199 cities in our sample. Figure 19 plots the HAI for each of these cities against the fitted value of τ. Clearly, housing affordability in the model is closely related negatively to the cost of producing a unit of housing quality.

In the model, city-level housing affordability matters significantly for *home-ownership* for lower income households only. Figure 20 plots home-ownership rates for households in the 20th, 50th and 80th income percentiles against the HAI for each city in the sample. In the figure it is clear that affordability is effectively uncorrelated with home-ownership for households in the 80th percentile, and shows only a slight positive correlation for those in the 50th. For households in the bottom quintile, however, the correlation of affordability with ownership is striking.

7.2   A Policy to Improve Housing Affordability

We now consider the effects of a simple policy designed to increase the affordability of housing, with emphasis on houses targeted by low income households. We have in mind a policy through which government subsidies reduce the cost of producing housing in the market segments targeted by these households. Specifically, consider a tax/subsidy scheme that changes the cost of construction from $T(q) = \tau q$ to

$$\hat{T}(q) = [\hat{\tau}_0 + \hat{\tau}_1(q - \hat{q})]\,\tau q \tag{44}$$

where $\hat{\tau}_0 \leq 1$, $\hat{\tau}_1 > 0$, and $\hat{q} > 0$. Under the policy, a developers building houses of quality $q < (1 - \hat{\tau}_0)\hat{q}/\hat{\tau}_1$ receive subsidies, while those building houses of quality $q >$
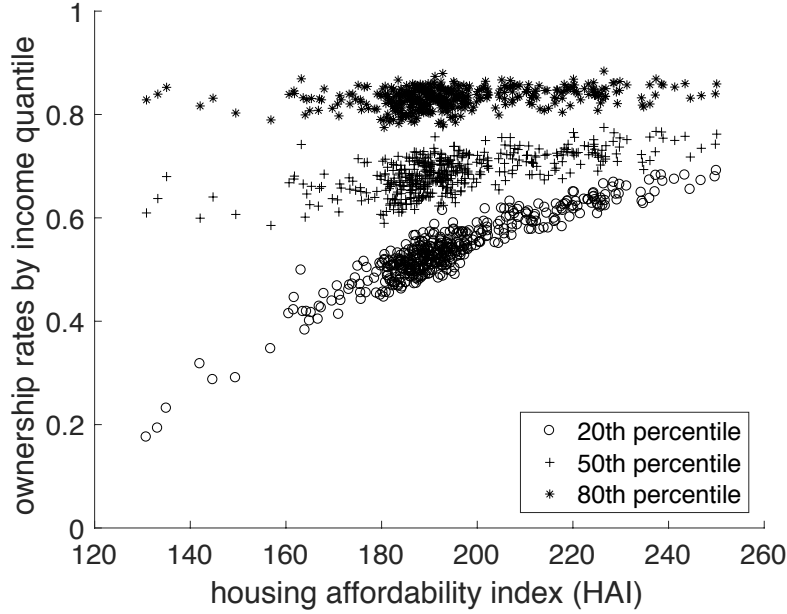
Figure 20: Ownership across cities by income quantile.

$(1-\hat{\tau}_0)\hat{q}/\hat{\tau}_1$ are taxed. This policy is chosen not to mimic any specific policy observed, but rather to illustrate the implications of policies of a general form. The construction cost changes it effects are passed through to home buyers through competition in the directed search equilibrium.

We think of (44) as representative of a number of different possible policies designed to make housing more affordable in the sense of raising the HAI. For example, alternatively we could specify a subsidy to construction overall, coupled with an appropriately progressive property tax on both homeowners and rental properties.[29] That said, it is not representative of all policies that change the relative costs of supplying and/or occupying houses of different qualities. For example, the policy treats rental and owner-occupied houses symmetrically, and as such neither encourages nor discourages home-ownership directly. In any case, the particular allocation of consumption, housing quality, ownership rates, etc. that arises in the presence of the policy clearly could be implemented via a number of different policies with observationally equivalent results.

Now, consider a relatively "unaffordable" city with a construction cost parameter

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[29]A proportional income tax will make no difference; such a tax is effectively already present in the baseline model.
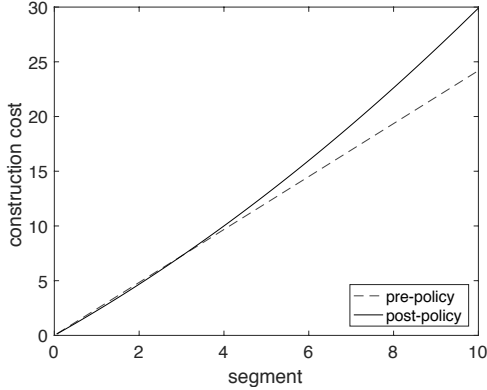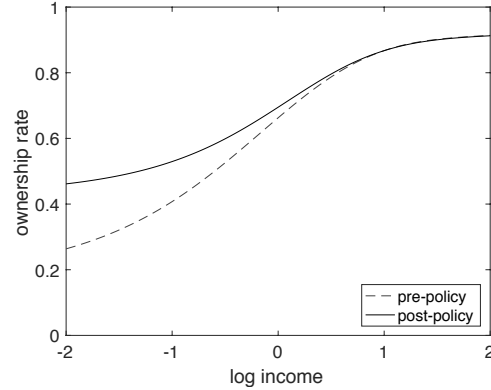
Figure 21: Construction costs.

Figure 22: Ownership rate vs. income.

$\tau = 2.4193$. This corresponds to an HAI one standard deviation below the average for the 199 simulated MSA's in Section 6.2. Specifically, the city has an HAI = 178.1923 whereas the average HAI = 200.9869. We construct a policy of the form (44) that

(i) raises the HAI for the city to the *average* affordability, and

(ii) balances the budget at the city level. That is, the total taxes collected in equilibrium equal the total subsidies paid out.

The following policy satisfies these requirements in the stationary equilibrium:

$$\{\hat{\tau}_0, \hat{\tau}_1, \hat{q}\} = \{0.9296, 0.0338, 0.9240\}. \tag{45}$$

Construction costs with and without the affordability policy are displayed in Figure 21. As may be seen, under this particular policy, the effective costs of relatively high quality housing must be increased substantially in order to achieve somewhat lower costs of relatively low quality housing. This reflects first the fact that given the distribution of households across housing quality levels (which depends on the distribution of income) the majority of households are effectively subsidized while relatively few are taxed. Second, it reflects, as we will see, substitution of relatively high income households into lower quality housing units.

The affordability policy affects the levels and distributions of home-ownership, housing quality, consumption, and welfare. It generally redistributes from upper to lower income households. It has, however, effects throughout the distribution.

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7.2.1 Home-ownership

While the net effect on construction costs is small (as the budget is balanced) overall home-ownership rises significantly, from 63.41% in the baseline to 68.36% under the policy. This occurs in spite of the fact that the policy does not favor home-ownership *per se*. The increase in home-ownership is concentrated overwhelmingly in the bottom half of the income distribution, as can be seen in Figure 22.

The increase in home-ownership is driven by the behavior of sellers. The cost of putting a house up for sale is forgone rent, which is proportional to the construction cost, and thus is changed directly by the policy. The benefit of selling lies in capturing the value of the ownership premium. For low quality housing, this premium is small and importantly, does not change much with q (see Figure 6). Consequently, even a small reduction in the construction cost lowers the rent sufficiently to induce a significant increase in the rate at which vacant houses of a wide range of qualities are put on the market for sale. This, together with the fact that significant share of the population lives in rental housing of these types, leads to a large increase in home-ownership for this segment of the population.

In contrast, houses occupied by above median income households are sold (rather than rented) at high rates even in the baseline. The policy *increases* rents on these houses, reducing the incentive to offer them for sale when vacant. Market tightness rises, sending prospective buyers into lower q sub-markets. This reduces the *quality* of their housing, but has only very minor effects on their rates of ownership.

7.2.2 Housing Quality and Costs

While the affordability policy changes home-ownership significantly only for households in the bottom 50% of the income distribution, it affects housing quality and both prices and rents throughout the distribution. Table 6 documents the average quality of housing consumed by a "representative" household at the midpoint of each quintile of the income distribution both before and after the policy change. As can be seen, the average quality of housing for the representative household rises for the bottom four quintiles and falls for the top quintile. The decline in housing quality is large for households in the top quintile.

The effects of the policy are not surprising given that it affects directly the construction cost and hence the rent and return to selling differentially for low and high
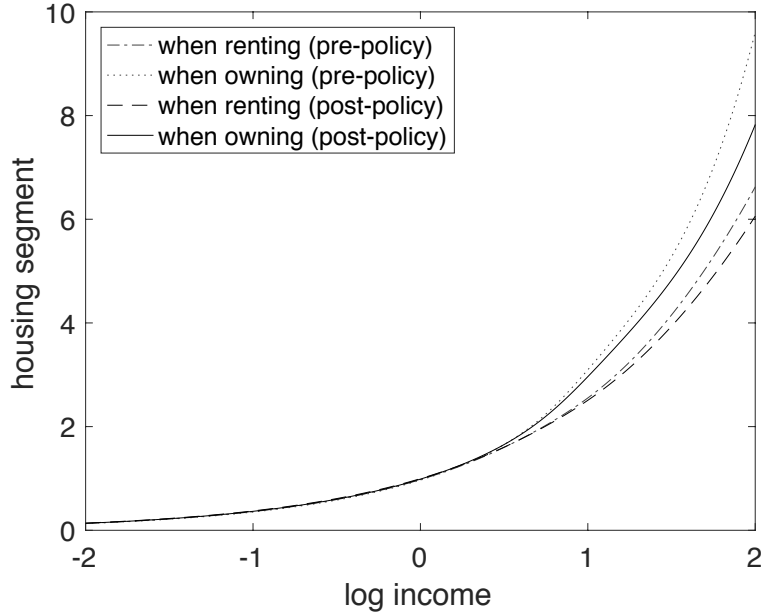
Figure 23: Market segment vs. income by housing tenure.

quality houses. As noted above, the rents on high quality houses (inhabited by high income households) rise, sending them into lower sub-markets as both renters (due to the direct increase in rent) and as prospective owners (due to the increase in tightness associated with diminished returns to selling in this quality range). Similar forces are at work (in reverse) for lower quality houses which see their construction costs reduced; rents fall, inducing lower income households to rent better houses and to search in higher quality sub-markets where tightness has declined.

Overall, the city-wide average quality of rented housing rises while that of owner-occupied housing declines. The former effect reflects mainly the increase in housing quality for low income households who rent at a high rate. The latter reflects the combined effects of the increase in ownership (of low quality housing) by low income households and the decline in quality for high income ones.

The affordability policy lowers the average sale price from 4.69 to 4.23, by reducing the cost of building low-quality homes. The policy, however, *raises* average rent slightly (from 0.18 to 0.19) by raising the average quality of homes rented, most commonly by low-income households. The policy reduces the average price/rent ratio from 25.81 to 21.84 and, as intended, raises the HAI from 178.19 to 200.99.

Table 6: Housing Quality under the Policy to Improve Housing Affordability

Household Income	q When Renting		q When Owning		Average q	
	Pre-	*Post-*	*Pre-*	*Post-*	*Pre-*	*Post-*
$F^{-1}(0.1)$	0.3354	0.3482	0.3282	0.3410	0.3326	0.3445
$F^{-1}(0.3)$	0.6313	0.6498	0.6196	0.6384	0.6250	0.6429
$F^{-1}(0.5)$	0.9769	0.9956	0.9669	0.9865	0.9703	0.9893
$F^{-1}(0.7)$	1.5048	1.5118	1.5397	1.5456	1.5318	1.5383
$F^{-1}(0.9)$	2.7227	2.6637	3.3545	3.1949	3.2752	3.1279
Agg. Average q	0.8730	0.9310	1.8935	1.6907	1.5213	1.3286

7.2.3 Consumption and Welfare

The affordability policy affects the consumption of both goods and housing services for all households. The bottom row of Table 7 measures the impact on welfare for the median household in each quintile using the percentage change in goods consumption that would have the effect on household utility equivalent to that of the policy. Overall, the policy increases welfare for the median household in each of the bottom four quintiles while reducing welfare for that of the top quintile. The percentage welfare gain is largest for the bottom quintile and falls as income rises.

The first seven rows of Table 7 report the welfare effects of the affordability policy emanating from changes in goods consumption (row one), consumption of housing services (overall, row two; while renting and owning respectively, rows three and four) and home-ownership (overall, row six; due to the ownership premium itself and the rate of ownership respectively, rows seven and eight).

For households in the bottom three quintiles, welfare gains come mainly from increases in housing quality. This is true even for the bottom quintile which experiences a large increase in home-ownership. Given the quality of the houses these households own, home-ownership *per se* does not contribute much to welfare.

For households in the top quintile, the effects of reduced housing quality are paramount. For these households reduced quality lowers welfare not only directly, but also through a reduced ownership premium, which of course depends on $g(q)$ as specified in (39) with the parameters of the baseline calibration. The overall welfare

losses to these households are mitigated significantly, however, by a relatively large increase in goods consumption. The results for the second quintile diverge from those of the first in that they gain from the affordability policy overall; and from those for the bottom three quintiles in that their gains are driven largely by increased goods consumption. These households experience a negligible effect on home-ownership and only a very small increase in housing quality.

Table 7: Percentage Welfare Effects (Expressed in Consumption Equivalents)

| Post-Policy Outcome(s) | Household Income | | | | |
	$F^{-1}(0.1)$	$F^{-1}(0.3)$	$F^{-1}(0.5)$	$F^{-1}(0.7)$	$F^{-1}(0.9)$
Consumption	0.1095	0.2017	0.3067	0.4922	1.3433
Housing Services	0.9731	0.7537	0.5036	0.1037	-1.1362
Quality While Renting	0.6366	0.3811	0.1897	0.0322	-0.0861
Quality While Owning	0.3343	0.3711	0.3133	0.0714	-1.0510
Home Ownership	0.0087	0.0154	0.0225	0.0246	-0.2862
Utility Premium	0.0005	0.0020	0.0047	0.0036	-0.2802
Ownership Rate	0.0080	0.0130	0.0176	0.0210	-0.0063
All Post-Policy Outcomes	1.0955	0.9743	0.8356	0.6209	-0.0945

8 Conclusion

We have developed a theory of the distribution of housing tenure based on search and frictional assignment. Vacant housing units that are supplied to the rental market can be occupied immediately whereas those supplied to the ownership market take time to sell because of the difficulty of matching them with buyers. Although all households have a preference for owning, many end up renting because they are unable to find an affordable home to buy in their preferred market segment. Higher income households demand higher quality houses and, due to complementarity between ownership and quality, are willing to pay more to own them. Consequently, developers and other owners of high quality homes are more likely to offer them for sale than are sellers of lower quality houses.

The model can be calibrated to replicate relevant features of the median U.S. city's

housing market; specifically, the average time to sell, ownership duration, price-rent ratio and distribution of ownership by income quintile. In the calibrated model, within the city average cost of renting vs. owning rises with income for individual households and by quality for houses of the same type. Across cities, we find that variation in median income, inequality, average age (population growth) and land/amenity values account well qualitatively, for variation in both rates of ownership and average price-rent ratios observed across U.S. MSA's.

We then use the model to study the effects of policies designed to increase the affordability of housing. Specifically, we consider the implications of a policy which taxes construction of high quality housing and uses the proceeds to subsidize that of low quality housing in a way which lowers the cost of the median quality house. The main impacts of such a policy are (1) to raise the quality of housing for low income households and lower it for high income ones and (2) to raise the rate at which housing is sold rather than rented to low income households. The particular policy modeled increases the welfare of households in the bottom four quintiles at the expense of those in the top quintile.

In future work, we intend to relax some of the theory's limiting assumptions. In particular, that household incomes are certain and permanent, moves both within the city and to/from outside are exogenous and that there is no aggregate uncertainty. At the same time, we plan to consider empirical applications of the current environment which take account of specific amenities exhibited and constraints existing in particular cities.

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A Additional Details for the Simplified Model

A.1 A directed search equilibrium

Definition 2. *A Directed Search Equilibrium is a household value $W(y)$ and associated optimal choices, $q_R(y)$, $q_S(y)$ and $p(y)$, for all $y \in [\underline{y}, \bar{y}]$; values $V(q)$ for all house types, $q \in \mathbb{R}_+$; collections of functions for tightness, $\theta(q, p)$, and rents $x(q)$; and measures of houses, renters and homeowners, $m(q)$, $r(y)$ and $n(y)$ such that:*

1. *$W(y)$, $q_R(y)$, $q_S(y)$ and $p(y)$ satisfy the household Bellman equation, (7) and (8), (9) and (10).*

2. *There is free entry into new housing construction and rental markets: that is, $V(q)$ and $x(q)$ satisfy $V(q) = x(q) = \tau q$.*

3. *The tightness functions reflect optimal price posting strategies by sellers in all sub-markets of each segment: that is, $\theta(q, p)$ satisfies (6).*

4. *For all $q \in \mathbb{R}_+$ and y in the support of F:*

$$r(y) = (1 - \lambda(\theta(q_S(y), p(y)))) f(y) \tag{A.1}$$

$$n(y) = \lambda(\theta(q_S(y), p(y))) f(y) \tag{A.2}$$

$$m(q) = r(q_R^{-1}(q)) + \frac{f(q_S^{-1}(q))}{\theta(q, p(q_S^{-1}(q)))} \tag{A.3}$$

A.2 Proof of Proposition 1

(i) This follows directly from (8) and (9).

(ii) Using the free entry condition, (6), the seller's matching probability is given by:

$$\gamma(\theta(y)) = \frac{\tau q_S(y)}{p(y)} = \Delta y^{-\frac{\psi}{1+\psi}}, \tag{A.4}$$

which is decreasing in y. Given the Cobb-Douglas matching function, (3), it follows that tightness in the sub-market target by a household of income y is

$$\theta(y) = \left(\frac{\Delta}{\kappa}\right)^{\frac{1}{\eta}} y^{-\frac{\psi}{\eta(1+\psi)}}, \tag{A.5}$$

and the matching probability for buyers is then increasing in y:

$$\lambda(\theta(y)) = \kappa^{\frac{1}{\eta}} \Delta^{\frac{\eta-1}{\eta}} y^{\frac{(1-\eta)\psi}{\eta(1+\psi)}} \tag{A.6}$$

(iii) The income of a household that searches for a house to buy of quality q_S can be derived from (9) as

$$y(q_S) = \left[\frac{(1+\alpha\psi)\,\tau q_S}{\alpha(1+\psi)\Delta} \right]^{1+\psi} \tag{A.7}$$

It follows from (10) that the price of a house of quality q_S is

$$p(q_S) = \left[\frac{1+\alpha\psi}{\alpha(1+\psi)} \right]^{\psi} \left[\frac{\tau q_S}{\Delta} \right]^{1+\psi}. \tag{A.8}$$

The price-rent ratio for such a house is then

$$\frac{p(q_S)}{x(q_S)} = \frac{p(q_S)}{\tau q_S} = \left[\frac{1}{\Delta} \right]^{1+\psi} \left[\frac{(1+\alpha\psi)\,\tau}{\alpha(1+\psi)} \right]^{\psi} q_S^{\psi}. \tag{A.9}$$

which is increasing in q_S.

A.3   Proof of Proposition 2

The city-wide home-ownership rate is:

$$N = \int_{\underline{y}}^{\bar{y}} \lambda(\theta(y))dF(y) = \kappa^{\frac{1}{\eta}} \Delta^{\frac{\eta-1}{\eta}} \int_{\underline{y}}^{\bar{y}} y^{\frac{(1-\eta)\psi}{\eta(1+\psi)}} dF(y) \tag{A.10}$$

where $\theta(y)$ is used as more concise notation for $\theta(q_S(y), p(y))$. If $(1-\eta)\,\psi < \eta\,(1+\psi)$, then N is the expected value of an increasing, concave function of income. Therefore aggregate ownership is increasing in average income and will decrease with a mean-preserving increase in inequality.

The average sale price and the average rental payment are

$$P = \frac{1}{N} \int_{\underline{y}}^{\bar{y}} p(y)\lambda(\theta(y))dF(y) = \frac{\alpha(1+\psi)}{1+\alpha\psi} \left[\frac{\int_{\underline{y}}^{\bar{y}} y^{1+\frac{(1-\eta)\psi}{\eta(1+\psi)}} dF(y)}{\int_{\underline{y}}^{\bar{y}} y^{\frac{(1-\eta)\psi}{\eta(1+\psi)}} dF(y)} \right] \tag{A.11}$$

$$X = \frac{1}{1-N} \int_{\underline{y}}^{\bar{y}} x(q_R(y)) \left[1 - \lambda(\theta(y))\right] dF(y) = \frac{1}{1-N} \left[\frac{\alpha\bar{Y}}{\tau} - \frac{(1+\alpha\psi)PN}{\tau(1+\psi)} \right]. \tag{A.12}$$

The numerator of the multiplicand in (A.11) is the expected value of an increasing convex, function of income. This dominates the denominator, and so the average sale price increases with average income. Similarly, because the numerator dominates the denominator, the average price increases with a mean-preserving increase in income inequality. The effect of an increase in average income on rent is ambiguous, as it depends on the response the response of the rate of home-ownership by income, $\lambda(\theta(y))$. The average rent paid will, however, fall with a mean-preserving increase in income inequality, as N increases, and PN decreases. It follows that the aggregate price-rent ratio inceases with a mean-preserving increase in income inequality.

B Full Model Details

B.1 The consumption-saving decision

Owners issue claims to achieve zero wealth in the event of death: $w'_D = -V(q_S)$. These claims work essentially as a reverse mortgage. The intertemporal Euler equations are

$$\phi_R(q_S, p) = \beta(1 - \lambda(\theta(q_S, p)))\frac{c_R}{c'_R} \tag{B.1}$$

$$\phi_B(q_S, p) = \beta\lambda(\theta(q_S, p))\frac{c_R}{c'_B} \tag{B.2}$$

$$\phi_N = \beta(1 - \pi)\frac{c_N}{c'_N} \tag{B.3}$$

$$\phi_S = \beta\pi\frac{c_N}{c'_S} \tag{B.4}$$

The no-arbitrage conditions are:

$$\phi_N = \beta(1 - \pi) \tag{B.5}$$

$$\phi_S = \beta\pi \tag{B.6}$$

$$\phi_D = \frac{\beta\delta}{1 - \delta} \tag{B.7}$$

$$\phi_R(q, p) = \beta(1 - \lambda(\theta(p; q))) \tag{B.8}$$

$$\phi_B(q, p) = \beta\lambda(\theta(p; q)) \tag{B.9}$$

$$\phi_D(q, p) = \frac{\beta\delta\lambda(\theta(p; q))}{1 - \lambda} \tag{B.10}$$

The Euler equations then imply a constant consumption stream. To achieve this, wealth positions, $\{w_R, w_B, w_N, w_S\}$, must satisfy $w_B = w_N + p$, $w_S = w_R - V(q_S)$ and the following budget constraints:

$$c + \beta\lambda(\theta)\left[w_N + p\right] + \beta(1 - \lambda(\theta))w_R + \frac{\beta\delta\lambda(\theta)}{1 - \delta}\left[p - V_S\right] + x_R = w_R \tag{B.11}$$

$$c + \beta\pi\left[w_R - V_S\right] + \beta(1 - \pi)w_N - \frac{\beta\delta}{1 - \delta}V_S + \zeta q = w_N \tag{B.12}$$

For convenience, the notation here has been simplified by replacing, for example, $\theta(q_S, p)$ with θ, $V(q_S)$ with V_S, and $x(q_R)$ with x_R. Combining both budget constraints to eliminate c yields

$$w_R - w_N = V_S + \frac{\beta\lambda(\theta)(p - V_S) + (1 - \delta)(x_R - x_S)}{(1 - \delta)\left[1 - \beta\left(1 - \pi - \lambda(\theta)\right)\right]}. \tag{B.13}$$

This relationship and a budget constraint determine the level of consumption:

$$c = (1 - \beta)w_R - x_R - \frac{\beta\lambda(\theta)}{1 - \delta}\left(\frac{\left[1 - \beta(1 - \pi)\right](p - V_S) - (1 - \delta)(x_R - x_S)}{1 - \beta\left(1 - \pi - \lambda(\theta)\right)}\right). \tag{B.14}$$

A household with permanent income y initially enters the city as a renter with wealth equal to the present discounted value of lifetime income: $w_R = y/(1 - \beta)$. The constant consumption level for this household is therefore

$$c = y - \frac{\left[1 - \beta(1 - \pi)\right]x_R + \beta\lambda(\theta)x_S}{1 - \beta\left(1 - \pi - \lambda(\theta)\right)} - \frac{\beta\lambda(\theta)\left[1 - \beta(1 - \pi)\right](p - V_S)}{(1 - \delta)\left[1 - \beta\left(1 - \pi - \lambda(\theta)\right)\right]}. \tag{B.15}$$

Note that (B.15) is (29) in the text. The financial wealth of a household changes with every transition to a different ownership status and in the event of death according to

$$w_R = \frac{y}{1 - \beta} \tag{B.16}$$

$$w_B = \frac{y}{1 - \beta} + p - V_S - \frac{\beta\lambda(\theta)(p - V_S) + (1 - \delta)(x_R - x_S)}{(1 - \delta)\left[1 - \beta\left(1 - \pi - \lambda(\theta)\right)\right]} \tag{B.17}$$

$$w_N = \frac{y}{1 - \beta} - V_S - \frac{\beta\lambda(\theta)(p - V_S) + (1 - \delta)(x_R - x_S)}{(1 - \delta)\left[1 - \beta\left(1 - \pi - \lambda(\theta)\right)\right]} \tag{B.18}$$

$$w_S = \frac{y}{1 - \beta} - V_S \tag{B.19}$$

$$w_D = -V_S \tag{B.20}$$

$$\tag{B.21}$$

B.2 The search decision

Households direct their search by choosing a particular sub-market (price) and market segment (quality). The search decision appears in the Bellman equation for a household that is currently renting:

$$W^R(w) = \max_{q_S,p} \left\{ \begin{array}{l} (1-\alpha)\ln c + \alpha \ln q_R \\ +\beta\lambda(\theta(q_S,p))W^N(q_S, w_B - p) \\ +\beta\left[1 - \lambda(\theta(q_S,p))\right]W^R(w_R) \end{array} \right\} \tag{B.22}$$

subject to

$$c = w - x(q_R) - \beta\lambda(\theta(q_S,p))w_B - \beta(1 - \lambda(\theta(q_S,p)))w_R - \frac{\beta\delta\lambda(\theta(q_S,p))}{1-\delta}(p - V(q_S))$$

and

$$\beta\gamma(\theta(q_s,p))\left[p - V(q_S)\right] = (1 - \delta - \beta)V(q_S). \tag{B.23}$$

where the latter constraint imposes that the searching household correctly anticipates that market tightness, θ, is determined by the free entry of sellers according to (23). By substituting this constraint into the household's objective function, the optimal choice of segment and sub-market solve the following:

$$W^R(w) = \max_{q_S,\theta} \left\{ \begin{array}{l} (1-\alpha)\ln c + \alpha \ln q_R + \beta\left[1 - \lambda(\theta)\right]W^R(w_R) \\ +\beta\lambda(\theta)W^N\left(q_S, w_B - V(q_S) - \frac{1-\delta-\beta}{\beta\gamma(\theta)}V(q_S)\right) \end{array} \right\} \tag{B.24}$$

subject to

$$c = w - x(q_R) - \beta\lambda(\theta)w_B - \beta(1 - \lambda(\theta))w_R - \frac{\delta(1-\delta-\beta)}{\theta(1-\delta)}V(q_S).$$

The first order condition with respect to q_S is

$$\left(\frac{1-\alpha}{c}\right)\frac{\delta[1-\delta-\beta]V'(q_S)}{1-\delta} + \beta\gamma(\theta)\left[\frac{\partial W^N}{\partial q} - \frac{\partial W^N}{\partial w}\left(1 + \frac{1-\delta-\beta}{\beta\gamma(\theta)}\right)V'(q_S)\right] = 0 \tag{B.25}$$

and the first order condition with respect to θ is

$$\beta\left[\frac{1-\alpha}{c}(w_R - w_B) + W^N - W^R\right]\frac{\theta\lambda'(\theta)}{\lambda(\theta)}$$
$$+ \frac{[1-\delta-\beta]V(q_S)}{\gamma(\theta)}\frac{\theta\gamma'(\theta)}{\gamma(\theta)}\frac{\partial W^N}{\partial w} + \left(\frac{1-\alpha}{c}\right)\frac{\delta[1-\delta-\beta]V(q_S)}{(1-\delta)\gamma(\theta)} = 0. \tag{B.26}$$

The value associated with home-ownership satisfies

$$W^N(q_S, w) = (1 - \alpha)\ln c + \alpha h(1, q_s) + \beta\pi W^R(w_s + V(q_S)) + \beta(1 - \pi)W^N(q_S, w_N) \tag{B.27}$$

subject to

$$c = w + \beta\pi w_s - \beta(1 - \pi)w_N + \frac{\beta\delta}{1 - \delta}V(q_S) - \zeta q_S.$$

The Benveniste-Scheinkman conditions are therefore

$$\frac{\partial W^N(q_S, w)}{\partial w} = \frac{1 - \alpha}{c} \tag{B.28}$$

$$\frac{\partial W^N(q_S, w)}{\partial q} = \frac{1}{1 - \beta(1 - \pi)} \left\{ \begin{array}{l} \alpha\dfrac{\partial h(1, q_S)}{\partial q} \\[2mm] + \dfrac{1 - \alpha}{c}\left[\left(\beta\pi + \dfrac{\beta\delta}{1 - \delta}\right)V'(q_S) - \zeta\right] \end{array} \right\} \tag{B.29}$$

Substituting (B.23), (B.28) and (B.29) into first order conditions (B.25) and (B.26) yields

$$\left(\frac{1 - \alpha}{c}\right)\left[\frac{p}{(1 - \delta)V(q_S)} - \frac{\delta}{1 - \delta}\right]V'(q_S)$$
$$= \frac{1}{1 - \beta(1 - \pi)}\left\{\alpha\frac{\partial h(1, q_S)}{\partial q} + \frac{1 - \alpha}{c}\left[\left(\beta\pi + \frac{\beta\delta}{1 - \delta}\right)V'(q_S) - \zeta\right]\right\} \tag{B.30}$$

and

$$\left[\frac{1 - \alpha}{c}(w_R - w_B) + W^N - W^R\right](1 - \eta(\theta)) = \frac{1 - \alpha}{c}\left[\eta(\theta) + \frac{\delta}{1 - \delta}\right](p - V(q_S)) \tag{B.31}$$

where $\eta(\theta) = \theta\gamma'(\theta)/\gamma(\theta) = \theta\lambda'(\theta)/\lambda(\theta) + 1$. Condition (B.30) simplifies to

$$\left(\frac{1 - \alpha}{c}\right)\left[\frac{[1 - \beta(1 - \pi)]p - (\beta\pi + \delta)V(q_S)}{(1 - \delta)V(q_S)}V'(q_S) + \zeta\right] = \alpha\frac{\partial h(1, q_S)}{\partial q}.$$

With substitutions from (B.16), (B.17), (B.24), (B.27), condition (B.31) simplifies to

$$p - V(q_S) = \frac{(1 - \delta)(1 - \eta(\theta))}{1 - \beta(1 - \pi - \eta(\theta)\lambda(\theta))}\left[x(q_R) - x(q_S) + \frac{\alpha[h(1, q_S) - h(0, q_R)]}{(1 - \alpha)/c}\right].$$

These correspond to conditions (31) and (32) in the text.

C Data Definitions, Sources and Calculations

C.1 Definitions and sources

All data on housing and households comes from the 2010 American Community Survey (ACS, 5-year estimates) accessed via the Census Bureau at http://factfinder.census.gov.

House price: To compute mean house prices by urban area, we use the data on housing value obtained from Housing Question 16 in the ACS. The question was asked at housing units that were owned, being bought, vacant for sale, or sold and not occupied at the time of the survey. The estimated value is the respondent's estimate of how much the property (house and lot, mobile home and lot, or condominium unit) would sell for if it were for sale. We also used the average price-asked on vacant housing for sale only and on housing sold but unoccupied. This was calculated by dividing the aggregate price-asked by the number for sale and sold but unoccupied.

Rent: To compute mean rent for each urban area, we use the data on gross rent obtained from answers to Housing Questions 11a-d and 15a in the ACS. Gross rent is the contract rent plus the estimated average monthly cost of utilities if these are paid by the renter.

Housing tenure: To compute the number of owning and renting households we use data obtained from Housing Question 14 in the ACS. The question was asked at occupied housing units. Occupied housing units are classified as either owner occupied or renter occupied:

Owner occupied – A housing unit is owner occupied if the owner or co-owner lives in the unit even if it is mortgaged or not fully paid for.

Renter occupied – All occupied housing units which are not owner occupied, whether they are rented or occupied without payment of rent, are classified as renter occupied.

Households – the number of households was computed as the sum of owner and renter occupied units.

Vacant housing units – A housing unit is vacant if no one is living in it at the time of interview. Units occupied at the time of interview entirely by persons who are staying two months or less and who have a more permanent residence elsewhere are considered to be temporarily occupied, and are classified as "vacant." New units not

yet occupied are classified as vacant housing units if construction has reached a point where all exterior windows and doors are installed and final usable floors are in place.

Median household income: The data on income during the last 12 months were derived from answers to Questions 47 and 48, which were asked of the population 15 years old and over. Household income includes the income of the householder and all other individuals 15 years old and over in the household. "Total income" is the sum of the amounts reported separately for wage or salary income; net self-employment income; interest, dividends, or net rental or royalty income or income from estates and trusts; Social Security or Railroad Retirement income; Supplemental Security Income (SSI); public assistance or welfare payments; retirement, survivor, or disability pensions; and all other income

Gini coefficient: The Gini index of income inequality for each urban area comes from the ACS and measures the dispersion of the household income distribution.

Population-weighted density for metropolitan and micropolitan areas is from http://www.census.gov/population/metro/data/pop_pro.html. It is calculated as a weighted-average of population densities for each census tract where the weights are the share of the total area population in each census tract. It is intended to reflect the population density experienced by the average person in the urban area.

Median age refers to the median age of the population taken from the 2010 ACS. (It is not the median age of the head of the household, which does not appear to be easily obtainable.)

Land values are taken from column 5 of Albouy *et al.* (2017) appendix Table A2. The MSA's for which they provide these estimates are based on the 1999 OMB definitions. The MSA's used in the current paper are from the 2010 census and are slightly different and greater in number. This is in part because some MSA's have been subdivided and also because there are some new MSA's.[30] Where MSA's have been subdivided, we use the same average land values for both. Where we could find no match, we dropped the MSA from the data set.

COLI: The local cost of living index is the ACCRA Cost of Living Index provided by Council for Community and Economic Research (https://www.c2er.org/). It measures relative price levels for consumer goods and services in participating areas for

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[30]The MSA's in 2010 are based on the concept of Core Based Statistical Areas.

a mid-management standard of living. Weights are based on the Bureau of Labor Statistics' 2004 Consumer Expenditure Survey. We use these weights to extract a COLI for non-housing expenditures only. Data are for selected urban areas within the larger MSA. In the few cases where there are multiple areas within a given MSA, we used a simple average.

Amenity controls: "Natural" amenities taken from Albouy (2016):

Heating and cooling degree days (Annual) – Degree day data are used to estimate amounts of energy required to maintain comfortable indoor temperature levels. Daily values are computed from each day's mean temperature, (max + min)/2. Daily heating degree days (**hdd65**) are equal to $\max\{0; 65 - meantemp\}$ and daily cooling degree days (**cdd65**) are $\max\{0; meantemp - 65\}$. Annual degree days are the sum of daily degree days over the year. The data here refer to averages from 1970 to 1999.

Sunshine – Average percentage of possible sunshine (**sun_a**). The total time that sunshine reaches the surface of the earth is expressed as the percentage of the maximum amount possible from sunrise to sunset with clear sky conditions.

Average slope (**slope_pct**) – The slope of the land in the metropolitan area (percent), using an average maximum slope technique based on a 30 arcsec x 30 arcsec grid.

Coastal proximity (**inv_water**) – Equal to the logarithm of the inverse distance in miles to the nearest coastline from PUMA centroid.

Latitude (**latitude_w**) – Measured in degrees from the equator.

C.2 Fitting construction costs (τ) to cross-city data

We compute MSA-level construction cost parameter values that best match the observed ownership rates and price-rent ratios. We allow median income, the income Gini and median age to match the characteristics of each MSA. For a given construction cost parameter value τ, the resulting simulated economies yield predicted ownership rates and aggregate price-rent ratios. For each MSA, we choose the value for τ that best matches the observed MSA-level statistics in the following sense: we minimize a weighted average of the squared differences between the simulated statistics and their empirical counterparts. For the weights, we use the inverse of the variances of the MSA ownership rates and price-rent ratios.

D Additional Empirical Results

Tables 8 and 9 contain full parameter estimates for Tables 3 and 4, respectively.

In Tables 10 and 11 we consider samples which also include smaller urban agglomerations. Table 8 includes *Micropolitan* areas which are urban agglomerations containing fewer than 50,000 households but more than 10,000. This increases the number of urban areas to 942 but we have no data for land values or amenities. Table 11 includes all urban areas with more than 1000 households, which increases the number of observations to 3360. Controlling for population density, the results documented in Table 10 are qualitatively similar to those of Table 3: (1) The price-rent ratio is positively correlated with median income, the Gini index and age and (2) the ownership rate is positively correlated with median income and age but negatively correlated with the Gini index. In Table 11, we have no data on density for smaller urban clusters and so cannot control for it. Nevertheless, the same qualitative correlations for median incomes, inequality and median age are obtained.

Table 8: Full results for Table 3

	Ownership Rate		Price-Rent Ratio			
			(price-asked)		(estimated-value)	
Log Median Income	0.076	(0.030**)	14.497	(2.833***)	13.088	(2.114***)
Gini index	-0.449	(0.138***)	90.286	(13.395***)	65.427	(8.523***)
Age	0.007	(0.001***)	0.032	(0.083)	0.122	(0.044***)
Log Density	-0.024	(0.006***)	-3.585	(0.787***)	-1.447	(0.336***)
Log Land Value	-0.010	(0.005**)	2.680	(0.736***)	2.321	(0.651***)
cdd65	-0.024	(0.006***)	2.710	(0.790***)	2.998	(0.455***)
hdd65	-0.013	(0.003***)	0.554	(0.319*)	0.903	(0.236***)
sun_a	-0.067	(0.035*)	18.869	(6.491***)	19.239	(5.357***)
slope_pct	-0.001	(0.002)	0.692	(0.184***)	0.717	(0.104***)
inv_water	-0.064	(0.056)	41.339	(8.294***)	28.713	(4.453***)
latitude_w	-0.003	(0.001*)	0.082	(0.182)	0.263	(0.114**)
Constant	-0.076	(0.302)	-174.27	(33.08***)	-168.5	(25.35***)
$\overline{R}^2$	0.63		0.56		0.79	
# obs	332		332		332	

Notes: Standard errors (in parentheses) are clustered at the state level.

Table 9: Full results for Table 4

	Ownership Rate		Price-Rent Ratio			
			(price-asked)		(estimated-value)	
Log Median Income	0.093	(0.030***)	5.874	(3.120*)	5.945	(2.720**)
Gini index	-0.448	(0.148***)	56.147	(17.899***)	46.824	(14.960***)
Age	0.008	(0.001***)	-0.066	(0.101)	0.097	(.062)
Log Density	-0.025	(0.006***)	-2.766	(0.808***)	-1.175	(0.468**)
Log Land Value	-0.012	(0.005***)	3.851	(0.769***)	3.979	(0.874***)
cdd65	-0.024	(0.005***)	3.216	(1.064***)	3.419	(0.537***)
hdd65	-0.013	(0.004***)	0.433	(0.449)	0.732	(0.374*)
sun_a	-0.028	(0.036)	14.454	(8.165*)	14.872	(6.111**)
slope_pct	-0.002	(0.001**)	0.745	(0.260***)	0.639	(0.111***)
inv_water	-0.095	(0.053*)	50.33	(10.33***)	29.017	(5.816***)
latitude_w	-0.003	(0.002)	-0.059	(0.165)	0.175	(0.150)
Constant	0.299	(0.002)	-40.138	(22.633*)	-60.04	(22.58**)
$\overline{R}^2$	0.65		0.49		0.77	
# obs	199		199		199	

Notes: Standard errors (in parentheses) are clustered at the state level.

Table 10: All U.S. Metropolitan and Micropolitan Areas

Dependent Variable	Ownership Rate	Price-Rent Ratio	
		(price-asked)	(estimated-value)
Log Median Income	0.011	14.315	21.396
Robust s.e.	(.012)	(2.369***)	(1.395***)
Clustered s.e.	(.018)	(2.579***)	(2.346***)
Gini index	-0.599	59.731	45.261
Robust s.e.	(.070***)	(13.009***)	(6.848***)
Clustered s.e.	(.113***)	(18.471***)	(15.951**)
Median age	0.006	0.120	0.101
Robust s.e.	(.000***)	(0.096)	(.038***)
Clustered s.e.	(.001***)	(0.126)	(.051**)
Log Density	-0.017	-1.078	-0.238
Robust s.e.	(.002***)	(0.442**)	(.262)
Clustered s.e.	(.003***)	(0.537**)	(.407)
Adjusted R^2	0.47	0.05	0.43
# obs	942	937	942

Table 11: U.S. Urban Areas with more than 1000 Households
(Median income not adjusted for local non-housing cost of living)

	Ownership Rate		Price-Rent ratio	
Log Median Income	0.103	0.119	13.328	13.222
Robust s.e.	(.006***)	(.008***)	(.522***)	(.615***)
Clustered s.e.	(.012***)	(.016***)	(1.013***)	(1.280***)
Gini index	-0.530	-0.438	37.639	37.023
Robust s.e.	(.038***)	(.040***)	(2.851***)	(3.328***)
Clustered s.e.	(.038***)	(.044***)	(4.185***)	(5.944***)
Median age	0.007	0.006	0.044	0.046
Robust s.e.	(.000***)	(.000***)	(.018**)	(.018**)
Clustered s.e.	(.000***)	(.000***)	(.035)	(.037)
Log Households	–	-0.007	–	0.050
Robust s.e.		(.001***)		(.0.091)
Clustered s.e.		(.002***)		(.183)
Adjusted R^2	0.42	0.43	0.29	0.29
# obs	3360	3360	3360	3360